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Linear wave theory in magnetized quantum plasmas

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Abstract. We study the spin and ion effects in quantum plasma, where the two-fluid model of electrons is being used which treats the spin-up and -down populations relative to the magnetic field as different species. We find the susceptibility of electrons and ions where the ions are classical, but strongly coupled. The general dispersion relation is derived for wave propagation in homogeneous magnetized plasmas for arbitrary direction of propagation. We discuss the applicability of our results.

1. Introduction

In 1960s, the quantum plasmas were first studied by Pines (1961, 1999) in regimes where we have a high density and a low temperature as compared to normal plasmas. The effects of quantum tunneling are believed to be important in such plasmas, which gives correction to the dispersion relation. Recently, the field of quantum plasmas has become an intense field of investigation, having applications in the high energy density physics, nanoelectronics, nanoscale devices (Craighead 2000), and ultracold plasmas (Li et al. 2005) as well as in the interiors of the compact objects such as white dwarfs, neutron stars, magnetars, and supernovas (Jung 2001; Opher et al. 2001; Chabrier et al. 2002), where the density can reach ten orders of magnitude that of ordinary solids. The description of such dense and/or strong magnetized systems makes electron spin properties (Brodin and Marklund 2007a, 2007b; Marklund and Brodin 2007), the Fermi pressure (Haas et al. 2000; Haas 2005; Garcia et al. 2005; Manfredi 2005; Shukla 2006b), and the Bohm–de Broglie potential (Haas et al. 2000; Haas 2005, 2007; Garcia et al. 2005; Manfredi 2005; Shukla and Stenflo 2006; Shukla 2006a, Shukla 2006b; Shukla and Eliasson 2006; Shukla et al. 2006a, 2006b) along with certain quantum electrodynamic effects (Lundström et al. 2006; Marklund and Shukla 2006; Brodin et al. 2007) important.

The usual regime where the quantum effects are important is high-density low-temperature plasmas (although exceptions to this rule are possible (Brodin et al. 2008a)), where either the thermal de-Broglie wavelength times plasma frequency is comparable to the thermal velocity or the Fermi pressure is comparable to the thermal pressure (Brodin et al. 2008a). In recent studies, several new models of plasmas incorporating various quantum spin effects have been developed, (see, e.g., (Marklund and Brodin 2007; Brodin et al. 2008a)) and in present study we consider the two-fluid model of electrons, where the spin-up and -down populations relative to the magnetic field are treated as different species where the aspects of the linear wave propagation are treated by using this model. The effects of the spin enter from the magnetization current, and the magnetic dipole force.

Electron spin effects can be important when the difference in energy between two spin states is larger than the thermal energy and the presence of large number of particles in the Debye sphere does not necessarily influence the significance of spin effects (Marklund and Brodin 2008). For such plasmas, the collective and quantum effects can be important at the same time (Lundin et al. 2007; Marklund et al. 2010).

We study the linear wave propagation in homogeneous magnetized plasma for an arbitrary direction of propagation where the wave frequencies may be larger than the spin precession frequency as well as below the spin precession frequency. When the Fermi temperature is larger than the (thermodynamics) temperature, the ions can be strongly coupled (and classical) although the electrons are quantum mechanical and only weakly coupled. Thus, we have also studied the case when ions are classical and strongly coupled (Shukla and Eliasson 2011). The study results in a general conductivity tensor including electrons and ion currents.

