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Spatially correlated rainfall in a protective irrigation system

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ABSTRACT. Allowing water to be traded in an irrigation system with *de facto* riparian rights can increase the efficiency of water use. With irrigation supplementing uncertain rainfall, both supply and demand for water are uncertain. When rainfall events are positively correlated, water supply and demand movements are negatively correlated, causing large price fluctuations. Using a stylized two crop, two region irrigation system, numerical integration over a bivariate rainfall distribution is used to demonstrate the impact of varying rainfall correlation on water price, expected returns, and return variability. A spot market for water captures the majority of the returns to water rights transfers, relative to a complete market. This spot market results in higher returns and reduced return volatility for farmers in the water deficit region, while farmers in the water surplus region see greater returns but also greater return volatility.

1. Introduction

Throughout the world, irrigation systems are not achieving performance anywhere near that projected by their design (Joshi and Agnihotri, 1984; Chambers, 1988; Sampath, 1992; Rosegrant, 1997). Some see the establishment of water user associations and empowering them with more control over water allocation as a way to improve efficiency (Nagaraj, 1999; Mollinga *et al*., 2000). In theory, using markets to trade water rights should improve the efficiency with which water is used (Burness and Quirk, 1979; Howe *et al*., 1986; Caswell *et al*., 1990; Rosegrant and Binswanger, 1994). Tradable water rights encourages the transfer of water from low value to high value uses, increasing economic surplus. Markets are particularly attractive to some in light of the many failures that have occurred with government involvement (Lam, 1996a,b). However, factors such as environmental externalities (Chakravorty *et al*., 1995), limited abilities to enforce water rights (Young, 1986; Rhodes and Sampath, 1988; Sampath, 1992; Tsur and Dinar, 1997), and distributional concerns (Howitt, 1994; Miller, 1996) may prevent a market for water rights from attaining all potential gains. In light of this work, it is important to extend the analyses of water markets with the addition of more realistic factors.

Empirical research generally finds that water markets increase efficiency. Early US studies suggest that the gains from market-driven water transfers could be large (Vaux and Howitt, 1984; Vaux, 1986). For Thobani (1997), experiences in Chile and Mexico suggest that substantial gains can occur,

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and that distributional effects may favor the poor. However, Romano and Leporati (2002) interpret the evidence as suggesting that in Chile poor farmers are worse off. Hearne and Easter (1996) find that efficiency gains have occurred in Chile, but that large, wealthy farmers are capturing most of the gains. For southern Australia, Bjornlund and McKay (2002) also find that markets do move water to higher value uses, and that rents are predominantly captured by larger farms.

Many developing country irrigation systems are protective, providing irrigation water when normal rains do not occur (Wallach, 1984; Mollinga, 1998). Such systems typically have among the worst performance. Water use is excessive in the head regions; water frequently fails to reach the tail regions (Chaudhary *et al*., 1989; Deshparde and Supe, 1989). Tradable water rights are seldom considered as a means of correcting this asymmetric water distribution. Where water markets are developing is on the fringes of these systems, where canal leakage is replenishing the water table, and farmers are able to install pumps and sell the water they raise (Chatterjee, 2000). As rainfall is an important water source, supplies and demands for water in protective systems are sensitive to weather events. The correlation of weather events will affect the distribution of economic surplus, the subject of the current analysis.

Where sufficient storage capacity exists, it can serve to manage supply uncertainty. With appropriate geologic structures, an aquifer can serve as a buffer for stochastic surface flows (Tsur, 1990; Provencher and Burt, 1993; Knapp and Olson, 1995). This stabilization benefit may be larger than the production impact of the water (Tsur, 1990). Optimal management of groundwater requires accounting for open access and pumping cost effects (Provencher and Burt, 1993). If water rights are clearly established, and sufficient storage capacity exists, water users can be permitted to 'bank' water by leaving water they are entitled to use in the system reservoir. In California, a water banking system was established to deal with the 1991 and 1992 droughts (Howitt, 1994; Isreal and Lund, 1995; Miller, 1996). Within the US context, an important role for water banks is to facilitate water transfers that do not threaten to violate 'beneficial use' requirements for security of water rights (Miller, 1996). Although between season transfers were possible, the bank served mainly as a way of trading water between users. Iglesias *et al*. (2003) show that for the Guadalquivir Basin in southern Spain, banking water between seasons, even without water trading, may enhance farmer benefits between 32 and 82 per cent. Unfortunately, where storage and pumping capacity are limited and non-riparian water rights unenforceable, banking is not practical.

In spite of the potential for market-based reforms, the political aspects are often daunting. Dinar *et al*. (1998) carefully document the various groups affected by institutional reforms. Using this information, they highlight the fact that the success of any proposed reform is critically dependent on political features. In a comprehensive review, Johansson *et al*. (2002) outline the theory of water markets, and highlight the fact that equity issues often play a major role in reforms. This review supports the work of Tsur and Dinar (1995), who hold that equity concerns are an important consideration. Non-efficiency concerns tend to manifest themselves as restrictions on

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trades. These frictions often interfere more with permanent rights transfers than spot trades (Bjornlund and McKay, 2002). A seasonal spot market that does not involve transfers of long-term water rights is probably easier to implement than tradable permanent water rights.

This paper uses numerical integration to explore the impact of rainfall variability in a stylized protective irrigation system. Farmer's choose from two different crops, one that is water intensive and another that requires little water. Farmers are also able to trade their water rights. Market transactions are restricted by two factors, gravity and participation. Water can only be sold by uphill farmers to downhill farmers, and farmers cannot credibly threaten to withhold water they would not use at a zero price. Comparisons are made between three situations – autarky, where water rights are not traded; partially informed, where farmers trade water rights after committing to a cropping pattern; and fully informed, where cropping and water rights trading occurs when future crop season rainfall is known. The fully informed situation is clearly not realistic, making the result that most of the gains occur in the move from autarky to a partially informed market especially encouraging.

The remainder of the paper is organized as follows. The next section sets out a two region, two crop model, using simple quadratic production functions. An appendix illustrates a more general multiple crop formulation, and develops the Kuhn–Tucker conditions. Results are presented for a parameterization of the model, together with a sensitivity analysis of several parameter changes. The results are followed by a discussion, where implications and extensions of the results are described. The paper ends with a brief conclusion.

