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Sheared-flow-driven vortices in a magnetized dusty electron–positron plasma

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Abstract. It is shown that sheared plasma flows can generate nonthermal electrostatic waves in a magnetized dusty electron–positron plasma. Linearly excited modes attain large amplitudes and start interacting among themselves. Nonlinearly coupled modes self-organize in the form of coherent vortices comprising a vortex chain and a double vortex. Conditions under which the latter appear are given. The relevance of our investigation to space, astrophysical, and laboratory plasmas is pointed out.

Dedication

This paper is dedicated to John Dougherty on the occasion of his 65th birthday.

1. Introduction

It is well known that electron–positron plasmas, which are composed of fully ionized charged particles with the same mass and opposite charge, are produced by high-energy particle and laser beams. Electron–positron plasmas are ubiquitous in the early universe (Misner et al. 1973: Rees 1983; Tajima and Taniuti 1990), in active galactic nuclei (Miller and Witta 1987), in pulsar magnetospheres (Goldreich and Julian 1969; Michel 1982), in the Van Allen radiation belts, and near the polar caps of rapidly rotating neutron stars (Lightman and Zdziarski 1987; Zdziarski 1987). Although in most astrophysical settings, pair plasmas are fully relativistic, recent laboratory experiments (Surko et al. 1989; Greaves and Surko 1995) show that non-relativistic electron–positron plasmas can be created. This possibility opens up new directions for investigating collective processes (Shukla et al. 1986; Yu et al. 1986; Rizzato 1988; Iwamoto 1993; Zhao et al. 1996; Sakai et al. 1999) that appear on time and space scales that are absent in electron–ion plasmas. Furthermore, both astrophysical and laboratory plasmas may contain high-Z ions (or charged dust particulates), which affect the dynamics of the pair plasma when it is embedded in an external magnetic field. For example, a number of authors (Rizzato 1988; Berezhiani et al. 1992; Jammalamadaka et al. 1996; Cho et al. 2000) have discussed the electromagnetic properties of a magnetized pair plasma containing heavy ions and stationary dust grains.

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In this paper, we investigate the properties of low-frequency (in comparison with the electron gyrofrequency) electrostatic waves in a strongly magnetized electron–positron plasma in the presence of sheared plasma flows and extremely massive charged dust grains. It is found that sheared plasma flows can generate electrostatic waves due to a Kelvin–Helmholtz-like instability. The condition under which the latter arises is given. Sheared-flow-driven modes interact among themselves. The corresponding nonlinear mode-coupling equations are derived. It is further shown that the stationary solutions of the nonlinear equations can be represented in the form of vortex streets and dipolar vortices. Criteria for the occurrence of the latter are presented. The applications of our work to space and laboratory plasmas are discussed.

2. Basic equations

We consider a strongly magnetized dusty electron–positron plasma embedded in an external magnetic field $B_0\hat{\mathbf{z}}$, where B_0 is the strength of the magnetic field and $\hat{\mathbf{z}}$ is the unit vector along the z direction. We assume that the micrometre-sized dust grains are negatively charged and stationary. Thus, at equilibrium, we have $n_{p0}(x) = n_{e0}(x)+Z_0(x)n_{d0}(x)$, where $n_{j0}(x)$ is the unperturbed, non-uniform, number density of the particle species j ($j = p$ for positrons, e for electrons, and d for dust grains) and Z_0 is the number of charges residing on the dust grain surface. The unperturbed plasma also contains sheared velocities $\hat{\mathbf{z}}V_{j0}(x)$, which have gradients along the x axis. Sheared flows, which are maintained by external sources, generate a negligibly small sheared magnetic field. In the presence of a low-frequency (in comparison with the gyrofrequency $\omega_c = eB_0/mc$, where e is the magnitude of the electron charge, m is the electron mass, and c is the speed of light) electrostatic wave, the perpendicular components of the electron and positron velocities in a cold plasma are

$$\mathbf{v}_{e\perp} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\phi + \frac{c}{B_0\omega_c} d_t^e \boldsymbol{\nabla}_\perp \phi \tag{1}$$

and

$$\mathbf{v}_{p\perp} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\phi - \frac{c}{B_0\omega_c} d_t^p \boldsymbol{\nabla}_\perp \phi, \tag{2}$$

where ϕ is the electrostatic potential and

$$d_t^{e,p} = \partial_t + \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\phi \cdot \boldsymbol{\nabla} + (V_{e0,p0} + v_{ez,pz})\partial_z.$$

