Ameliorating Majority Decisiveness through Expression of Preference Intensity

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In pairwise voting, when a simple majority rule produces a winner, that winner is robust to the minority's preferences. The typical means of protecting the minority from the decisiveness of the majority is by increasing the required majority or by augmenting the simple majority rule with constitutional constraints. In the former case the required majority q becomes larger than one-half, and this implies that the q-majority rule becomes biased in favor of one of the alternatives, usually the status quo. In the latter case the augmented rule becomes biased in favor of the minority. The main issue examined in this paper is whether the amelioration of majority decisiveness can be attained by unbiased voting rules that allow some restricted expression of preference intensities. Our results clarify that the use of scoring rules provides a positive answer to the above question when voters resort to variable degrees of coordinated strategic voting. The results are illustrated in the special cases of the plurality and the Borda rules.

In elections where the outcome is determined by a pairwise simple majority rule, a Condorcet (1785) winner need not exist. In other words, the simple majority rule is often unstable, and therefore, in general, this rule is not a well-defined voting rule (Austen-Smith and Banks 1999). Another weakness of the simple majority rule is that whenever the majority happens to share the same view regarding the best candidate, the majority consensus candidate is always selected, regardless of the minority preferences and, in particular, intensity of preferences (Sen 1970). This second problem of majority decisiveness, namely, the universal robustness of the selection of the majority consensus to the intensity of the minority preferences, can be resolved by increasing the required simple majority to a special or supramajority $q, q > 1/2$, or by augmenting the simple majority rule with constitutional constraints that protect the minority. In the former case the resulting q-majority rule is biased in favor of one of the alternatives, usually the status quo. As established by Greenberg (1979), a sufficient increase in $q, q \geq (k-1)/k$, also resolves the instability problem. In the latter case the resulting rule becomes biased in favor of the minority. The main objective of this paper is to show that the amelioration of the simple-majority decisiveness can be attained by unbiased voting rules that allow some restricted expression of preference intensities, *viz*., by scoring rules. As is well known, these rules are also not vulnerable to the first stability problem, that is, they are well-defined voting rules.

The notion of majority decisiveness is related to the classical notion of majority tyranny, which has drawn a lot of attention, especially since the second half of the eighteenth century. The *locus classicus* is the *Federalist* No. 10 (Madison 1787) and the Madisonian arguments elaborated by Dahl (1956). For Madison, tyranny of the majority was an instance of factionalism—of a majority group pursuing goals contrary to the rights of others or to "the permanent and aggregate interests of the community." For John Stuart Mill, a tyrannical majority violates someone's liberty to do as pleased when not harming others. Tyranny of a majority is thus characterized by two basic elements: (i) factionalism, that is, effective implementation of goals pursued by a specific majority group; and (ii) "unjustness" of at least one favored alternative x imposed by that majority group. Using modern terminology, tyranny of a particular majority group T means that the set of groups that are decisive with respect to those undesirable alternatives consists of T and all the groups that contain it. The point of departure of this study is the different notion of *decisiveness of the majority*, any majority, which is an important property of preference aggregation rules or of voting rules that has been extensively studied by social choice theorists (Arrow 1963; Sen 1970). In the context of voting theory, majority decisiveness is defined as the ability of any majority group to impose its will whenever its members share a common view regarding the desirable collective decision.[1]

Although majority decisiveness and tyranny of the majority are different notions and one does not imply the other, they share a common problematic aspect, namely, their existence entails a large set of losers (the members of the permanent minority group or the members of any minority group) likely to be deeply alienated from the political system. Majority decisiveness is nevertheless a weaker, a less menacing, and, perhaps, even an acceptable property of voting rules relative to the tyranny of the majority for two reasons. First, the incidence of effective concentration of decision-making power implied by majority decisiveness is not restricted to a specific permanent majority group, so it does not imply factionalism. Second, the incidence of majority

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[1] This ability depends on whether the majority-coalition members resort to sincere or to coordinated strategic voting.

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decisiveness is not restricted to "unjust" alternatives, so it is not necessarily bad or undesirable.

In the context of voting systems, the attempts to resolve the problem of a specific tyrant majority and the lesser problem of majority decisiveness have been based on the application of biased voting rules that do not allow the expression of preference intensities. Such biased rules discriminate in favor of voters, the members of the minority group, and/or alternatives, the "just" ones.[2] By definition, unbiasedness toward voters, the anomymity condition, ensures that tyranny of a (specific) majority does not exist. However, it does not necessarily rule out decisiveness of any majority. As already noted, the main objective of this paper is to examine whether the amelioration of majority decisiveness can be attained by applying *unbiased (anonymous and neutral) voting rules* that *allow expression of preference intensities*.[3] Our results clarify how the use of scoring rules can provide a positive answer to the above question. Surprisingly, the role of expression of preference intensities through unbiased rules, in particular, of scoring rules as a means of preventing majority decisiveness, is hardly discussed either in the vast literature on the issue of majority tyranny or in the more formal (social choice) literature on scoring rules. Filling this gap, our main novel contribution is the clarification of the effectivity of scoring rules in alleviating the problem of majority decisiveness.[4]

Scoring rules can be defined in terms of fixed or flexible scores. Under a flexible unconstrained scoring rule voters are assigned an equal number of scores and no constraint is imposed on the allocation of the initial endowment of the scores among the alternatives. One may think that a flexible scoring rule that enables perfect expression of preference intensities better serves the purpose of avoiding decisiveness of the majority relative to any scoring rule based on fixed scores. Our results imply that this expectation is unfounded. The problem of majority decisiveness can be resolved only by scoring rules that allow a limited common degree of expression of preference intensities.

We present a general definition of majority decisiveness that allows variable degrees of coordinated strategic voting. The two extreme cases of zero and an unlimited degree of such coordinated strategic voting that are encountered in discussions on majority decisiveness can be viewed as the two canonical special cases. Suppose that the severity of the problem of majority decisiveness is directly related to the size of the dominated minority and is inversely related to the difficulty for the majority to coordinate its strategic voting. Then the severity of the problem of majority decisiveness of size α resorting to coordinated strategic voting of degree d decreases with an increase in α and apparently with an increase in d, if such an increase implies a harder coordination task.

Our first result clarifies to what extent a given scoring rule alleviates the problem of majority decisiveness when voters sincerely reveal their preferences. The second and main general result clarifies the effectivity of a given scoring rule in resolving the problem of majority decisiveness when voters resort to strategic voting. Assuming that voters do not take advantage of strategic voting or that they fully take advantage of such voting, the results are illustrated in the special cases of the well-known plurality and Borda rules. The main result is also used to identify the scoring rules that prevent all possible degrees of majority decisiveness under unconstrained coordinated strategic voting. It turns out that scoring rules can eliminate all possible decisive majorities.