2. The model

To illustrate the implications of rainfall variability in an irrigation system, a stylized two region, two crop model is developed. A multi-crop generalization is presented in the appendix. Region x is upstream and region y is downstream. The two regions are identical in all respects except for water availability. The two crops have quadratic per unit land area yield functions of the form

$$g_i(w) = \delta_i(w - \alpha_i)(\beta_i - w) \tag{1}$$

where $\alpha_i \leq w \leq \beta_i$ defines the range of water application rates over which total growth is positive. δ_i is a scaling factor that determines the maximum yield. With $\delta_i = 4\bar{g}_i/(\beta_i - \alpha_i)^2$, where $\bar{g}_i$ is the maximum yield, maximum yield is held constant, while α_i and β_i are varied. This definition results in $\partial\delta_i/\partial\alpha_i = -\partial\delta_i/\partial\beta_i$. This formulation is consistent with Doorenbos and Kassam (1979), among others, who observe a minimum water threshold. The quadratic form is a simple version of the polynomial yield functions reported by Solomon (1983).

Letting q_i be the price for crop i, the total value product of water applied to crop i is $q_i g_i(w)$, with the marginal value product being

$$q_i g_i'(w) = q_i \delta_i(\alpha_i + \beta_i - 2w) \tag{2}$$

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With a competitive water market, price equals marginal value product, so that the above can be rearranged to generate a demand curve for water for crop i

$$w_i(p) = \begin{cases} \frac{1}{2}(\alpha_i + \beta_i) - \frac{1}{2}\frac{p}{q_i\delta_i} & 0 < p < \bar{p}_i \\ [0, \sqrt{\alpha_i\beta_i}] & p = \bar{p}_i \\ 0 & p > \bar{p}_i \end{cases} \tag{3}$$

where p is the water price and $\bar{p}_i$ is the choke price for crop i. The choke price, $\bar{p}_i = q_i\delta_i(\alpha_i + \beta_i - 2\sqrt{\alpha_i\beta_i})$, is determined by the choke quantity, $\bar{w}_i = \sqrt{\alpha_i\beta_i}$, which satisfies $g_i(w) = wg_i'(w)$. This latter condition follows from the requirement that the profit earned by growing the crop must at least equal the cost of watering it, and the competitive water market condition $p = g_i'(w)$. It corresponds to finding the application rate that equalizes marginal and average product. Finally, if the planted area is divisible into an irrigated and an unirrigated area, demand will be perfectly elastic at the choke price over the interval $[0, \bar{w}_i]$.

This demand curve responds as expected to parameter changes, with demand decreasing in water price, increasing in crop price, and increasing in a yield curve shift. The first two results are confirmed by the fact that $\partial w_i/\partial p \leq 0$, $\partial w_i/\partial q_i \geq 0$ and $\partial \bar{p}_i/\partial q_i = 0$. The third result follows by looking at the effect of equal changes in α_i and β_i, with δ_i adjusted so that maximum yield is held constant. For prices far enough from the choke price, demand increases proportionately when the interval $[\alpha_i, \beta_i]$ shifts out, as $\partial w_i/\partial\alpha_i + \partial w_i/\partial\beta_i = 1$. At the choke price, $\partial \bar{p}_i/\partial\alpha_i + \partial \bar{p}_i/\partial\beta_i < 0$, so that shifting out the interval $[\alpha_i, \beta_i]$ reduces the choke price.

With predetermined areas planted to each crop, total water demand is the sum of the demands for each crop. Since each crop has a perfectly elastic segment at its choke price, the demand curve for water will be a combination of downward-sloping and horizontal segments, each horizontal segment corresponding to the choke price of the most recently introduced crop. With areas x_1 and x_2 planted to crops 1 and 2 respectively, the total demand for water is

$$W^x(p) = \begin{cases} 0 & p > \bar{p}_1 \\ [0, \bar{w}_1]x_1 & p = \bar{p}_1 \\ w_1(p)x_1 & \bar{p}_2 < p < \bar{p}_1 \\ w_1(\bar{p}_2)x_1 + [0, \bar{w}_2]x_2 & p = \bar{p}_2 \\ w_1(p)x_1 + w_2(p)x_2 & 0 \leq p < \bar{p}_2 \end{cases} \tag{4}$$

where $\bar{w}_i = \sqrt{\alpha_i\beta_i}$ is the choke quantity for crop i, $\bar{p}_i$ is the corresponding choke price, and the demand curve for each crop, $w_i(p)$, is defined as above.

If there is perfect foresight, then the planting decision and the water use decision are made jointly. In this case only the most profitable crop will be grown. The owner will be indifferent between two crops when their profitability is the same, with the water price where this occurs marking the transition between the two crops. This equality relation is defined by

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the condition

$$g(w_1(\bar{p}_{12})) - pw_1(\bar{p}_{12}) = g(w_2(\bar{p}_{12})) - pw_2(\bar{p}_{12}) \tag{5}$$

which can be solved for the transition price $\bar{p}_{12}$, where profit per unit land is equal for both crops. This price is the analog for the crop 2 choke price when plantings are predetermined. The demand curve with perfect foresight has the same form as equation (4), with $\bar{p}_2$ replaced by $\bar{p}_{12}$. In general, $\bar{p}_{12} < \bar{p}_2$, as the crop being replaced earns positive profit, rather than the zero profit condition used when the land choice is predetermined.

Now let the supply of water in regions x and y be given by $S^x + u^x$ and $S^y + u^y$, with S^x and S^y being the certain portion of the supply, and u^x and u^y being the uncertain portions. With uphill region x in a position to sell water to downhill region y, the market clearing equilibrium is determined by

$$S^x + u^x - W^x(p) = W^y(p) - (S^y + u^y)$$

with the gravity constraint satisfied if $S^x + u^x - W^x(p) \geq 0$ and $W^y(p) - (S^y + u^y) \geq 0$. If u^x and u^y are inversely correlated, then the change in the total supply is less than the total variation, while, if they are positively correlated, the supply shocks tend to be relatively large. Therefore, negative correlation is associated with small price swings, while positive correlation with large ones. In the two stage problem, x_i and y_i are chosen prior to the realization of u^x and u^y. Assuming risk neutrality, an equilibrium in the two stage problem is a 4-tuple $\{x_1, x_2, y_1, y_2\}$ such that

$$\begin{aligned} &E[g_1(w_1(p^e)) - p^e w_1(p^e)]x_1 + E[g_2(w_2(p^e)) - p^e w_2(p^e)]x_2 \\ &\geq E[g_1(w_1(\tilde{p})) - \tilde{p}w_1(\tilde{p})]\tilde{x}_1 + E[g_2(w_2(\tilde{p})) - \tilde{p}w_2(\tilde{p})]\tilde{x}_2 \end{aligned}$$

for region x, with an analogous condition for y. p^e is the equilibrium price in the second stage given $\{x_1, x_2, y_1, y_2\}$, while $\tilde{p}$ is the second stage equilibrium price when x chooses $\tilde{x}_1$ and $\tilde{x}_2$. Since $W^x(p)$ and $W^y(p)$ are nonlinear and the binding or lack thereof of the constraints depends on the realization of u^x and u^y, the expected price and profit are not simple linear functions of the distributional parameters. It is therefore not possible to use Jensen's inequality or other relations that depend on well-behaved functions to derive general results. The impact of different parameters on earnings and volatility measures is therefore explored with a numerical example.

