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Vorticity reconnection during vortex cutting by a blade

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A computational study of vorticity reconnection, associated with the breaking and reconnection of vortex lines, during vortex cutting by a blade is reported. A series of Navier–Stokes simulations of vortex cutting with different values of the vortex strength are described, and the different phases in the vortex cutting process are compared to those of the more traditional vortex tube reconnection process. Each of the three phases of vortex tube reconnection described by Melander & Hussain (*Phys. Fluids* A, vol. 1(4), 1989, pp. 633–635) are found to have counterparts in the vortex cutting problem, although we also point out numerous differences in the detailed mechanics by which these phases are achieved. Of particular importance in the vortex cutting process is the presence of vorticity generation from the blade surface within the reconnection region and the presence of strong vortex stretching due to the ambient flow about the blade leading edge. A simple exact Navier–Stokes solution is presented that describes the process by which incident vorticity is stretched and carried towards the surface by the ambient flow, and then interacts with and is eventually annihilated by diffusive interaction with vorticity generated at the surface. The model combines a Hiemenz straining flow, a Burgers vortex sheet and a Stokes first problem boundary layer, resulting in a nonlinear ordinary differential equation and a partial differential equation in two scaled time and distance variables that must be solved numerically. The simple model predictions exhibit qualitative agreement with the full numerical simulation results for vorticity annihilation near the leading-edge stagnation point during vortex cutting.

Key words: flow–structure interactions, vortex flows, vortex interactions

1. Introduction

A blade approaching a vortex tube whose axis is oriented orthogonal to the blade leading edge will penetrate into the vortex core. In an inviscid fluid, Helmholtz's laws require that vortex lines remain material lines, and consequently the vortex tube will deform around the blade leading edge, but vortex lines originating within the vortex tube will remain within the tube (Marshall & Grant 1996; Marshall 2001). In a viscous fluid, the Helmholtz restriction no longer applies, and vortex lines originating within the tube are observed to break and reconnect to vortex lines originating within the blade boundary layer (Liu & Marshall 2004). The term 'vorticity reconnection'

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refers to the topological change associated with vortex lines originating within the vortex tube breaking and reconnecting to vortex lines originating within a different vorticity region (Kida & Takaoka 1994). The term 'vortex cutting' refers to a specific type of vorticity reconnection process in which a solid object (such as the blade in the example above) passes through the vortex core, forcing vortex lines originating within the vortex tube to break and reconnect to those within the boundary layer of the solid object. Vortex cutting differs from traditional vortex reconnection problems by the fact that a vorticity generation surface (the surface of the solid body) lies in the reconnection region.

Vortex cutting is commonly observed when a propeller or airfoil blade passes through a vortex structure for cases where the blade width is small or of the same order as the vortex core diameter. In the canonical vortex cutting problem, the ambient vortex axis is orthogonal to the symmetry plane of the blade, and the relative translation velocity of the blade and the vortex is in the direction of the blade chord (Coton *et al.* 2004). Vortex cutting is common in helicopter flows under slow flight conditions, where the main rotor vortices are swept backwards and chopped by the tail rotor (Leverton, Pollard & Wills 1977; Sheridan & Smith 1980). The problem also arises in pumps, where intake vortices are cut by the pump impeller (Nagahara, Sato & Okamura 2001). In torpedo and submarine flow fields, the streamwise hull vortices and vortices shed from upstream control surfaces can become ingested into the propeller intake and cut by the propeller blades (Felli, Roberto & Guj 2009; Felli & Falchi 2011). Similarly in turbomachinery flows, upstream vortices shed from the stator blades are advected downstream and can be cut by the rotor blades (Binder 1985), leading to enhanced turbulence generation. In cases where the vortex possesses a non-zero axial flow, vortex cutting events can lead to exertion of a sudden force on the blade, which can cause performance degradation, material fatigue or pitting (particularly in the presence of local cavitation), and noise and vibration generation (Paterson & Amiet 1979; Ahmadi 1986; Cary 1987).

Vortex cutting is part of the more general process of orthogonal vortex–blade interaction, a review of which is given by Coton *et al.* (2004). A series of detailed experimental studies of orthogonal vortex–blade interaction and vortex cutting at high Reynolds number have been performed in the wind tunnel at Glasgow University using blade pressure measurements, flow visualization and particle image velocimetry (Doolan, Coton & Galbraith 1999, 2001; Green, Doolan & Cannon 2000; Early, Green & Coton 2002; Wang *et al.* 2002; Green, Coton & Early 2006). Experiments of vortex cutting in water at low Reynolds numbers have also been conducted, which allow for improved visualization using techniques such as laser-induced fluorescence, as reported by Johnston & Sullivan (1992), Marshall & Krishnamoorthy (1997) and Krishnamoorthy & Marshall (1998). The effect of vortex cutting on the ambient axial flow within the vortex core was examined using simplified numerical and analytic models by Marshall (1994), Marshall & Yalamanchili (1994), Marshall & Krishnamoorthy (1997) and Lee, Burggraf & Conlisk (1998) and experimentally by Krishnamoorthy & Marshall (1994). A key feature in many of these papers concerns wave motion induced on the vortex core by the sudden blocking of axial motion within the core during the vortex cutting process.

Detailed simulation of blade penetration into a vortex core was reported by Marshall & Grant (1996) for inviscid flow and by Liu & Marshall (2004) for viscous flow, in which the blade plane is oriented orthogonal to the vortex core at the point of impact. Cases with varying blade angle of attack were examined in viscous flow simulations by Filippone & Afgan (2008). These simulations exhibit a series of stages of the

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vortex cutting process. In the early stages of vortex cutting, as the blade leading edge is just starting to penetrate into the vortex core, the vortex responds in an almost inviscid fashion by reorienting the vortex lines originating within the vortex core to wrap around the blade leading edge. In the inviscid problem, the vortex lines within the vortex cannot be cut in accord with the second Helmholtz vortex law (Marshall 2001), and they consequently bend around the blade leading edge and stretch, creating a strong vortex sheet (Marshall & Grant 1996). In a viscous flow, the vorticity within the vortex near the leading edge diffusively interacts with vorticity of the opposite sign from the blade boundary layer, where the latter is generated by the induced velocity along the blade span generated by the approaching vortex. The middle stage of vortex cutting is dominated by this diffusive cancellation of vorticity of opposite sign between the vortex core and the blade boundary layer, which is what allows vortex lines in the vortex to be cut and to reconnect with vortex lines in the blade boundary layer, in a manner analogous to classic vortex reconnection problems (Saffman 1990; Kida & Takaoka 1994).

When the blade penetrates a sufficient distance into the vortex core, the spanwise velocity induced by the columnar vortex changes direction (Liu & Marshall 2004). This change in spanwise velocity direction leads to a change in sign of the vorticity orthogonal to the blade plane at the blade leading edge, so that after this point the blade vorticity in this orthogonal direction is of the same sign as that within the vortex core and diffusive cancellation can no longer occur. As a result, the reconnection process between the vortex lines originating within the vortex core and those originating within the blade boundary layer is delayed, and the remaining parts of the vortex wrap around the blade leading edge in the form of a thin sheet, similar to what is observed in the inviscid problem. This situation constitutes the late stage of vortex cutting.

The objective of the current paper is to determine whether the same parameters and time scales govern the breaking and rejoining of vortex lines in both the vortex cutting problem and the classical vortex tube reconnection problem, and whether the different phases of the vortex reconnection problem (see e.g. Shelley, Meiron & Orszag 1993; Kida & Takaoka 1994) have analogues in the vortex cutting problem. This investigation is conducted using two different approaches. In the first approach, we conduct a series of high-resolution simulations of vortex cutting by a blade with no ambient axial flow within the vortex and with different values of the impact parameter I, defined in terms of the vortex–blade relative velocity U, the vortex core radius σ and circulation Γ by $I = 2\pi\sigma U/\Gamma$. We examine the temporal and spatial variation of pressure, vorticity, surface vorticity flux, and stretching and reconnection of vortex lines to understand how these measures compare to similar measures in the literature on colliding vortex tubes. In the second approach, we present a highly simplified model that examines the vorticity diffusive cancellation process between an incident vortex (a stretched vortex sheet) and vorticity generated from a no-slip surface. The model is made analytically tractable by 'unwrapping' the blade surface, so that the vortex–blade interaction very close to the blade leading edge is represented by the problem of a stretched vortex sheet interacting with a flat surface in the presence of a straining flow. While this model is a bit too simplified to provide accurate quantitative comparison with the full Navier–Stokes vortex cutting simulation, as an exact Navier–Stokes solution it is useful for suggesting parameter scalings and for illustrating the physics of the vorticity cross-diffusion process in the presence of a no-slip surface.

We start with a brief review of classical vortex reconnection in § 2, which is used to compare the vortex cutting results to classical vortex tube reconnection later in the

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paper. The numerical method used in the study is described in § 3. Results of a series of numerical simulations with different impact parameter values are described in § 4, with discussion of the similarities and differences between vortex cutting and vortex tube reconnection problems. In § 5, we present a new exact Navier–Stokes solution that describes, in a qualitative way, some aspects of how the deformed impinging vortex interacts with the blade surface within the cutting region. Conclusions are given in § 6.

2. Highlights of vortex reconnection

As described by Melander & Hussain (1989), vortex reconnection processes can be characterized by a series of three phases: (1) inviscid induction, which leads to alignment of the vortex cores into an antiparallel formation (in which vortex axes are parallel with opposite axial vorticity sign) followed by core flattening and stretching, (2) bridging of the vortex cores, which occurs via cross-diffusion and cancellation of opposite-sign vorticity between the cores and subsequent linking of vortex lines, and (3) threading, or formation of fine threads from remnants of vorticity that have not yet completed reconnection before the cores are advected away from each other as a result of the high curvature of the reconnected vorticity sections.

In the first phase, two vortices driven towards each other will deform to adopt an antiparallel configuration in the region near the reconnection position (Siggia 1985). In many reconnection problems, the vortices are initially placed so that they will interact in this antiparallel orientation, but in some cases, such as the problems of orthogonally offset vortices examined by Zabusky & Melander (1989) and Boratav, Pelz & Zabusky (1992), significant distortion of the vortices is required to attain the antiparallel configuration. Once in this antiparallel configuration, each vortex will induce a two-dimensional straining flow on the opposing vortex, and as a consequence the core of each vortex will become deformed and elongated. The curvature of the three-dimensional vortices along their axes serves both to drive the vortex cores into each other and to induce an additional background straining flow on the vortex pair, which causes each deformed core to develop a head–tail structure (Kida, Takaoka & Hussain 1991). As shown in figure 1(*a*), the 'head' is a region of increased thickness along the outside of the curved vortex core, and the inner part of the core stretches out to become the 'tail'.

