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PAY-AS-YOU-GO PENSIONS AND ENDOGENOUS RETIREMENT

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This paper revisits two classic results in standard overlapping generations models with a pay-as-you-go (PAYG) pension system. Firstly, a PAYG system crowds out private savings and reduces the overall capital stock. Secondly, in dynamically efficient economies, a PAYG system will reduce steady-state welfare. These classic results have been derived and exposed in models with no retirement decision. However, by allowing for endogenous retirement, and without taking recourse to any frictions, uncertainty, or myopia, it is shown that a PAYG system may be neutral and may even increase the capital–labor ratio. In particular, it is shown that the effect of the pension contribution rate on capital intensity depends on the elasticity of substitution between consumption and leisure. If the elasticity of substitution is between 0 and 1, an increase in the contribution rate will increase capital intensity. Moreover, it is shown that the result regarding welfare may also be overturned. An increase in the PAYG contribution rate may increase steady-state welfare.

Keywords: Endogenous Retirement, Pensions, Social Security, Welfare

1. INTRODUCTION

The purpose of this paper is to revisit some of the classic results in overlapping generations (OLG) models with a pay-as-you-go (PAYG) old-age pension system, that is, pension benefits are financed through contributions of contemporary workers. In a standard OLG model with non-working elderly individuals, several well-established results can be derived. (i) A fully funded pension system neither affects capital accumulation nor welfare, that is, the system is neutral. (ii) A PAYG pension system will reduce the incentive to save and thereby reduce capital accumulation. Thus, the PAYG system is not neutral. (iii) A PAYG pension

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system will reduce the long-run welfare if the economy is dynamically efficient. These results were originally deduced by Diamond (1965), Aaron (1966), and Samuelson (1975). Besides, they are now standard in the textbook literature [Blanchard and Fischer (1989), Azariadis (1993), and de la Croix and Michel (2002)]. Since most developed countries are considered dynamically efficient, it seems that PAYG pension programs are not desirable from a welfare standpoint [Aaron (1966), Abel et al. (1989), and Barbie et al. (2004)]. However, most countries have implemented pension programs with a substantial PAYG component. This leaves the following puzzle open: Why are PAYG pension systems so popular, when several well-established results suggest they do more harm than good in dynamically efficient economies?

Not surprisingly, a large literature has emerged with a focus on the rationale for such pension programs. For example, Krueger (2006) highlights the role played by unfunded pension systems in ameliorating idiosyncratic risks in worlds with incomplete financial markets. Andersen and Bhattacharya (2011) and Caliendo and Gahramanov (2011) study the importance of agent myopia in this context. Fuster et al. (2007) introduce bidirectional altruism along with mortality and earnings risks in a framework similar to one adopted by Conesa and Krueger (1999). Cooley and Soares (1999) explore a PAYG model with elements from political economy. All of these papers introduce new elements and are able to justify a PAYG system.

In this paper, we reconsider the results regarding the PAYG system and ask whether these classic and established results are robust to a more general description of the original Diamond (1965) model. Is it possible to overturn these results without introducing new frictions or uncertainty or non-classical preferences? The generalization is done by letting the retirement decision be optimally chosen. Recall that in the standard OLG model, and in much of the ensuing literature, old-age labor supply is arbitrarily fixed at zero implying that retirement is exogenously imposed. This assumption is especially troublesome since it is readily apparent that retirement decisions are not made in a vacuum and clearly respond to the pension system itself, that is, old-age pensions and the retirement decision are inextricably linked. In this paper, we introduce endogenous retirement in the second period of life.[1] Thus, the setting is identical to the original setting in Diamond (1965), except we allow for a natural, endogenous retirement decision.

An early contribution that studies the relation between endogenous retirement and capital accumulation is Hu (1979). Hu shows within a general equilibrium framework with a bequest motive that the impact of an increase in retirement benefits on capital accumulation is ambiguous. He also shows that more generous pension benefits stimulate old agents to retire earlier. In Michel and Pestieau (2013), an OLG model with logarithmic utility and Cobb–Douglas technology is applied. They show how the effect of the pension contribution on the steady-state level of capital depends on the values of parameters in the model. Specifically, they demonstrate that the result on the capital stock per worker is ambiguous, as two opposing effects are at work. An increase in the PAYG contribution rate

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reduces the participation rate, which, all other things equal, increases the steady-state capital stock per capita. But, an increase in the contribution rate also reduces savings and the steady-state capital stock. Hence, the total effect on steady-state capital stock per capita is inconclusive. We will return to these results and compare them against ours in the conclusion.

In the present paper we follow this line of research and study how a PAYG pensions system will affect the steady-state capital intensity and welfare when endogenous retirement is allowed. The analysis is conducted in an equilibrium framework with a constant elasticity of substitution (CES) utility function, a neoclassical production function, and a PAYG governmental budget constraint. Several results regarding the effects of introducing a PAYG system on the steady-state capital–labor ratio and welfare are derived. First, it is shown that with endogenous retirement, the effect of the PAYG contribution rate on capital–labor ratio is ambiguous, may be decreasing, increasing, or even neutral. The standard results on capital intensity in Diamond (1965) model may no longer occur. Particularly, if the elasticity of substitution between consumption and leisure is equal to one, that is, with a logarithmic utility function, implementing a PAYG system is perfectly neutral in terms of the capital–labor ratio. Moreover, it is shown that there may be long-run welfare gains from introducing a PAYG pension even if the interest rate is greater than the population growth rate. The welfare effects crucially depend on how the capital–labor ratio changes which depends on the elasticity of substitution. Abel et al. (1989) provide a test of whether a country should introduce a PAYG system or not. In this paper, the results regarding welfare effects offer a similar but new test.

The plan for the rest of the paper is as follows. Section 2 outlines the environment of the model, a generalization of Diamond (1965) model allowing for endogenous retirement. In Section 3 we study the behavior of the consumers. Section 4 describes the equilibrium and studies the effects of the PAYG system on savings, labor supply, and the capital–labor ratio. In Section 5 we analyze the welfare effects, and Section 6 provides concluding remarks. Proofs of all major results can be found in the appendices.

2. THE MODEL

2.1. Preliminaries

The description of the economy closely follows that in Andersen and Bhattacharya (2013). The economy is closed and consists of firms, an infinite sequence of two-period lived OLG, an initial old generation, and an infinitely lived government. Let $t = 1, 2, \ldots$ index time. At each date t, a new generation composed of a continuum of identical young agents appears. Population is assumed to be stationary and the number of individuals in each generation is normalized to one.

Identical firms hire the available labor force L_t, and the aggregate capital stock K_t, to produce the aggregate output. Production occurs according to a neoclassical

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production function $F(K_t, L_t)$, exhibiting constant returns to scale. The homogenous output can be used both as a consumption good and as an investment good. The capital stock is assumed to depreciate completely in each period.[2] Hence, investment in period t determines the capital stock in period $t+1$, that is, $I_t = K_{t+1}$. The initial old agents are endowed with K_1 units of capital. Let $k_t \equiv K_t/L_t$ denote the capital–labor ratio. Output per unit of labor is $f(k_t) \equiv F(K_t/L_t, 1)$, where $f(k_t)$ is the intensive production function. The standard neoclassical assumptions for the production function apply, that is, for all $k > 0$, $f_k > 0, f_{kk} < 0$, and the Inada conditions hold.