In the next section we introduce our setting and the two assumptions of voting rules unbiasedness. The notion of majority decisiveness is then presented, followed by the results and their implications. Concluding remarks are contained in the last section. Formal definitions and proofs appear in the Appendix.

ASSUMPTIONS AND SETTING

Let $N=\{1,\ldots,n\}, n\geq 3$, denote a finite set of voters and A a finite set consisting of k distinct alternatives, $k\geq 3$. Individual preference relations are defined over A and are assumed to be strict (indifference is not allowed). In this study we focus on unbiased voting rules that are usually referred to as scoring rules.[5] A voting rule specifies for any given preference profile a nonempty set of alternatives in A. Unbiasedness has two aspects: unbiasedness toward voters and unbiasedness toward alternatives. Unbiasedness toward voters (anonymity) requires invariance of the voting rule with respect to permutations of voters' preferences; if the preference relations of the voters are permuted, then

[2] Biased voting rules violate the two standard symmetry properties of voting rules, namely, neutrality and anonymity.

[3] Recently, Saari and Sieberg (2001) explain why it can be quite likely that a majority loses over most of the issues in a different setting of pairwise voting.

[4] There are two main classes of studies of scoring rules and, in particular, of the Borda (1781) rule. On the one hand, there are those studies that seek to expose the weakness of these rules. The most notable disadvantages of scoring rules are their demanding informational requirements, their susceptibility to manipulation (Nitzan 1985; Smith 1999), and their vulnerability to some disturbing anomalies, the so-called voting paradoxes (Brams 1976; Fishburn 1974; Nurmi 1999; Saari 1989, 2000). On the other hand, there are those studies that can be conceived as proponents of scoring rules. These studies emphasize five types of advantages of scoring rules: first, their stability advantages, *viz*., their success in securing voting equilibria and, thus, ensuring political stability (Moulin 1988; Mueller 2003; Saari 1995; Sen 1970); second, their axiomatic advantages, namely, their uniqueness in satisfying alternative sets of desirable properties of social choice functions (there are more than 40 axiomatizations of scoring rules [see Chebotarev and Shamis 1998 and references therein]); third, their success in selecting an alternative that is the closest to being unanimously preferred according to some reasonable metric between preference profiles (Lerer and Nitzan 1985); fourth, their important advantage of allowing expression of intensity of preferences, in marked contrast to the commonly used majoritarian rules; and, finally, their success in implementing proportional representation (Chamberlin and Courant 1983).

[5] In the literature scoring rules are sometimes referred to as positional rules (Gardenfors 1973) or as point-voting schemes (Mueller 2003; Nitzan 1985).

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the outcome of the voting rule is not affected. Unbiasedness toward alternatives (minimal neutrality)[6] requires appropriate variance of the voting rule with respect to permutations of the alternatives in A; if the alternatives are permuted in the preferences of the voters on A, then the alternative(s) selected by the voting rule changes accordingly. This property ensures that the labeling of the alternatives is immaterial, all that matters is the voters' preferences.

Definition. Let $\{S_1, S_2, \ldots, S_k\}$ be a monotone sequence of real numbers, $S_1 \leq S_2 \leq \cdots \leq S_k$ and $S_1 < S_k$. Each of the n voters ranks the candidates, assigning S_1 points to the one ranked last, S_2 points to the one ranked next to the last, and so on. Under a *scoring rule* a candidate with a maximal total score is elected. If the sequence $\{S_1, S_2, \ldots, S_k\}$ is strictly monotone, that is, $S_1 < S_2 < \cdots < S_k$, the scoring rule is called a *strict scoring rule*.

Scoring rules allow intrapersonal preference intensity because scores are assigned to the alternatives according to their ranking. The intensity of preferring one alternative to another one can be represented by the difference in their scores.[7] Scoring rules also allow a special restricted form of interpersonal preference comparisons because under these rules preference intensities of all voters are implicitly assumed to be comparable and identical, as long as the relative rankings of the compared alternatives are identical. The most common scoring rules are the plurality and the Borda rules.[8]

Definition. The *plurality rule* is a scoring rule where $\{S_1, S_2, \ldots, S_k\} = \{0, 0, \ldots, 1\}$. Hence, the candidate who is ranked first by the largest number of voters is elected. The plurality rule is the most commonly used scoring rule.[9]

Definition. The *Borda rule* is a strict scoring rule where $\{S_1, S_2, \ldots, S_{k-1}, S_k\} = \{0, 1, \ldots, k-2, k-1\}$.

Under the Borda rule each voter reports his preferences by ranking the k candidates from top to bottom (ties are not allowed), assigning no points to the candidate being ranked last, one point to the one being ranked next to the last, and so on, up to $k-1$ points to the most preferred candidate. A candidate with the highest total score, called a Borda winner, is elected.

The Borda rule is the most well-known strict scoring rule.[10] Notice that under this rule the preference intensities of all individuals are comparable and equal as long as the difference between the relative rankings of the compared alternatives in the voters' preference ordering is the same (the relative rankings of the compared alternatives need not be identical).

DECISIVENESS OF THE MAJORITY

When the size of a decisive majority group is T, $|T| = \alpha n$, we say that there exists an α-majority decisiveness, $1/2 < \alpha < 1$. Henceforth α is assumed to be a fraction with denominator n. As noted in the introduction, the severity of the decisiveness problem depends on the size of the majority, namely, on α, as well as on the extent of strategic voting manipulations performed by the members of the majority coalition. *Ceteris paribus*, the severity of the problem decreases with an increase in the majority size. A decisiveness of 90% of the voters is less of a problem than a decisiveness of 55%.[11] A less obvious characteristic of majority decisiveness is the degree of coordinated strategic voting exercised by the members of the majority coalition. In the nonstrategic voting theory voters are assumed sincerely to reveal their (true) preferences. That is, voting manipulations of any sort are ruled out. However, in the more recent strategic-voting theory, typically, any conceivable preference manipulation is allowed, which clearly facilitates the attainment of decisiveness. The existence of a decisive majority in the former case of nonstrategic voting is more menacing and severe than the existence of a decisive majority in the latter case of unconstrained strategic voting, which hinges on more demanding requirements (stricter informational requirements and possibly a higher degree of coordination among the majority-coalition members).

In the current study we allow the above two extreme degrees of manipulation (i.e., sincere voting and unconstrained coordinated strategic voting) as well as intermediate degrees of strategic voting that may characterize the political–economic environment. These degrees represent an essential characteristic of the political culture of the voters, reflecting, for example, the limit to the voters' self restraint in trying to enhance their interest. The degree of strategic voting d is the maximal deviation between the ranking of any

[6] The definition of neutrality follows that of Moulin (1988, 233).