3. Example

The following numerical results explore the implications of the relationship between u^x and u^y. The numerical calculations and graphics were generated using R (Ihaka and Gentleman, 1996), with the market equilibrium and expected returns evaluated using routines written in C. The parameters were chosen to be loosely consistent with those used by Janmaat (2004), and are shown in table 1, with the total value product curves shown in figure 1, panel (a), and the water demand curve for both predetermined and flexible cropping shown in panel (b).

The water demand curves reflect the profit maximizing water use for a particular water price, in the cases when land use decisions are predetermined and when they are flexible. Figure 2 shows the total value

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Table 1. *Numerical example parameter values, $q_i\bar{g}_i$ calculated assuming $\delta_i = 4\bar{g}_i/(\beta_i - \alpha_i)^2$. For numerical example, $\bar{p}_{12} = 0.4356$*

Crop	α_i	β_i	$q_i\bar{g}_i$	$\bar{w}_i = \sqrt{\alpha_i\beta_i}$	$\bar{p}_i$	$w_i(\bar{p}_{12})$
1	0	150	100	0	2.6667	62.75
2	50	350	150	132.28	0.9028	167.33

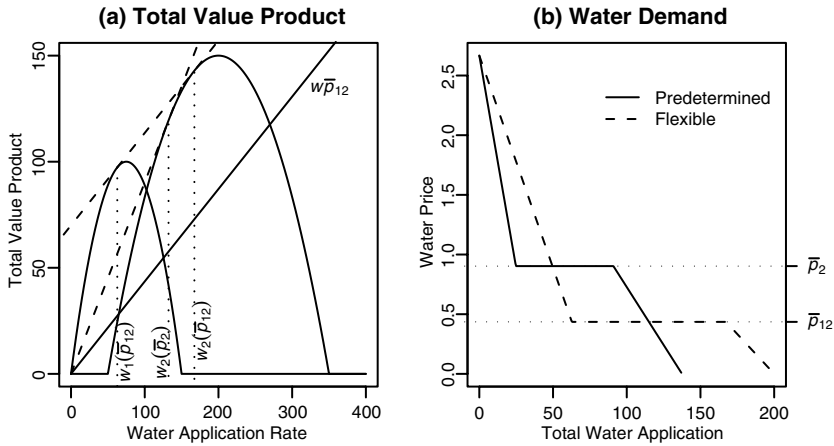


Figure 1. *Total value product and water demand for two crops. In panel (a) dashed lines identify tangencies. $w_1(\bar{p}_{12})$ and $w_2(\bar{p}_{12})$ mark quantities of water where for price $\bar{p}_{12}$, profit for each crop is equal. $w_2(\bar{p}_2)$ is the application rate $\bar{p}_2$, the water price when it is first profitable to irrigate crop 2. Predetermined water demand curve shows water demand when one unit of land is divided equally between both crops. Entry price for second crop is $\bar{p}_2$. Flexible water demand is demand when crop area can be chosen optimally given water price. Above transition price $\bar{p}_{12}$, all land is planted to crop 1, while below $\bar{p}_{12}$, all land is planted to crop 2.*

product curves for the two crop example with the land use fixed at three different combinations and the unrestricted case. The unrestricted TVP curve is an outer envelope for the restricted TVP curve, a result of the fact that relaxing a restriction cannot reduce profits in a constrained optimization. In the two stage optimization, farmers are choosing which of the restricted TVP curves to locate on. The realized TVP depends on the realized water level, with the expected TVP lying inside the unrestricted and restricted TVP curves for all but a degenerate distribution of available water.

The greatest profit is earned when crop and water choice are made jointly. The smallest profit occurs when farmers are restricted to autarkic choices in a two stage environment. The equilibrium with a spot market after the planting decision is made lies between these two. Figure 3 displays contour maps of the total value product for increasing water levels in the two regions, as the system moves from an autarkic equilibrium to an equilibrium

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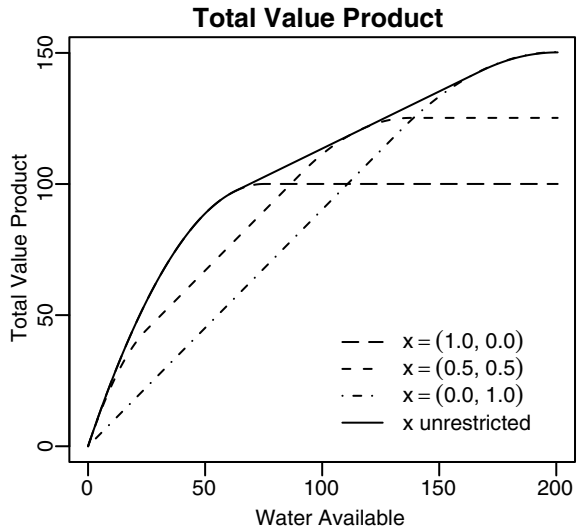


Figure 2. *Total value product with fixed land allocations and free land use variable. Excess water assumed removable from system.*

with full flexibility. The largest gains from the introduction of a water market occur when water is relatively more plentiful in the uphill region and relatively scarce in the lower. In such situations, downhill farmers, with a higher marginal productivity of water, are able to purchase it from the upper farmers. Overall productivity therefore increases. The maximum gain from moving to an environment where water use and crop choice decisions are made jointly also occurs when water is relatively abundant in the uphill region and relatively scarce in the lower region. However, these gains are more modest, no more than half the gains available when the system moves from an autarkic equilibrium to an equilibrium with a spot market. Thus, about two-thirds of the gains available in moving from an autarkic two stage equilibrium to a fully flexible equilibrium are captured in a post planting spot market. The participation constraint effect is also evident in these figures. When there is a large amount of water in region 1, then the threat of withholding it is not credible. The region 2 farmer takes this into account, and does not purchase water.

It is clear from figure 3 that the greatest gains occur along the lower margin of the maps, where the region 1 farmers have enough water to be in the transition region between the low water and high water crop, and farmers in region 2 have very little water. This result is not particularly surprising, as in this region the marginal value product of water is very high, while it is high enough in region 1 that no water is actually released to region 2. Introducing a water market allows farmers in region 2 to purchase water from region 1, increasing overall efficiency. These results hold for the certainty case, with a restriction on the cropping decision. In what follows, results for uncertain water availability are described.

