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Trajectory optimization of flexible mobile manipulators

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SUMMARY

A computational algorithm is developed to find a dynamic motion trajectory of a mobile manipulator with flexible links and joints that will allow the robot to carry a maximum load between two specified end positions. A compact form of the linearized state space dynamic equations is organized as well as constraint equations. Then, the problem of finding a maximum load carrying capacity on flexible mobile manipulators is formulated as a trajectory optimization problem.

KEYWORDS: Trajectory optimization; Mobile manipulators; Linearized trajectory.

I. INTRODUCTION

Finding the full load motion for a specified point-to-point task can increase the productivity and economic usage of mobile manipulators. In a classical fixed base manipulator, the dynamic load carrying capacity (DLCC) is defined as a maximum load, which a manipulator can carry repeatedly in its fully extended configuration, while the dynamics of the load and manipulator itself must be taken into account.[1] In a mobile manipulator with lighter weight and higher motion speed, the existence of the flexible structure on joints and links degrades the motion accuracy. Accordingly, an additional constraint must be imposed, when the DLCC is determined for flexible mobile manipulators. The constraint should limit the end effector tracking error due to the deflection or oscillation of the mobile manipulator.

Wang and Ravani[1,2] presented an algorithm to determine the load carrying capacity on fixed base robots. The limitation of the rigid joint and link assumption in the formulation and analysis of flexible manipulators were investigated by the other researchers.[3–7]

This paper, presents a new method to determine the load carrying capacity for mobile manipulators with flexible links and joints during the motion between two end positions. For an analysis of flexibility, the Lagrangian assumed mode is employed. Then, the problem of finding the maximum load carrying capacity for flexible mobile manipulators is converted into a trajectory optimization problem. An objective function is defined and dynamic equations are linearized, as well as other constraints on its state space form to solve with Iterative linear Programming (ILP) method.

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II. LINEARIZED STATE SPACE REPRESENTATION OF DYNAMIC EQUATIONS

The Lagrangian assumed mode formulation is used to model the dynamics of the link flexibility on a mobile manipulator[8] (Figure 1). Also, the elastic mechanical coupling between the ith joint and link is modeled as a linear torsional spring with a constant stiffness coefficient k_{ti} (Figure 2).

Then, the compact form of extended equation of motion,[9] can be represented as:

$$[M]_e \ddot{\vec{q}}_e + C(\vec{q}_e, \dot{\vec{q}}_e) + G(\vec{q}_e) + [K_t]_e [\Delta \vec{q}]_e = \vec{\tau}_e \qquad (1)$$

If the load is treated as a variable, the dynamic Equation (1) is rearranged as:

$$\begin{aligned} \ddot{\vec{q}}_e &= [M(\vec{q}_e, m_L)]_e^{-1}(\vec{\tau}_e - C(\vec{q}_e, \dot{\vec{q}}_e, m_L) - G(\vec{q}_e, m_L) \\ &\quad - [K_t]_e \Delta \vec{q}_e) = \vec{f}(\vec{q}_e, \dot{\vec{q}}_e, \vec{\tau}_e, m_L) \end{aligned} \qquad (2)$$

Defining the state vector $\vec{X} = [x_1\ x_2]^T$, where $\vec{x_1} = \vec{q}_e$ and $\vec{x_2} = \dot{\vec{q}}_e$, the state space representation of Equation (2) can be rewritten in the following form:

$$\begin{aligned} \dot{\vec{X}} &= \begin{bmatrix} \dot{\vec{x_1}} \\ \dot{\vec{x_2}} \end{bmatrix} = \begin{bmatrix} \vec{x_2} \\ \hline \vec{f}(\vec{X}(j), \vec{\tau}_e(j), m_L) \end{bmatrix} \\ &= \vec{F}(\vec{X}(j), \vec{\tau}_e(j), m_L) \end{aligned} \qquad (3)$$

Discretized form of the equation (3) is:

$$\frac{\vec{X}(j+1) - \vec{X}(j)}{h} = \vec{F}(\vec{X}(j), \vec{\tau}_e(j), m_L), \quad j = 1, 2, \ldots, p \qquad (4)$$

where, $h = \frac{\Delta T}{p}$ and ΔT is the overall motion time and p is the total number of time intervals. The nonlinear function f at the $(k+1)$th trajectory is expanded in Taylor series about the kth trajectory. After neglecting the higher order (non linear) terms the following equation is obtained in a matrix form:

$$\vec{X}(j+1) = [G_j]\vec{X}(j) + [H_j]\vec{\tau}_e(j) + \vec{B}_j m_L + \vec{D}_j \qquad (5)$$

The elements of matrices $[G_j]$, $[H_j]$ and $\vec{B}_j$, $\vec{D}_j$ are found in Reference [10]. $\vec{X}(j+1)$ can be written as a linear

