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Failure recovery for wrench capability of wire-actuated parallel manipulators

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SUMMARY
Wire-actuated parallel manipulators and their failures are studied in this paper taking into consideration their failure modes. A methodology for investigating the effect of wire/actuator failures on the force/moment capability of manipulators is presented, and the criteria for full and partial recovery from these failures are established. The methodology is also applicable for the cases that the minimum norm solution for the vector of wire tensions gives a negative value for tension by treating the corresponding wire as failed. The proposed criteria are also valid for the manipulators that utilize hybrid actuation of wires and joints. Three planar wire-actuated parallel manipulators are used as the case study to illustrate the proposed methodology and criteria.

KEYWORDS: Failure analysis; Failure recovery; Wire-actuated parallel manipulators.

1. Introduction

In parallel manipulators, the mobile platform (end effector) is connected to the base by several legs/limbs/branches/wires. Therefore, considering the actuation, parallel manipulators could be categorized as solid-link manipulators (Fig. 1(a)) and wire-actuated manipulators (Fig. 1(b)). The solid-link parallel manipulators consist of kinematic chains of links with actuated (active) and passive joints; for example, in Fig. 1(a), the prismatic joints are actuated and revolute joints are passive. In the wire-/cable-actuated manipulators (also referred to as the wire-suspended or cable-driven manipulators), the motion of the mobile platform is controlled by wires/cables. As wires act in tension (could pull but not push) and cannot exert forces in both directions along their lines of action, to manipulate an m degrees of freedom (DOF) rigid body suspended by wires and ensure that it could resist/apply an external force/moment (wrench), generally at least $m + 1$ wires are needed. Hence, wire-actuated manipulators usually require some means of redundant actuation (e.g., redundant wires, external force/moment or gravity).

Wire-actuated parallel manipulators are excellent candidates considering the applications that require large workspace, low mass and low cost, and these could be designed to have high load capacity and dynamic characteristics. Similar to other types of parallel devices, the actuators of wire-actuated manipulators could be mounted at the base. Also, the wires act in parallel on the mobile platform. Therefore, the potential applications of wire-actuated manipulators include both the terrestrial applications, such as manufacturing, entertainment, medical and service sectors, and the space applications. For some of these applications, fail-safe manipulators are crucial, e.g., when the device is used in surgery or in high-speed operation. For tasks in hazardous environments and space/remote operations, human access to the manipulator could be very difficult, dangerous or impossible, while in some applications the downtime needs to be minimized.

Failure analysis of serial manipulators has been investigated for decades,[1] and has received more attention compared to parallel manipulators. In ref. [2] post-failure recovery from the full or partial loss of actuator torque in the closed-loop manipulators was investigated utilizing actuation redundancy to compensate for the lost torque and modifying the task time to reduce the overall actuator torque requirement. In ref. [3] the failure mode and effect analysis was performed to study the failure modes of parallel manipulators with their effects on the degrees of freedom, actuation and constraint. Redundancy types, such as redundant degrees of freedom, redundant sensing, redundant actuation and redundant legs/branches, have been suggested for fault tolerant designs. The effect of redundancy in joint displacement sensing for parallel manipulators has been investigated to reduce the number of forward displacement solutions/assembly modes,[4,5] to allow the fixtureless calibration of manipulators[6] and to facilitate the joint sensor fault detection, isolation and recovery.[7] Redundancy in actuation has been proposed to reduce the uncertainty/singularity configurations of parallel manipulators.[8,9] In ref. [10] the task space was partitioned into major and secondary tasks in order to complete the major task and optimize a secondary goal such as actuator fault tolerance. The reduced motion of parallel manipulators due to active joint jam and the design modification to compensate for the accuracy degradation was investigated in ref. [11, 12]. In ref. [13] the effect of losing a wire on the null space of the Jacobian matrix of a planar wire-actuated manipulator was considered. Fault tolerance of wire-actuated parallel manipulators needs further investigation in order to develop methodologies to improve their performance after failure.

In this paper, the failure of wire-actuated parallel manipulators is studied. Failure modes of these manipulators

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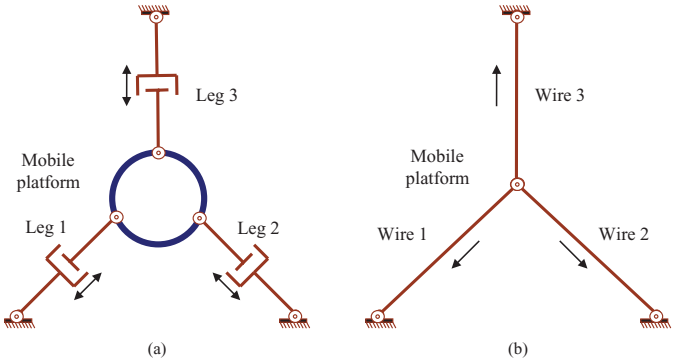


Fig. 1. (Colour online) Planar parallel manipulators: (a) 3-DOF solid-link manipulator, and (b) 2-DOF wire-actuated manipulator.