[7] The fact that scoring rules use information on preference intensity and not just information on the voters' ordinal preferences implies that they do not satisfy the independence of irrelevant alternatives (IIA) property. Arrow's (1963) IIA condition is stated as a property of an aggregation function that transforms a preference profile to a social preference relation. This condition is naturally adapted to our context, where the aggregation function is a voting rule that transforms a preference profile to a subset of alternatives; see, for example, Austen-Smith and Banks 1999.

[8] Two related voting rules are "approval voting" (Brams and Fishburn 1978) and the "unconstrained point-voting scheme" (Dodgson 1876; Nitzan 1985). Both of these rules can be considered variants of a scoring rule because they are individual-specific "flexible scoring rules." Approval voting is defined by an individually specific sequence $\{S_1, S_2, \ldots, S_k\}$. Each voter assigns t candidates $(1 \leq t \leq k-1)$ ranked highest a score of 1, where t can vary across voters. This rule is not a scoring rule because, although the sequence $\{S_1, S_2, \ldots, S_k\}$ is of the general form $\{0, 0, \ldots, 1, 1\}$, the rule allows individual flexibility regarding the choice of the sequence. Under the unconstrained point-voting scheme, each voter has complete flexibility in allocating his initial endowment of points. The initial point endowments of the voters are equal and an elected candidate is one who accumulates a maximal number of points.

[9] For an axiomatic characterization of the plurality rule, see Richelson 1978.

[10] For axiomatic characterizations of the Borda rule, see Nitzan and Rubinstein 1981, Saari 1990, and Young 1974.

[11] Certainly the problem does not exist when $\alpha = 1$. When $\alpha = (n-1)/n$ the problem is minimal. In fact, in such a case "decisiveness" might be considered desirable and it might be referred to as "minimal democracy" (Austen-Smith and Banks 1999).

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alternative in the true and reported preferences of the voters.

Example 1. Suppose that individual i's true and reported preferences on the five alternatives a, b, c, e, and f are as in Table 1. The fourth column specifies the deviation in the ranking of the reported alternative. The maximal deviation is obtained for alternative f. In the reported preferences this alternative is ranked second, while according to the true preferences it is ranked fifth. Hence, in this example of a single voter the degree of strategic voting is three.

TABLE 1. Example 1

Rank	True Preferences	Reported Preferences	Deviation in the Ranking of the Reported Alternative
1st	*a*	*b*	1
2nd	*b*	*f*	3
3rd	*c*	*a*	2
4th	*e*	*e*	0
5th	*f*	*c*	2

The degree of strategic voting exercised by the members of the majority coalition can be represented by d. In the literature the two commonly assumed values of d are the extreme values 0 and $k-2$. When $d=0$, the voters report their true preferences. When $d=k-2$, strategic voting is unconstrained. The case $d=k-1$ is disregarded because the definition of decisiveness is based on majority consent regarding the most preferred (top-ranked) alternative. In such a case manipulation is performed over the remaining $k-1$ alternatives, which implies that the degree of strategic voting does not exceed $k-2$.

In this study the voting rules are assumed to satisfy the unbiasedness requirements, namely, anonymity and neutrality. Unbiasedness toward voters implies that if there is an α-majority decisiveness of some specific group, then there is an α-majority decisiveness of any group of size α. That is, due to the anonymity property, decisiveness is contagious: The incidence of effective concentration of decision-making power is not restricted to a specific majority group. In other words, permanent factionalism is impossible. This already rules out the classical tyranny of a specific permanent majority. Unbiasedness toward alternatives implies that if there exists an α-majority decisiveness of some group that can impose a particular ("unjust") alternative, then it can impose the selection of any alternative. That is, due to the neutrality property, decisiveness is contagious in another sense: The incidence of the effectivity of the decisive majority coalition is not restricted to a specific alternative or subset of alternatives but is unlimited.

By definition, then, under anonymous and neutral voting rules the classical tyranny of the majority is impossible. However, the lesser problem of majority decisiveness may still exist. Such decisiveness means that *every* majority group resorting to sincere or coordinated strategic voting can *always* impose its will (its most desired alternative). That is, it can ensure the selection of the majority consensus under any preference profile.[12] In general, elimination of the classical tyranny of the majority and amelioration of majority decisiveness are two different issues. In our setting, however, where the voting rules are unbiased toward voters and toward alternatives, the only relevant issue is the amelioration of majority decisiveness.

By definition, a decisive coalition T resorting to a degree of manipulation d can impose its will without having any knowledge of the preferences or the votes of the individuals outside T. This means that whenever T has a consensus candidate its members can adopt contingency-free voting strategies that ensure the selection of the consensus candidate regardless of the voting strategies adopted by the members of the remaining minority. Whether or not majority decisiveness exists depends on the voting rule V applied by the voters. If there exists a decisive majority of size α, we say that the voting rule V is vulnerable to an α-majority decisiveness.

The following example illustrates the possible existence of majority decisiveness under the unbiased Borda rule.

Example 2. Suppose that there are three voters whose true preference relations over the four alternatives a, b, c, and e are presented in Table 2. In this case, alternative b is selected by the Borda rule because it receives the largest number of points (seven), where alternative a, which is preferred by a majority of 66%, receives only six points. However, voters 1 and 2 can guarantee the selection of alternative a, even without knowing voter 3's preferences. Suppose that voters 1 and 2 report the preferences presented in Table 3. In this case, independent of voter 3's preferences, the selection of alternative a is guaranteed, since it already receives six points from voters 1 and 2, where every other alternative receives only two points. The maximal score that can be assigned by voter 3 to any of these alternatives is three points, which cannot change the sure selection of alternative a. Notice that in this example the majority group consisting of voter 1 and voter 2 delivering a d-message ($d=2$) is decisive. Since similar examples can be constructed for majority coalitions that consist of voters 1 and 3 or of voters 2 and 3, there exists a two-thirds-majority decisiveness.

Resolution of the problem of majority decisiveness requires that such decisiveness does not exist. In our setting the problem is therefore resolved if there exists no coalition T, $|T|=\alpha n$, $1>\alpha>1/2$, which is d-decisive, $0\le d\le k-2$.

The following three notable attempts have been made to resolve the problem of majority tyranny. Unlike this study, these attempts resort to biased voting rules that are not neutral or not anonymous. The first attempt is based on the application of special-majority

[12] Usually (Arrow 1963; Sen 1970), a coalition is called decisive if it can always impose the common preference relation of its members on the social preference relation. Here we define majority decisiveness in terms of the ability of the majority-group members to impose the selection of their common most preferred alternative.