Firms are assumed to maximize profits, given by $[F(K_t, L_t) - w_t L_t - R_t K_t]$, where w_t and R_t are, respectively, the competitive wage rate and the gross real return between $t-1$ and t. As markets are perfectly competitive, the input factors are rewarded by their marginal products:

$$w_t \equiv w(k_t) = f(k_t) - k_t f_k(k_t), \tag{1}$$

$$R_t \equiv R(k_t) = f_k(k_t). \tag{2}$$

Notice that $w_k(k_t) > 0$, and that $R_k(k_t) < 0$.

Now, let us turn to the consumers. Individuals live for two periods and they are endowed with one unit of time in each period. In the first period they are young and inelastically supply one unit of labor. In the second period they are old and care about leisure. Hence, old agents make a trade-off between leisure and consumption. The portion of their old age devoted to leisure is called retirement. Let $c_{1,t}$ $\left(c_{2,t+1}\right)$ denote the consumption of the final goods at date t (date $t+1$) by generation born at t, and let $l_{t+1} \in [0, 1]$ denote the labor supply by an old agent at time $t+1$. The length of leisure time, $1 - l_{t+1}$, is termed retirement and is endogenously determined. Preferences are given by a time-separable CES utility function:

$$U(c_{1,t}, c_{2,t+1}, l_{t+1}) \equiv \frac{c_{1,t}^{1-\gamma}}{1-\gamma} + \beta \frac{c_{2,t+1}^{1-\gamma}}{1-\gamma} + \beta\mu \frac{(1-l_{t+1})^{1-\gamma}}{1-\gamma}, \tag{3}$$

where $\beta \in (0, 1)$ is a discount factor, μ measures the importance of leisure when old, that is, the preference for retirement, and $\gamma > 0$ is the elasticity of substitution between consumption and leisure.

2.2. Pensions

The PAYG pension scheme is run by the government. At any date t, it levies a payroll tax τ_t on each young and old worker to finance a lump-sum transfer to each of the currently retired. In this model economy, retirement is imposed as a precondition to receive pensions, so the total pension benefit is $(1 - l_t)B_t$, depending on both the per unit benefit level B_t and the length of retirement $(1 - l_t)$. This specification fits well with current practice in a number of countries where there are double taxes on continued work: the payroll tax $\tau_t w_t l_t$ and the forgone pension

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benefits $B_t l_t$. In every period, the government balances the tax income and the pension benefits in a PAYG program. The balanced budget scheme is thus given by

$$(1-l_t)B_t = (1+l_t)\tau_t w_t. \tag{4}$$

The left-hand side of equation (4) represents the pension benefits paid to current retirees and the right-hand side captures the total social security tax revenue, $\tau_t w_t$, from young agents and $\tau_t w_t l_t$ from current old workers. A similar formulation is found in Michel and Pestieau (2013) and Miyazaki (2017).

Here, the old agent contributes to his own pension. This makes the formulation a bit different from the standard one where only the current young contribute to the pension for the current, retired old. Also, as all these are happening within the same period, the agent does not earn any interest on his old-age contributions.

To understand how the PAYG pension system affects the economy, it is crucial to understand how the consumers behave. In the next section we therefore solve the agent's problem and explore how the agent's optimal savings and labor supply respond to ceteris-paribus changes in the PAYG system.

3. INDIVIDUAL'S PROBLEM

Recall that an individual born in period t inelastically supplies one unit of labor when young. The resulting labor income is allocated to young-age consumption, savings, and pension contribution. To simplify notation we define the effective young-age wage rate as $w_t^y = (1-\tau_t)w_t$. Hence, the budget constraint in period t can be written as

$$c_{1,t} = w_t^y - s_t. \tag{5}$$

In period $t+1$, the individual consumes out of accumulated savings, the income earned while working when old, and the pension benefits while retired. That is, $c_{2,t+1} = R_{t+1}s_t + (1-\tau_{t+1})w_{t+1}l_{t+1} + B_{t+1}(1-l_{t+1})$. Because we assume an individual gets pensions only after he leaves the labor market, his old-age labor supply actually has two costs: (i) leisure (ii) potential pensions that he would get if he is not in the labor market. Similarly, we define the effective old-age wage rate as $w_{t+1}^o = (1-\tau_{t+1})w_{t+1} - B_{t+1}$. The budget constraint in period $t+1$ can thus be written as

$$c_{2,t+1} = R_{t+1}s_t + w_{t+1}^o l_{t+1} + B_{t+1}. \tag{6}$$

The problem of an agent born at time t, taking τ_t, τ_{t+1}, w_t, w_{t+1}, R_{t+1}, and B_{t+1} as given, is to choose $c_{1,t} \geq 0, c_{2,t+1} \geq 0$ and $l_{t+1} \in [0,1]$ to maximize utility (3) subject to budget constraints (5) and (6). It is assumed that the individuals do not perceive a link between their social security contributions and the benefits they will receive.

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We can rewrite the individual's problem as follows. The present value of implicit quantity of disposable lifetime income of generation t is given by

$$W_t \equiv w_t^y + \frac{w_{t+1}^o + B_{t+1}}{R_{t+1}} = (1-\tau_t)w_t + \frac{(1-\tau_{t+1})w_{t+1}}{R_{t+1}}. \tag{7}$$

W_t is the total lifetime income an individual would get if he supplies one unit of labor in each period. The implicit lifetime income is spent on consumption goods in both periods and leisure when old.

Maximizing utility (3) subject to (7) gives the standard interior solutions with CES utility functions:

$$c_{1,t} = \frac{1}{1+\frac{1}{R_{t+1}}(\beta R_{t+1})^{\frac{1}{\gamma}} + \frac{w_{t+1}^o}{R_{t+1}}\left(\beta\mu\frac{R_{t+1}}{w_{t+1}^o}\right)^{\frac{1}{\gamma}}} W_t, \tag{8}$$

$$c_{2,t+1} = \frac{(\beta R_{t+1})^{\frac{1}{\gamma}}}{1+\frac{1}{R_{t+1}}(\beta R_{t+1})^{\frac{1}{\gamma}} + \frac{w_{t+1}^o}{R_{t+1}}\left(\beta\mu\frac{R_{t+1}}{w_{t+1}^o}\right)^{\frac{1}{\gamma}}} W_t, \tag{9}$$

$$1-l_{t+1} = \frac{\left(\beta\mu\frac{R_{t+1}}{w_{t+1}^o}\right)^{\frac{1}{\gamma}}}{1+\frac{1}{R_{t+1}}(\beta R_{t+1})^{\frac{1}{\gamma}} + \frac{w_{t+1}^o}{R_{t+1}}\left(\beta\mu\frac{R_{t+1}}{w_{t+1}^o}\right)^{\frac{1}{\gamma}}} W_t. \tag{10}$$

Using equations (8) and (10), we can find expressions for optimal savings and old-age labor supply, denoted by $s_t = w_t^y - c_{1,t} \equiv s(w_t^y, w_{t+1}^o, R_{t+1}, B_{t+1})$ and $l_{t+1} \equiv l(w_t^y, w_{t+1}^o, R_{t+1}, B_{t+1})$.[3]

The corner solution is given by

$$s_t = \frac{w_t^y - (\beta R_{t+1})^{-\frac{1}{\gamma}} B_{t+1}}{1+(\beta R_{t+1})^{-\frac{1}{\gamma}} R_{t+1}}, \tag{11}$$

$$l_{t+1} = 0. \tag{12}$$

The corner solution represents the standard Diamond model, that is, without labor supply in the second period of life. In this paper, we only focus on the case with interior solutions.