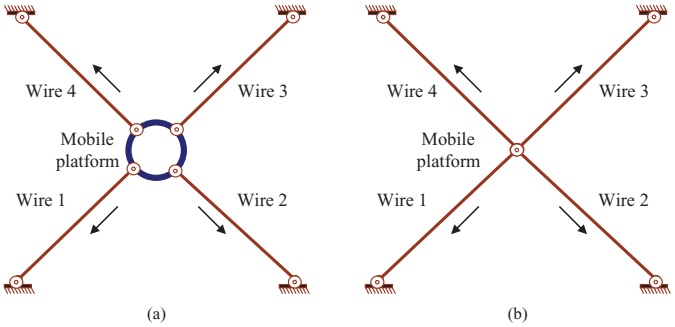


Fig. 2. (Colour online) Planar wire-actuated parallel manipulators with four wires: (a) 3-DOF manipulator, and (b) 2-DOF manipulator.

are discussed in Section 2. A methodology for recovering the lost force/moment capability due to the failure of wires and actuators is presented in Section 3. The kinematics and static modelling of planar wire-actuated manipulators and simulation results for the loss of wire/actuator force/torque are reported in Section 4. The paper concludes with Section 5.

2. Manipulator Failure Modes

Wire-actuated parallel manipulators could fail because of the failure of their components (e.g., wires, pulleys, spools and motors), subsystems (end effector) and systems (mechanical, electrical, software and controller). If any malfunctioning in the wire actuating mechanism affects the performance of manipulator such that the task cannot be completed as desired, then the manipulator is considered failed. Considering the mechanical system, wire-actuated parallel manipulators could fail because of the failure of a wire (wire breakage, wire jam or undesired flexibility of wire), sensor failure, actuator failure and transmission failure. These failures, in addition to loss of information, could result in the loss of degrees of freedom, loss of actuation and loss of motion constraint; please refer to ref. 3 for detailed discussion.

From the force point of view, the failure of manipulator occurs if the wire does not provide the required force/torque, for example, when the actuator force/torque is lost partially or fully or the actuator is saturated. This could also happen when the wire is broken or slack (zero tension), wire is jammed (constant length) or its actuating mechanism malfunctions such that a different (constant or limited) force is provided by the wire. It should be noted that based on the task, the wire could be intentionally kept slack, i.e., its tension is set to zero, for example, to avoid entangling of the wire with an obstacle or when the tension of the wire is not required.

3. Failure Recovery Methodology

In parallel manipulators, the mobile platform is connected to the base by a number of legs/branches/wires, for example, refer to Fig. 1. As wires act in tension, to manipulate an m DOF rigid body suspended by wires, i.e., to fully constrain these manipulators, in the absence of gravity and external wrench, the number of wires/actuators of the manipulator should be larger than the degrees of freedom of manipulator (Figs. 1(b) and 2). Hence, wire-actuated manipulators usually require some means of redundant actuation (e.g., redundant wires and/or gravity). In this paper, when the number of wires n is equal to $m + 1$, the manipulator will be referred to as "non-redundant" (Fig. 2(a)). For redundant actuation, one or more additional wires could be included, for example, the 2-DOF translational manipulator of Fig. 2(b) with four wires.

In order to provide fail-safe operation and fault tolerance, redundant wires could be incorporated in the design, or anchors could be mounted on slides, or combinations of these two could be used. After the failure of a wire, by adjusting the anchor positions of the remaining wires to new locations, the necessary platform wrench could be achieved for successful termination of the task. Alternatively, if the design supports the continuous motion of anchors along the slides, the required wrench of mobile platform will be provided by the wires and the linear actuators used for moving the anchors.

For the n wire-actuated parallel manipulators, the $n \times 1$ vector of wire forces, $\tau = [\tau_1 \cdots \tau_n]^{\mathbf{T}}$, is related to the $m \times 1$ vector of forces and moments (wrench) applied by the mobile platform $\mathbf{F} = [F_x \ F_y \ F_z \ M_x \ M_y \ M_z]^T$, with the $m \times n$ transposed Jacobian matrix $\mathbf{J}^T$ as

$$\mathbf{F} = \mathbf{J}^T \boldsymbol{\tau} = \left[\mathbf{J}_1^T \, \mathbf{J}_2^T \cdots \mathbf{J}_i^T \cdots \mathbf{J}_{n-1}^T \, \mathbf{J}_n^T\right] \tau = \sum_{j=1}^{n} \mathbf{J}_j^T \tau_j, \quad (1)$$

where $m \leq 6$ depending on the dimension of task space, e.g., $m = 3$ for planar motion. Column j of $\mathbf{J}^T$, $\mathbf{J}_j^T$, corresponds to the wrench applied on the mobile platform by the jth wire/actuator.