The second 'bridging' phase of vortex reconnection is dominated by the diffusive cross-cancellation of vorticity between the two antiparallel vortex cores. A simple model for this cross-cancellation process was proposed by Saffman (1990), the predictions of which were compared to results of numerical simulations by Shelley *et al.* (1993). As the opposite-sign vorticity cancels out due to diffusion between the two touching vortex cores, the vortex lines passing through the annihilated vorticity reconnect to those from the opposing vortex as a consequence of the requirement that the strength of a vortex tube remains uniform along the tube. These resulting bridges between the two vortex structures continue to grow stronger as more vorticity is diffusively annihilated within the vortex cores. However, since the vortex lines within these bridges are highly curved, their self-induced velocity increases as the bridges grow stronger, eventually causing them to propagate away from each other and discontinuing the vortex reconnection process before all vorticity within each vortex core has had a chance to diffusively interact with that in the opposing core.

The third phase of vortex reconnection deals with the remnants of vorticity, called threads, which are left behind as the bridges pull away from each other. As shown

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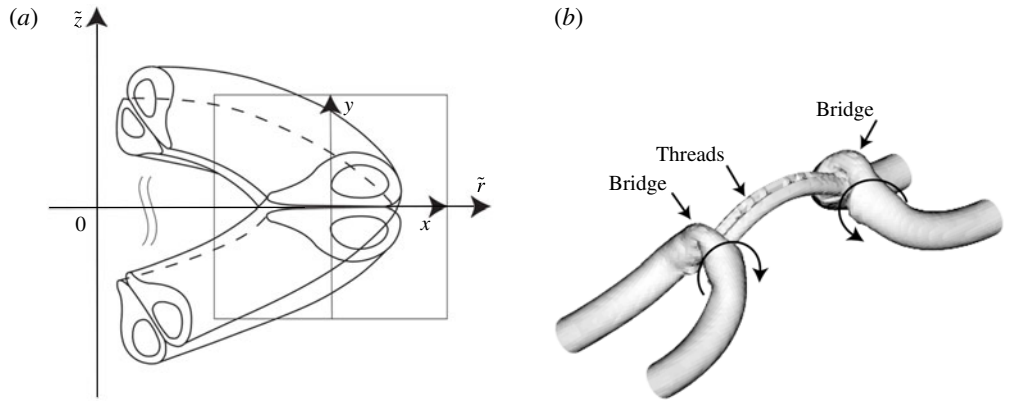


FIGURE 1. (*a*) Deformation of vortex cores into a head–tail configuration (from Kida *et al.* 1991). (*b*) Simulation of the reconnection of a vortex pair performed using a triply periodic spectral method (from Marshall 2001), showing the direction of the induced velocity from cross-linked regions of the vortex cores and the vorticity threads left over from remnants of the core as the reconnected vortices move apart.

in figure 1(*b*), the threads have the form of a curved vortex pair, with a strength much less than that of the original vortex pair. The thread curvature is a result of both the self-induced velocity of the threads on each other and the velocity induced by the bridges. The thread curvature leads to a weak self-induced velocity that drives the threads towards each other, but this motion is also influenced by the straining flow induced by the bridges. The velocity induced by the bridges also causes strong stretching of the threads, which intensifies the vorticity within the thread cores. As a consequence of these various effects, the threads appear to remain in contact over a long time but the cross-diffusion between them is slow.

3. Numerical model

The Navier–Stokes equations were solved in primitive-variable form using a finite-volume method (Lai 2000) and a block-structured mesh with hexahedral elements. The domain consisted of four implicitly coupled blocks and was designed to achieve high spatial resolution along the leading edge of the blade and along the blade boundary layer (figure 2). The simulation algorithm stores all dependent variables at the cell centres and calculates second-order-accurate approximations of the diffusive and convective fluxes on the cell boundaries. The momentum and continuity equations are coupled together using the PISO algorithm (Issa 1985). To achieve additional numerical stability, the time derivative is weighted between a second-order derivative approximation and a first-order upwind with a 90:10 ratio.

The computations were performed with a columnar vortex convected towards a fixed blade by a uniform upstream flow U. The Cartesian coordinate system used for these simulations was oriented such that the uniform flow was in the x direction, the normal vector of the blade centre plane was in the y direction and the blade span was in the z direction (figure 3). Standard inflow and outflow boundary conditions were used for the boundary planes in the x direction, and symmetry boundary conditions were used in the y and z directions. The blade was a NACA0012 airfoil with chord length c and with leading edge lying on the line $x = 0$. The computational domain spanned

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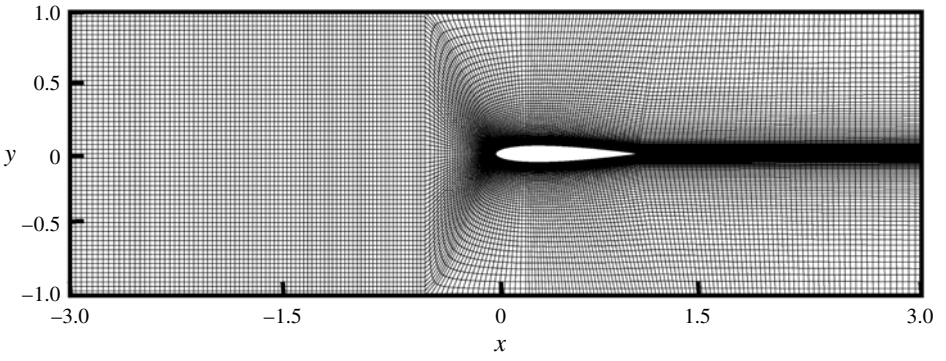


FIGURE 2. Cross-sectional view of the computational grid in the plane $z = 0$. The inlet and outlet planes are at $x = -3$ and $x = 3$, respectively, and the blade span length is equal to unity.

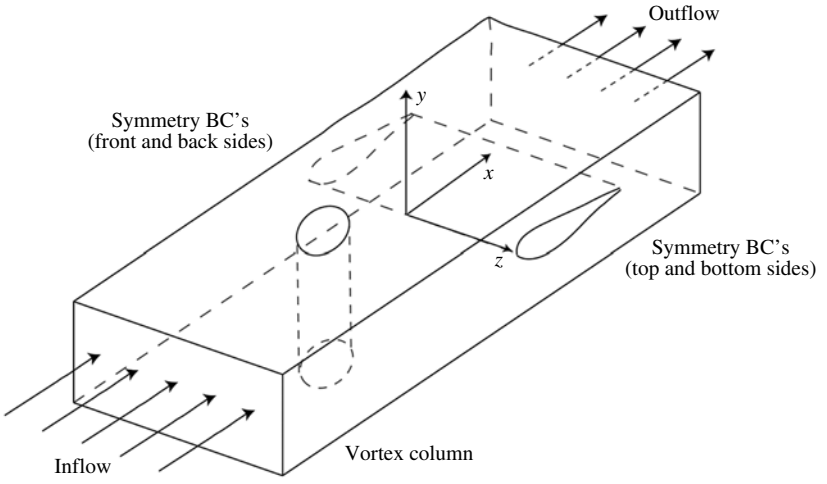


FIGURE 3. Schematic diagram showing coordinate system and boundary conditions used for the numerical computations.

the region $-3 \leqslant x/c \leqslant 3$, $-1 \leqslant y/c \leqslant 1$ and $-1.25 \leqslant z/c \leqslant 1.25$, where the blade centre plane is given by the intersection of the blade with the $y = 0$ plane. The initial velocity field was evaluated using a columnar Rankine vortex with uniform vorticity distribution, strength Γ and core radius $\sigma_0/c = 0.25$, along with two similar vortices in neighbouring domains in the spanwise direction. The vortex axis was initialized on the central plane ($z_0 = 0$) at a location $x_0/c = -1.1$ upstream of the blade leading edge, and the inlet plane was located upstream of the blade leading edge at $x_{inlet}/c = -3$.

Several dimensionless parameters can be defined that govern orthogonal vortex–blade interactions. The vortex and blade Reynolds numbers are defined as $Re_v \equiv \Gamma/\nu$ and $Re_B \equiv Uc/\nu$, where ν is the kinematic viscosity. The free-stream velocity U can be used to define the impact parameter as $I \equiv 2\pi\sigma_0 U/\Gamma$, which is the ratio of the relative vortex–blade velocity to the maximum swirl velocity within the vortex. The thickness parameter T/σ_0, which is the ratio of the blade thickness T to the vortex core radius σ_0, is important for determining the extent and type of vortex

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Case	Re_v	$2\pi\sigma_0 U/\Gamma$
1	3142	0.5
2	667	2.4
3	157	10
4	79	20

TABLE 1. Values of the dimensionless parameters for the reported computations. The blade thickness parameter $T/\sigma_0 = 0.8$ and blade Reynolds number $Re_B = 1000$ for all cases examined.

core deformation as the vortex approaches the blade. The ambient vortex axial flow was set to zero for all cases examined in the paper. The computations reported were selected to compare cases in the regime of high impact parameter and low thickness parameter. The values of these parameters for the cases considered in the paper are given in table 1. The thickness parameter is fixed at $T/\sigma_0 = 0.8$ and the blade Reynolds number is set at $Re_B = 1000$ for all cases examined. The impact parameter varies from 0.5 to 20. In all instances, the impact parameter is sufficiently high that no separation of the boundary layer vorticity is observed prior to impact with the vortex. The vortex Reynolds number varies inversely with the impact parameter, ranging from 79 to 3142 for the different cases examined.