In the following lemma we derive several results regarding individual's first-order responses of savings and labor supply.

LEMMA 1.

(*a*) $\frac{\partial s_t}{\partial w_t^y} \in (0,1)$, $\frac{\partial l_{t+1}}{\partial w_t^y} < 0$, $\frac{\partial s_t}{\partial B_{t+1}} < 0$, $\frac{\partial l_{t+1}}{\partial B_{t+1}} < 0$ *for all* $\gamma > 0$;

(*b*) $\frac{\partial s_t}{\partial R_{t+1}} > 0$, $\frac{\partial l_{t+1}}{\partial R_{t+1}} < 0$, $\frac{\partial s_t}{\partial w_{t+1}^o} < 0$, $\frac{\partial l_{t+1}}{\partial w_{t+1}^o} > 0$ *for* $0 < \gamma \leq 1$,

and the signs of $\frac{\partial s_t}{\partial R_{t+1}}$, $\frac{\partial l_{t+1}}{\partial R_{t+1}}$, $\frac{\partial s_t}{\partial w_{t+1}^o}$ *and* $\frac{\partial l_{t+1}}{\partial w_{t+1}^o}$ *are ambigious for* $\gamma > 1$.

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The proof of Lemma 1 is in Appendix A. The results in part (a) follow from a pure income effect. Accordingly, an increase in the young-age wage income (w_t^y) will stimulate them to smooth consumption. By saving more and working less as old, individuals have more consumption of final goods in both periods and more consumption of leisure when old. Similarly, with a higher pension level (B_{t+1}) in old age, individuals save less and retire earlier to smooth consumption between two periods and between final goods and leisure.

The results in part (b) show the effects of the interest rate (R_{t+1}) and the effective old-age wage rate (w_{t+1}^o). In general, the effects of changes of these two variables on savings and old-age labor supply are ambiguous since both income effect and substitution effect exist. However, for $0 < \gamma \leq 1$, as in the standard Diamond model, the substitution effect dominates the income effect.[4] Hence, an increase in the interest rate will increase savings. Also, as the total returns from savings at the beginning of old-age ($R_{t+1}s_t$) increase, the income effect induces individuals to consume more leisure. A similar intuition applies to the effect of the effective old-age wage rate. When substitution effect dominates the income effect, an increase in the effective old-age wage rate will induce the old to work longer. Then the old-age labor income ($w_{t+1}^o l_{t+1}$) will increase, and savings will be reduced in order to smooth consumption between two periods.

Thus far we have studied individual's behavior and how savings and old-age labor supply respond to different changes in the environment. This will be important when we go on to study the effects of the pension system in equilibrium in the next section.

4. EQUILIBRIUM

In this section, we study the effects of increasing the size of a PAYG pension system in an equilibrium. There are three markets in this economy. Equilibrium in the labor market in period t is given by $L_t = 1 + l_t$, as the young individuals supply one unit of labor, and the old individuals supply l_t units. We assume labor is homogeneous, so the wage rates are the same for young and old workers at any point in time.

In the capital market, the aggregate savings of the young in any period become the start-of-period capital for the next period. Hence, the equilibrium condition in the capital market is given by $K_{t+1} = s_t$, where s_t is aggregate savings. Using the definition of k, we can rewrite this condition as $k_{t+1}L_{t+1} = s_t$. By combining this with the equilibrium condition in the labor market, the equilibrium condition in the capital market is

$$(1+l_{t+1})k_{t+1} = s_t. \tag{13}$$

Equation (13) relates capital intensity, savings, and optimal labor supply by the elderly. According to Walras law in period t, equilibrium in the labor market and in the capital market implies equilibrium in the output market, which is given by $Y_t = c_{1,t} + c_{2,t} + K_{t+1}$.

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An increase in the contribution rate will affect the government's budget constraint, given by equation (4). Solving for B, the one-period forward formulation becomes

$$B_{t+1} = \frac{\tau_{t+1} w_{t+1}(1+l_{t+1})}{1-l_{t+1}}. \tag{14}$$

The fact that the old get pensions after retirement, not from the beginning of the old-age, makes the problem much more complicated compared with the standard Diamond model: the pension level depends on the labor supply, while the labor supply is also affected by the pension level.

We analyze the effects of a PAYG system in a defined benefit scheme. Specifically, we assume a constant tax rate policy, $\tau_t = \tau$ for all t, and study how an increase in the contribution rate will affect capital intensity, savings, labor supply, and welfare in the equilibrium. We first consider the effects of the PAYG system in a partial equilibrium given wages and interest rates, and then the general equilibrium effects.

4.1. Partial Equilibrium

We start by studying the partial equilibrium case where the factor prices are fixed. Accordingly, the effect of the pension program depends on the responses of the individuals and the government's budget constraint. Before we try to solve the model, let us look more closely at the setup and discuss some of the intuition of the model. With a constant tax policy, the budget constraint of the government and the net returns to labor supply become[5]

$$B_{t+1} = \frac{\tau w_{t+1}(1+l_{t+1})}{1-l_{t+1}}, \tag{15}$$

$$w_t^y = (1-\tau) w_t, \tag{16}$$

$$w_{t+1}^o = (1-\tau) w_{t+1} - B_{t+1} = \left(1 - \frac{2\tau}{1-l_{t+1}}\right) w_{t+1}. \tag{17}$$

Obviously, as the contribution rate τ increases, the immediate impact is that pension level (B_{t+1}) increases, and the effective wage rates, w_t^y and w_{t+1}^o, decrease. According to Lemma 1, an increase in B_{t+1} will reduce savings and old-age labor supply, and a decrease in w_t^y will reduce savings but increase labor supply. It follows that the total effect on old-age labor supply is ambiguous. When $0 < \gamma \leq 1$, a decrease in w_{t+1}^o will increase savings and reduce labor supply. Hence, the total effect on savings is ambiguous as well. These are direct effects from an increase in contribution rate. However, as labor supply changes, the pension level also changes. As we have discussed above, with a balanced government budget, the pension level and the old-age labor supply affect each other, which makes the equilibrium effects complicated.

To proceed, we look at the present value of implicit lifetime income:

$$W_t = w_t^y + \frac{w_{t+1}^o + B_{t+1}}{R_{t+1}} = (1-\tau)\left(w_t + \frac{w_{t+1}}{R_{t+1}}\right). \tag{18}$$

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With fixed factor prices and a constant contribution rate, the implicit lifetime income is $(1-\tau)$ proportion of the level without PAYG pension program (henceforth laissez-faire).