2. Plasma system

2.1. System equations

We will focus on the collective properties of the quantum electron plasma and the effects of entanglement are not considered. Neglecting spin–spin interaction, spin flip is not allowed by taking frequency greater than spin flip frequency (Brodin et al. 2008a). For the electrons, we have used the system of equations (Brodin et al. 2008a) of which the continuity equation

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{v}_s) = 0, \tag{1}$$

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where the index s denotes the considered species $+,-$ of plasma particles, n_s is the particle density, and $\mathbf{v}_s$ is the particle velocity. The momentum equation

$$m_s n_s \left\{ \frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s \right\} = q_s n_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B})$$

$$-\nabla P + \frac{\hbar^2 n_s}{2m_s} \nabla \left(\frac{1}{\sqrt{n_s}} \nabla^2 \sqrt{n_s} \right) + \frac{2\mu_s n_s}{\hbar} \nabla (\mathbf{B} \cdot \mathbf{S}). \quad (2)$$

The spin evolution equation for electrons is

$$(\partial_t + \mathbf{v}_s \cdot \nabla)\mathbf{S}_s = -\frac{2\mu_s}{\hbar} \mathbf{B} \times \mathbf{S}_s. \quad (3)$$

For P, we use an equation of state corresponding to the Fermi pressure, i.e., $\nabla P = ((3\pi^2)^{2/3})\hbar^2 \nabla n_s^{5/3}/5m_s$ (Landau and Lifshitz 1959). The quantum non-locality effects are included in the fluid momentum equation by the term $(\hbar^2 n_s/2m_s)\nabla(1/\sqrt{n_s}\nabla^2\sqrt{n_s})$ and is called the Bohm–de Broglie potential. Spin degree of freedom introduced by the spin magnetic moment is $\mu_s = \mu_B$, where μ_B is Bohr magneton and magnetization due to spin sources is $\mathbf{M} = \mathbf{M}_+ + \mathbf{M}_- = -2n_s\mu_B \mathbf{S}/\hbar = \pm \mu_B n_\pm \hat{\mathbf{B}}$. Here, $\hat{\mathbf{B}}$ is a unit vector in the direction of magnetic field B.

The ions are assumed to be classical but may be strongly coupled (Shukla and Eliasson 2011). The dynamics of ions are obtained by solving a set of continuity and momentum equations given by

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0, \quad (4)$$

and the momentum equation

$$\left(1 + \tau_m \frac{\partial}{\partial t}\right) \left[\left\{ \frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i \right\} - \frac{q_+}{m_i} (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \right.$$

$$\left. + \frac{\gamma_i k_B T_i}{m_i n_i} \nabla n_i \right] - \frac{\eta}{m_i n_i} \nabla^2 \mathbf{v_i} - \frac{(\zeta + \frac{\eta}{3})}{m_i n_i} \nabla (\nabla . \mathbf{v_i}) = 0, \quad (5)$$

where index i represents the considered ion species and q_+, v_i, n_i, and m_i are the charge, velocity, density, and mass of ions, respectively. γ_i is the adiabatic index for the ion fluid. We took $\gamma_i = 1$. ζ, η are the bulk ion viscosities, τ_m is the viscoelastic relaxation time for ions.

The system is closed via Maxwell's equatios. Hence, we use Amperes law

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{j} + \nabla \times \mathbf{M}) + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}, \quad (6)$$

together with Faraday's law

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t. \quad (7)$$

2.2. Susceptibility for electrons

The aim is to study the linear wave propagation in homogeneous magnetized plasma for an arbitrary direction of propagation where the wave frequencies may be larger than the spin precession frequency. We linearize and Fourier decompose the system (1)–(3). We choose the coordinates such that the unperturbed magnetic field is $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$, while taking the wave vector $\mathbf{k} = k_y \hat{\mathbf{y}} + k_z \hat{\mathbf{z}}$, without loss of generality. The velocity, number density, electric field, and magnetic field are linearized as: $\mathbf{v}_s = \mathbf{v}_{1s}, n_s = n_{0s} + n_{1s}, \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1, \mathbf{E} = \mathbf{E}_1$, respectively, and we write the spin as $\mathbf{S}_s = \mathbf{S}_{0s} + \mathbf{S}_{1s}$, where $\mathbf{S}_{0s} = S_{0s}\hat{\mathbf{z}}$ such that, $S_{0s} = S_\pm = 1$. Here, quantities with subscript 0 are in equilibrium and with subscript 1 are behaving sinusoidally as, $\mathbf{S}_{1s} = \tilde{\mathbf{S}}_{1s} \exp(i\mathbf{k}.\mathbf{r} - i\omega t)$. The perturbed quantities are assumed to be small compared to the equilibrium parts $|\mathbf{B}_{1s}| \ll |\mathbf{B}_{0s}|$ etc. We assume that the unperturbed plasma is neutral and isotropic. Thus, we have $\nabla n_{0s} = 0$. Linearizing the pressure and Bohm–de-Broglie potential term in (2), the density proportional terms can be written as