Typical values of blade and vortex Reynolds numbers in vortex cutting problems depend upon the application. Taking helicopter flow as an example system, for instance, the typical blade Reynolds number Re_B is estimated by Leishman (2006) as about 5×10^6 for retreating blades and 2×10^7 for advancing blades. The typical vortex strength is estimated by Leishman as being approximately equal to the bound vortex strength on the blade surface, which, for typical values of the blade solidity and thrust coefficient, gives helicopter vortex Reynolds numbers Re_v of about 20 % of the blade Reynolds number. While these Reynolds-number values are much higher than those used in the current computations, it was shown by Liu & Marshall (2004), and is also the case for vortex reconnection problems in general, that the value of the Reynolds number has only a minor influence on the vortex cutting flow results. For instance, Liu & Marshall (2004) obtained computational results for blade lift coefficient during chopping of a vortex with axial flow which agree well with experimental values obtained at a Reynolds number three orders of magnitude higher than the computational value.

All length variables are non-dimensionalized using the blade chord c, velocity variables are non-dimensionalized by the free-stream velocity U, time is non-dimensionalized by the advective time c/U, and vorticity is non-dimensionalized by the inverse time scale U/c. Pressure and shear stress are non-dimensionalized by ρU^2, where ρ is the fluid density. The blade surface vorticity flux, defined by

$$\boldsymbol{q} = -\nu \frac{\partial \boldsymbol{\omega}}{\partial n}, \tag{3.1}$$

where $\boldsymbol{n}$ is the outward unit normal of the blade surface and ν is the kinematic viscosity, is non-dimensionalized by $\nu U/c^2$.

Grid independence was examined by repeating the calculation in case 2 for three different meshes, where the total number of grid points varied by a factor of 4.5 between the coarsest and finest meshes. The meshes are labelled as mesh A (865 536 grid points), mesh B (1900 701 grid points), and mesh C (3883 238 grid points).

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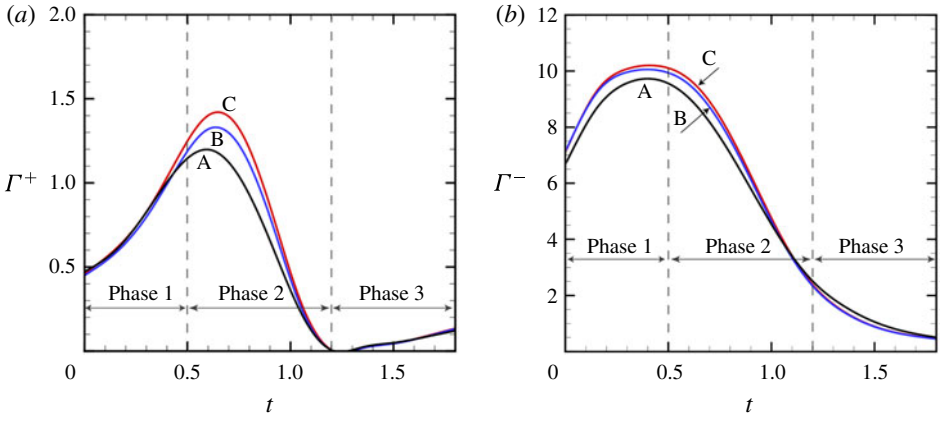


FIGURE 4. (Colour online) Positive and negative circulation measures, Γ^+ and Γ^-, versus dimensionless time. The circulation was calculated along a line extending out from the blade front in the x direction over the interval $-1.3 \leqslant x \leqslant 0$ for three different meshes: mesh A, 865 536 grid points (black); mesh B, 1900 701 grid points (blue); and mesh C, 3883 238 grid points (red). The three phases of vortex reconnection are identified using dashed vertical lines.

All computations in this comparison were performed with the same parameter values and with the same computational domain size. Grid independence was demonstrated by computing the positive and negative circulation measures, defined by

$$\left.\begin{gathered} \Gamma^+ = \int_L \omega_y^+ \,\mathrm{d}x, \quad \Gamma^- = \int_L \omega_y^- \,\mathrm{d}x, \\ \text{where } \omega_y^+ = \begin{cases} \omega_y & \text{if } \omega_y > 0 \\ 0 & \text{if } \omega_y \leqslant 0 \end{cases} \quad \text{and} \quad \omega_y^- = \begin{cases} 0 & \text{if } \omega_y > 0 \\ -\omega_y & \text{if } \omega_y \leqslant 0. \end{cases} \end{gathered}\right\} \tag{3.2}$$

The line L lies on the blade symmetry plane, extending over the interval $-1.3 \leqslant x \leqslant 0$ upstream of the blade leading edge and passing through the ambient position of the vortex axis. The results are plotted as functions of time for each mesh type in figure 4. The peak negative circulation measure has about 4 % difference between meshes A and B, and less than 1 % difference between meshes B and C. The peak positive circulation measure has about 10 % difference between meshes A and B, and less than 5 % difference between meshes B and C. The negative values of ω_y lie within the vortex core, and the positive values of ω_y are induced within the thin blade boundary layer at the leading edge in response to the induced velocity by the vortex, which explains why the positive circulation measure is more sensitive to grid resolution than is the negative circulation measure. All subsequent computations in the paper were performed using the grid in mesh C. The three different phases of vortex cutting indicated in figure 4 are discussed in the next section.

The effect of domain size was also considered to ensure the accuracy of the numerical simulations. The most important parameter was found to be the distance between the initial position of the vortex core and the inlet plane. If this distance was set too small, the induced velocity from the vortex caused weak positive y vorticity

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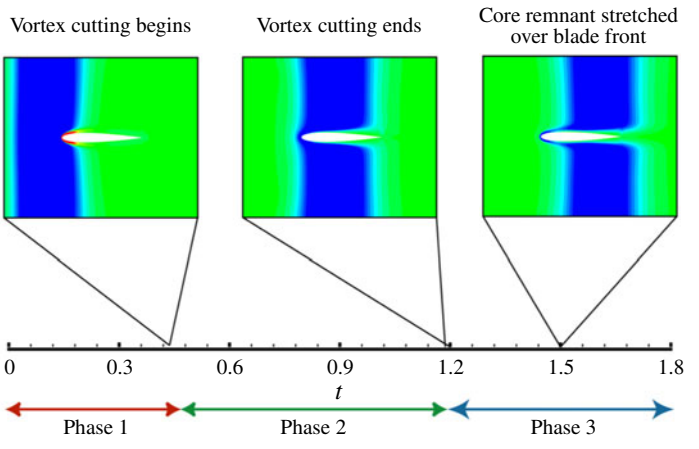


FIGURE 5. Timeline of the vortex cutting process, showing the three phases of vortex reconnection.

to be generated on the inlet plane, which propagates towards the blade behind the vortex. When this vorticity reaches the blade leading edge, it diffusively cancels with the uncut portion of the vortex core that was stretched around the blade leading edge. Several domains were examined, and the mesh in figure 2 was chosen in order to ensure that the strength of this inlet vorticity was small and that it did not reach the blade until very late in the computation. In addition, the effect of time step was considered and a dimensionless time step of 0.015 was chosen to ensure a Courant–Friedrichs–Lewy (CFL) number of less than unity.

4. Vortex cutting simulation results

The ambient vorticity within the vortex is in the negative y direction. As the vortex approaches the blade, it induces a spanwise velocity that is associated with generation of vorticity in the positive y direction along the blade leading edge. It is the diffusive cross-cancellation of the y-oriented vorticity within the vortex with that in the boundary layer along the blade leading edge that controls the vortex cutting process, through which vortex lines within the vortex are cut and reconnect to vortex lines within the blade boundary layer.

4.1. *Vorticity dynamics during vortex cutting*

Throughout this section, detailed results are shown for case 2, and then comparisons of selected results for computations at other values of the impact parameter (listed in table 1) are given in § 4.2. A timeline of the basic vortex cutting process is given in figure 5, in which contour plots of ω_y are shown in the x–y plane for three times to illustrate the position of the vortex core relative to the blade during different phases of the vortex cutting process. Both figure 5 and the positive and negative circulation plots in figure 4 indicate three phases of vortex cutting, which correspond to the three phases of vortex reconnection discussed by Melander & Hussain (1989).

A close-up plot is given in figure 6, showing the ω_y values within a region near the blade leading edge, where regions with high negative values of ω_y are shown in blue and regions with high positive values of ω_y are shown in red. In the first phase

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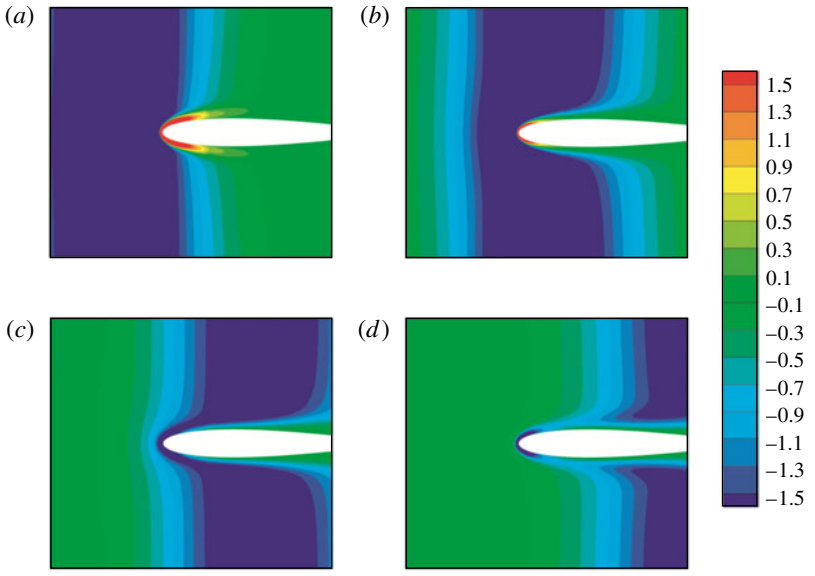


FIGURE 6. Contour plots from case 2 showing a close-up of ω_y near the blade from a slice along the blade centre span in the x–y plane for (*a*) $t = 0.75$, (*b*) 1.05, (*c*) 1.35 and (*d*) 1.65.

of vortex cutting, the vortex interacts with the inviscid flow induced by the blade, and the front part of the vortex deforms slightly, bending around the blade. Also during this phase, the vortex induces formation of a region of positive ω_y within the blade leading-edge boundary layer. Phase 2 begins when the vortex impacts with the blade boundary layer, so that the regions with positive and negative values of ω_y, within the blade boundary layer and the vortex core, respectively, interact by diffusion and partially annihilate each other. This viscous cross-diffusion leads to a rapid decrease in both the negative and positive circulation measures, as shown in figure 4 during the phase 2 time interval, and causes the vortex lines in the vortex core to break and reconnect to those in the blade boundary layer. Phase 3 occurs for times greater than about $t = 1.2$, for which the value of ω_y within the blade boundary layer changes from positive to negative. As demonstrated by Liu & Marshall (2004), this change in sign is associated with the fact that, as the blade leading edge passes through the core, the induced spanwise velocity along the leading edge changes direction. After the change in sign of ω_y within the blade boundary layer occurs, the y component of vorticity within the core is of the same sign as that within the blade boundary layer and diffusive annihilation can no longer occur. A series of plots showing how the value of ω_y along the blade leading edge changes sign is given in figure 7, for a similar time frame as in figure 6 but viewed looking along the x direction, directly towards the blade front region. From this figure, we see that the change in sign of ω_y first occurs at the ends of the blade span and then moves towards the blade centre as time progresses.