Using equations (8) and (10), we can easily obtain the following implicit expressions for savings and old-age labor supply:

$$s_t = w_t^y - \frac{1}{1+\beta^{\frac{1}{\gamma}} R_{t+1}^{\frac{1}{\gamma}-1} + (\beta\mu)^{\frac{1}{\gamma}} R_{t+1}^{\frac{1}{\gamma}-1} \left(w_{t+1}^o\right)^{1-\frac{1}{\gamma}}} W_t, \tag{19}$$

$$l_{t+1} = 1 - \frac{(\beta\mu R_{t+1})^{\frac{1}{\gamma}} \left(w_{t+1}^o\right)^{-\frac{1}{\gamma}}}{1+\beta^{\frac{1}{\gamma}} R_{t+1}^{\frac{1}{\gamma}-1} + (\beta\mu)^{\frac{1}{\gamma}} R_{t+1}^{\frac{1}{\gamma}-1} \left(w_{t+1}^o\right)^{1-\frac{1}{\gamma}}} W_t. \tag{20}$$

As contribution rate changes, although factor prices (w_t, w_{t+1}, R_{t+1}) are fixed in the partial equilibrium, the effective wage rates $(w_t^y,\ w_{t+1}^o)$ change and will affect savings and the old-age labor supply. According to equation (17), the effective old-age wage rate is also a function of labor supply and contribution rate. If we can solve $l_{t+1} \equiv \hat{l}(\tau, w_t, w_{t+1}, R_{t+1})$ from equation (20), then $s_t \equiv \hat{s}(\tau, w_t, w_{t+1}, R_{t+1})$ will also be solved from equation (19). But for a general γ, it is impossible to solve l_{t+1} explicitly from equation (20), and it is hard to tell how an increase in contribution rate is going to affect the old-age labor supply.

Thus, we look at a special case where $\gamma = 1$. Using equations (16), (17), (19), and (20) and $L_{t+1} = 1 + l_{t+1}$, we obtain

$$s_t = (1-\tau)\frac{\beta(1+\mu)w_t - \frac{w_{t+1}}{R_{t+1}}}{1+\beta+\beta\mu}, \tag{21}$$

$$L_{t+1} = (1-\tau)\frac{2+2\beta+\beta\mu-\beta\mu R_{t+1}\frac{w_t}{w_{t+1}}}{1+\beta+\beta\mu}. \tag{22}$$

Since capital stock in each period comes from aggregate savings of previous period, we require $s_t > 0$ for all t. Also, $L_{t+1} > 0$. Hence, equations (21) and (22) imply that in the partial equilibrium,

$$\frac{\partial s_t}{\partial \tau} < 0,\ \frac{\partial l_{t+1}}{\partial \tau} = \frac{\partial L_{t+1}}{\partial \tau} < 0.$$

The partial derivative signs indicate that the wage rate and interest rate are held fixed. We then see that with a logarithmic utility function, the effects on savings and labor supply can be signed. The intuition behind the results is as follows. With a logarithmic utility function, equation (8) becomes

$$c_{1,t} = \frac{1}{1+\beta+\beta\mu} W_t. \tag{23}$$

As τ changes, $c_{1,t}$ remains the same proportion of W_t as in the laissez-faire case. Also, the effective young-age wage rate is $(1-\tau)$ proportion of the wage rate in the laissez-faire. Combining equations (16), (18), and (23), we know that

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the savings by young agents, $s_t = w_t^y - c_{1,t}$, are exactly $(1-\tau)$ proportion of the saving amount in the laissez-faire case. This gives equation (21).

For the labor supply, if $\gamma = 1$, equation (10) becomes

$$1-l_{t+1} = \frac{\beta\mu \frac{1}{w_{t+1}^o/R_{t+1}}}{1+\beta+\beta\mu} W_t = \frac{\beta\mu \frac{1}{w_{t+1}^o/R_{t+1}}}{1+\beta+\beta\mu} (1-\tau)\left(w_t + \frac{w_{t+1}}{R_{t+1}}\right). \tag{24}$$

As contribution rate increases, both the lifetime income (W_t) and the price of leisure (w_{t+1}^o/R_{t+1}) change. According to equation (24), this is equivalent to the case where lifetime income remains the same as in the laissez-faire case ($w_t + \frac{w_{t+1}}{R_{t+1}}$), but the price of leisure becomes

$$\frac{\frac{w_{t+1}^o}{R_{t+1}}}{1-\tau} = \frac{1-\frac{2\tau}{1-l_{t+1}}}{1-\tau}\frac{w_{t+1}}{R_{t+1}}, \tag{25}$$

which is a decreasing function of τ since $\frac{\partial}{\partial\tau}\frac{1-\frac{2\tau}{1-l_{t+1}}}{1-\tau} = -\frac{1}{(1-\tau)^2}\frac{1+l_{t+1}}{1-l_{t+1}} < 0$. So as contribution rate increases, the price of leisure decreases, which induces individuals to consume more leisure and reduce old-age labor supply.

Using equations (21) and (22), we can further calculate the capital–labor ratio:

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = \frac{s_t}{L_{t+1}} = \frac{\beta(1+\mu)w_t - \frac{w_{t+1}}{R_{t+1}}}{2+2\beta+\beta\mu-\beta\mu R_{t+1}\frac{w_t}{w_{t+1}}}. \tag{26}$$

Equation (26) shows that with a logarithmic utility function, the capital intensity will not be affected by the PAYG pension program in the partial equilibrium. Recall that in a standard Diamond model, the capital accumulation (savings) also decreases as the contribution rate increases, but the labor supply is fixed at 1, so the capital intensity must decrease. With endogenous retirement, the old-age labor supply is also reduced, and the capital–labor ratio keeps constant in this special case.

These are only partial equilibrium effects. The changes in savings and labor supply will change the capital–labor ratio and affect factor prices. The changes in wage rates and interest rates will in turn affect savings and old-age labor supply. To find the general equilibrium effects, we proceed to the next part.

4.2. General Equilibrium

In a general equilibrium, factor prices change as well. The capital accumulation equation (13), together with the factor market equilibrium conditions (1) and (2), implies the equilibrium law of motion for the capital–labor ratio, given $k_1 > 0$:

$$k_{t+1}[1+\hat{l}(\tau, w(k_t), w(k_{t+1}), R(k_{t+1}))] = \hat{s}(\tau, w(k_t), w(k_{t+1}), R(k_{t+1})), \tag{27}$$

where $\hat{s}(\cdot)$ and $\hat{l}(\cdot)$ are defined above by equations (19) and (20).

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In the remainder of this paper, we focus on the long-run steady-state equilibrium. Specifically, for a given τ, the capital–labor ratio k is stationary and satisfies:

$$k[1+\hat{l}(\tau, w(k), R(k))] = \hat{s}(\tau, w(k), R(k)). \tag{28}$$

It is assumed that a unique steady-state k exists. The conditions needed to ensure the existence are straightforward and discussed in Nourry (2001). From equation (28), it is clear that a PAYG pension program can affect the steady-state capital–labor ratio through several channels. The changes in contribution rate will affect young agents' savings and old agents' labor supply—these are direct effects. The indirect effects work through the factor prices. As the capital–labor ratio changes, both the wage rates and the interest rates change, which in turn affect the agents' behavior.