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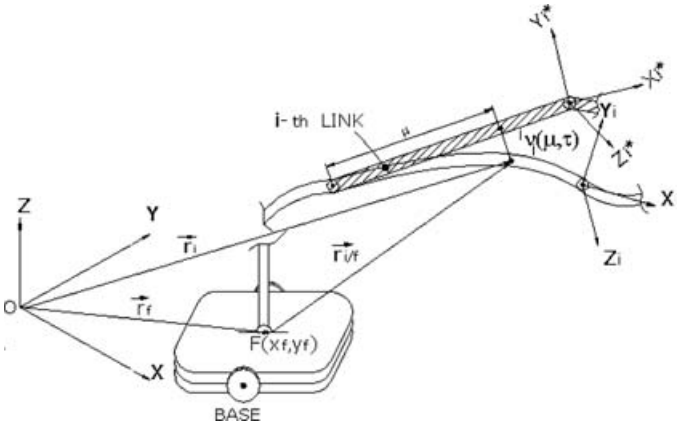


Fig. 1. A schematic view of a flexible mobile manipulator.

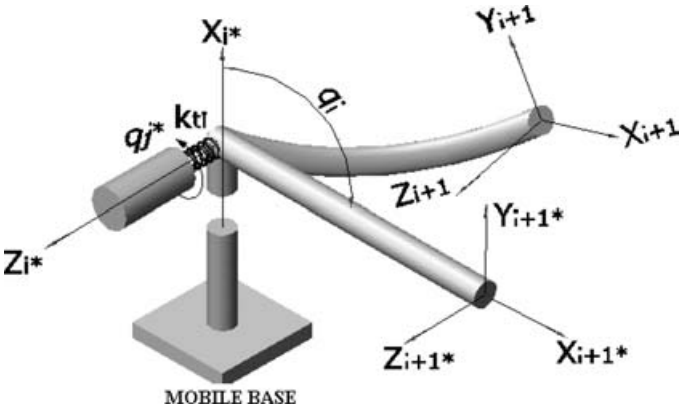


Fig. 2. Flexible joint-link assembly on a mobile manipulator.

combination of the payload m_L and the torque control $\vec{\tau}_e(i), i = 1, 2, \ldots\ldots, j$. Equation (5) then becomes:

$$\vec{X}(j+1) = \vec{X}_h(j+1) + \vec{\beta}_j m_L + \sum_{j=1}^{j} [\alpha_{ji}]\vec{\tau}_e(i) \qquad (6)$$

This equation is the basic linearized dynamic equation, where details of terms $\vec{X}_h(j+1)$, $\vec{\beta}_j$ and, $[a_{ji}]$ are shown in Reference [10].

III. THE PROBLEM FORMULATION

Considering point-to-point motions with actuator, the complete formulation can be written to maximize:

The $\boldsymbol{m}_{Load}$

The initial and final states are:

$$\begin{aligned} \vec{x}_1(t_i) &= \vec{q}(t_i) = \vec{x}_{1i}, \quad \vec{x}_2(t_i) = 0 \\ \vec{x}_1(t_f) &= \dot{q}(t_f) = \vec{x}_{1f}, \quad \vec{x}_2(t_f) = 0 \end{aligned} \qquad (7)$$

The problem is subject to the state space Equation (6), and actuator constraint:

$$\vec{\tau}_{\min}(\vec{X}(t)) \le \vec{\tau}_e(t) \le \vec{\tau}_{\max}(\vec{X}(t)) \qquad (8)$$

The bounds $\vec{\tau}_{\min}(\vec{X}(t))$ and $\vec{\tau}_{\max}(\vec{X}(t))$ are in general nonlinear functions representing the torque-speed characteristics of the joint actuators.

The joint variable constraints are:

$$\vec{x}_1^- \le \vec{x}_1(t) \le \vec{x}_1^+ \qquad (9)$$

where $\vec{x}_1^+$, $\vec{x}_1^-$ are the upper and lower bounds of the joint variables, respectively. The final constraint is the upper bound of the payload m_L. Where, m_L is the smaller value of the static load carrying capacity at the two end positions. The optimum control trajectory $\vec{\tau}_e(t)$ and optimum state variable trajectory $\vec{X}(t)$ must be found to maximize the dynamic load carrying capacity m_L. In the above formulation, the objective function consists only of a single variable m_L, which is time independent, and a simple linear function. However, most of the constraint functions of this problem are nonlinear. The nonlinearity of the state variable constraints is due to the nonlinear state space Equation 3. The nonlinearity of the control variable constraints is due to the nonlinear torque-speed characteristic curve of the joint actuators. The optimal solution of this problem, in general, cannot be obtained analytically and numerical optimization techniques, such as the Iterative linear Programming (ILP) method[2,4] must be used.

IV. CONCLUSION

A computational algorithm has been presented to find optimal trajectories for flexible joint and link mobile manipulators. The algorithm synthesizes a dynamic motion trajectory that allows the mobile manipulator to carry the maximum load between two given end positions in a specified time. Dynamic equations are formulated in the state space form and then are linearized, as well as constraint equations. Then, a suitable algorithm is addressed to solve this problem as a trajectory optimization problem. The computational algorithm developed in this paper is conceptually straightforward and has the ability to handle a variety of joint and control variables.

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