The x component of vorticity also undergoes a sign change as the vortex core passes over the leading edge of the blade. Initially, the induced spanwise velocity from the vortex causes a region of positive ω_x to form on the blade top and a region of negative ω_x on the blade bottom. As the vortex passes over a given point on the blade surface,

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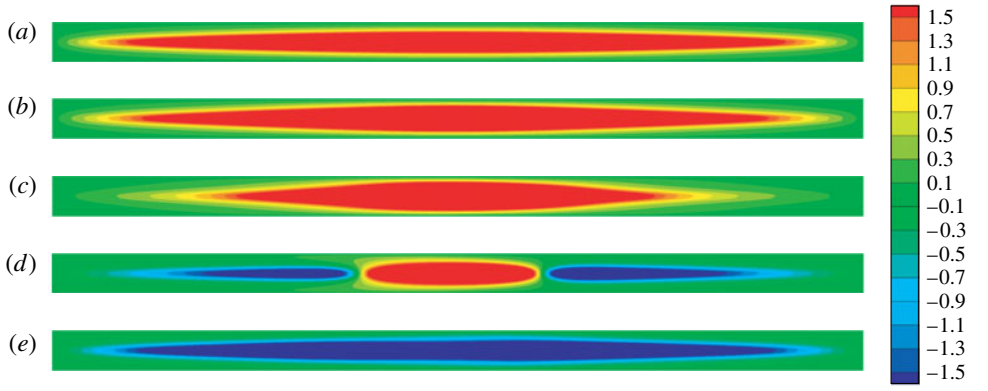


FIGURE 7. Contour plots from case 2 of ω_y on the front of the blade for (*a*) $t = 0.15$, (*b*) 0.45, (*c*) 0.75, (*d*) 1.05 and (*e*) 1.35.

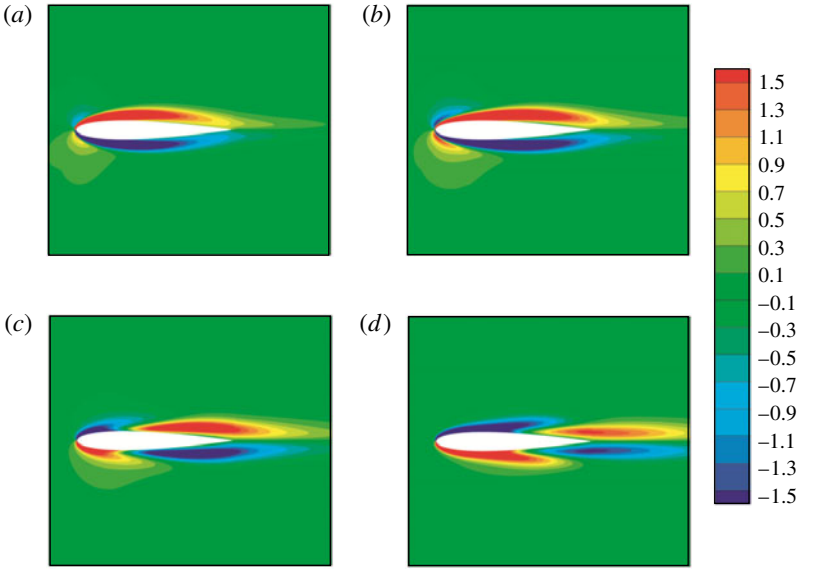


FIGURE 8. Contour plots from case 2 of ω_x from a slice along the blade centre span in the x–y plane for (*a*) $t = 0.75$, (*b*) 1.05, (*c*) 1.35 and (*d*) 1.65.

the induced spanwise velocity at that point changes direction, resulting in a change in the sign of ω_x. Plots illustrating this effect using a time series of contours of ω_x are shown on the cross-sectional plane $z = 0$ (figure 8) and on projections of the blade top and bottom surfaces looking along the y axis (figure 9). The change in sign of ω_x closely follows the path of the vortex core.

The essence of vortex reconnection involves cutting of vortex lines originating from one vorticity region and reconnection to vortex lines originating from another vorticity region via diffusion-regulated annihilation of vorticity between the two regions. This vortex line reconnection process for the vortex cutting problem is illustrated in figure 10 for a time series as the blade penetrates into the vortex core. In order to

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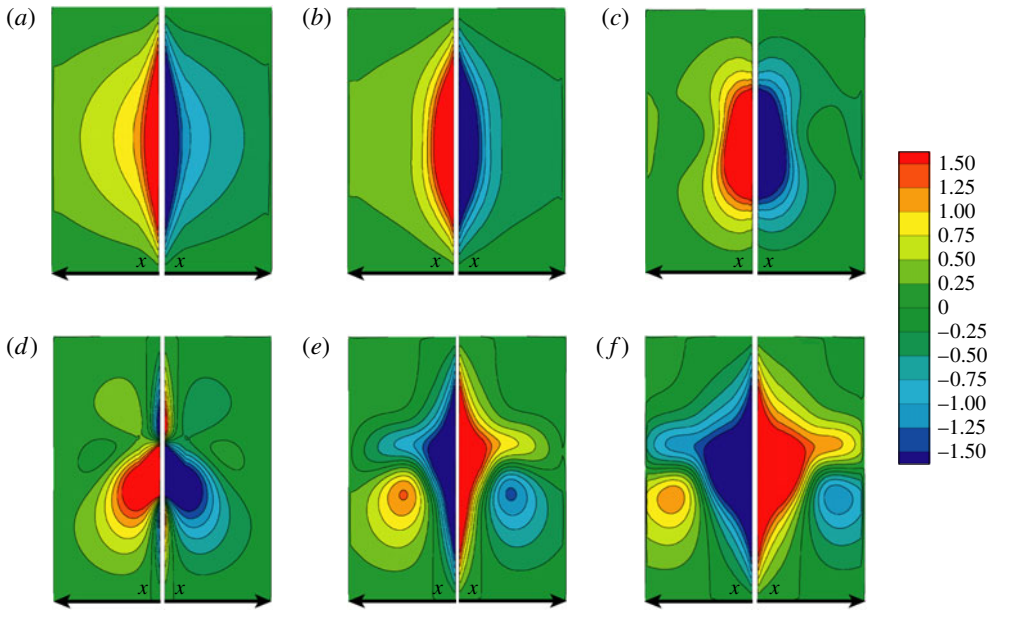


FIGURE 9. Contour plots from case 2 of ω_x on the blade top (left) and bottom (right) at (*a*) $t = 0.15$, (*b*) 0.45, (*c*) 0.75, (*d*) 1.05, (*e*) 1.35 and (*f*) 1.65.

illustrate the vortex reconnection process, we colour the vortex lines red to indicate vortex lines that originate within the vortex and remain within the vortex, black to indicate vortex lines that originate within the blade boundary layer and remain within the boundary layer, and blue to indicate vortex lines that originate within the vortex and cross over to join those within the blade boundary layer (or vice versa). Near the beginning of this sequence, the sign of ω_y is positive at the blade leading edge, and vorticity annihilation between the boundary layer and the vortex allows the vortex lines within the core to cut and reconnect to those within the boundary layer, increasing the number of blue lines. As the vortex core centre passes the position of the blade leading edge, the value of ω_y at the leading edge changes sign. Vortex lines from the vortex (in red) now begin to deform and to stretch in the spanwise direction instead of being cut. The pattern change of the black vortex lines illustrates how the vorticity orientation in the boundary layer along the front section of the blade changes as the vortex core passes over the blade leading edge.

The pressure change along the vortex core caused by the presence of the blade is another important aspect of the vortex cutting process. For reconnection of two antiparallel vortex tubes, Saffman (1990) proposed that the localized increase in vortex core pressure creates a positive feedback loop to drive the vortex reconnection process. We examined how the pressure in the vortex core changed for vortex cutting in order to understand if pressure plays a similar role in the vortex cutting problem. Figure 11 illustrates the pressure in the vortex core for a contour plot in the $z = 0$ plane, passing through the centre span of the blade. Initially there is a large region of low pressure in the vortex core as well as on the top and bottom surfaces of the blade and a region of high pressure around the blade leading edge. As the vortex moves closer to the blade, the pressure in the vortex core increases close to the impingement region and a pressure gradient forms. This phenomenon is similar to the one Saffman (1990)

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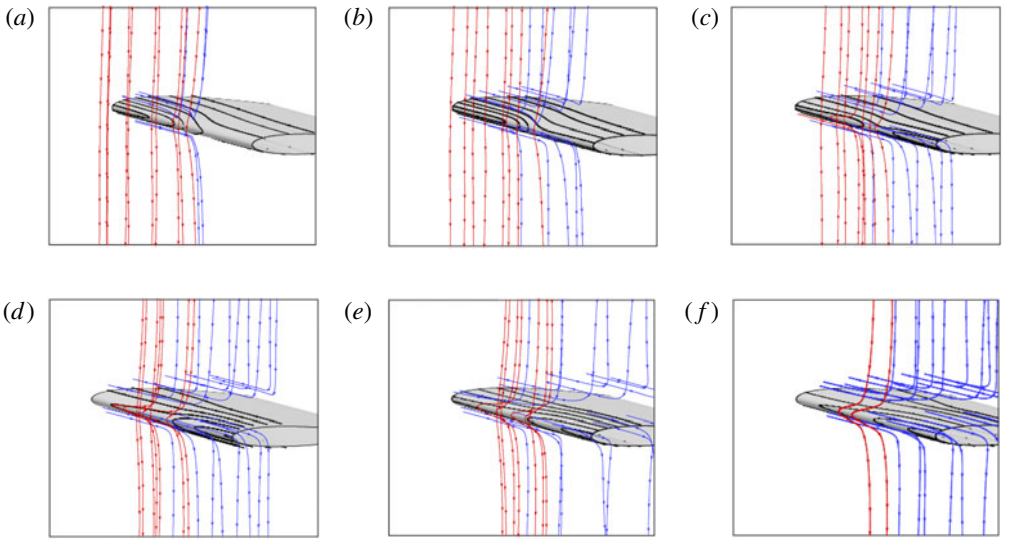


FIGURE 10. Oblique view of the vortex cutting process. Vortex lines originating in the vortex core can either remain within the vortex core (red) or be cut and reconnect to vortex lines in the boundary layer (blue). Similarly, vortex lines originating within the blade boundary layer can either stay in the boundary layer (black) or join to those originating within the core (blue). Images are shown at times (*a*) $t = 0.75$, (*b*) 0.90, (*c*) 1.05, (*d*) 1.20, (*e*) 1.35 and (*f*) 1.50. The (uncut) red vortex lines near the blade leading edge become deflected in the spanwise direction as they near the blade.