$$\nabla P + (\hbar^2 n_s/2m_s)\nabla(1\sqrt{n_s}\nabla^2\sqrt{n_s})$$

$$= \left[k_B T/m + \nabla \hbar^2 \nabla^2 / 4m_e^2 n_0 \right] \nabla n_1 = v_E^2 \nabla n_1, \quad (8)$$

where v_E^2 is an effective velocity and m_e is mass of an electron. Here, in (8), we have used the classical isothermal equation of state. This applies for $T > T_F$, and the effective velocity term can be written as $v_E^2 = v_T^2 + \hbar^2 k^2 / 4m_e^2$. Here, $v_T^2 = k_B T/m$ is the thermal velocity, $T_F = m v_F^2 / k_B$ is the Fermi temperature, and $v_F = \hbar(\pi^2 n_s/3^2)^{1/3}/m_e$ is the Fermi velocity. For $T < T_F$, the effective velocity term can be written as $v_E^2 = v_F^2/3 + \hbar^2 k^2 / 4m_e^2$. Thus, all the effects of the Fermi pressure as well as the particle dispersion can be calculated exactly as a classical thermal correction, and are incorporated in a generalized (k-dependent) thermal velocity v_E. By solving $\mathbf{v}$ in terms of $\mathbf{E}$, we can compute the free current density $\mathbf{J} = q_e n_0 \mathbf{v}_e$. The Magnetization currents are calculated by solving (3) for $\mathbf{S}$ in terms of $\mathbf{B}$, then $\mathbf{B}$ in terms of $\mathbf{E}$ from Faraday's law. Next, we write the magnetization current by taking the curl of the magnetization due to different spin sources, $\mathbf{J}_M = \nabla \times \mathbf{M}_+ + \nabla \times \mathbf{M}_-$. The (free) conductivity tensor is $\sigma_{ij(f)}^{(f)}$, where $J_i = \sigma_{ij}^{(f)} E_j$ and magnetic conductivity tensor is $\sigma_{ij}^{(m)}$, where $J_i = \sigma_{ij}^{(m)} E_j$. We have calculated susceptibility by using the relation ${\chi^{(e)}}_{ij} = i({\sigma^{(f)}}_{ij} + {\sigma^{(m)}}_{ij})/\epsilon_0 \omega$. Here, Einstein's summation convention is used and ϵ_0 is the permitivity of free space. The susceptibility for electrons $\chi^{(e)}$ have the following components:

$$\chi_{11}^{(e)} = \frac{-q_e^2 n_{\pm 0}(\omega^2 - v_E^2 k^2)}{\epsilon_0 m_e \omega_d^4} - \frac{2q_e \omega_{ce} \mu_B (\Delta n_0) k_y^2}{\epsilon_0 m_e \omega_d^4}$$

$$- \frac{4\mu_B^3 B_0 (\Delta n_0) k_z^2}{\epsilon_0 \omega^2 \omega_B^2 \hbar^2} + \mu_B^2 n_0 \left\{ \frac{\omega^2 k_y^4 + (\omega^2 - \omega_{ce}^2) k_y^2 k_z^2}{\epsilon_0 m_e \omega^2 \omega_d^4} \right\},$$