The parallel components of the electron and positron fluid velocities are determined from

$$D_t v_{ez} - \frac{c}{B_0} V'_{e0}\partial_y\phi = \frac{e}{m_e}\partial_z\phi \tag{3}$$

and

$$D_t v_{pz} - \frac{c}{B_0} V'_{p0}\partial_y\phi = -\frac{e}{m_e}\partial_z\phi, \tag{4}$$

where we have assumed that

$$\left|\partial_t + \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\phi \cdot \boldsymbol{\nabla}\right| \equiv |D_t| \gg (V_{e0,p0} + v_{ez,pz})\partial_z,$$

and introduced the notation $V'_{e0,p0} = \partial V_{e0,p0}/\partial x$.

Inserting the perpendicular fluid velocities (1) and (2) into the equation that

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follows from the subtraction of the ion continuity equation from the electron continuity equation, we obtain, after using Poisson's equation,

$$D_t\left(\nabla^2+\frac{\omega_p^2}{\omega_c^2}\nabla_\perp^2\right)\phi+\frac{\omega_p^2}{\omega_c}\kappa_d\partial_y\phi+4\pi e(n_{e0}\partial_z v_{ez}-n_{p0}\partial_z v_{pz})=0, \tag{5}$$

where

$$\omega_p^2=\omega_{pe}^2+\omega_{pp}^2\equiv\frac{4\pi(n_{e0}+n_{p0})e^2}{m},\qquad \kappa_d=(n_{e0}+n_{p0})^{-1}\frac{\partial(Z_0n_{d0})}{\partial x}.$$

We note that (5) has been obtained by assuming that the dust-grain dynamics is not important since we are looking for wave phenomena on a time scale much shorter than the dust plasma period.

Equations (3)–(5) are the governing equations for nonlinearly coupled electrostatic waves in a non-uniform electron–positron magnetoplasma containing charged dust grains and sheared plasma flows.

3. Instability and nonlinear solutions

In order to investigate the instability of the pair plasma under consideration, we neglect the nonlinear terms in (3)–(5) and combine them to obtain

$$\partial_t^2\left(\nabla^2+\frac{\omega_p^2}{\omega_c^2}\nabla_\perp^2\right)\phi+\frac{\omega_p^2}{\omega_c}\kappa_d\partial_{ty}^2\phi+\omega_p^2\partial_z^2\phi+\frac{\omega_{pe}^2}{\omega_c}\left(V_{e0}'-\frac{n_{p0}}{n_{e0}}V_{p0}'\right)\partial_{yz}^2\phi=0. \tag{6}$$

Supposing that ϕ is proportional to $\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, where ω and $\mathbf{k}=k_y\hat{\mathbf{y}}+k_z\hat{\mathbf{z}}$ are the wave frequency and the wave vector respectively, we obtain from (6)

$$\omega^2-\omega\omega_*-a\omega_p^2\left[1+\frac{k_y}{\omega_c k_z}\frac{n_{e0}}{n_{e0}+n_{p0}}\left(V_{e0}'-\frac{n_{p0}}{n_{e0}}V_{p0}'\right)\right]=0, \tag{7}$$

where

$$\omega_*=-\frac{(\omega_p^2/\omega_c)\kappa_d k_y}{k^2+k_y^2\omega_p^2/\omega_c^2},\qquad k^2=k_y^2+k_z^2,\qquad a=\frac{k_z^2}{k^2+k_y^2\omega_p^2/\omega_c^2}.$$

The roots of (7) are

$$\omega=\frac{\omega_*}{2}\pm\frac{1}{2}\left\{\omega_*^2+4a\omega_p^2\left[1-\frac{b(\delta V_{p0}'-V_{e0}')}{\omega_c}\right]\right\}^{1/2}, \tag{8}$$

where

$$b=\frac{k_y n_{e0}}{k_z(n_{e0}+n_{p0})}\qquad \delta=\frac{n_{p0}}{n_{e0}}.$$

Equation (8) predicts an oscillatory instability for

$$\frac{V_{p0}'}{\omega_c}>\frac{1}{b\delta}+\frac{\omega_*^2}{4ab\delta\omega_p^2}+\frac{V_{e0}'}{\delta\omega_c}.$$

For $V_{p0}'\approx V_{e0}'\equiv V_0'$, instability occurs for

$$V_0'>\omega_c\frac{n_{e0}}{bZ_0n_{d0}}\left(1+\frac{\omega_*^2}{4a\omega_p^2}\right).$$

The maximum growth rate of that instability is roughly $(ab)^{1/2}(Z_0n_{d0}/n_{e0})^{1/2}(V_0'/\omega_c)^{1/2}\omega_p$.