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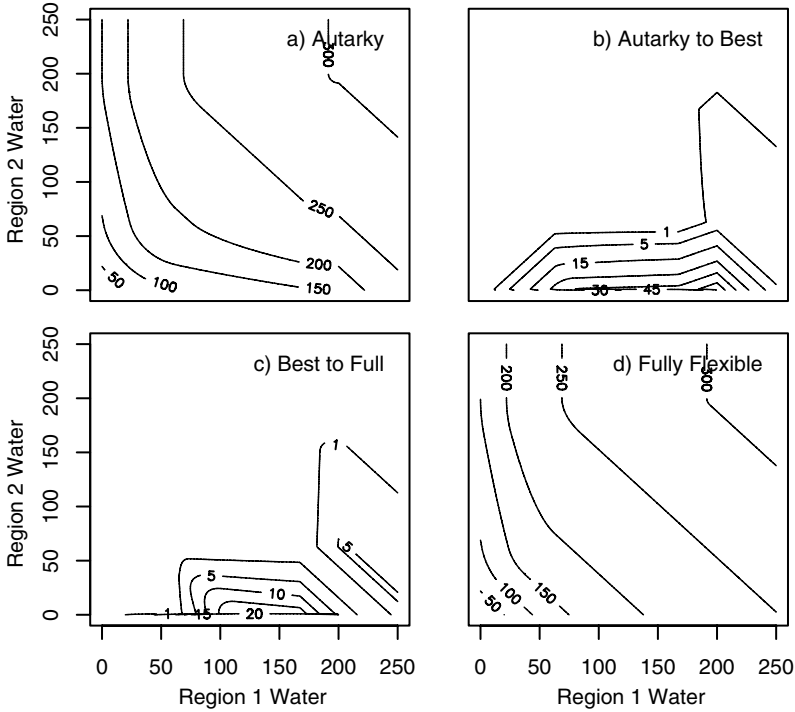


Figure 3. *Change in total value product with movement from autarky to full flexibility in cropping choice and water use choice. Intermediate state 'Best' refers to best crop choice being made with a second period spot market for water.*

Numerical integration of a bivariate rainfall distribution in this system was used to assess the impact of uncertainty on expected profit and prices. A truncated bivariate normal distribution over water availabilities $\{0, 1, \ldots, 249, 250\} \times \{0, 1, \ldots, 249, 250\}$, with $\sigma_1 = \sigma_2 = 20$ and various values of μ_1, μ_2, and ρ were used. The truncated distribution was scaled to ensure that the total probability mass was equal to one. The truncation of the normal distribution leads to non-constant marginal means and variances for distributions estimated near the bounds. To accommodate this, the same probability matrix was used to calculate the market equilibrium and autarky cases. To facilitate comparisons, proportional differences were used to compare the autarky case and the best response case.

Four cases were evaluated, exploring the region near the middle of the lower margin in figure 3. This is where water is relatively abundant in region 1 and relatively scarce in region 2 and market-based reforms show the greatest promise. μ_1 was set at 150 or 200, so that in autarky region 1 either grows both crops or exclusively crop 2. When $\mu_1 = 150$, μ_2 was either 20 or 50, so that it normally purchases water, irrespective of the crop chosen. When $\mu_1 = 200$, μ_2 either 20 or 100. The latter case represents a situation where water is relatively abundant in both regions.

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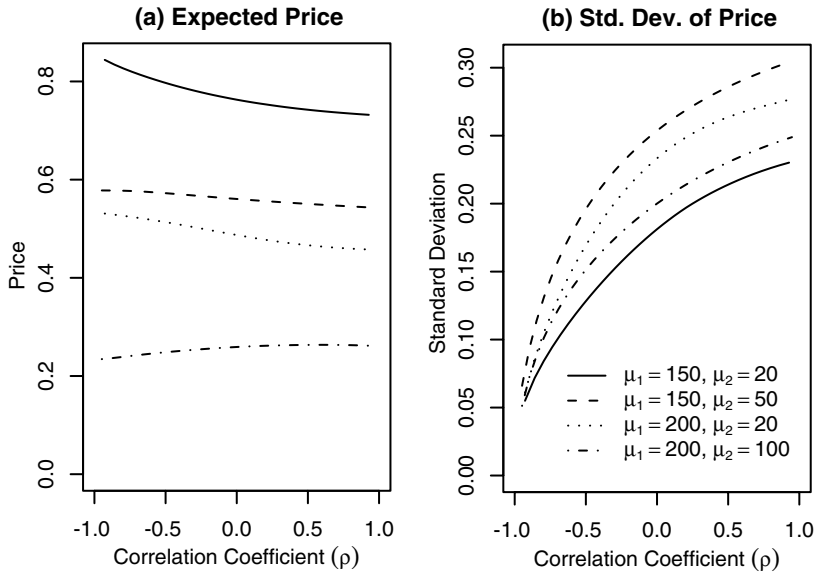


Figure 4. *Expected value and standard deviation of price.*

Figure 4 shows the expected value of the water price and the standard deviation of that price for the four cases. For the three cases where water was scarce in region 2, $\mu_2 \leq 50$, the water price fell with increasing correlation in rainfall events. The expected hierarchy also prevails, with a lower water price as the total water in the system increases. The standard deviation of the price, shown in panel (b), is increasing with increasing correlation. This result is consistent with the fact that, with increasing correlation, the supply of water that region 1 is willing to sell is increasingly negatively correlated with the demand for water from region 2. As such, price variations will be magnified. As noted above, the rainfall variance was not constant as ρ was varied. However, the variances and means were approximately the same for $\rho \to -1$ and $\rho \to 1$, suggesting that the overall trend is accurate.

The price fluctuations have different impacts on region 1 and region 2. Figure 5 reports the equilibrium results, in terms of share of land planted to the drought tolerant crop (panel a), the change in expected return (panel b), and the change in standard deviation (panel c) for the four cases. The cropping pattern was only modestly affected by the correlation between rainfall events. When $\mu_1 = 150$ and $\mu_2 = 20$, there is a negative relationship between rainfall correlation and area planted to crop 1. The equilibrium cropping pattern for region 1 follows the same pattern as the price. When the expected price for water is greater, farmers in region 1 reduce their planting of the high water crop, leaving more water available for sale. When $\mu_1 = 150$ and $\mu_2 = 50$, there is almost no response to rainfall correlation in terms of region 1's behavior. For both of these cases, region 2 farmers plant all land to crop 1.