At this stage, it is instructive to pause briefly and revisit a classical result from the standard Diamond model. In such a model, where young agents work one unit of time and old agents are retired the entire second period, a well-known result is that a PAYG system will always reduce the incentive to save and thereby reduce capital accumulation at a stable steady state [see Blanchard and Fischer (1989)]. But in the presence of endogenous retirement, a PAYG system will not only affect savings but also the old-age labor supply. And as shown above, there are countervailing effects in this setup, it is by no means clear-cut how the capital–labor ratio will be affected. All sorts of possibilities, including full neutrality, may emerge.

In the following section, we study how an increase in the contribution rate is going to affect the capital accumulation (savings), labor supply, and capital–labor ratio in the steady state. We first consider the logarithmic utility case and then the general CES utility case. For simplicity, we assume a Cobb–Douglas production function:

$$F(K, L) = AK^{\alpha}L^{1-\alpha}. \tag{29}$$

The first-order conditions of firms given by equations (1) and (2) now become

$$w_t = (1-\alpha)\, A\left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{30}$$

$$R_t = \alpha A\left(\frac{K_t}{L_t}\right)^{\alpha-1}. \tag{31}$$

4.2.1. Logarithmic utility case. Evaluating equations (21) and (22) in the steady state and plugging in factor prices (30) and (31), we solve for L, K, and thus k in the steady state:

$$L = (1-\tau)\,\frac{(1-\alpha)\,(2+\mu)}{1-\alpha+\mu}, \tag{32}$$

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$$K = \left[\frac{1+2\beta+\beta\mu+\frac{1}{\alpha}}{A\beta\,(1-\alpha+\mu)}\right]^{\frac{1}{\alpha-1}} L, \tag{33}$$

$$k = \frac{K}{L} = \left[\frac{1+2\beta+\beta\mu+\frac{1}{\alpha}}{A\beta\,(1-\alpha+\mu)}\right]^{\frac{1}{\alpha-1}}. \tag{34}$$

In this special case, both the capital accumulation and labor supply decrease in the steady state. However, if we look at the capital–labor ratio in equation (34), we see that it is independent of the contribution rate. Hence, if preferences conform to the commonly studied logarithmic utility, a PAYG pension system is exactly neutral in the steady state.

PROPOSITION 1. *In a two-period OLG model with endogenous retirement, if we assume a Cobb–Douglas production function and a logarithmic utility function, an increase in the PAYG contribution rate reduces both capital accumulation and labor supply. However, capital intensity is independent of the contribution rate. Hence, a PAYG pension system is neutral, that is,* $dk/d\tau = 0$ *if* $\gamma = 1$.

The intuition has been discussed in the partial equilibrium analysis. Actually equation (26) has already implied the results in Proposition 1. In the general equilibrium, both wage rate and interest rate are only functions of capital–labor ratio, and will not be affected by contribution rate, so according to equation (26), the steady-state capital–labor ratio will not be affected by contribution rate either.

4.2.2. CES utility case. Here we turn to a more general case with a CES utility function and a Cobb–Douglas production function. Evaluating all the equilibrium conditions in the steady state, we get two equations in k and l:

$$l = (1-\alpha)\,(1-\tau)\left[1+\frac{Ak^{\alpha-1}-1}{1+(\beta\alpha)^{-\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}k^{(\alpha-1)\left(1-\frac{1}{\gamma}\right)}}\right]-1, \tag{35}$$

$$k^{\alpha\left(\frac{1}{\gamma}-1\right)} = \mu^{\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}(1-\alpha)^{-\frac{1}{\gamma}}\frac{\left(1-\frac{2}{1-l}\tau\right)^{-\frac{1}{\gamma}}(\alpha+(1-\alpha)\tau+l)}{1-l}. \tag{36}$$

Based on equations (35) and (36), we have the following proposition:

PROPOSITION 2. *In a two-period OLG model with endogenous retirement, if we assume a Cobb–Douglas production function and a CES utility function with the elasticity of substitution between consumption and leisure less than 1, an increase in the PAYG contribution rate will increase capital intensity and reduce the old-age labor supply, that is,*

$$\frac{dk}{d\tau} > 0 \text{ and } \frac{dl}{d\tau} < 0 \text{ if } \gamma \in (0,1).$$

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The proof of Proposition 2 is in Appendix B. In the standard Diamond model, a PAYG system is never neutral. An increase in the size of the system necessarily reduces capital formation. This is the original concern with PAYG systems if the economy is underaccumulating. Here, if endogenous retirement is taken into account, we find that implementing a PAYG pension can be neutral, as we have shown in the logarithmic case, or rather than being a source of concern, a PAYG system can even increase the capital–labor ratio in the economy. Proposition 2 identifies the conditions under which a PAYG system will increase capital intensity. We do not have a conclusion on the changes of capital accumulation (savings) in this model, but since total labor supply unambiguously decreases as the contribution rate increases, even though the capital stock shrinks (which is the case in the numerical example in the next section), the decrease of the labor supply plays a more important role for the increase of capital–labor ratio in the economy. Notice that the result only holds for $\gamma \in (0, 1)$. The rationale for confining the analysis to this case is that it contrasts the classical result.

In Diamond (1965), a PAYG system reduces the capital–labor ratio and via that channel, it reduces steady-state welfare if the economy is dynamically efficient. Since most countries satisfy this condition, the PAYG system weakens the welfare. The potential to overturn this line of logic is now possible. In the model with endogenous retirement, a PAYG system may help to increase the capital–labor ratio. As pointed out in Hu (1979), there are two countervailing effects as the size of a PAYG pension program increases. On one hand, the PAYG pension program causes the marginal rate of substitution between old-age consumption and leisure (retirement) to diverge from the wage rate, and this results in static inefficiency. On the other hand, the possibility of increasing steady-state capital intensity may help to increase the dynamic efficiency. The total welfare effect depends on which one prevails, and that is the subject of the next section.

5. WELFARE

In this section, we assume $R > 1$[6] and study the effect of implementing a PAYG pension system on the steady-state utility of a representative agent. With $R > 1$, a PAYG system can never be Pareto-improving in the standard Diamond model. The initial generation gains, but all future generations lose. Each future generation incurs a loss because it receives a return 1 on the social security contributions which is less than the return R it would earn by investing those funds in capital. Once endogenous retirement is introduced, matters are far more complicated because of the presence of another effect: agents modify their old-age labor supply in response to the pension program. While capital–labor ratio increasing becomes possible, the possibility of welfare improvement also emerges, as long as the dynamic efficiency gains from capital–labor ratio increasing dominate the static inefficiency caused by price distortion.

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The analysis of welfare effects is conducted in a general setup without assuming either CES utility or Cobb–Douglas technology. The utility function takes the following form:

$$U(c_{1,t}, c_{2,t+1}, l_{t+1}) = u(c_{1,t}) + \beta[v(c_{2,t+1}) + \mu h(1-l_{t+1})], \tag{37}$$

where $u(\cdot)$, $v(\cdot)$, and $h(\cdot)$ are each twice continuously differentiable. Moreover, $u' > 0 > u''$, $v' > 0 > v''$, and $h' > 0 > h''$. From the above analysis, we know that for any given τ, the steady-state indirect utility function can be written as

$$U(\tau) = u(c_1(\tau)) + \beta v(c_2(\tau)) + \beta\mu h(1-l(\tau)), \tag{38}$$

with

$$\begin{aligned} c_1(\tau) &= (1-\tau)w(k(\tau)) - \hat{s}(\tau, w(k(\tau)), R(k(\tau))), \\ c_2(\tau) &= R(k(\tau))\hat{s}(\tau, w(k(\tau)), R(k(\tau))) \\ &\quad + w(k(\tau))\hat{l}(\tau, w(k(\tau)), R(k(\tau))) + \tau w(k(\tau)), \\ l(\tau) &= \hat{l}(\tau, w(k(\tau)), R(k(\tau))), \end{aligned}$$

where $\hat{s}(\cdot)$ and $\hat{l}(\cdot)$ are defined above but now we further write k as a function of τ.