When wire i is failed, its tension τ_{ci} will be different (instantaneously or permanently) than the desired force value, τ_i, for example, it may have zero force, constant force, lower or higher force than that is required. If this level of wire tension does not affect the required force/moment of mobile platform, then the failed wire will not result in the failure of the manipulator. Otherwise, if the remaining wires cannot provide the lost wrench after the failure of wire i, the manipulator will be considered as failed. Likewise, the calculated tension for a wire might be a negative value, which is not feasible. These cases will be examined in the following subsections.

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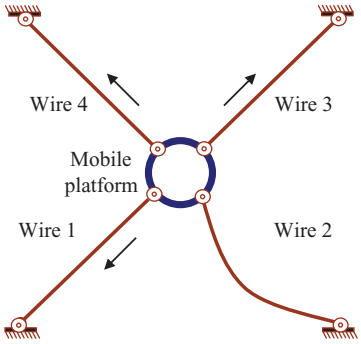


Fig. 3. (Colour online) Four-wire-actuated parallel manipulator with slack wire 2.

3.1. *Different wire forces*

The force of wire i will be zero, $\tau_{ci} = 0$, when wire i is broken or slacked (Fig. 3), or the actuator of wire i is failed. Wire i could also have a non-zero input, $\tau_{ci} \neq 0$, different than the required force value τ_i, for example, when the wire tension and/or the actuator torque reaches the limit (maximum value). Then, the mobile platform wrench will be

$$\mathbf{F}_f = \left[\mathbf{J}_1^T\, \mathbf{J}_2^T \cdots \mathbf{J}_i^T \cdots \mathbf{J}_{n-1}^T\, \mathbf{J}_n^T\right]\boldsymbol{\tau}_f = \sum_{j=1}^{n} \mathbf{J}_j^T \tau_j - \mathbf{J}_i^T(\tau_i - \tau_{ci}), \tag{2}$$

where $\boldsymbol{\tau}_f = [\tau_1\, \tau_2 \ldots \tau_{ci} \ldots \tau_{n-1} \tau_n]^T$ and the change in the wrench of mobile platform will be

$$\Delta\mathbf{F}_f = \mathbf{F} - \mathbf{F}_f = \mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) = \mathbf{J}_i^T(\tau_i - \tau_{ci}). \tag{3}$$

In other words, if at this pose the required input (force) from wire i is τ_{ci}, i.e., $\tau_i = \tau_{ci}$, there will be no change in the force/moment capability of the manipulator. However, if the required input from wire i is different, i.e., $\tau_i \neq \tau_{ci}$, then the manipulator would be considered as failed unless the remaining wires could provide the lost wrench due to failed wire i.

3.2. *Correctional input from healthy wires*

When the tension of the failed wire i is $\tau_{ci} \neq \tau_i$, to fully recover the lost wrench and maintain the force/moment capability $\mathbf{F}$ of the manipulator, the remaining wires must provide the required wrench for the mobile platform. If the correctional force, $\Delta\boldsymbol{\tau}_{\text{corr}} = [\Delta\tau_1 \Delta\tau_2 \ \ldots\ 0\ \ldots\ \Delta\tau_{n-1} \Delta\tau_n]^T$, to be provided by the remaining (healthy) wires compensates for the lost force/moment partially or completely, then the recovered wrench of platform will be

$$\mathbf{F}_r = \mathbf{J}^T\boldsymbol{\tau}_f + \mathbf{J}_f^T\Delta\boldsymbol{\tau}_{\text{corr}}, \tag{4}$$

where entry i of $\Delta\boldsymbol{\tau}_{\text{corr}}$ and column i of the reduced Jacobian matrix $\mathbf{J}_f^T$ are, respectively, replaced by a zero and a zero vector, i.e., the Jacobian matrix after failure of wire i is

$$\mathbf{J}_f^T = \left[\mathbf{J}_1^T\, \mathbf{J}_2^T\ \cdots\ \mathbf{0}\ \cdots\ \mathbf{J}_{n-1}^T\, \mathbf{J}_n^T\right]. \tag{5}$$

Then, the change in the wrench of mobile platform will be

$$\Delta\mathbf{F}_r = \mathbf{F} - \mathbf{F}_r = \mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) - \mathbf{J}^T\Delta\boldsymbol{\tau}_{\text{corr}}. \tag{6}$$