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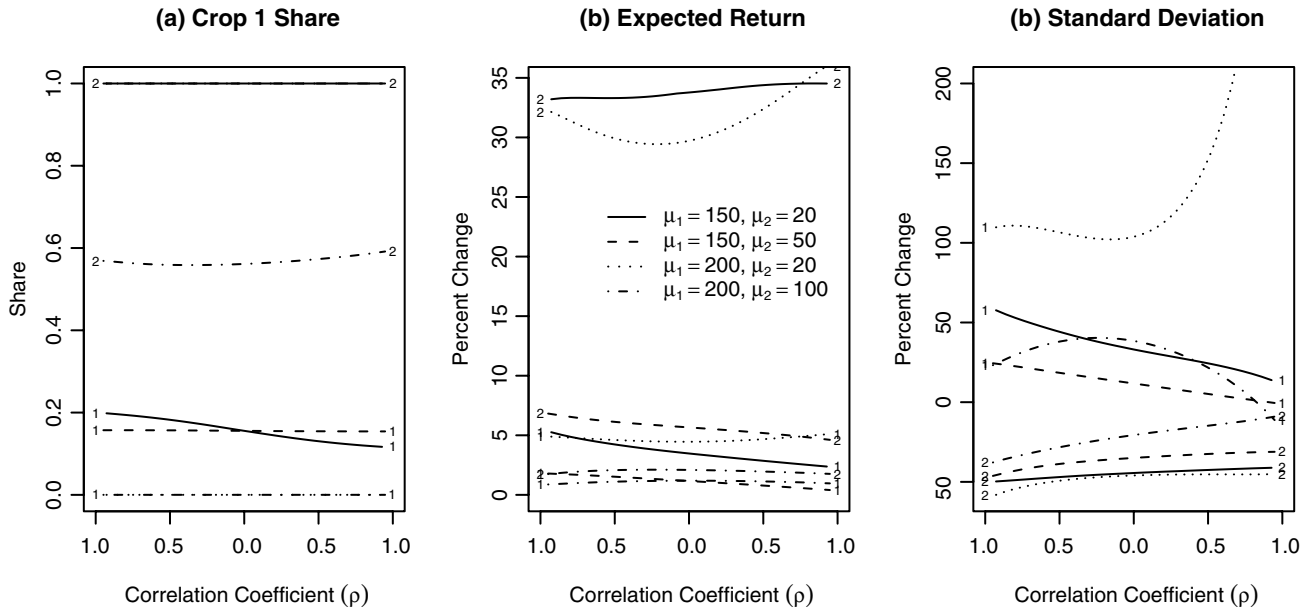


Figure 5. *Land allocation, expected return, and standard deviation of profit earned by representative farmer in each region. Lines terminated with the number 1 refer to region 1, while those terminated with a 2 refer to region 2.*

When $\mu_1 = 200$, farmers in region 1 plant all their land to the high water crop. With $\mu_2 = 20$, farmers in region 2 continue to plant all of their land to the low water crop. When $\mu_2 = 100$, region 2 farmers split their land between the two types of crops. As the correlation in rainfall increases, the area planted to crop 1 increases. This response is inconsistent with the price pattern, which would predict greater planting to the low water crop when the price for water is greater. However, when $\mu_1 = 200$, there is a high probability that region 1 will not be able to credibly threaten to withhold water. When rainfall events are negatively correlated, surplus water will exist in region 1 when there is a shortage in region 2, and *vice versa*. When rainfall events are positively correlated, surpluses and shortages occur at the same time in both regions. Therefore, region 2 farmers are more likely to benefit from region 1 surpluses when the correlation is negative, increasing the benefit from investing in the high water crop.

For the change in expected return, the greatest gain occurs in region 2 when $\mu_1 = 20$, whether $\mu_1 = 150$ or $\mu_1 = 200$. With water being scarce in region 2, the marginal benefit of an extra unit is large. Profit can therefore be substantially increased by purchasing water, even as it has to be paid for. Since region 2 farmers are planting all land to the low water crop, their water demand is downward sloping and they are able to retain some of the rents when there is a water market. This contrasts with larger water volumes, when region 2 farmers are in the perfectly elastic portion of their water demand curve. When this is the case, all rents are extracted by water sellers, leading to a relatively small change in profit. This result augments that suggested by Thobani (1997), who observes that establishing clear water rights gives small farmers with low quality land something to sell.

The relative profit change for region 1 is small, and generally decreases as the rainfall correlation increases. Two factors drive this result. One is the fact that farmers in region 1 are operating on a portion of their MVP curve where change is slow. When $\mu_1 = 150$, most transactions occur where region 1 farmers are on the perfectly elastic region of their water demand curve, resulting in no difference between profit from extra water used in region 1 or sold. Thus, the relative benefit to farmers in region 1 is small. The relative gain for farmers in region 1 is also lower than that in region 2, in part because the absolute profit in region 1 is higher (between 132 and 150 currency units under autarky) than that in region 2 (between 44 and 63 currency units for $\mu_2 = 20$ cases under autarky). Therefore, an extra currency unit received in region 2 generates three times the relative gain in region 2 as compared to region 1. However, since the relative increase for region 2 is between four and six times that in region 1, this scaling effect is only part of the explanation.

Panel (c) shows the relative change in the standard deviation of profit. In all cases, the standard deviation of returns in region 2 falls. In general, the gain to region 2 is greater the lower the correlation between rainfall in the two regions. This is in line with the price volatility, which falls with reduced correlation in rainfall. The standard deviation for region 1 is increased except for highly correlated rainfall when $\mu_1 = 200$ and $\mu_2 = 100$. In all other cases, the standard deviation of profit is greater with a water market than without it. In general, the standard deviation of region 1's profit increases

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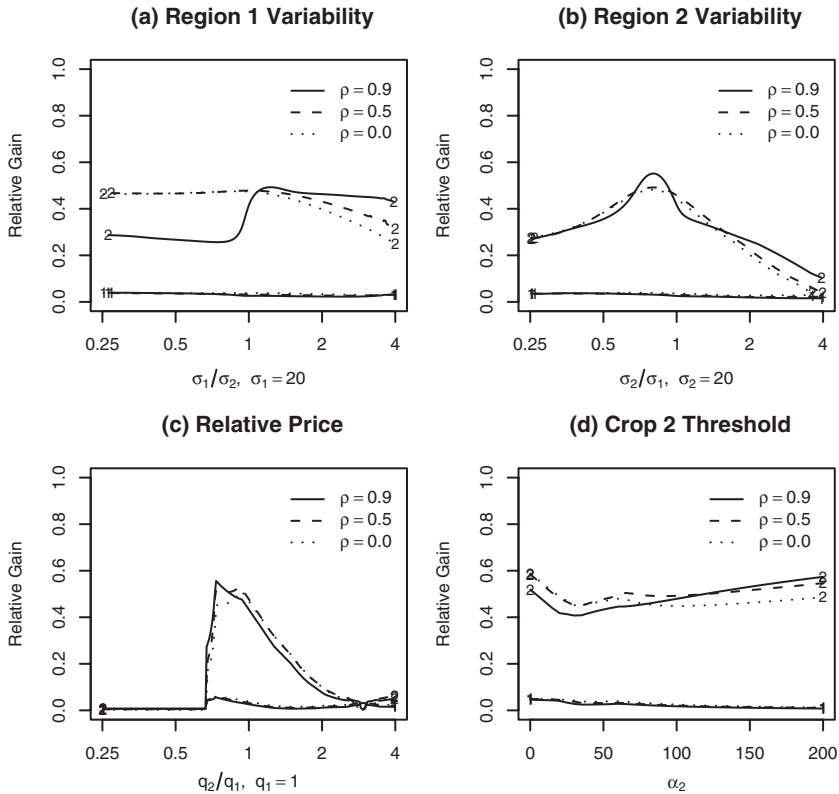