The government is assumed to be benevolent. It chooses τ to maximize the utility of an agent in a steady-state equilibrium. Differentiating the indirect utility in equation (38) with respect to τ and reorganizing, we get

$$\begin{aligned} \tfrac{dU}{d\tau} &= -wu'(\cdot) + \beta w v'(\cdot) + \left[(1-\tau)\, w_k u'(\cdot) + \beta v'(\cdot)\left(R_k\hat{s} + w_k\hat{l} + \tau w_k\right)\right]\tfrac{dk}{d\tau} \\ &\quad + \left[\beta R v'(\cdot) - u'(\cdot)\right]\tfrac{d\hat{s}}{d\tau} + \beta\left[w v'(\cdot) - \mu h'(\cdot)\right]\tfrac{d\hat{l}}{d\tau}, \end{aligned}$$

where

$$\begin{aligned} \frac{d\hat{s}}{d\tau} &= \hat{s}_\tau + \left(\hat{s}_w w_k + \hat{s}_R R_k\right)\frac{dk}{d\tau}, \\ \frac{d\hat{l}}{d\tau} &= \hat{l}_\tau + \left(\hat{l}_w w_k + \hat{l}_R R_k\right)\frac{dk}{d\tau}. \end{aligned}$$

By solving the individual's problem (choosing $c_{1,t}, c_{2,t+1}, l_{t+1}$ to maximize utility (37) subject to budget constraints (5) and (6)) and evaluating equilibrium conditions in the steady state, we have $u'(c_1) = \beta R v'(c_2)$, and $w\left(1 - \frac{2\tau}{1-\hat{l}}\right)v'(c_2) = \mu h'(1-\hat{l})$. Also, $w_k = -kR_k$ and $\hat{s} = (1+\hat{l})k$. By exploiting these relations, the effect of contribution rate on utility can be written as

$$\frac{dU}{d\tau} = \beta w(1-R)v'(\cdot) + (1-\tau)(1-R)\beta k R_k v'(\cdot)\frac{dk}{d\tau} + \beta\frac{2\tau}{1-\hat{l}} w v'(\cdot)\frac{d\hat{l}}{d\tau}.$$

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Evaluated near $\tau = 0$, it all boils down to a simple-looking condition:

$$\left.\frac{dU}{d\tau}\right|_{\tau=0} = \underbrace{\beta\,(1-R)\,v'(\cdot)}_{(1)}\underbrace{\left(w - w_k\frac{dk}{d\tau}\right)}_{(2)}\Bigg|_{\tau=0}. \tag{39}$$

Since $R > 1$, factor (1) is negative, while the sign of factor (2) is indeterminate. The sign of factor (2) is partly determined by Proposition 1. Accordingly, whether a PAYG system is desirable or not does not solely depend on comparison between interest rate and population growth rate; it also depends on how the capital–labor ratio responds to the pension program. Indeed, the possibility arises that there may be long-run welfare gains from introducing a PAYG pension. As stated earlier, if the pension system can increase capital–labor ratio and if the corresponding positive effect dominates, there will be welfare improvement in the long run.

Based on Proposition 1 and equation (39), we deduce the following proposition.

PROPOSITION 3. *In a two-period OLG model with endogenous retirement, the effect of introducing a PAYG pension program on the steady-state welfare is*

$$\left.\frac{dU}{d\tau}\right|_{\tau=0} = \beta(1-R)v'(c_2)\left(w - w_k\frac{dk}{d\tau}\right)\Bigg|_{\tau=0}. \tag{40}$$

That is, given $R > 1$,

If $\frac{dk}{d\tau} < \frac{w}{w_k}$, there is no long-run welfare case for introducing a PAYG pension;

If $\frac{dk}{d\tau} > \frac{w}{w_k}$, introducing a PAYG pension will be beneficial in terms of steady-state welfare.

Since $\frac{w}{w_k} > 0$, if introducing a PAYG pension program is going to reduce capital–labor ratio in the long run, the steady-state welfare must be hurt. However, $dk/d\tau$ being positive is not enough to ensure a welfare gain. To generate welfare gains from implementing a PAYG pension program, the capital intensity increasing should be high enough ($\frac{dk}{d\tau} > \frac{w}{w_k}$) to overcome the static inefficiency. Andersen and Bhattacharya (2013) show in a model with endogenous young-age labor supply that there is no case for introducing a PAYG pension. The reason is that in their setup with CES utility functions, even if $\gamma \leq 1$, introducing a pension system will cause the capital–labor ratio to decrease. Thus, the role of the capital–labor ratio is very essential when measuring the welfare. Also notice that in a standard setup, if the pension is neutral in capital–labor ratio, it is neutral in welfare; this is not the case here. With a logarithmic utility function ($\gamma = 1$), introducing a PAYG pension is neutral in capital–labor ratio, but according to Proposition 3, the long-run welfare decreases.

To extend the results outside of $\tau = 0$, we close this section with a numerical example, not intended to be a calibration exercise. Suppose $U = \frac{1}{1-\gamma}c_1^{1-\gamma} + \beta\left[\frac{1}{1-\gamma}c_2^{1-\gamma} + \mu\frac{1}{1-\gamma}(1-l)^{1-\gamma}\right]$ and $F = A\left(\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right)^{-\frac{1}{\rho}}$, $\rho \geq -1$.

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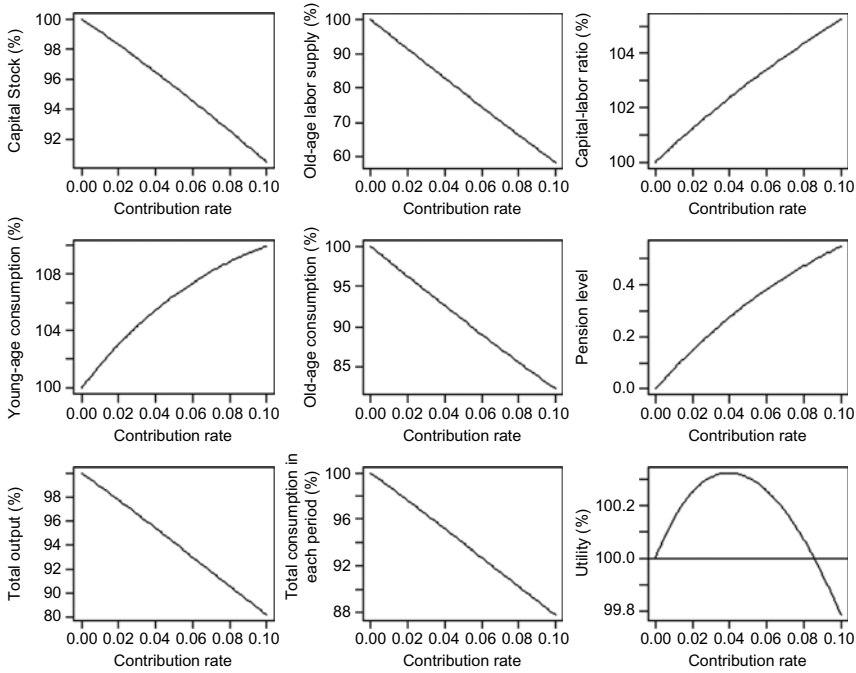


FIGURE 1. The effects of introducing a PAYG pension in the steady state.