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TABLE 2. Example 2, Part I

Rank	Voter 1's Preferences	Voter 2's Preferences	Voter 3's Preferences	Borda's Rank
1st	*a*	*a*	*b*	*b* (7 points)
2nd	*b*	*b*	*e*	*a* (6 points)
3rd	*c*	*e*	*c*	*e* (3 points)
4th	*e*	*c*	*a*	*c* (2 points)

TABLE 3. Example 2, Part II

Rank	Voter 1's Preferences	Voter 2's Preferences	Voter 3's Preferences	Borda's Rank
1st	*a*	*a*	?	*a* (at least 6 points)
2nd	*b*	*e*	?	? (max. 5 points)
3rd	*c*	*c*	?	?
4th	*e*	*b*	?	?

rules.[13] Such rules are anonymous, that is, the alternatives chosen by these rules are insensitive to permutations of the voters' preferences. Hence, if tyranny of some T-majority coalition is avoided, then tyranny of any S-majority coalition of equal size, $|T| = |S|$, is also avoided. However, these rules are not neutral. As noted above, control over α, *viz*., an increase in the required special majority, directly reduces the severity of the tyrant majority problem but does not eliminate tyranny.[14,15]

The second attempt is based on the relaxation of the anonymity property to guarantee respect for the preferences of some predetermined subsets of society, namely, the preferences of some minority groups. The protection of rights of some minority groups is typically secured by constitutional means. The issue of majority tyranny often comes up in the context of the debate about simple majoritarian concepts of democracy versus concepts of liberal democracy that focus on the protection of minority rights (Dahl 1956; Riker 1982).

The third attempt is based on the incorporation of an anonymous veto function into the voting rule. Such a veto function allows every individual, or some coalitions of individuals, to veto one or more alternatives, where the larger the coalition is, the higher the number of alternatives the coalition may veto. The function leaves at least one alternative that is not vetoed by anyone.

Respect for the minority principle requires that *every* coalition (and, in particular, small ones) has some minimal effect on the voters' selected alternatives. Note that while the first two attempts give up respect for the minority principle, the third one does respect this principle.

The main claim of this study is that there exists an alternative effective means of alleviating the problem of majority decisiveness. Allowing any degree of strategic voting, our proposed protection of minority rights is based on the use of the well-known family of anonymous and neutral voting rules, namely, the scoring rules. This alternative resolution of the problem differs from the existing solutions. First, by definition, the use of scoring rules does not imply the requirement for a special majority.[16] Second, scoring rules do satisfy the anonymity and neutrality properties. And, third, scoring rules do not imply the existence of any veto function.

RESULTS

In this section we examine the effectivity of scoring rules as a means of resolving the problem of majority decisiveness, first under sincere voting, $d = 0$, and then under coordinated strategic voting, $0 < d \leq k - 2$. The implications of the results are derived for the two extreme cases of sincere voting and unconstrained coordinated strategic voting, focusing on the two most commonly used scoring rules: the plurality and the Borda rules. For a given number of alternatives k, the following result characterizes the values of α for which a particular scoring rule is not vulnerable to an α-majority decisiveness.

Theorem 1. Under sincere voting, $d = 0$, a scoring rule is immune to an α-majority decisiveness, $1/2 < \alpha < 1$, if and only if

$$\alpha(S_k - S_{k-1}) < (1 - \alpha)(S_k - S_1). \qquad \textbf{(1)}$$

[13] Such rules are sometimes referred to as supramajority or qualified-majority rules.

[14] Notice that the severity of the problem of majority decisiveness can sometimes be indirectly reduced by control of d, *viz*., by reducing the exercised degree of manipulation.

[15] Instability of supramajority rules also reduces the problematics of majority tyranny. According to this interesting observation (Miller 1983), "The pluralist political process leads to unstable political choice and such instability of choice in fact fosters the stability of pluralist political systems."

[16] In a binary decision context, the requirement for a special majority is usually justified when there exists asymmetry in the payoffs of the two possible alternatives. The use of such a rule makes the barrier to change from a status quo alternative commensurate with the significance (payoff implications) of the change. The use of such rules may, however, entail a significant cost because it may stultify any choice other than the status quo, thus greatly limiting the ability of the group to act.

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Proof. See the Appendix.

The proof of this theorem is based on the following idea: First, identify the most unfavorable circumstances (preference profile) for the majority and, then, derive the condition that ensures that, even in those circumstances, the α- majority can impose its will. This condition certainly ensures that in any other more favorable situation the majority is capable of imposing its will. Hence, if this condition is not satisfied, the scoring rule is immune to an α-majority decisiveness. The most unfavorable preference profile for the majority has the following two characteristics. First, the members of the majority coalition share the same view regarding the first and second most preferred alternatives. Second, the most preferred alternative for the majority is the worst for the minority members and the second-best alternative for the majority is the best alternative for the minority members. The idea of the proof is illustrated in the following example.

Example 3. Suppose that the voting rule is the Borda rule and that there are four voters and five alternatives. In this case, $S_k - S_{k-1} = 1$ and $S_k - S_1 = 4$. By substituting these values in inequality (1), we obtain that under sincere voting, when $n = 4$ and $k = 5$, the Borda rule is immune to an α-majority decisiveness if and only if $\alpha < 4(1-\alpha)$, from which we obtain that $\alpha < 0.8$. In this case, then, even a majority of three of the four individuals is not tyrant. A typical most unfavorable profile for the three-member majority coalition consisting of individuals 1, 2, and 3 is presented below; a and b are the first and second-best alternatives of the majority group, and voter 4's best and worst alternatives are, respectively, b and a. The preferences of the four individuals over the five-alternative set $\{a, b, c, e, f\}$ are presented in Table 4.

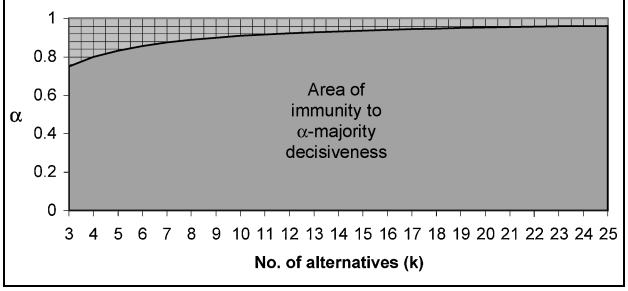
TABLE 4. Example 3

Rank	Preferences of Voters 1–3	Voter 4's Preferences	Borda's Rank
1st	*a*	*b*	*b* (13 points)
2nd	*b*	*c*	*a* (12 points)
3rd	*c*	*e*	*c* (9 points)
4th	*e*	*f*	*e* (5 points)
5th	*f*	*a*	*f* (1 point)

In this most unfavorable unfortunate case for the majority, the majority coalition cannot impose its will (alternative a) and alternative b is selected, because the difference between the total points given by the majority to alternatives a and b does not exceed the largest possible difference in the points assigned to these two alternatives by the minority group member, voter 4. Figure 1 presents the combinations of α and k resulting in immunity of the Borda rule to an α-majority decisiveness, under sincere voting.