Figure 6. *Sensitivity of relative gain to changing water availability volatility in each region, changing relative price of crop, and changing threshold values. Unless otherwise stated, all parameter values are as reported in table 1.*

with a reduction in rainfall correlation, except for the $\mu_1 = 200$ and $\mu_2 = 20$. There is a scale impact on these results as well. For region 1, the standard deviation was much lower (between 8 and 10 units for the $\mu_1 = 150$ cases) than for region 2 (as high as 37 units for the $\mu_2 = 20$ cases). The large relative increase in return variability in region 1 is not a large absolute increase. These results suggest that for region 2 farmers, introducing a spot market for water after planting is complete reduces profit variance and increases expected profit. However, for region 1 farmers, having a spot market increases expected profit, but almost always increases profit variance as well.

Variation in several key parameters was also explored. Figure 1 shows the increase in profit over autarky for four parameter changes. Panel (a) shows the impact of changing the variation of water availability in region 1, holding water variation in region 2 constant at $\sigma_2 = 20$. This change has relatively little impact on the region 1 returns. For low correlations, the relative gain to region 2 is essentially constant when $\sigma_1 < \sigma_2$, subsequently

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falling for further increases in σ_1. When the correlation is high, the relative gain to region 2 is not as high when $\sigma_1 < \sigma_2$, sharply increasing in the neighborhood of $\sigma_1 = \sigma_2$, and then slowly declining. The planting pattern (not shown) has region 2 always planting the lightly irrigated crop, while region 1 plants over 20 per cent of the heavily irrigated crop throughout. The rise in the relative return to region 2 with high correlation follows from the presence of a market enabling region 2 farmers to purchase water when it is scarce, a situation that does not occur with any significant probability until the variance of region 1's water availability is sufficiently large.

Panel (b) shows the relative gain when the variance of water availability in region 2 increases, holding the variability in region 1 constant. The relative gain to region 1 is low and changes little. Region 2's relative gain is maximized when water availability is slightly less volatile in region 2 than in region 1. The gain is larger for higher correlations, and, after an initial rapid drop, declines more slowly at the higher correlation. At the maximum relative gain, region 1 has increased its planting of the lightly irrigated crop. This will lower the equilibrium price and make more water available to region 2. As volatility further increases, region 2 begins planting the heavily irrigated crop. This occurs because planting the heavily irrigated crop is analogous to purchasing an option on high water availability, and the value of the option increases as volatility increases. Becker (1999) reports this behavior for Israeli farmers. However, the relative gain falls, as with higher volatility the ability of a water market to offset unfavorable realizations diminishes.

Panel (c) shows the relative gains as the price ratio between the crops changes. With a high price for crop 1, neither region plants crop 2, and the presence of a water market provides little relative benefit. When the MVP for crop 2 attains a maximum greater than that of crop 1, then planting some of the area to crop 2 becomes rational. After this point, $q_2/q_1 = 2/3$ in the example, the gains to region 2 from a spot market become large. The gains are great when the prices are relatively close, which results in region 1 planting most of its land to the heavily irrigated crop, while region 2 plants only the lightly irrigated crop. When the price of the heavily irrigated crop becomes high enough, then both regions plant mostly the heavily irrigated crop, and the gains from a spot market for water diminish.

Panel (d) shows the relative gain as the threshold water required for the heavily irrigated crop increases. The relative gain for region 1 remains small, and declines as the threshold increases. The relative gains for region 1 initially fall, and then increase as the threshold increases. The fall is greater when the correlation is higher, as with a lower threshold, region 1 will have less water to sell during an unfavorable realization. As the threshold increases, region 1 reduces its planting of the heavily irrigated crop. This increases the amount of water that region 1 has available to sell, and the frequency of events when surplus water will simply flow to region 2. Consequently, the gain to region 2 increases the greater the threshold.

4. Discussion

In this paper, a two region irrigation system was modeled, with and without a market for water. One of the two regions was 'uphill' and generally

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received a large volume of water, while the other region was 'downhill', generally facing a water scarcity. With multiple crops, water demand will have perfectly elastic segments where crop switches occur. The price at which the switch occurs satisfies a zero profit condition when the cropping decision is made before available water is known, and an equal profit condition if crop and water decisions are made jointly. When water volumes are certain, introducing a water spot market, after the planting decision, captures about two-thirds of the gains in the move from a two stage autarky situation to a fully integrated cropping and water decision for a stylized example.

In protective irrigation systems, irrigation water supplements rainfall. The numerical example was extended to an environment with uncertain water availability. To simulate a protective irrigation system with riparian water rights, the expected water availability in the uphill region was high relative to the downhill region. Four different combinations of expected water availability were explored over a range of correlations in water availability events between the two regions. As expected, introducing a spot market for water increases the mean profit for farmers in both regions. The relative gain is greater for farmers in the low water region, in part due to their lower autarky profitability, and in part due to the higher marginal productivity of water in the scarcity region. In contrast to this result, farmers in the uphill region experience an increase in profit variability, while farmers in the downhill region see their profit variability fall. The increase in variability, as a percentage, is considerably larger for the uphill farmers than the decrease experienced by the downhill farmers. This relative impact is largely due to the relatively low profit variability with autarky in the uphill region. However, variability of returns does increase, a consequence of the fact that in some situations water can be sold at a high price, while in others there is no credible way for uphill farmers to withhold water, keeping the system at the autarky situation.

That the relative benefit from a water market flows predominantly to the farmers in the water scarce region is somewhat surprising. It is often supposed that a water market will primarily benefit those with abundant water, who will be able to extract all rents from those that need the water. However, in this case those with a shortage of water are able to realize greater gains as they have a higher marginal value product for water. This is not altogether inconsistent with the distributional results observed in Chile and Australia (Bjornlund and McKay, 2002; Hearne and Easter, 1996; Romano and Leporati, 2002), as the buyers in those studies also captured a larger share of the gains. In the model presented here, two facts mitigate against this rent extraction. First, a downward-sloping marginal value product together with no price discrimination allows downhill farmers to retain some of the rents. Second, a participation constraint prevents uphill farmers from charging for water that they would otherwise allow to flow away unused. This latter fact reduces potential efficiency gains for the system by preventing the equalization of marginal value products between the regions. However, it also prevents the downhill farmers from facing a reduction of profit with a water market, relative to the situation without one.