With $\gamma = 0.4$, $\beta = 0.5$, $\mu = 1$, $\alpha = 0.35$, $\rho = 3.5$, and $A = 6$, Figure 1 plots the effects of increasing the pension contribution rate on the economy. To see relative changes, the vertical axis depicts the values of variables in percent as compared with the laissez-faire case ($\tau = 0$), except for pension level.

As Figure 1 illustrates, in an economy with $R > 1$, as the contribution rate increases, both the capital stock and labor supply in this economy decrease, thus the total output is also reduced. Investment and total consumption level also decrease. But as the contribution rate increases in the range of $(0, 4\%)$, the steady-state capital–labor ratio and welfare both increase. This is because within this range, the dynamic efficiency gains from capital–labor ratio increase dominate the static inefficiency from price distortion. However, as capital–labor ratio keeps increasing, the dynamic efficiency gains may become weaker or even disappear, and the steady-state welfare starts to decrease.

6. CONCLUSION

A classic result in dynamic public economics states that a PAYG system will reduce capital intensity, and if the economy is dynamically efficient, the system will reduce welfare as well. This result can be shown in a standard Diamond OLG model with exogenous labor supply. The main channel for this result is the effect

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on savings through the pensions system. In this paper, we study these established results in a setting where endogenous retirement is included. The relation between the pension system, capital intensity, and welfare becomes significantly more complex. The reason is that a second channel emerges. It relates the pension system and labor supply, and thus the capital–labor ratio, along with the relation between savings, factor prices, and capital accumulation.

Within the environment applied in this paper, several results are obtained. Firstly, with logarithmic utility and Cobb-Douglas technology, in the steady state, the pension system will not affect the capital–labor ratio and thus a PAYG system is neutral in terms of capital intensity. Secondly, it is shown that with CES utility, the effect of the PAYG system on the steady-state capital intensity depends on the elasticity of substitution between consumption and leisure. If the elasticity of substitution is between 0 and 1, the PAYG pension scheme will increase the capital–labor ratio. Thirdly, introducing a PAYG system may increase steady-state welfare. These results are at odds with the classic results in a standard Diamond OLG model.

As mentioned in the Introduction, Hu (1979) derives a similar model. However, he includes bequest and assumes a lump-sum tax while we use distortionary taxes. More importantly, Hu (1979) is unable to characterize the conditions under which implementing a PAYG system may increase the steady-state capital–labor ratio and welfare. The current paper, in effect, offers a test of whether a country can gain from a PAYG system, depending on the elasticity of substitution between consumption and leisure, and the response of capital intensity to the pension system.

In conclusion, it is important that a caveat be recorded. The current analysis, following the literature, is restricted to steady states. Our justification for using steady-state welfare as the yardstick for judging desirability is simple: if a policy fails to generate long-run welfare gains, it is unlikely to be ever adopted. However, despite our demonstration of the steady-state welfare gains, it may be questioned whether such an arrangement can ever be implemented in the short run. After all, it is quite possible that some initial generations are hurt in the process of getting to that long-run outcome. There may be ways to leave the welfare of these generations unchanged even after the policy is introduced. Presumably there are mechanisms—similar in spirit to ones studied in Rangel (2003), Boldrin and Montes (2005), and Andersen and Bhattacharya (2017)—which allow some of the future welfare gains to be brought forward to compensate the initial generations. Such matters are worthy of future study.

NOTES

1. See, for example, Andersen and Bhattacharya (2013) for a study of young-age endogenous labor supply and social security. By allowing for endogenous labor supply in the first period of life, they show that, under a sufficient condition that the old will be no less risk-averse than the young, the Aaron–Samuelson results alluded to earlier still hold.

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2. As one period represents about 35 years, it is reasonable to assume that the depreciation rate is 1.

3. Using equation (10), given R_{t+1}, w_t, w_{t+1}, τ_t, τ_{t+1}, and B_{t+1}, we can find the condition for interior solutions ($l_{t+1} > 0$): $1 + \frac{(\beta R_{t+1})^{\frac{1}{\gamma}}}{R_{t+1}} - \left(\beta\mu \frac{R_{t+1}}{(1-\tau_{t+1})w_{t+1} - B_{t+1}}\right)^{\frac{1}{\gamma}} \left[(1-\tau_t)w_t + \frac{B_{t+1}}{R_{t+1}}\right] > 0$.

4. For the logarithmic case ($\gamma = 1$), a well-known result in the standard Diamond model is that as interest rate increases, the income effect and substitution effect cancel each other, and savings remain unchanged. However, even in the standard model, this result won't hold if there is pension or other forms of income in the old age, as shown in equation (11). With $\gamma = 1$, equation (11) becomes $s_t = \left(\beta w_t^y - \frac{B_{t+1}}{R_{t+1}}\right)/(1+\beta)$. So if $B_{t+1} > 0$, as interest rate increases, agents will save more.

5. Here we keep the time subscripts on w and R, although in the partial equilibrium, they are held constant.

6. In the standard Diamond model, the condition for dynamic efficiency in a steady state is interest rate being greater than the population growth rate, which is $R > 1$ if there is no population growth. When endogenous retirement is introduced in the model, the conditions for dynamic efficiency are not discussed in this paper. Instead, we compare the interest rate and population growth rate directly and require $R > 1$.

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APPENDIX A: PROOF OF LEMMA 1

The proof of Lemma 1 is straightforward and follows from the derivatives. The proof of Proposition 2 is as follows. Since

$$s_t = w_t^y - \frac{(\beta R_{t+1})^{-\frac{1}{\gamma}}\left(R_{t+1}w_t^y + w_{t+1}^o + B_{t+1}\right)}{1+(\beta R_{t+1})^{-\frac{1}{\gamma}}R_{t+1} + \mu^{\frac{1}{\gamma}}\left(w_{t+1}^o\right)^{1-\frac{1}{\gamma}}},$$

and

$$l_{t+1} = 1 - \frac{R_{t+1}w_t^y + w_{t+1}^o + B_{t+1}}{\left(1+\beta^{-\frac{1}{\gamma}}R_{t+1}^{1-\frac{1}{\gamma}}\right)\left(\frac{w_{t+1}^o}{\mu}\right)^{\frac{1}{\gamma}} + w_{t+1}^o},$$

it is obvious when $0 < \gamma \leq 1$ $\left(\Leftrightarrow 1-\frac{1}{\gamma} \leq 0\right)$, we have $\frac{\partial s_t}{\partial w_{t+1}^o} < 0$ and $\frac{\partial l_{t+1}}{\partial R_{t+1}} < 0$. Also, s_t and l_{t+1} can be written as