By definition, the problem of majority decisiveness is resolved if the scoring rule is immune to decisiveness of any α- majority, $1/2 < \alpha < 1$. In such a case the rule must therefore be immune to majority decisiveness for $\alpha = (n-1)/n$. Immunity to majority decisiveness implies not that every voter is a vetoer, but that every voter has a veto power under at least one preference profile. Theorem 1 has the following direct consequences.

FIGURE 1. Vulnerability of the Borda Rule to an α-Majority Decisiveness for k Alternatives Under Sincere Voting ($d = 0$)



Under sincere voting, $d = 0$:

(i) A scoring rule is immune to majority decisiveness if and only if

$$(n-1)(S_k - S_{k-1}) < (S_k - S_1). \qquad \textbf{(2)}$$

This is directly verified by substituting $\alpha = (n-1)/n$ into inequality (1).

(ii) Under the plurality rule, where $S_k = 1$ and $S_{k-1} = S_1 = 0$, inequality (2) reduces to $n - 1 < 1$, which can never be satisfied. Hence, independent of n and k, the above inequality cannot be satisfied. This implies that the plurality rule is always vulnerable to majority decisiveness. In fact, inequality (1) cannot be satisfied for $S_k = 1$, $S_{k-1} = S_1 = 0$, and any α, $1/2 < \alpha < 1$. This means that, independent of n and k, under sincere voting, the plurality rule is vulnerable to any α-majority decisiveness.

(iii) Under the Borda rule, where $S_k = k - 1$, $S_{k-1} = k - 2$, and $S_1 = 0$, inequality (2) reduces to $n < k$. Thus, the Borda rule is vulnerable to decisiveness of the majority if and only if $n \geq k$. In fact, independent of n, the Borda rule is immune to an α-majority decisiveness if $\alpha < (k-1)/k$. An alternative form of the condition ensuring the immunity of the Borda rule to an α-majority decisiveness is $k > 1/(1-\alpha)$. That is, a sufficient increase in the number of alternatives is an effective means of preventing the decisiveness of an α-majority.

Theorem 1 and its consequences focus on majority decisiveness under sincere voting. Let us turn to the generalization of the theorem in the context of coordinated strategic voting.

Theorem 2. Let $\alpha n = m(d+1)$ for some integer m. Under coordinated strategic voting of degree d, a scoring rule is vulnerable to an α-majority decisiveness, $1/2 < \alpha < 1$, if

$$\alpha > \frac{S_k - S_1}{2S_k - S_1 - \bar{S}}, \quad \text{where} \quad \bar{S} = \frac{1}{d+1} \sum_{j=k-(d+1)}^{k-1} S_j. \qquad \textbf{(3)}$$

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Proof. See the Appendix.

The condition that guarantees the vulnerability of a scoring rule to an α-majority decisiveness requires that α should be larger than an expression that depends on three factors: (i) the score assigned by every individual to the worst alternative, S_1; (ii) the score assigned by every individual to the most preferred alternative, S_k; and (iii) the average score $\bar{S}$ assigned by the individuals exercising a degree of strategic voting d to the $d+1$ alternatives whose rank was strategically coordinated. The proof of this theorem is based on the following idea: First, identify the most favorable circumstances (preference profile) for the majority and, then, derive the condition that ensures that, even in those circumstances, the α- majority cannot impose its will. This condition certainly ensures that under any other less favorable situation the majority is unable to impose its will. Hence, if this condition is not satisfied, the scoring rule is vulnerable to an α-majority decisiveness. The most favorable preference profile for the majority has the following two characteristics. First, the members of the majority coalition share the same view regarding their most preferred alternative. Second, the coordinated voting strategy of the majority group attempts to spread the total scores left to the group equally over the other alternatives and thus minimize the maximal total score assigned to any of the remaining alternatives. The following example clarifies the idea of the proof.

Example 4. The true preferences of three voters over the four alternatives a, b, c, and e are given in Table 5. Under sincere voting the Borda rule would select alternative b. However, the first two voters can report preferences as in Table 6, which would result in the selection of alternative a, provided that the reported preferences of voter 3 do not change. The alternative voting strategy of voters 1 and 2 might be plausible only if they form some particular beliefs regarding the reported preferences of voter 3, since the latter voter might as well report that his preference relation is $e \succ c \succ b \succ a$, in which case candidate e is selected, again, preventing the selection of a. Nevertheless, the two voters have an optimal coordinated voting strategy that guarantees the selection of alternative a, regardless of the reported preferences of the third voter. According to this strategy the two voters sincerely report a as their best alternative and spread their remaining total points equally over the other three alternatives. That is, voter 1 reports the preference relation $a \succ c \succ e \succ b$ and voter 2 reports the preference relation $a \succ b \succ e \succ c$. In this case, regardless of the preferences of the third voter, alternative a gains enough points to secure its selection (even if it receives no points from the third voter).

Under sincere voting, $d=0$, $\bar{S}=S_{k-1}$, and $\alpha > (S_k - S_1)/(2S_k - S_1 - S_{k-1})$ is the necessary and sufficient condition for the vulnerability of a scoring rule to an α-majority decisiveness. This result is an alternative statement of Theorem 1.

Under unconstrained coordinated voting strategies, that is, when $d=k-2$, $\bar{S}=[1/(k-1)]\sum_{j=1}^{k-1} S_j$ is the lower bound of the maximal score that can be assigned by the majority coalition to a candidate differing from the majority consensus. If $\alpha n = m(d+1)$ for some integer m, then this bound is equal to that maximal score that is, in fact, the score assigned by the majority coalition to all $k-1$ candidates who are not the majority consensus. Theorem 2 has the following direct consequences.

(i) Under unconstrained coordinated strategic voting, $d=k-2$, a scoring rule is immune to an α-majority decisiveness, $1/2 < \alpha < 1$, if $\alpha < (S_k - S_1)/(2S_k - S_1 - \bar{S})$.

(ii) If $\alpha n = m(d+1)$ for some integer m, then a scoring rule is immune to an α-majority decisiveness, $1/2 < \alpha < 1$, if and only if $\alpha < (S_k - S_1)/(2S_k - S_1 - \bar{S})$.