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The result that uphill farmers experience an increase in mean profit and an increase in profit variability suggests that farmers in regions with abundant water may be somewhat ambivalent towards a plan to allow water trading. Thus, they may not be as enthusiastic as one might expect to suggestions such as those of Rosegrant and Binswanger (1994) and Zilberman and Lipper (1999) that the current distribution be established as the basis for tradable water rights. In the system modeled in this paper, farmers cannot be forced to sell water that they would otherwise choose to use. Given the planting decision, uphill farmers are better off with a spot market for water than without. However, the planting pattern is selected prior to knowing the water availability, so that in some realizations the autarky cropping pattern may have generated higher returns. This is analogous to purchasing an option, which may never be in the money. If the trading plan is based on pre-planting contracts for water delivery, then states of nature may exist where uphill farmers are worse off with such contracts as compared to not having any option to trade water. It is therefore not surprising that regions – states or parts of irrigation systems – with abundant water are inclined to implement barriers to trading that water.

In this study, it is also shown that the majority of the gains from tradable water rights are captured with a spot market. Thus, if the numerical example used reasonably approximates reality, then we do not need to wait for institutional reforms that can enforce long-term water contracts before introducing market style reforms. Simply entrenching existing *de facto* water rights, as suggested by Rosegrant and Binswanger (1994), and allowing spot trades can capture a substantial share of the efficiency gains. Such reforms should be much easier to accomplish than efforts to implement a complete set of markets through futures and/or forward contracts and like mechanisms.

The results presented are for a partial equilibrium analysis. A general equilibrium approach would consider the impact of reallocating water on factor markets. In a developed country context where factors are highly mobile and agriculture is a small share of the economy, general equilibrium effects are probably small. In spite of this though, they are able to command considerable attention when trading water affects income distributions (Howitt, 1994). In developing country irrigation systems, factor mobility may be more limited and output markets more localized. Factor market adjustments in response to water trades will have further distributional implications, and may further change the distribution of gains.

A number of possible extensions of this work can be identified. One is the inclusion of water contracts that are traded before water availability is realized. Such contracts would be expected to increase the efficiency of the system. However, they may create certain states where some market participants are worse off. How prevalent are those states? Alternatively could contingent contracts – options to purchase or sell water – increase efficiency and limit the occurrence of 'regret' states? Martin *et al*. (2001) suggest precipitation options as a vehicle whereby farmers can insure against weather shocks. What difference does it make if the options are based on the price of water or the realization of the water availability variable?

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Additional decision-makers could also be added. One way to do so would be to include a third region, between the two in the current model. In some situations, this region would purchase water, while in others it would sell water. This intermediate may capture the gains that in the current model are captured by the tail-end. The gravity constraint could also be relaxed, if property rights are well enough established for a banking system to be implemented. With a bank, uphill farmers can purchase water from downhill farmers, as the water is physically still in the reservoir. Relaxing the gravity constraint would benefit uphill farmers, increasing their relative gain. However, in many developing country systems the physical and institutional infrastructure is inadequate to support a water banking system, making riparian rights and a gravity constraint a more realistic representation.

The results of this paper are generated using a stylized model that is only loosely related to actual crops. The model could be parameterized to an actual system with appropriate crop production functions and rainfall data. Crop water response functions have been estimated many times, and thus parameters should be readily available. Rainfall data could be used to estimate a parameterized bivariate distribution or used as an empirical distribution. Since rainfall data are reasonably common, such an extension would be relatively easy to perform. However, since these specifics would tend to mask the relationships illustrated by the example used here, this parameterization is left to future work.

A number of additional features could also be added to the model. In general, one would expect transmission losses between the uphill and downhill regions. This is a transaction cost that could be incorporated. With high transmission losses, water transactions would decrease, and the gains from a water market would fall. If there is uncertainty about the size of those losses – weather-related, theft, measurement problems, etc. – then a further reduction in the use of water markets would be expected. These issues, not included in the current model but clearly important, highlight the fact that implementing a water market is not simply a matter of announcing that such a market exists. Mechanisms for timely delivery for agreed-upon volumes of trade and the enforcement of contracts must be put in place. Where infrastructure is primitive, such as in developing country systems, simply assigning ownership rights and the right to trade may not accomplish much of a change in the water use pattern.

Finally, externalities were not considered, some of which are thought to have substantial impacts (Joshi and Agnihotri, 1984; Singh and Singh, 1995; Kijne, 1996; O'Hara, 1997). The strong result of the numerical example was that the greatest gains from a water market occur when water is scarce in the downhill region. Such scarcity is typical of arid and semi-arid regions where irrigation projects bring former deserts into agricultural production. However, irrigation waters that contain dissolved salts can lead to salt accumulation problems. Subsurface water flow, both near the surface and via the water table, can move these salts, such that the location of the damage is not the location where the water was applied. A related problem is too much water. In the model, excess water simply flowed down the hill or out of the system. Excess water did not adversely affect the total

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value product. This may be unrealistic. If application of excessive water in the uphill region raises the water table in the downhill region, crops can 'drown', depressing yield. Likewise, large rainfall events may lead to flooding, which can also depress yield. A water market may generate opportunities for downhill farmers to purchase water from uphill farmers to reduce these externalities, an extra gain that is not part of the current model (Janmaat, 2005). However, a market itself may not be capable of attaining efficiency without the opportunity to trade rights associated with the source of the externality (Weber, 2001).

The results of this analysis have relevance in other situations where investment decisions must be made before the availability of a critical input is known. The unique aspect of this analysis is the correlation in movements of the water supply and water demand curves, which can enhance price volatility. Financial markets have similar features, in that information shocks are often interpreted in a similar way by buyers and sellers. This correlation leads to larger price fluctuations, as information that increases demand also reduces supply (sellers more reluctant to sell). Aspects of this analysis may also apply to other agricultural industries, such as livestock, where herd sizes are chosen before feed market conditions are known. Since livestock producers often grow some feed as well as purchasing feed, their demand for feed likely increases at the same time that supplies are short. This would enhance price volatility in the same way that water price volatility was enhanced by rainfall correlations in the present analysis.

5. Conclusion

Relaxing the regulatory barriers preventing the trading of water rights is often advocated as a means of increasing the efficiency with which water can be used. For a stylized example, a spot market for water after the crop planting decision is made captures approximately two-thirds of the potential efficiency gains, relative to a fully coordinated crop and water optimization. This suggests that efforts to implement a spot market for water transfers should be pursued, even if it is unclear whether it can be extended to incorporate longer-term contracting. When uncertainty is present, farmers in both a water rich and a water scarce region are better off in expected value terms if they are able to trade water, with the relative gains being greatest for those in the water scarce region. Return variability is reduced for those in the water scarce region, while it is increased in the water rich region. Although both water rich and water poor farmers can expect to gain, the gains to the water rich are relatively small, suggesting that they may be the most difficult to convince of the potential to gain from a water market.