$$\chi_{22}^{(e)} = \frac{-q_e^2 n_{\pm 0}(\omega^2 - v_E^2 k_z^2)}{\epsilon_0 m_e \omega_d^4} - \frac{4\mu_B^3 (\Delta n_0) k_z^2}{\epsilon_0 \omega^2 \omega_B^2 \hbar^2},$$

$$\chi_{33}^{(e)} = \frac{-q_e^2 n_{\pm 0} \left\{ (\omega^2 - \omega_{ce}^2) - v_E^2 k_y^2 \right\}}{\epsilon_0 m_e \omega_d^4} - \frac{4B_0 \mu_B^3 (\Delta n_0) k_y^2}{\epsilon_0 \omega^2 \omega_B^2 \hbar^2},$$

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$$\chi_{12}^{(e)} = \frac{-iq_e^2\omega_{ce}n_{\pm 0}(\omega^2 - v_E^2k_z^2)}{\epsilon_0 m_e\omega\omega_d^4} - \frac{2i\mu_B^2(\Delta n_0)k_z^2}{\epsilon_0\omega\omega_B^2\hbar}$$
$$- \frac{i\mu_B q_e\omega(\Delta n_0)k_y^2}{\epsilon_0 m_e\omega_d^4} = -\chi_{21}^{(e)},$$

$$\chi_{13}^{(e)} = \frac{-iq_e^2\omega_{ce}n_{\pm 0}v_E^2k_yk_z}{\epsilon_0 m_e\omega\omega_d^4} + \frac{2i\mu_B^2(\Delta n_0)k_yk_z}{\epsilon_0\omega\omega_B^2\hbar}$$
$$- \frac{i\mu_B q_e(\Delta n_0)(\omega^2 - \omega_{ce}^2)k_yk_z}{\epsilon_0 m_e\omega\omega_d^4} = -\chi_{31}^{(e)},$$

$$\chi_{32}^{(e)} = \frac{-q_e^2n_{\pm 0}v_E^2k_yk_z}{\epsilon_0 m_e\omega_d^4} + \frac{4B_0\mu_B^3(\Delta n_0)k_yk_z}{\epsilon_0\omega^2\omega_B^2\hbar^2} = \chi_{23}^{(e)}.$$

We have used $q_e = -e$ as the charge of an electron, $k^2 = \sqrt{k_\perp^2 + k_\parallel^2}$, where $k_\perp$ is the perpendicular part of the wave vector and $k_\parallel$ is the parallel part of the wave vector. Here, $\Delta n_0 \equiv n_{0+} - n_{0-}$, $\mu_+ = \mu_- = -\mu_B$, $\omega_d^4 = \omega^4 - \omega_{cs}^2(\omega^2 - v_E^2k_z^2) - \omega^2k^2v_E^2$, $\omega_B^2 = \omega^2 - \omega_{cr}^2$, and $\omega_{ce} = B_0q_e/m_e$ is the electron cyclotron frequency. Note that the components of the susceptibility that change sign when interchanging indices, are real, whereas the imaginary components have a sign change. Thus, the electron susceptibility fulfills $\chi_{ij} = \chi_{ji}^*$, i.e., it is Hermitian, reflecting the absence of dissipation. Here, the star denotes the complex conjugate.

2.3. Susceptibility for ions

We have linearized (4) and (5) to find the susceptibility for non-relativistic and non-degenrate ions in a magnetized quantum plasma. We have neglected the ion spin contribution to the susceptibility as compared to electrons, due to their smaller magnetic moment. Solving (4) and (5) to find the ion velocity as a function of the electron field, we can calculate the ion contribution to current density. Using ${\chi^{(i)}}_{ij} = i{\sigma^{(i)}}_{ij}/\epsilon_0\omega$, the ion susceptibility is found to be

$$\chi_{11}^{(i)} = \frac{-aq_+^2n_{0i}(\tilde{\omega}^3 - \tilde{\omega}\tilde{v}_T^2k^2)}{\epsilon_0 m_e\omega\tilde{\omega}_d^4},$$