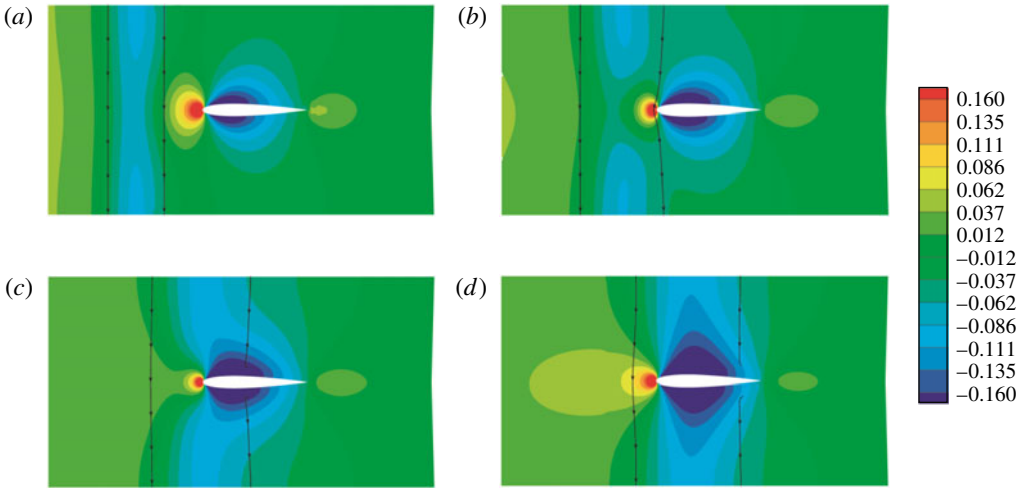


FIGURE 11. Pressure contours in the x–y plane passing through the centre span of the blade for (*a*) $t = 0.3$, (*b*) 0.6, (*c*) 0.9 and (*d*) 1.2. The outlines of the vortex are shown by plotting vortex lines on the two sides of the vortex.

described, but it occurs for a different reason. In the case of vortex cutting by a blade, the impingement of the vortex upon the ambient high pressure near the blade leading edge no doubt plays a significant role in driving the localized pressure gradient along the vortex core, and any role that core deformation might play in this process is

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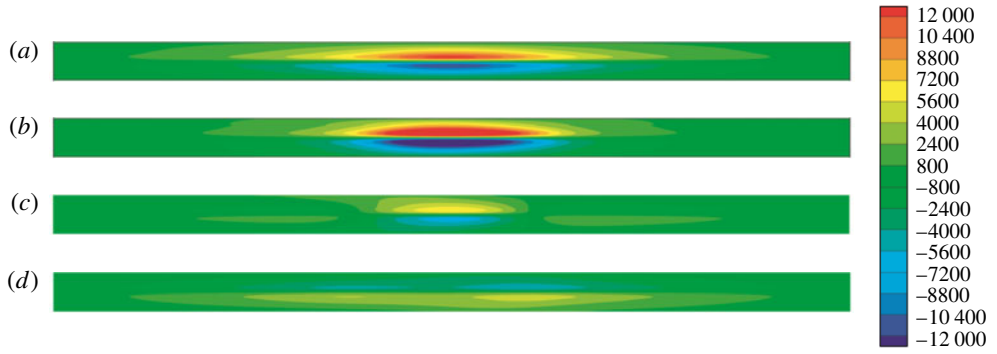


FIGURE 12. Contour plots of the x component of the vorticity flux, q_x, on the blade leading edge, at times (*a*) $t = 0.45$, (*b*) 0.75, (*c*) 1.05 and (*d*) 1.35.

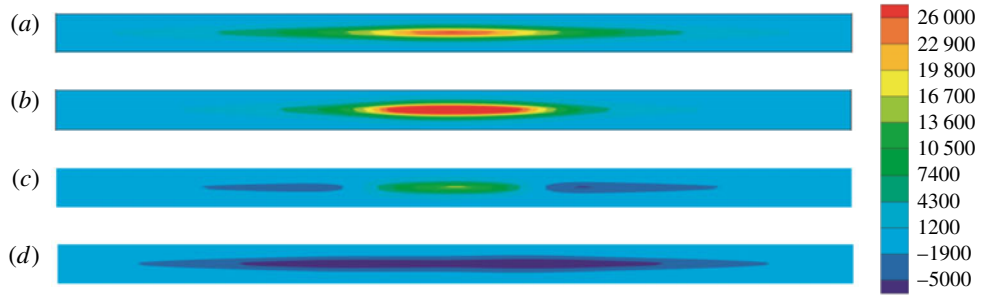


FIGURE 13. Contour plots of the y component of the vorticity flux, q_y, on the blade leading edge, at times (*a*) $t = 0.45$, (*b*) 0.75, (*c*) 1.05 and (*d*) 1.35.

obscured. As the vortex passes over the blade, the low-pressure region in the vortex core connects with the low-pressure regions on the top and bottom surfaces of the blade, as shown in figure 11(*c*,*d*).

The key difference between vortex cutting by a blade and more traditional vortex reconnection is the fact that vorticity is generated on the blade surface, which lies within the region dominated by vorticity diffusive interaction. The x and y components of the surface vortex flux, q_x and q_y, are shown within a projection of the front region of the blade (looking along the x axis) in figures 12 and 13. The vorticity flux contour plots exhibit a change in sign around $t = 1.05$, which corresponds with the change in sign of ω_x and ω_y during vortex passage previously discussed in this section. The contour plots show that the vorticity flux is primarily generated within the leading-edge region of the blade, and not on the blade top and bottom surfaces away from the leading edge. This region is exactly where the breaking and rejoining of vortex lines occur, and further demonstrates the importance of considering the role of surface vorticity flux in a blade–vortex cutting problem.

4.2. *Effect of impact parameter*

Plots of the time variation of the maximum and minimum values of vorticity and surface vorticity flux are shown for all four cases examined in figures 14 and 15 in order to better understand the effect of impact parameter on the temporal changes in

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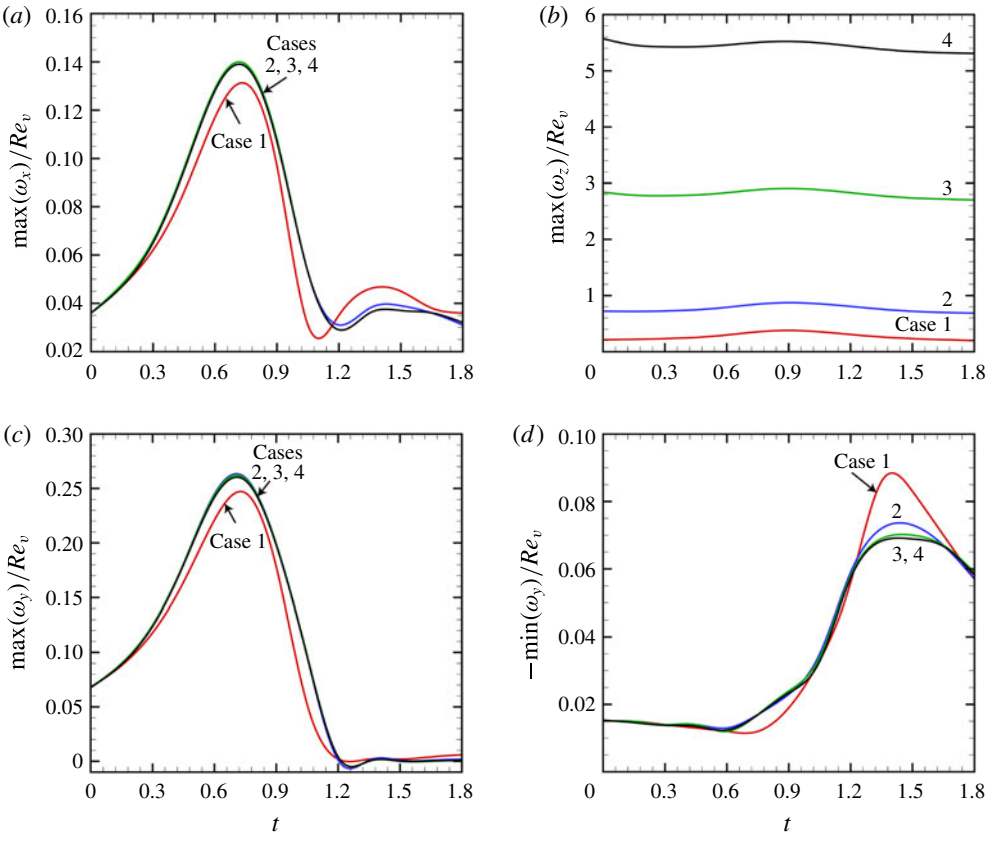


FIGURE 14. (Colour online) Time variation plots of (*a*) maximum ω_x, (*b*) maximum ω_z, (*c*) maximum ω_y and (*d*) minimum ω_y, normalized with respect to the vortex Reynolds number, $Re_v = \Gamma/\nu$. Results are shown for cases 1 (red), 2 (blue), 3 (green) and 4 (black).

these quantities throughout the vortex cutting process and the manner in which the vortex cutting process changes with respect to the impact parameter. In interpreting these plots, it is helpful to keep in mind the time intervals for major transitions in the cutting process, namely, the onset of penetration of the blade leading edge into the vortex core ($t \cong 0.45$), the passage of the blade leading edge out of the opposite side of the vortex position ($t \cong 1.2$) and stretching of vortex remnants over the blade leading edge ($t > 1.2$). These three time intervals correspond approximately to the three phases of traditional vortex tube reconnection as described in § 2.