(iii) By inequality (3) it can be readily verified that when $d>0$, the plurality rule (where $S_k=1$ and $S_1=0$) is vulnerable to an α-majority decisiveness for any α, $1/2 < \alpha < 1$.

(iv) Under unconstrained coordinated strategic voting, $d=k-2$, and a sufficiently large number of voters, the Borda rule is immune to the decisiveness of an α- majority as long as $\alpha \leq \frac{2}{3}$. Figure 2

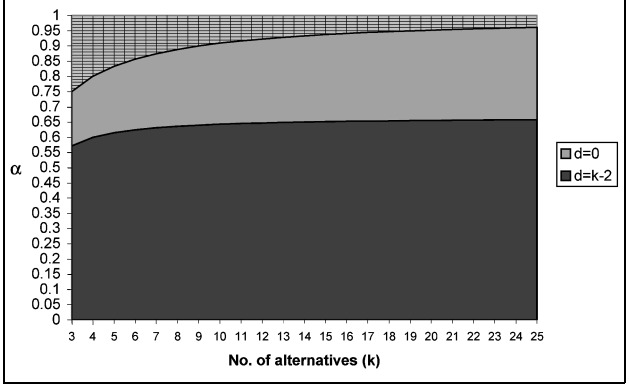
TABLE 5. Example 4, Part I

Rank	Preferences of Voters 1 and 2	Voter 3's Preferences	Borda's Rank
1st	*a*	*b*	*b* (7 points)
2nd	*b*	*c*	*a* (6 points)
3rd	*c*	*e*	*c* (4 points)
4th	*e*	*a*	*e* (1 point)

TABLE 6. Example 4, Part II

Rank	Preferences of Voters 1 and 2	Voter 3's Preferences	Borda's Rank
1st	*a*	*b*	*a* (6 points)
2nd	*e*	*c*	*e* (5 points)
3rd	*c*	*e*	*c* (3 points)
4th	*b*	*a*	*b* (3 points)

FIGURE 2. Vulnerability of the Borda Rule to an α-Majority Decisiveness for *k* Alternatives When *d* = 0 and *d* = *k* − 2



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presents the combinations of α and k resulting in immunity of the Borda rule to an α-majority decisiveness under unconstrained strategic voting ($d = k - 2$). The area of immunity in this case is contained in the area representing immunity under sincere voting ($d = 0$).

The plurality rule is a scoring rule that can be considered as a restricted form of approval voting.[17] Theorem 2 implies that the vulnerability of the Borda rule to majority decisiveness is equal to the vulnerability of the scoring rule defined by the scores $\{S_1, S_2, \ldots, S_{k-1}, S_k\} = \{0, 0, \ldots, 0, 0, 1, 1, \ldots, 1, 1\}$. This rule is another restricted form of approval voting. In fact, the vulnerability of the restricted forms of approval voting to majority decisiveness can be equal to that of other representative scoring rules. Restricting the voters to the use of these rules may therefore preserve the potential of standard scoring rules such as the Borda rule to ameliorate majority decisiveness while considerably reducing the informational requirements of the elections. Put differently, majority decisiveness can be ameliorated by the simple and easy-to-implement restricted forms of approval voting.

When voting is sincere, the size of a decisive majority is relatively large. Under coordinated strategic voting the size of a decisive majority is reduced. However, coordinated strategic voting might be realistic in small committees but it is highly unrealistic when the number of voters is large. It can be shown that majority decisiveness is sustained by strong Nash equilibrium strategies.[18] The decisiveness of the majority could therefore be based not on the requirement of coordinated strategic voting, but on the notion of strong Nash equilibrium. This implies that the implications of Theorem 2 are relevant for strategic voting by few or by many voters.

The last question we examine is whether effective unconstrained manipulations that result in an α-majority decisiveness without requiring any knowledge or even any beliefs regarding the minority preferences can be prevented by the use of an appropriate scoring rule. The above four consequences of Theorem 2 directly imply that the answer to this question is positive.

Theorem 3. Under unconstrained strategic voting, $d = k - 2$, and any number of voters n, $k \geq n$, there always exists a scoring rule that is immune to majority decisiveness.

Proof. See the Appendix.

CONCLUSION

The resolution of the problem of majority decisiveness is usually based on the application of voting rules that neither are unbiased toward the voters and/or toward the alternatives nor allow the expression of preference intensities. The question motivating this study is whether the amelioration of majority decisiveness can be attained by unbiased voting rules that do allow expression of preference intensities. The affirmative answer to this question is the main message of our analysis. It implies that the amelioration of majority decisiveness need not be based on a voting rule that is biased in favor of alternatives (the "just" ones) or in favor of voters (the minority-group members). Specifically, scoring rules are the unbiased voting rules that enable the resolution of the majority-decisiveness problem by allowing the expression of preference intensities. The effectivity of these rules as a means of resolving the problem of tyrant majorities has been demonstrated under sincere voting as well as under any degree of coordinated strategic voting.

Scoring rules are defined in terms of fixed scores. Our analysis implies that a possible rationale for this rigidity is that the restricted common pattern of expression of preference intensities imposed on voters who use a scoring rule is precisely the means that enables the resolution of the majority-decisiveness problem. The unrestricted point-voting scheme is a flexible scoring rule where voters are assigned an equal number of scores and no constraint is imposed on the allocation of the initial endowment of the scores among the candidates.[19] Under this flexible scoring rule, the optimal strategy for every member of a majority coalition interested in securing the selection of the majority consensus, the most preferred candidate from the viewpoint of the majority is to overload his vote points on the majority consensus. Such a strategy is always effective in ensuring the selection of the candidate preferred by the majority. In other words, it results in decisiveness of the majority. The fixed scoring rules impose a fixed uniform pattern of expression of preference intensities on all voters. Despite this restriction, voters can still vote strategically and assign the enforced fixed scores in a way that best serves their true interest. Under a scoring rule, the most effective strategy for every member of a majority coalition interested in securing the selection of the majority consensus candidate is to report sincerely that candidate as his most preferred one. As for the remaining candidates, effective strategic majority voting requires that the members of the majority coalition coordinate their reported preferences to avoid overloading of vote points as much as possible. Specifically, the interest of the majority coalition is to spread the coalition's assigned points equally over the remaining candidates, subject to the constraint of the point allocation permitted by the scoring rule. Whether or not such a strategy is effective depends on the scoring rule in use and on the size of the majority. Our main result states the condition ensuring the effectivity of a scoring rule in preventing decisiveness of the majority.

As is well known, the use of scoring rules involves payment of a price because these rules are highly manipulable and vulnerable to some embarrassing voting

[17] See footnote 8.