$$\chi_{22}^{(i)} = \frac{-aq_+^2n_{0i}(\tilde{\omega}^2 - \tilde{v}_T^2k_z^2)}{\epsilon_0 m_e\tilde{\omega}_d^4},$$

$$\chi_{33}^{(i)} = \frac{-aq_+^2n_{0i}\left\{(\tilde{\omega}^3 - \tilde{\omega}\tilde{\omega}_{ce}^2) - \tilde{v}_T^2k_y^2\right\}}{\epsilon_0 m_e\omega\tilde{\omega}_d^4},$$

$$\chi_{12}^{(i)} = \frac{-iaq_+^2n_{0i}\tilde{\omega}_{ce}(\tilde{\omega}^2 - \tilde{v}_T^2k_z^2)}{\epsilon_0 m_e\omega\tilde{\omega}_d^4} = -\chi_{21}^{(i)},$$

$$\chi_{13}^{(i)} = \frac{-iaq_+^2n_{0i}\tilde{\omega}_{ce}\tilde{v}_T^2k_yk_z}{\epsilon_0 m_e\omega\tilde{\omega}_d^4} = -\chi_{31}^{(i)},$$

$$\chi_{23}^{(i)} = \frac{-aq_+^2n_{0i}\tilde{\omega}\tilde{v}_T^2k_yk_z}{\epsilon_0 m_e\omega\tilde{\omega}_d^4} = \chi_{32}^{(i)}.$$

Here, $\tilde{\omega} = \omega(1 - i\omega\tau_m + i\eta k^2/m_sn_{0s}\omega)$, $\tilde{v}_T^2 = \{(1 - i\omega\tau_m)v_T^2 - i\omega(\zeta + \frac{\eta}{3})/m_in_{0i}\}\tilde{\omega}/\omega$, $a = 1 - i\omega\tau_m$, $\tilde{\omega}_d^4 = \tilde{\omega}^4 - \omega_{ci}^2(\tilde{\omega}^2 - v'^2_Tk_z^2) - \tilde{\omega}^2k^2v'^2_T$, and $\tilde{\omega}_{ci} = (1 - i\omega\tau_m)B_0q_+/m_i$ is the ion cyclotron frequency. Note that the ions are Hermitian only if all dissipiative parameters are put to zero, i.e., $\tau_m = \eta = \zeta = 0$.

2.4. General dispersion relation

Maxwell's equation gives dispersion relation if currents can be expressed in terms of **E**. On solving (6) and (7) with the given current sources, we get matrix D which is written by using Einstein's summation convention

$$D_{ij} = \delta_{ij}\left(1 - \frac{k^2c^2}{\omega^2}\right) - \frac{k_ik_jc^2}{\omega^2} + \chi_{ji}^{(e)} + \chi_{ji}^{(i)}. \tag{9}$$

Here, D is a 3×3 dispersion matrix including ions as well as electron contribution. The components of D are then given by

$$D_{11} = 1 - \frac{c^2k^2}{\omega^2} - \frac{a\tilde{\omega}^2\omega_{pi}^2(\tilde{\omega}^2 - k^2\tilde{v}_T^2)}{\omega^2\tilde{\omega}_d^4} - \frac{\omega_{pe}^2(\omega^2 - k^2v_E^2)}{\omega_d^4}$$
$$- \frac{4B_0^2(\Delta\omega_{pe}^2)\mu_B^3k_\parallel^2}{e\omega_{ce}\omega^2\omega_B^2\hbar^2} + \frac{\mu_B\omega_{ce}(\Delta\omega_{pe}^2)k_\perp^2}{e\omega^2\omega_d^4}$$
$$+ \frac{\mu_B^2\left(\omega_{pe}^2\right)\left\{k_\perp^4\omega^2 + k_\parallel^2k_\perp^2(\omega^2 - \omega_{ce}^2)\right\}}{e^2\omega^2\omega_d^4}, \tag{10}$$