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Appendix: Multiple crop, two region model

A multiple crop, two region version of the model is developed in this appendix. It is assumed that the behavior in each region can be modeled with a representative farmer that is a price taker in all markets. Let crops be indexed by $i \in I$, where I is an index set. The land area planted to crop i by farmer $j \in \{1,2\}$ is $x_{i,j}$, and $w_{i,j} \geq 0$ is the per unit area rate at which water is applied to crop i. The production function for crop i is $g_i(w)$, which is zero everywhere outside $[\alpha_i, \beta_i]$, positive inside, with an interior maximum. Finally, let $s_{i,j} \in [0, x_{i,j}]$ be the share of land planted to crop i that farmer j is applying water to, d_j is the water purchased by j, $d_j < 0$ if j is a supplier of water, and W_j is the water available in region j. With these definitions, the objective function for the representative farmer in region j with $x_{i,j}$ predetermined is

$$\max_{d_j, \{w_{i,j}, s_{i,j}\}_{i \in I}} \left\{ \sum_{i \in I} q_i s_{i,j} g_i(w_{i,j}) - p d_j \right\} \tag{6}$$

subject to

$$\sum_{i \in I} s_{i,j} w_{i,j} \leq W_j + d_j \tag{7}$$

where q_i is the price of crop i and p is the price of water. Equilibrium requires that either $\sum d_j = 0$ or $p = 0$, and gravity demands that $d_i \leq 0$, where region 1 is the uphill region.

The Kuhn–Tucker conditions for farmer j are

$$\begin{array}{lll} -p + \lambda_1 \leq 0 & \lambda_1 \geq 0 & \lambda_1(-p + \lambda_1) = 0 \\ q_i s_{i,j} g_i' - \lambda_1 s_{i,j} \leq 0 & w_{i,j} \geq 0 & w_{i,j}(q_i s_{i,j} g_i' - \lambda_1 s_{i,j}) = 0 \\ q_i g_i - \lambda_1 w_{i,j} - \lambda_{2.i} \leq 0 & \lambda_{2,i} \geq 0 & \lambda_{2,i}(q_i g_i - \lambda_1 w_{i,j} - \lambda_2) = 0 \\ & s_{i,j} \geq 0 & s_{i,j}(q_i g_i - \lambda_1 w_{i,j} - \lambda_2) = 0 \end{array} \tag{8}$$

with λ_1 the multiplier on the water budget constraint and λ_2 the multiplier on the upper share constraint $w_{i,j} \leq x_{i,j}$. From the first equation in (8), it follows that $p = \lambda_1$ if any water is used. The second equation gives $q_i g_i' = \lambda_1$ for each crop where $w_{i,j} > 0$, which together with $p = \lambda_1$ implies that for all irrigated crops, $q_i g_i' = q_k g_k' = p$. The final two equations together imply that, if $0 < s_{i,j} < x_{i,j}$, then $\lambda_{2,i} = 0$ and $q_i g_i = p w_{i,j}$, demonstrating that water is not applied to crop i until the revenues generated by watering crop i equal the cost of watering it.

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The per unit land water demand for each crop i by farmer j is

$$w_{i,j}(p) = \begin{cases} 0 & p \geq \bar{p}_i \\ [0, \bar{w}_i] & p = \bar{p}_i \\ g_i'^{-1}(p/q_i) & 0 \leq p < \bar{p}_i \end{cases} \tag{9}$$

where $\bar{w}_i$ is the water application rate that solves $g_i(w)/w = g_i'(w)$ and $\bar{p}_i = q_i g_i'(\bar{w}_i)$. If $g_i'' < 0$, then demand is decreasing in p and increasing in q_i in the range where demand is nonzero, and $\bar{p}_i$ is increasing in q_i. The aggregate water demand by j is the sum of the demands for each crop that j has planted

$$w_j(p) = \sum_{i \in I} w_{i,j}(p) \tag{10}$$

For each crop, there will be a perfectly elastic segment at the entry water price for that crop.

At planting time, farmer j chooses the values of $x_{i,j}$. These values solve

$$\max_{\{x_{i,j}\}i \in I} \left\{ \sum_{i \in I} q_i x_{i,j} g_i(w_{i,j}(p^*)) - p^* \sum_{i \in I} x_{i,j} w_{i,j}(p^*) \right\} \tag{11}$$

subject to $\sum_{i \in I} x_{i,j} \leq x_j$ with p^* as the equilibrium price and x_j the total land that farmer j controls. With ϕ as the multiplier on the land budget, the Kuhn–Tucker condition for this optimization is

$$q_i g_i(w_{i,j}^*) - p^* w_{i,j}^* - \phi \leq 0 \quad \phi \leq 0 \quad \phi(q_i g_i(w_{i,j}^*) - p^* w_{i,j}^* - \phi = 0) \tag{12}$$

where $w_{i,j}^* = w_{i,j}(p^*)$. It follows from this that it is only optimal to plant more than one crop if two crops are equally profitable per unit land area, given the equilibrium water price p^*. Otherwise, only one crop is planted.

If the planting and water application decision can be made jointly, the problem is solved with $x_{i,j}$ replacing $s_{i,j}$ and the constraints $0 \leq s_{i,j} \leq x_{i,j}$ replaced by $\sum_{i \in I} x_{i,j} \leq x_j$ in equations (6) and (7). The Kuhn–Tucker conditions for this optimization are

$$\begin{array}{lll} -p + \lambda_1 \leq 0 & \lambda_1 \geq 0 & \lambda_1(-p + \lambda_1) = 0 \\ q_i x_{i,j} g_i' - \lambda_1 x_{i,j} \leq 0 & w_{i,j} \geq 0 & w_{i,j}(q_i x g_i' - \lambda_1 x_{i,j}) = 0 \\ q_i g_i - \lambda_1 w_{i,j} - \lambda_2 \leq 0 & \lambda_2 \geq 0 & \lambda_2(q_i g_i - \lambda_1 w_{i,j} - \lambda_2) = 0 \\ & x_{i,j} \geq 0 & x_{i,j}(q_i g_i - \lambda_1 w_{i,j} - \lambda_2) = 0 \end{array} \tag{13}$$

From these conditions it can be shown that $p = q_i g_i'$ and that $0 < x_{i,j} < x_j$ can be true only when $g_i - w_{i,j} g_i' = g_k - w_{k,j} g_k'$, when two crops are equally profitable. In all other cases it is optimal to grow only one crop. The water demand in this situation is equal to $g_i'^{-1}(p/q_i)$ over the region where crop i is the most profitable, and is perfectly elastic over the interval $[g_i'^{-1}(p/q_i), g_k'^{-1}(p/q_k)]$ when crops i and k are equally profitable given water price p.

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