Figure 14 shows the time variation of the maximum values of ω_x and ω_z, normalized with respect to the vortex Reynolds number, Re_v. The minimum values of these vorticity components follow nearly identical profiles as the maximum values owing to the symmetry on the top and bottom surfaces of the blade, and so are not plotted. The different cases considered differ from each other by the value of the vortex strength, leading to different values of the vortex Reynolds number and vortex–blade impact parameter. However, the plots of maximum and minimum normalized ω_x values for these different cases appear to nearly collapse onto a single curve, although with a slight deviation for case 1. The plot in figure 14(*a*) exhibits an increase in ω_x with time until around $t = 0.75$, which corresponds to the time at which the vortex core

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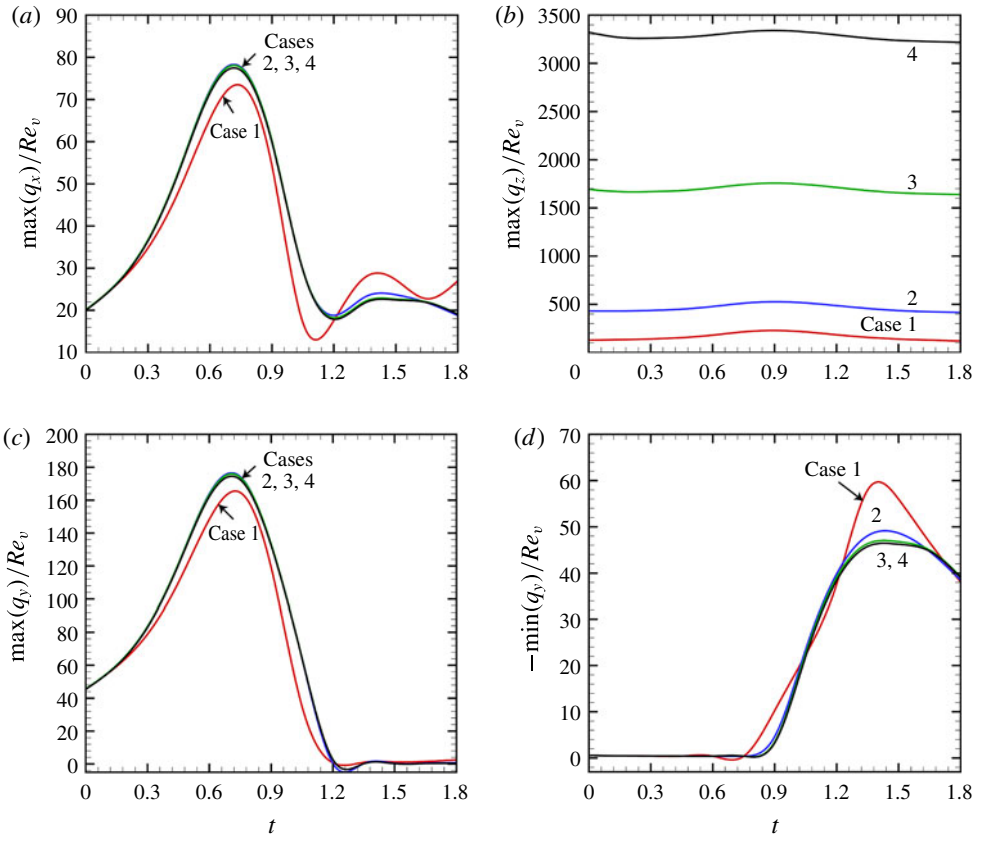


FIGURE 15. (Colour online) Time variation plots of the maximum values of the surface vorticity flux components (*a*) q_x, (*b*) q_z and (*c*) q_y, and minimum values of (*d*) q_y, normalized with respect to the vortex Reynolds number. Results are shown for cases 1 (red), 2 (blue), 3 (green) and 4 (black).

axis passes over the blade leading edge. Following this time, the value of ω_x gradually decreases until it approaches its initial value as the vortex moves past the tail end of the blade. The maximum value of ω_x occurs in the centre of the blade, slightly above the blade centre plane but within the blade boundary layer. From figure 14(*a*), we conclude that the maximum value of ω_x is dominated by the initial cutting of the vortex, and that this value increases approximately linearly with the vortex strength.

The maximum value of ω_z/Re_v, shown in figure 14(*b*), clearly does not collapse onto a single curve for the different cases examined. The results for the different cases appear to be qualitatively similar, but the maximum value of ω_z/Re_v increases (decreases) as the vortex Reynolds number increases (decreases). The maximum in ω_z also occurs at a somewhat later time than does the maximum in ω_x, although it seems to occur at the same time for all of the cases examined. In contrast to the maximum of ω_x, the maximum value of ω_z occurs to the left of the centre of the blade span and slightly below the blade centre plane. This shift is probably due to the induced velocity of the vortex creating a slightly higher free-stream velocity on the left side of the blade and slightly lower velocity on the right side, looking downwind towards the blade leading edge.

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The maximum and minimum values of ω_y, normalized by the vortex Reynolds number, are plotted in figure 14(*c*,*d*) for the four cases listed in table 1, each with a different value of the impact parameter. We again see that the curves nearly coincide for the different cases examined, with the exception of a slight deviation for case 1. This data collapse indicates that the value of ω_y varies approximately linearly with vortex strength. The maximum positive value of ω_y (which occurs within the boundary layer at the blade leading edge) initially increases as the blade penetrates into the vortex core until a time of about $t = 0.75$, and after which it decreases to a value of nearly zero as the vortex core passes over the leading edge of the blade and the positive vorticity on the blade front induced by the vortex changes sign. The maximum negative value of ω_y, which initially occurs within the vortex core, exhibits a gradual decay with time during the initial part of the computation, which is due to viscous diffusion of the vortex core. The maximum negative value of ω_y is not significantly effected by the vortex cutting process until the vortex core moves past the blade leading edge and the value of ω_y within the blade boundary layer changes from a positive to a negative sign. From this time until the end of the computation, the point of maximum negative value of ω_y is located at the blade leading edge. The maximum negative value of ω_y gradually increases from the time ($t = 0.75$) of change in sign of ω_y within the blade boundary layer to a time at which the blade leading edge has entirely penetrated through the vortex core (about $t = 1.5$), after which the maximum negative value of ω_y remains approximately constant for the remainder of the computation. Both the generation of negative ω_y due to induced spanwise velocity from the vortex and the increase in vorticity due to stretching of the remaining portions of the vortex wrapped about the blade leading edge contribute to producing the maximum negative ω_y value. The fact that the peak value of the maximum negative value of ω_y occurs quite late, when the vortex core is well past the centre of the blade, suggests the presence of significant uncut vorticity remnants from the impinging vortex. These uncut vorticity remnants wrap around the blade surface in the form of a thin vortex sheet, for which the value of the negative-vorticity component ω_y continuously intensifies by stretching about the blade leading edge, while at the same time it is regulated by viscous diffusion, eventually approaching a constant value in a manner analogous to the classical Burgers vortex (Burgers 1948).

The time variation in the positive and negative components of the surface vorticity flux is shown in figure 15. The x and z vorticity fluxes, q_x and q_z, exhibit a functional form very similar to the corresponding maximum values of the vorticity components ω_x and ω_z. As was the case for the vorticity, the maximum positive and negative values of the vorticity flux in these directions follow the same curve, and the maximum values of q_x for the different cases examined collapse nearly onto a single curve, while those for q_z do not. We again see an initial increase in the positive-vorticity flux component q_y, which then drops off to zero as the cutting process stops, and a late increase in the negative component of vorticity flux as negative y vorticity is generated on the blade leading edge.

4.3. *Comparison to vortex tube reconnection*

To conclude this section, we return to the discussion in § 2 of the different physical processes that occur during vortex reconnection, and compare these processes to those occurring during vortex cutting. We had identified three distinct phases of vortex reconnection. The first phase is dominated by inviscid interaction of the vortices and has two parts – antiparallelization and core deformation. The vortex

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cutting process appears at first view to be most similar to the orthogonally offset vortex reconnection problem, of the type examined by Zabusky & Melander (1989) and Boratav *et al.* (1992), since the ambient blade boundary layer vorticity is in the z direction and the ambient vorticity within the vortex is in the y direction. However, unlike in the orthogonal vortex tube reconnection problem, the impinging vortex does not twist around to orient itself antiparallel to the ambient boundary layer vorticity as it approaches the blade. Instead, the induced velocity from the vortex generates new vorticity in the antiparallel direction to the incident vortex on the blade leading edge as the vortex approaches. Instead of an inviscid vortex twisting process, in the vortex cutting case the antiparallel configuration is achieved via viscous vorticity generation on the blade surface. The extent of inviscid core deformation in the vortex cutting process depends greatly on the value of the thickness parameter, T/σ_0. For sufficiently large values of this parameter, significant core deformation can occur as the impinging vortex interacts inviscidly with its image over the blade surface.

The second phase of vortex reconnection entails viscous annihilation of antiparallel vorticity between the two vorticity regions, leading to vortex line cutting and reconnection between the regions. This process occurs in much the same way in the vortex tube reconnection and vortex cutting problems, with the difference that in classical vortex tube reconnection all of the vorticity is present at the start of the computation, whereas for vortex cutting vorticity is continually being generated at the blade surface during the reconnection process.

Both the classical vortex tube reconnection and the vortex cutting processes are incomplete, leading to the formation of uncut remnants, or threads, in the third phase of reconnection. Despite this similarity, the reasons that the vortex tube reconnection and the vortex cutting are incomplete are quite different. For vortex tube reconnection, the incomplete reconnection occurs due to the velocity induced by the highly curved bridge regions connecting the two vortex tubes. As these bridges grow in strength, the self-induced velocity gets larger and it is eventually sufficient to move the cores away from one another before all opposite-sign vorticity in the reconnection region has been annihilated. For the vortex cutting process, the incomplete cutting occurs due to the change in sign of the vorticity flux in the antiparallel (y) direction on the blade leading edge as the leading edge penetrates sufficiently far into the vortex core. Regardless of the different mechanism leading to incomplete reconnection, the end result for both processes is the formation of uncut vorticity threads that linger for long time periods after the primary reconnection or cutting process is complete.