$$D_{22} = 1 - \frac{c^2k_\parallel^2}{\omega^2} - \frac{a\tilde{\omega}^2\omega_{pi}^2(\tilde{\omega}^2 - k_\parallel^2\tilde{v}_T^2)}{\omega^2\tilde{\omega}_d^4} - \frac{\omega_{pe}^2(\omega^2 - k_\parallel^2v_E^2)}{\omega_d^4}$$
$$- \frac{4\left(\Delta\omega_{pe}^2\right)\mu_B^3B_0^2k_\parallel^2}{e\omega_{ce}\omega^2\omega_B^2\hbar^2}, \tag{11}$$

$$D_{33} = 1 - \frac{c^2k_\perp^2}{\omega^2} - \frac{a\tilde{\omega}^2\omega_{pi}^2(\tilde{\omega}^2 - \omega_{ci}^2) - k_\perp^2\tilde{v}_T^2}{\omega^2\tilde{\omega}_d^4}$$
$$- \frac{\omega^2\omega_{pe}^2(\omega^2 - \omega_{ce}^2) - k_\perp^2v_E^2}{\omega^2\omega_d^4} - \frac{4\left(\Delta\omega_{pe}^2\right)\mu_B^3B_0^2k_\perp^2}{e\omega_{ce}\omega^2\omega_B^2\hbar^2}, \tag{12}$$

$$D_{12} = \frac{-ia\tilde{\omega}\tilde{\omega}_{ci}\omega_{pi}^2(\tilde{\omega}^2 - k_\parallel^2\tilde{v}_T^2)}{\omega^2\omega_d^4} - \frac{i\omega_{ce}\omega_{pe}^2(\omega^2 - k_\parallel^2v_E^2)}{\omega\omega_d^4}$$
$$+ \frac{i\omega\left(\Delta\omega_{pe}^2\right)\mu_Bk_\perp^2}{e\omega_d^4} - \frac{2i\left(\Delta\omega_{pe}^2\right)\mu_B^2B_0k_\parallel^2}{e\omega\omega_{ce}\omega_B^2\hbar} = -D_{21}, \tag{13}$$

$$D_{13} = \frac{-ia\tilde{\omega}\tilde{\omega}_{ci}\omega_{pi}^2k_\perp k_\parallel\tilde{v}_T^2}{\omega^2\tilde{\omega}_d^4} - \frac{i\omega_{ce}\omega_{pe}^2k_\perp k_\parallel v_E^2}{\omega\omega_d^4}$$
$$+ \frac{2i\left(\Delta\omega_{pe}^2\right)\mu_B^2B_0k_\perp k_\parallel}{e\omega\omega_{ce}\omega_B^2\hbar}$$
$$+ \frac{i\left(\Delta\omega_{pe}^2\right)(\omega^2 - \omega_{ce}^2)\mu_Bk_\perp k_\parallel}{e\omega\omega_d^4} = -D_{31}, \tag{14}$$

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$$D_{23} = c^2 k_\perp k_\parallel - \frac{a\tilde{\omega}^2 \omega_{pi}^2 k_\perp k_\parallel \tilde{v}_T^2}{\omega^2 \tilde{\omega}_d^4} - \frac{\omega_{pe}^2 k_\perp k_\parallel v_E^2}{\omega_d^4}$$

$$+ \frac{4\left(\Delta\omega_{pe}^2\right)\mu_B^3 B_0^2 k_\perp k_\parallel}{e\omega_{ce}\omega^2\omega_B^2\hbar^2} = -D_{32}. \quad (15)$$