[18] We are indebted to Steve Brams for calling our attention to this point. Note that our analysis is confined to the existence of a decisive α majority, that is, to situations in which members of an α-majority group prefer the selection of the same candidate. In such situations there are multiple strong Nash equilibria characterized by inequality (3), given d and the scores $S_1, \ldots, S_k$.

[19] See footnote 8.

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anomalies. The question is whether their well known axiomatic advantages and their ability to resolve the decisiveness majority problem, an advantage that has been the main concern of this study, justify the payment of such a price.

In the literature voting rules have been compared in terms of their axiomatic properties, in terms of the degree they satisfy a particular desirable property such as the Condorcet (1785) criterion or decisiveness (stability), in terms of their informational requirements, or in terms of their vulnerability to strategic manipulations or some undesirable voting anomalies. In this paper we proposed a new criterion for comparing voting rules, namely, their susceptibility to decisiveness of the majority. We have applied this criterion in the comparison of two notable voting rules, the plurality rule and the Borda rule. A comprehensive comparative study of other voting rules based on the proposed criterion, which is beyond the scope of our study, will certainly constitute a further contribution to the theory of voting.

APPENDIX

Let $N=\{1,\dots n\}, n\geq 3$, denote a finite set of voters; A, a finite set consisting of k distinct alternatives, $k\geq 3$; and $L(A)$, the set of linear orderings (complete, transitive, and asymmetric relations) over A. Voter i's preferences, $i\in N$, are denoted $u^i\in L(A)$. A preference profile is an n-tuple $u=\{u_i\}_{i\in N}\in L(A)^n$, the set of linear profiles on A.[20] A voting rule V is a mapping from $L(A)^n$ to the set of nonempty subsets of A. $V(u)$ describes the alternatives selected by the voters at the preference profile u. In this study we focus on voting rules that are usually referred to as scoring rules.[21]

Definition. *The degree of strategic voting*, d, an integer that satisfies $0\leq d\leq k-2$, is the maximal deviation between the true and the reported ranking of any alternative by any voter.

Definition. A *d-message* is a preference profile in which the degree of strategic voting is equal to d.

Definition. Given a voting rule V, a coalition of voters T is *d-decisive* if at every preference profile $u^{a(T)}$ where alternative a is the T-majority consensus, T has a d-message in $L(A)^t$, $t=|T|$, that ensures the selection of a.[22]

Majority decisiveness can now be defined in terms of d-decisiveness.

Definition. *$\alpha-$Majority decisiveness* means that there exists a coalition T, $|T|=\alpha n$, $1>\alpha>{}^1\!/_2$, and a degree of strategic voting d, $0\leq d\leq k-2$, such that T is d-decisive.

Definition. *Majority decisiveness* means that an α-majority decisiveness exists for some α and d, $1>\alpha>{}^1\!/_2$, $0\leq d\leq k-2$.

A tyrant α-majority is represented by the parameters α and d, namely, by the size of the decisive majority and by the degree of strategic voting exercised by its members. Recall that the severity of the decisiveness problem decreases with both of these parameters.

A veto function is defined as follows (Moulin 1988, 72).

Definition. Given the set of alternatives A and a group of voters N with respective cardinalities k and n, an *anonymous veto function* is a nondecreasing function v from $\{1,\dots,n\}$ to $\{0,\dots,k-1\}$, where $v(t)=m$ means that any coalition of size t can veto (prevent the election of) any subset with at most m alternatives.

Theorem 1. Under sincere voting, $d=0$, a scoring rule is immune to an *α-majority* decisiveness, ${}^1\!/_2<\alpha<1$, if and only if,

$$\alpha(S_k-S_{k-1})<(1-\alpha)(S_k-S_1).$$

Proof. Consider a preference profile where all αn members of an *α-majority* coalition, ${}^1\!/_2<\alpha<1$, share the same preference regarding the best (most preferred) alternative a and regarding the second best alternative b.[23] Also suppose that in this profile the minority voters' best alternative is b and their least preferred alternative is a. Notice that if the majority consensus alternative a is selected under such a most unfavorable profile, namely, under a profile where the majority consensus a gets minimal support from the minority and the challenger b receives maximal support from the members of both the majority and the minority coalitions, then the majority consensus a is selected under any other profile, so, by definition, the majority is decisive. Hence, an α-majority decisiveness does not exist if, under the assumed most unfavorable profile, the total score of alternative a is less than the total score of b. That is, the condition ensuring immunity to decisiveness of an α majority is

$$\alpha nS_k+(1-\alpha)nS_1<\alpha nS_{k-1}+(1-\alpha)nS_k,$$

which is equivalent to (1). The proof is therefore complete. ■

Theorem 2. Let $\alpha n=m(d+1)$ for some integer m. Under coordinated strategic voting of degree d, a scoring rule is vulnerable to an α-majority decisiveness, ${}^1\!/_2<\alpha<1$, if

$$\alpha>\frac{S_k-S_1}{2S_k-S_1-\bar{S}},\qquad \text{where}\qquad \bar{S}=\frac{1}{d+1}\sum_{j=k-(d+1)}^{k-1}S_j.$$

Proof. Consider a preference profile where all αn members of an α-majority coalition, ${}^1\!/_2<\alpha<1$, share the same preference regarding the best alternative a.[24] To impose the selection of its consensus alternative a, that is, to be d-decisive, the α-majority coalition must prevent the selection of *any* alternative other than candidate a at *all* possible preference profiles. An optimal strategy for attaining this goal must have two components. First, such a strategy requires that every member of the majority coalition sincerely reports the majority consensus a as his most preferred candidate. Second, such a strategy requires the minimization of the maximal total score assigned by the majority coalition to one of the remaining $k-1$ alternative candidates, that is, to the "coalition's reported second-best" candidate. This means that the majority-coalition members need to coordinate their reported preferences to avoid overloading of scores as much as possible. That is, they have to spread the coalition's assigned scores equally over the remaining $k-1$ candidates, subject to

[20] The notation and the definitions of scoring rules follow those of Moulin (1988).
[21] See footnote 5.
[22] d-Decisiveness is a special case of the widely used notion of winning (see, e.g., Moulin 1988). Given a voting rule V, a coalition of voters T is a winning coalition if, for every candidate a, coalition T has a message $u_T\in L(A)^t$ that ensures the selection of a. That is, $\forall u_{N-T}\in L(A)^{n-t}$, $V(u_T,u_{N-T})=a$.
[23] To simplify the proof and with no loss of generality, we select α such that αn is an integer.
[24] Again, to simplify the proof and with no loss of generality, we select α such that αn is an integer.