5. Model for vortex sheet in a straining flow near a surface

A key component of the vortex cutting process involves diffusive vorticity annihilation between the incident vortex and vorticity generated on the blade boundary layer. The incident vorticity initially has the form of a tube, but, as it wraps about the blade leading edge, it deforms into more of a sheet-like structure (Marshall & Grant 1996). Finally, the stretching of the impinging vortex imposed by the ambient flow about the blade leading edge plays a critical role in determining the vorticity evolution within the cutting region. In this section we describe a new exact Navier–Stokes solution that contains these three elements – antiparallel vorticity diffusive annihilation, vorticity generation on a surface, and vortex stretching. While the model to be presented incorporates these three key elements, to make the problem analytically tractable, we must dramatically modify certain other elements of the vortex cutting problem. In particular, the model does not deal with vorticity bending

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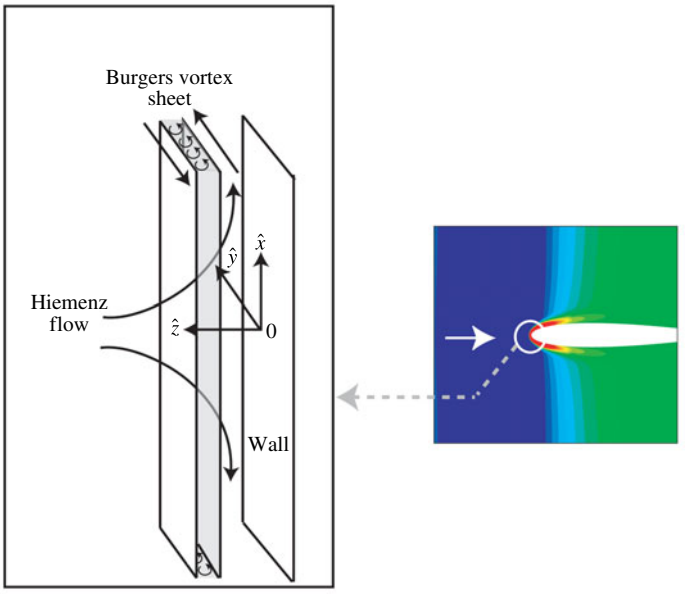


FIGURE 16. (*a*) Schematic diagram of the model flow field, consisting of a Burgers vortex sheet (shaded) immersed in a Hiemenz straining flow. (*b*) Illustration of vertical vorticity contours during vortex–blade interaction, showing the relationship between the model flow and vorticity dynamics occurring at the blade leading edge during the vortex cutting process.

around the highly curved blade leading edge, but instead considers vorticity above a flat surface that is stretched by an imposed viscous straining flow (Hiemenz 1911), as indicated in the schematic diagram in figure 16. Secondly, the vortex sheet is assumed to be infinitely wide, whereas the incident vortex in the vortex cutting problem has a finite width. Were it not for the presence of the solid surface, this stretched vortex sheet would be identical to a Burgers sheet, where the vorticity within the sheet is oriented along the direction of the straining flow (Burgers 1948; Gibbon, Fokas & Doering 1999). Because of these various simplifications incorporated into the model, we do not expect the model solutions to provide an accurate quantitative approximation of the vorticity field in the problem of vortex cutting by a blade. Nevertheless, the model contains many of the key ingredients of the vortex cutting problem and it may be helpful in suggesting physically motivated scaling for the vortex cutting problem. As an exact Navier–Stokes solution, the model is also of interest in its own right.

The model deals with a combination of a Hiemenz straining flow and a vortex sheet over a flat surface. The model problem is solved using the Cartesian coordinate system $(\hat{x}, \hat{y}, \hat{z})$ shown in figure 16, in which $\hat{z}$ points in the upstream direction, $\hat{x}$ is oriented in the direction of the Heimenz straining flow close to the wall, and $\hat{y}$ is in the direction of the flow induced by the vortex sheet. Figure 16 illustrates the fact that this model is intended to be representative not of the entire vortex–blade interaction problem, but only of the flow very close to the point of vortex impingement on the blade leading edge. The velocity and pressure fields are assumed to satisfy

$$\boldsymbol{u} = \hat{u}(\hat{x}, \hat{z})\boldsymbol{e}_{\hat{x}} + \hat{v}(\hat{z}, t)\boldsymbol{e}_{\hat{y}} + \hat{w}(\hat{z})\boldsymbol{e}_{\hat{z}}, \quad p = p_1(\hat{x}) + p_2(\hat{z}). \tag{5.1a,b}$$

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Under these conditions, the continuity and Navier–Stokes equations reduce to

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{w}}{\partial \hat{z}} = 0, \tag{5.2a}$$

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{w}\frac{\partial \hat{u}}{\partial \hat{z}} = -\frac{1}{\rho}\frac{\partial p_1}{\partial \hat{x}} + \nu\left(\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{z}^2}\right), \tag{5.2b}$$

$$\frac{\partial \hat{v}}{\partial t} + \hat{w}\frac{\partial \hat{v}}{\partial \hat{z}} = \nu\frac{\partial^2 \hat{v}}{\partial \hat{z}^2}, \tag{5.2c}$$

$$\hat{w}\frac{\partial \hat{w}}{\partial \hat{z}} = -\frac{1}{\rho}\frac{\partial p_2}{\partial \hat{z}} + \nu\,\frac{\partial^2 \hat{w}}{\partial \hat{z}^2}. \tag{5.2d}$$

We denote the straining rate of the external Hiemenz flow by s, the strength of the Burgers vortex sheet by γ, a scaled height variable by $\eta = \hat{z}\sqrt{s/\nu}$, and a scaled time by $\tau = st$. The velocity components can be written as

$$\hat{u} = s\hat{x}F'(\eta), \quad \hat{v} = \gamma G(\eta, \tau), \quad \hat{w} = -F(\eta)\sqrt{s\nu}. \tag{5.3a–c}$$

These expressions for velocity satisfy the continuity equation (5.2a) identically, and the momentum equations (5.2b) and (5.2d) reduce to an ordinary differential equation for $F(\eta)$ as

$$F''' + FF'' - (F')^2 + 1 = 0, \tag{5.4}$$

subject to the boundary conditions $F(0) = F'(0) = 0$ and $F'(\infty) = 1$. Substituting (5.3a–c) into (5.2c) yields a partial differential equation for $G(\eta, \tau)$ as

$$\frac{\partial G}{\partial \tau} - F(\eta)\frac{\partial G}{\partial \eta} = \frac{\partial^2 G}{\partial \eta^2}. \tag{5.5}$$

Equation (5.5) is subject to the boundary conditions

$$G(0, \tau) = 0, \quad G(\infty, \tau) = \alpha, \tag{5.6a,b}$$

where α is a prescribed parameter representing the ratio of the velocity far away from the plate to the vortex sheet strength.

Equation (5.4) is equivalent to the classic Hiemenz problem, which was solved using a fourth-order Runge–Kutta method, yielding $F''(0) = 1.23259$. Equation (5.5) was solved using the Crank–Nicolson method with $\Delta\eta = 0.005$ and $\Delta\tau = 0.001$. The primary dimensionless parameter is the ratio α of velocity $\hat{v}_\infty$ in the $\hat{y}$ direction far away from the surface to the initial vortex sheet strength γ. Other parameters include the initial position η_0 of the vortex sheet and the initial thickness Δ of the Stokes first problem boundary layer on the wall. A simple example calculation is considered using this model for the problem of a vortex sheet being driven into a flat wall by a Hiemenz straining flow. The initial condition for this example problem has the form of an equilibrium Burgers vortex sheet centred at η_0 and a Stokes first problem boundary layer with dimensionless thickness Δ, given by

$$G(\eta, 0) = G_0(\eta) = (1 + \alpha)\,\mathrm{erf}(\eta/2\Delta) - \tfrac{1}{2} - \tfrac{1}{2}\,\mathrm{erf}[(\eta - \eta_0)/\sqrt{2}], \tag{5.7}$$

where $\mathrm{erf}(\cdot)$ is the error function. For this example problem, we chose $\eta_0 = 10$ and $\Delta = 1$. Results for this flow are shown in figure 17 for the dimensionless velocity

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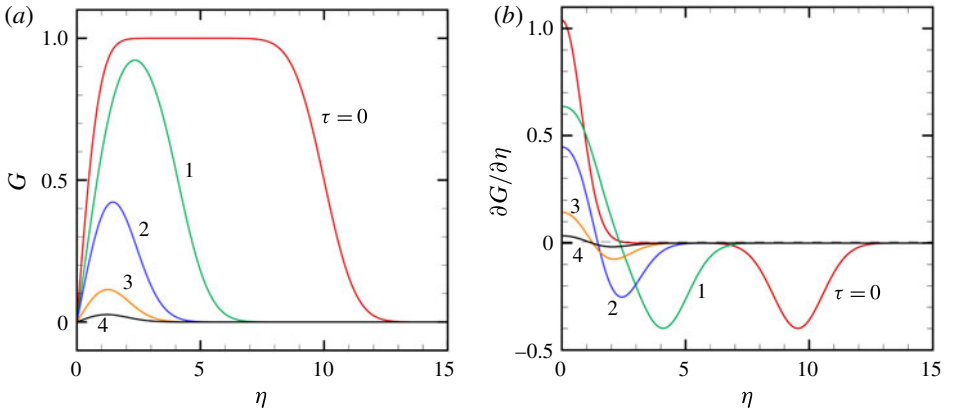


FIGURE 17. (Colour online) Variation of (*a*) the dimensionless velocity G and (*b*) the dimensionless vorticity $\partial G/\partial\eta$ as functions of η for a case with $\alpha = 0$. Plots are shown for $\tau = 0$ (red), 1 (green), 2 (blue), 3 (orange), and 4 (black).