Here, $\omega_{pe} = \sqrt{n_{\pm 0} q_e^2/\epsilon_0 m_e}$ is the plasma frequency of electrons, $\omega_{pi} = \sqrt{n_{0i} q_+^2/\epsilon_0 m_i}$ is the plasma frequency for ions, $\Delta\omega_{pe}^2 \equiv \omega_{p+}^2 - \omega_{p-}^2$, $\omega_{pe}^2 \equiv \omega_{p+}^2 + \omega_{p-}^2$ is the electron contribution to the plasma frequency. Here, ω_{p+}^2 and ω_{p-}^2 come from spin-up and -down population of electrons, respectively. The plasma is quasineutral such that $n_{0e} = n_{0+} + n_{0-} = n_{0i}$. The determinant of $\mathbf{D}$ will give the dispersion relation where the direction is taken relative to the $\hat{\mathbf{z}}$ axis. Some terms (the magnetization in particular) depend on $\mathbf{S}_0 = \pm\hat{\mathbf{z}}$. As a result, we have contributions $\propto n_{0+} + n_{0-} = n_{0e} \propto \omega_{pe}^2$ as well as contributions $\propto n_{0+} - n_{0-} \propto \Delta\omega_{pe}^2$. In thermodynamic equilibrium, $n_{0+} - n_{0-}/n_{0+} + n_{0-} = \tanh(B_0\mu_B/k_B T$, provided $T > T_F$. We will use this here, but other choices could be made, corresponding to $T < T_F$ or in a situation where the background is in a thermodynamic non-equilibrium.

3. Comparison with fluid theory

To validate, the results are compared for the case when the wave frequency is lower than the electron cyclotron frequency and the wave propagation is along the magnetic field with some previous results where strong coupling effects are ignored, i.e., $\tau_m = \eta = \zeta = 0$. Specifically, we have studied the MHD limit which can be obtained under the following assumptions:

1. The plasma is quasi-neutral, which holds for $\omega_{pi}^2/\omega_{ci}^2 \gg 1$.
2. Only electron thermal motion is of importance, which is possible if $k^2 v_{ti}^2 \ll \omega^2$.
3. The condition $k^2 c^2 \gg \omega^2$ is used, which is closely related to the condition 1.

For MHD regimes, multifluid equations including the magnetization effects due to electron spin can be found in Craighead (2000), and for the dynamics on a time scale much slower than the spin precession frequency, the magnetization is then given by

$$\mathbf{M} = (\mu_B \rho_0/m_i) tanh(\mu_B B/k_B T)$$

$$\hat{\mathbf{B}} \approx tanh\left(B_0\mu_B/k_B T\right) \propto \Delta n_0/n_0.$$

In this limit, the dispersion relation contains the Alfvén, fast magnetosonic, and slow magnetosonic wave modes and we obtain the agreement with the dispersion relation found in Lundin et al. (2008).

The second limit compared is the strong magnetic field effects. We then assume $\omega_c \gg \omega, |k|c$, and the resultant dispersion relation agrees with the dispersion relation found in Lundin (2009). A final indication that the algebra has been performed correctly is that the full conductivity tensor is Hermitian, which gives a useful test on all off-diagonal elements of the full conductivity tensor.

Furthermore, if we omit the spin effects from the dispersion relation, the obtained classical dispersion relation agrees with that found in Lundin et al. (2007), in the corresponding limit.

On performing the large number of tests discussed above, we thus deduce that our main results, (10)–(15), are applicable.

4. Comparison with kinetic theory

Linear results derived from spin kinetic theory have previously been derived by Lundin and Brodin (2010). In order to know the strengths as well as the limitations of the current theory, a detailed comparison with that work is in order. First, we pointed out that finite Larmor radius (FLR) effects are clearly not contained, and neither wave particle resonances. However, in classical theory, the fluid results are resolved from kinetic theory if the Bessel functions are expanded to first order in the argument, and a rectangular-shaped distribution function is chosen (i.e., we let $f_0(v_z) = 1$ for $-v_t < v_z < v_t$ and $f_0 = 0$ elsewhere). To a large extent, this procedure gives an agreement with Lundin and Brodin (2010) also when spin effects are included. However, one more physical effect is missing from fluid theory in the presence of spin. This has to do with spin-precession cyclotron resonances, which result in terms with denominators $1/(\omega - kv_z - \omega_{cg} \pm \omega_c)$ in kinetic theory. Such terms can be captured in fluid theory by including the spin-velocity correlation tensor (Zamanian et al. 2010), but at the cost that the theory becomes much more complex.