$$s_t = w_t^y - \underbrace{\frac{w_t^y}{\beta^{\frac{1}{\gamma}}R_{t+1}^{\frac{1}{\gamma}-1}\left(1+\mu^{\frac{1}{\gamma}}\left(w_{t+1}^o\right)^{1-\frac{1}{\gamma}}\right)+1}}_{\text{decreasing in } R_{t+1} \text{ if } \frac{1}{\gamma}-1>0} - \underbrace{\frac{w_{t+1}^o + B_{t+1}}{(\beta R_{t+1})^{\frac{1}{\gamma}}\left(1+\mu^{\frac{1}{\gamma}}\left(w_{t+1}^o\right)^{1-\frac{1}{\gamma}}\right)+R_{t+1}}}_{\text{decreasing in } R_{t+1}}, \tag{41}$$

$$l_{t+1} = 1 - \underbrace{\frac{R_{t+1}w_t^y + B_{t+1}}{\left(1+\beta^{-\frac{1}{\gamma}}R_{t+1}^{1-\frac{1}{\gamma}}\right)\left(\frac{w_{t+1}^o}{\mu}\right)^{\frac{1}{\gamma}} + w_{t+1}^o}}_{\text{decreasing in } w_{t+1}^o} - \underbrace{\frac{1}{\left(1+\beta^{-\frac{1}{\gamma}}R_{t+1}^{1-\frac{1}{\gamma}}\right)\mu^{-\frac{1}{\gamma}}\left(w_{t+1}^o\right)^{\frac{1}{\gamma}-1}+1}}_{\text{decreasing in } w_{t+1}^o \text{ if } \frac{1}{\gamma}-1>0}. \tag{42}$$

So when $0 < \gamma \leq 1$, $\frac{\partial s_t}{\partial R_{t+1}} > 0$ and $\frac{\partial l_{t+1}}{\partial w_{t+1}^o} > 0$. ■

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APPENDIX B: PROOF OF PROPOSITION 2

Based on equations (35) and (36), we define two functions:

$$G(k,l,\tau)=(1-\alpha)\,(1-\tau)\left(1+\frac{Ak^{\alpha-1}-1}{1+(\beta\alpha)^{-\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}k^{(\alpha-1)\left(1-\frac{1}{\gamma}\right)}}\right)-1-l=0,$$

$$H(k,l,\tau)=\mu^{\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}(1-\alpha)^{-\frac{1}{\gamma}}\frac{\left(1-\frac{2}{1-l}\tau\right)^{-\frac{1}{\gamma}}(\alpha+(1-\alpha)\,\tau+l)}{1-l}-k^{\alpha\left(\frac{1}{\gamma}-1\right)}=0.$$

Using $\frac{\partial G}{\partial k}dk+\frac{\partial G}{\partial l}dl+\frac{\partial G}{\partial \tau}d\tau=0$ and $\frac{\partial H}{\partial k}dk+\frac{\partial H}{\partial l}dl+\frac{\partial H}{\partial \tau}d\tau=0$, we can solve:

$$\frac{dk}{d\tau}=-\frac{\frac{\partial G}{\partial \tau}\frac{\partial H}{\partial l}-\frac{\partial H}{\partial \tau}\frac{\partial G}{\partial l}}{\frac{\partial G}{\partial k}\frac{\partial H}{\partial l}-\frac{\partial H}{\partial k}\frac{\partial G}{\partial l}};\ \frac{dl}{d\tau}=-\frac{\frac{\partial G}{\partial k}\frac{\partial H}{\partial \tau}-\frac{\partial H}{\partial k}\frac{\partial G}{\partial \tau}}{\frac{\partial G}{\partial k}\frac{\partial H}{\partial l}-\frac{\partial H}{\partial k}\frac{\partial G}{\partial l}},$$

where

$$\frac{\partial G}{\partial k}=-(1-\alpha)^2\,(1-\tau)\,\frac{Ak^{\alpha-2}+\left(\frac{1}{\gamma}\left(Ak^{\alpha-1}-1\right)+1\right)(\beta\alpha)^{-\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}k^{(\alpha-1)\left(1-\frac{1}{\gamma}\right)-1}}{\left(1+(\beta\alpha)^{-\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}k^{(\alpha-1)\left(1-\frac{1}{\gamma}\right)}\right)^2}<0,$$

$$\frac{\partial G}{\partial l}=-1<0,$$

$$\frac{\partial G}{\partial \tau}=-\,(1-\alpha)\left(1+\frac{Ak^{\alpha-1}-1}{1+(\beta\alpha)^{-\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}k^{(\alpha-1)\left(1-\frac{1}{\gamma}\right)}}\right)=-\frac{1+l}{1-\tau}<0,$$

$$\frac{\partial H}{\partial k}=-\alpha\left(\frac{1}{\gamma}-1\right)k^{\alpha\left(\frac{1}{\gamma}-1\right)-1}<0\text{ for }\gamma<1,$$

$$\frac{\partial H}{\partial l}=\mu^{\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}(1-\alpha)^{-\frac{1}{\gamma}}\left(1-\tfrac{2}{1-l}\tau\right)^{-\frac{1}{\gamma}}\frac{\frac{2\tau\,(\alpha+(1-\alpha)\,\tau+l)}{\gamma\,(1-l-2\tau)}+1+\alpha+(1-\alpha)\,\tau}{(1-l)^2}>0,$$

$$\frac{\partial H}{\partial \tau}=\frac{\mu^{\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}(1-\alpha)^{-\frac{1}{\gamma}}\left(1-\frac{2}{1-l}\tau\right)^{-\frac{1}{\gamma}}}{1-l}\left(\frac{2\,(\alpha+(1-\alpha)\,\tau+l)}{\gamma\,(1-l-2\tau)}+1-\alpha\right)>0.$$

Thus, for $\gamma<1$, $\frac{\partial G}{\partial k}\frac{\partial H}{\partial l}-\frac{\partial H}{\partial k}\frac{\partial G}{\partial l}<0$, that is, the denominator of $\frac{dk}{d\tau}$ and $\frac{dl}{d\tau}$ is positive. Also, $\frac{\partial G}{\partial k}\frac{\partial H}{\partial \tau}-\frac{\partial H}{\partial k}\frac{\partial G}{\partial \tau}<0$, that is, the numerator of $\frac{dl}{d\tau}$ is negative, so we always have $\frac{dl}{d\tau}<0$ for $\gamma<1$.

The numerator of $\frac{dk}{d\tau}$ is

$$\frac{\partial G}{\partial \tau}\frac{\partial H}{\partial l}-\frac{\partial H}{\partial \tau}\frac{\partial G}{\partial l}=\frac{2(1-\gamma)\mu^{\frac{1}{\gamma}}A^{1-\frac{1}{\gamma}}(1-\alpha)^{-\frac{1}{\gamma}}\left(1-\frac{2}{1-l}\tau\right)^{-\frac{1}{\gamma}}(\alpha+(1-\alpha)\,\tau+l)}{\gamma\,(1-\tau)\,(1-l)^2},$$

which implies:

$$\operatorname{sign}\left(\frac{dk}{d\tau}\right)=\operatorname{sign}(1-\gamma)\text{ for }\gamma<1,$$

so $\frac{dk}{d\tau}>0$ for $\gamma<1$. ■

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