$G = \hat{v}/\gamma$ and dimensionless vorticity $\zeta \equiv \partial G/\partial\eta$ as functions of dimensionless time τ. Based on the coordinate system shown in figure 16, the velocity is in the $\hat{y}$ direction and the vorticity is in the $\hat{x}$ direction. Dimensionless circulation measures Γ^+ and Γ^- are defined by

$$\left.\begin{gathered} \Gamma^+ = \int_L \zeta^+ \,\mathrm{d}\eta, \quad \Gamma^- = \int_L \zeta^- \,\mathrm{d}\eta, \\ \text{where } \zeta^+ = \begin{cases} \zeta & \text{if } \zeta > 0 \\ 0 & \text{if } \zeta \leqslant 0 \end{cases} \quad \text{and} \quad \zeta^- = \begin{cases} 0 & \text{if } \zeta > 0 \\ -\zeta & \text{if } \zeta \leqslant 0, \end{cases} \end{gathered}\right\} \tag{5.8}$$

and L denotes the range $(0, \eta_{max})$ of η. Plots of time variation of Γ^+ and Γ^-, given for the example problem in figure 18, provide a quantitative measure of the extent of vorticity diffusion-driven annihilation occurring between the incident vortex sheet and the boundary layer vorticity.

In the numerical simulation of vortex cutting by a blade discussed in § 4, the assumption that the ambient flow past the blade has the form of a plane straining flow would only be relevant in a region close to the blade leading edge. While we do not expect this simple model to provide an accurate quantitative description of the full numerical simulation, it is nevertheless of interest to compare the model results with those of the full numerical simulation within this near-blade region. In comparing these results, we note the relationships $(\hat{x}, \hat{y}, \hat{z}) = (y, -z, -x)$ and $(\hat{u}, \hat{v}, \hat{w}) = (v, -w, -u)$ between coordinate and velocity components in the model problem and the numerical simulation. The value of the spanwise velocity component w was extracted from the numerical simulations along a line corresponding to the x axis from $x = 0$ to $x = -15\sqrt{s/\nu}$, corresponding to the interval $0 \leqslant \eta \leqslant 15$. The straining rate s was obtained by extracting the velocity component v in the y direction along this same line and fitting a tangent line near $\eta = 0$, which for case 2 yields $s = 26.7$. The negative spanwise velocity $(-w)$ extracted along this line is comparable to the velocity component $\hat{v}$ in the simple theoretical model, and it is plotted as a function of η in figure 19(*a*) for different values of the scaled time variable $\tau = st$. Comparison results for the theoretical model in figure 19(*b*) were obtained using

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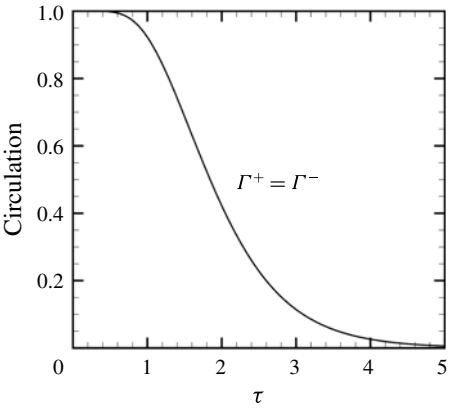


FIGURE 18. Time variation of dimensionless circulation measures, Γ^+ and Γ^-, as functions of τ for the example problem shown in figure 17. For the case with $\alpha = 0$, $\Gamma^+ = \Gamma^-$.

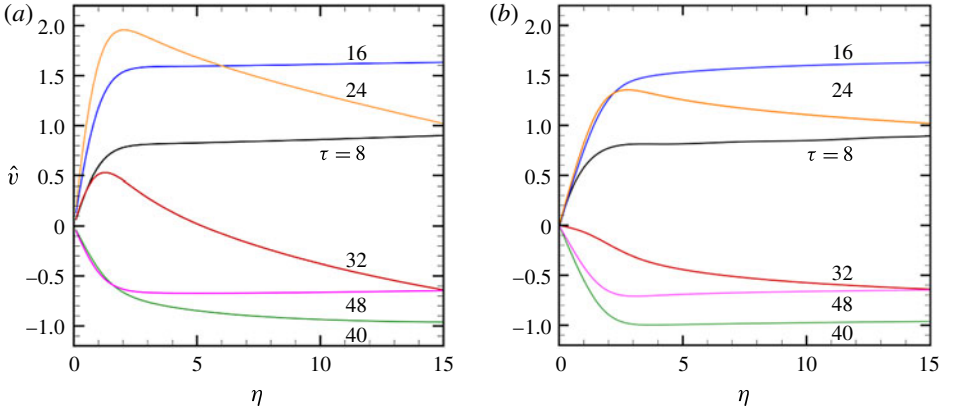


FIGURE 19. (Colour online) Comparison of profiles of dimensionless velocity $\hat{v}$ as a function of η at $\tau = 8$ (black), 16 (blue), 24 (orange), 32 (red), 40 (green) and 48 (purple). Plots are shown for (*a*) the vortex cutting simulation described in § 4 and (*b*) the simple model described in § 5.

the same initial velocity profile as shown in figure 19(*a*) for the vortex cutting simulation, and by varying the boundary condition α in (5.6*a*,*b*) as a function of time in accordance with the values of $\hat{v}$ at the position $\eta = 15$ along the extraction line obtained from the vortex cutting simulation. The model calculation was initialized at $\tau = 8$ in order to give the vortex time to approach the blade surface. The results from the model calculations are plotted in figure 19(*b*) for the velocity profile. The two sets of results do not exactly match up, but they have sufficiently good qualitative similarities to suggest that the temporal and spatial scaling suggested by the model are valid also with more general vortex cutting problems. To quantify the vorticity annihilation in the vortex cutting simulations and the simple model, we plot in figure 20 the time variation of the positive and negative circulation measures, Γ^+ and Γ^-, which are computed in this case using integration over the interval $0 \leqslant \eta \leqslant 15$, corresponding to the interval $-0.09 \leqslant x \leqslant 0$ in the numerical simulations of vortex

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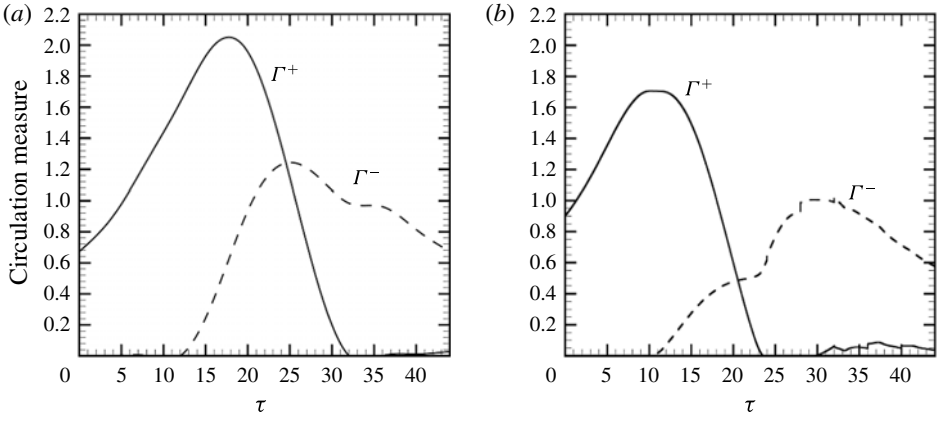


FIGURE 20. Time variation of the dimensionless circulation measures, Γ^+ and Γ^-, as functions of τ for the vortex cutting problem shown in figure 19. Plots are shown for (*a*) the vortex cutting simulation described in § 4 and (*b*) the simple model described in § 5.

cutting by a blade. We observe higher peaks of both the positive and negative circulation for the vortex cutting simulations compared to the simple model, which is probably an effect of the strong curvature of the vortex around the blade leading edge in the vortex cutting simulations. The qualitative features of the two plots, including the vorticity variation on the time scale τ and the manner in which the stretched vorticity within the flow interacts with and is annihilated by vorticity generated at the surface, appear to be similar between the model and the full simulations.

6. Conclusions

Computational results are reported for the cutting of a columnar vortex by a blade without axial flow for different values of the impact parameter, with a particular focus on comparison of the vortex cutting problem to the classic vortex tube reconnection problem. Each of the three phases of vortex tube reconnection were found to have a close analogy in the vortex cutting problem. The first vortex reconnection phase, involving inviscid response to the core resulting in both antiparallel orientation of the vorticity and core shape deformation, takes the form in the vortex cutting problem of generation of vorticity at the leading edge of the blade due to the spanwise velocity induced by the impinging vortex. While the diffusion of this vorticity into the blade boundary layer is a viscous process, the generation of vorticity at the blade surface is controlled by the inviscid slip velocity along the blade span. The resulting vorticity within the boundary layer wrapping about the blade leading edge is antiparallel to that within the approaching vortex core.

The second phase of vortex reconnection involves diffusion and annihilation of vorticity within the two antiparallel vorticity regions. This process occurs in the vortex cutting problem between the vortex core and the boundary layer at the blade leading edge. However, unlike the traditional vortex tube reconnection problem, for vortex cutting the boundary layer vorticity is generated at the blade surface as the vorticity annihilation occurs. A simple exact Navier–Stokes solution illustrating the key processes involved during this second phase of vortex cutting is described in § 5, in which we examine the interaction of a vortex sheet above a flat surface in

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the presence of a straining flow. In this model, the straining flow is in the direction of the vorticity vectors, similar to a Burgers vortex, and represents the effect of the ambient flow in the stagnation-point region near the blade leading edge. While this model is greatly simplified in order to reduce the problem to a manageable form, and in particular the effect of high curvature at the blade leading edge is ignored, it nevertheless is found to provide a good description of the qualitative features of vortex cutting and to indicate the appropriate temporal and spatial scaling for the problem.

Both the vortex tube reconnection problem and the vortex cutting problem result in incomplete reconnection of the vortex lines across the two vorticity regions; however, the reason that this occurs in the two problems is different. In vortex cutting, the incomplete cutting occurs when the blade leading edge passes sufficiently deeply into the vortex core that the induced spanwise velocity along the leading edge changes sign. After this sign change occurs, the generation of antiparallel vorticity at the leading-edge surface is stopped and the new vorticity generated is parallel to that within the vortex core. When the antiparallel vorticity in the boundary layer is depleted and replaced by parallel vorticity, the diffusive annihilation of vorticity between the vortex and the boundary layer can no longer occur, effectively stopping the vortex cutting process. What follows corresponds with the third phase of vortex reconnection, in which remnants of the uncut vortex remain for long time, wrapping around the blade boundary layer and stretching under the influence of the ambient flow around the blade.

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