5. Summary and conclusion

The analysis has been performed through a linearization of the perturbed parameters using a two-fluid description of electrons to calculate susceptibility and dispersion relation for the ions and electron spin plasmas. Here, electrons with spin-up and spin-down relative to the magnetic field formally constitute different species. The effect of the spin enters from the (spin) magnetization current, and the magnetic dipole force. Specifically, we find $J_i = \sigma_{ij} E_j$, where $\sigma_{ij}^{(e)} = \sigma^{(f)}{}_{ij} + \sigma^{(m)}{}_{ij}$ is due to free current and the magnetization current. Furthermore, $\sigma^{(f)}{}_{ij}$ is divided into Lorentz and spin parts that give the classical and spin contribution to the free conductivity tensor. For ions, we find free current contribution. Next, we will discuss a few of these features resulting from the electron spin. If the ratio of Zeeman energy to the thermal energy is small we have $\mu_B B_0/k_B T \ll 1$. This will limit the magnitude of most spin terms, as they are proportional to $\Delta\omega_p^2$, which in the case of thermodynamic equilibrium is

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given by $\Delta\omega_p^2 = \tanh\left(\mu_B B_0/k_B T\right)\omega_p^2 \approx (\omega_p^2)\mu_B B_0/k_B T$ (Zamanian et al. 2009) (where the last term equality holds for $\mu_B B_0/k_B T \ll 1$). Apart from the environment close to the pulsars and magnetars, $\mu_B B_0/k_B T \ll 1$ is generally fulfilled in existing plasmas, limiting the influence of the spin terms. Applications to the astrophysical regimes have been considered by, e.g., Lundin (2009), and we will here briefly discuss some potentially important spin effects that may contribute also for $\mu_B B_0/k_B T \ll 1$.

As regards the D_{11} contribution, from (10), the last spin term is proportional to ω_{pe}^2 rather than $\Delta\omega_{pe}^2$. The dispersion relation of compressional Alfvén waves is given by $D_{11} = 0$, and the spin influence on this mode has been discussed by Brodin et al. (2009). In the denominator of many spin terms, we have $\omega_B^2 = \omega^2 - \omega_{cr}^2$. At frequencies close to the resonance $\omega \approx \omega_{cr}$, the spin terms may clearly be influential. where for electrons, ω_{cr} differs slightly from the cyclotron frequency according to $\omega_{cr} = (g/2)\omega_{ce}$.

Earlier, we have only pointed to a few possibilities for spin effects to be important. Already, the classical theory for linear waves in a magnetized plasma contains a huge number of wave modes, described by $det\mathbf{D} = 0$ and the spin terms add considerable complexity. Thus, a full evolution of (10)–(15) is not discussed in the present work.

In the inertial confinement regime, ions are typically classical and strongly coupled. Therefore, strong coupling of ions could play an important role in laser and particle beam compressed layered targets. The susceptibility tensor for strongly coupled ions given in Section 2.3 can then prove to be useful.

6. Final remarks

The model used here is simpler than some other models that have appeared in the literature providing a kinetic description (Lundin and Brodin 2010) or containing higher order fluid moments (Zamanian et al. 2010). As a result, certain physical effects are absent in (10)–(15), as described in Section 4. Nevertheless, most linear effects are contained in (10) –(15) and the physics that is left out is relatively well-defined. When physical phenomenon of more complicated nature are investigated, e.g., nonlinear and/or inhomogeneous problems, more elaborate models may be too cumbersome to use. Simpler models like the one used here may then be appropriate, but it is important to know both the strengths as well as the limitations of such models. The analysis made in this paper has the aim of making both the strong and weak points of the model described by (1)–(5) as clear as possible.

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