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Next, we present the stationary solutions of (3)–(5) in a stationary frame $\xi = y + \alpha z - Ut$ by supposing that all the physical variables are functions of x and ξ only, where α and U are constants. Thus, in the stationary frame, (3) and (4) are exactly satisfied by

$$v_{ez} = -\frac{c}{B_0 U} V'_{e0}\phi - \frac{e\alpha}{mU}\phi \tag{9}$$

and

$$v_{pz} = -\frac{c}{B_0 U} V'_{p0}\phi + \frac{e\alpha}{mU}\phi. \tag{10}$$

Transforming (5) into the stationary frame, and eliminating v_{ez} and v_{pz} by means of (9) and (10), we obtain

$$D_{\xi\phi}(\nabla_\perp^2 \phi - \gamma\phi) = 0, \tag{11}$$

where we have assumed that $\nabla_\perp^2 \equiv \partial_x^2 + \partial_\xi^2 \gg \alpha^2 \partial_\xi^2$,

$$D_{\xi\phi} = \partial_\xi - \frac{c}{B_0 U}(\partial_x \phi \partial_\xi - \partial_\xi \phi \partial_x),$$

and

$$\gamma = \frac{\omega_c^2}{(\omega_p^2 + \omega_c^2)U^2}\left[\frac{\omega_p^2 \kappa_d U}{\omega_c} - \omega_p^2 \alpha^2 - \frac{\omega_{pe}^2 \alpha}{\omega_c}\left(V'_{e0} - \delta V'_{p0}\right)\right]. \tag{12}$$

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For $\gamma = 0$, which occurs if

$$U = U_0 \equiv \frac{\omega_c}{\kappa_d}\left[\alpha^2 + \frac{\alpha\omega_{pe}^2(V'_{e0} - \delta V'_{p0})}{\omega_c \omega_p^2}\right] > 0,$$

(11) is satisfied by

$$\nabla_\perp^2 \phi = \frac{4\phi_0 K^2}{a_0^2} \exp\left[-\frac{2}{\phi_0}\left(\phi - \frac{UB_0 x}{c}\right)\right], \tag{13}$$

which has a solution

$$\phi = \frac{UB_0}{c}x + \phi_0 \ln[2\cosh(Kx) + 2(1 - a_0^{-2})\cos(K\xi)]. \tag{14}$$

For $a_0 > 0$, (14) represents a row of identical travelling vortices, resembling the Kelvin–Stuart cat's eyes (Petviashvili and Pokhotelov 1992; Shukla et al. 1995). The parameter ϕ_0 characterizes the amplitude of the vortex street and $2\pi/K$ is the vortex size. For $a_0 = 1$, (14) becomes a solution in the form of a zonal flow,

$$\phi = \frac{UB_0}{c}x + \phi_0 \ln[2\cos(Kx)]. \tag{15}$$

On the other hand, for $\gamma \neq 0$, (11) is satisfied by

$$\nabla_\perp^2 \phi = C_1 \phi + C_2 x, \tag{16}$$

where $C_1 = \gamma - C_2 c/B_0 U$ and C_2 are constants. Equation (16) admits a double vortex (Stenflo 1987) for $\gamma > 0$ or $U > U_0$. Typical double-vortex profiles are exhibited in Farid et al. (2000).

4. Conclusions

In this paper, we have considered the linear and nonlinear propagation of low-frequency electrostatic waves in a non-uniform dusty electron–positron plasma

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containing equilibrium density and velocity gradients. It is found that sheared plasma flows in a dusty pair plasma can produce instability even when the electron and positron velocity gradients are equal and in the same direction. The differential flow gradient can, however, be produced by the presence of non-uniform dust-charge and dust number-density gradients. The physical mechanism of the instability is similar to the Kelvin–Helmholtz instability, in which the slippage is produced by non-uniform flows in two adjacent layers. Sheared-flow-driven electrostatic fluctuations interact nonlinearly and self-organize in the form of either a vortex street or a double vortex. The formation of a vortex street appears in the parameter regime where double-vortex formation is forbidden. The results of the present investigation should be helpful in understanding the salient features of sheared-flow-driven turbulence and associated nonlinear structures in astrophysical and laboratory pair plasmas that contain extremely massive charged dust grains and equilibrium inhomogeneities.

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