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the constraint of the scores $\{S_1, S_2, \ldots, S_{k-1}\}$ that must be assigned to these candidates and subject to the constraint on the permitted degree of manipulation d. Under these constraints, even if the coalition members share the same preference relation (not only the same best alternative), where the score assigned by the coalition to its true second-best alternative under sincere voting is the maximal possible score ($\alpha n S_{k-1}$), the α-majority coalition can ensure that the average score $\bar{S}$ assigned by its members to the coalition's reported second-best candidate is only equal to $1/(d+1)\sum_{j=k-(d+1)}^{k-1} S_j$. Since, by assumption, $\alpha n = m(d+1)$ for some integer m, this average score is achieved when the coalition members' reported preferences are cyclical over the $d+1$ alternatives that are ranked as second, third, and so on, up to the $d+2$ position (the scores assigned to these alternatives range from $S_{k-(d+1)}$ to S_{k-1}). Such a strategy is effective, that is, the α-majority coalition is decisive if the lowest possible total score assigned to candidate a under any preference profile, $\alpha n S_k + (1-\alpha)nS_1$, is greater than the highest possible total score that can be assigned to any other alternative under any preference profile, $\alpha n\bar{S} + (1-\alpha)nS_k$. That is, a sufficient condition for an α-majority decisiveness is

$$\alpha n S_k + (1-\alpha)nS_1 > \alpha n\bar{S} + (1-\alpha)nS_k,$$

which is equivalent to the requirement that $\alpha > (S_k - S_1)/(2S_k - S_1 - \bar{S})$. ■

Theorem 3. Under unconstrained voting manipulations, $d = k-2$, and any number of voters n, $n \geq k$, there always exists a scoring rule that is immune to majority decisiveness.

Proof. Consider the strict scoring rule defined by the strictly monotone sequence $\{S_1, S_2, \ldots, S_k\}$ that satisfies $S_k < (n-1)/(n-2)\bar{S}$, where $\bar{S} = [1/(k-1)]\sum_{j=1}^{k-1} S_j$.[25] By implications (i) and (ii) of Theorem 2 and the definition of majority decisiveness, this scoring rule is immune to majority decisiveness. The existence of the inequality is guaranteed by the possibility to control $\bar{S}$, making it as close as needed to S_k. ■

REFERENCES

Arrow, Kenneth J. 1963. *Social Choice and Individual Values*. 2nd ed. New Haven, CT: Yale University Press.

Austen-Smith, David, and Jeffrey S. Banks. 1999. *Positive Political Theory I: Collective Preferences*. Ann Arbor: University of Michigan Press.

Borda, Jean C. de. 1781. "Memoire Sur les Elections au Scrutiny." *Historoire de l'Academie Royals des Sciences*, Paris.

Brams, Steven J. 1976. *Paradoxes in Politics: An Introduction to the Non-obvious in Political Science*. New York: Free Press.

Brams, Steven J., and Peter C. Fishburn. 1978. "Approval Voting." *American Political Science Review* 72 (September): 831–47.

Chamberlin, John R., and Paul N. Courant. 1983. "Representative Deliberations and Representative Decisions: Proportional Representation and the Borda Rule." *American Political Science Review* 77 (September): 718–33.

Chebotarev, Pavel U., and Elena Shamis. 1998. "Characterizations of Scoring Methods for Preference Aggregation." *Annals of Operations Research* 80: 299–332.

Condorcet, Marquis de. 1785. *Essai sur l'application de l'analyse a la Probabilite des Decisions Rendues a la Pluralite des Voix*, Paris.

Dahl, Robert A. 1956. *A Preface to Democratic Theory*. Chicago: University of Chicago Press.

Dodgson, Carroll L. 1876. "A Method of Taking Votes on More than Two Issues." Reprinted in *Black* 1958: 224–34.

Fishburn, Peter C. 1974. "Paradoxes of Voting." *American Political Science Review* 68 (June): 537–46.

Gardenfors, Peter. 1973. "Positional Voting Functions." *Theory and Decision* 4: 1–24.

Greenberg, Joseph. 1979. "Consistent Majority Rules over Compact Sets of Alternatives." *Econometrica* 47: 627–36.

Lerer, Ehud, and Shmuel Nitzan. 1985. "Some General Results on the Metric Rationalization for Social Decision Rules." *Journal of Economic Theory* 37 (1): 191–201.

Madison, James. 1787. *Federalist*, paper No. 10.

Miller, Nicholas R. 1983. "Pluralism and Social Choice." *American Political Science Review* 77 (September): 734–47.

Moulin, Herve. 1988. *Axioms of Cooperative Decision Making*. Econometric Society Monograph. Cambridge: Cambridge University Press.

Mueller, Dennis C. 2003. *Public Choice III*. Cambridge: Cambridge University Press.

Nitzan, Shmuel. 1985. "The Vulnerability of Point-Voting Schemes to Preference Variation and Strategic Manipulation." *Public Choice* 47 (2): 349–70.

Nitzan, Shmuel, and Ariel Rubinstein. 1981. "A Further Characterization of Borda Ranking Method." *Public Choice* 36 (1): 153–58.

Nurmi, Hannu. 1999. *Voting Paradoxes and How to Deal with Them*. Berlin: Springer-Verlag.

Richelson, Jeffrey T. 1978. "A Characterization Result for the Plurality Rule." *Journal of Economic Theory* 19 (2): 548–50.

Riker, William H. 1982. *Liberalism against Populism: A Confrontation between the Theory of Democracy and the Theory of Social Choice*. San Francisco: Freeman.

Saari, Donald G. 1989. "A Dictionary for Voting Paradoxes." *Journal of Economic Theory* 48 (2): 443–75.

Saari, Donald G. 1990. "The Borda Dictionary." *Social Choice and Welfare* 7 (4): 279–317.

Saari, Donald G. 2000. "Mathematical Structure of Voting Paradoxes II: Positional Voting." *Economic Theory* 15 (1): 55–102.

Saari, Donald G., and Katri K. Sieberg. 2001. "The Sum of the Parts Can Violate the Whole." *American Political Science Review* 95 (June): 415–33.

Sen, Amartya K. 1970. *Collective Choice and Social Welfare*. San Francisco: Holden Day.

Smith, David A. 1999. "Manipulability Measures of Common Social Choice Functions." *Social Choice and Welfare* 16 (4): 639–61.

Young, Peyton H. 1974. "An Axiomatization of Borda's Rule." *Journal of Economic Theory* 9 (1): 43–52.

[25] Notice that $S_k < (n-1)/(n-2)\bar{S}$ is equivalent to $(n-1)/n < S_k/(2S_k - \bar{S})$.

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