In order to fully compensate for the lost mobile platform wrench, i.e., for $\Delta\mathbf{F}_r = \mathbf{0}$, the correctional force provided by the remaining wires should be[14]

$$\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_f^{\#T}\mathbf{J}_i^T(\tau_i - \tau_{ci}) = \mathbf{J}_f^{\#T}\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f), \tag{7}$$

where $\boldsymbol{\tau} - \boldsymbol{\tau}_f = [0\ 0\ \ldots\ (\tau_i - \tau_{ci})\ \ldots\ 0\ 0]^T$ is the lost wire force due to failure of wire i.

If the forces/torques of k wires/actuators are different than the corresponding desired values, then the lost wrench is $\sum \mathbf{J}_i^T(\tau_i - \tau_{ci}) = \mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f)$, where the summation is taken over the failed wires. The correctional force of the remaining wires will be,

$$\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_f^{\#T}\sum \mathbf{J}_i^T(\tau_i - \tau_{ci}) = \mathbf{J}_f^{\#T}\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f). \tag{8}$$

In this case, k columns of $\mathbf{J}^T$, corresponding to the wires/actuators with different inputs, are replaced by zeros resulting in $\mathbf{J}_f^T$. It should be noted that even though the lost wire/actuator inputs might not be zero, the mobile platform wrench corresponding to the correctional force from the remaining wires/actuators will be zero when $\sum \mathbf{J}_i^T(\tau_i - \tau_{ci}) = \mathbf{0}$, i.e., the wrench produced by the failed wires/actuators cancel out, or when $\sum \mathbf{J}_i^T(\tau_i - \tau_{ci})$ is in the null space of $\mathbf{J}_f^{\#T}$.

In Eqs. (7) and (8), the generalized inverse $\mathbf{J}_f^{\#T}$ maps the lost wrench, $\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f)$, to the orthogonal complement of the null space of the reduced Jacobian matrix $\mathbf{J}_f^T$. Then, $\mathbf{J}_f^T\Delta\boldsymbol{\tau}_{\text{corr}}$ maps this force/torque to the range space of $\mathbf{J}^T$ (generally a non-zero wrench). If $\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_f^{\#T}\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) = \mathbf{0}$, i.e., when $\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) = \mathbf{0}$ or $\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f)$ belongs to the null space of $\mathbf{J}_f^{\#T}$ (which is equivalent to the orthogonal complement of the range space of $\mathbf{J}_f^{\#T}$), then there is no need for correctional force from the remaining wires.

If $\mathbf{J}_f^T$ has full row-rank, i.e., $\mathbf{F}$ belongs to the range space of $\mathbf{J}_f^T$, $\mathbf{F} \in \Re(\mathbf{J}_f^T)$, the generalized (Moore–Penrose) inverse of $\mathbf{J}_f^T$ is

$$\mathbf{J}_f^{\#T} = \mathbf{J}_f\left(\mathbf{J}_f^T\mathbf{J}_f\right)^{-1}, \tag{9}$$

as the vector of wire forces is physically consistent (all entries have the same dimension of force, i.e., mass times acceleration).

When a combination of wires and joints are used, depending on the type of joint, the input vector might not be physically consistent. For example, if the manipulator has a combination of wires and active revolute joints in order to constrain the mobile platform, for example, the manipulator of ref. [15], then, when $\mathbf{J}_f^T$ has full row-rank, a weighting metric would be required for calculating the generalized inverse of $\mathbf{J}_f^T$ as

$$\mathbf{J}_f^{\#T} = \mathbf{W}_\tau\mathbf{J}_f\left(\mathbf{J}_f^T\mathbf{W}_\tau\mathbf{J}_f\right)^{-1}. \tag{10}$$

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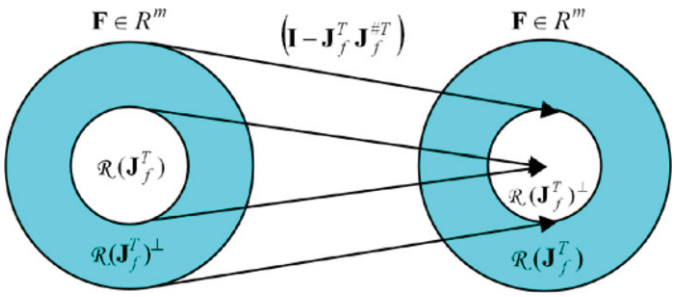


Fig. 4. (Colour online) Projection of mobile platform wrench onto the orthogonal complement of range space of $\mathbf{J}_f^T$.

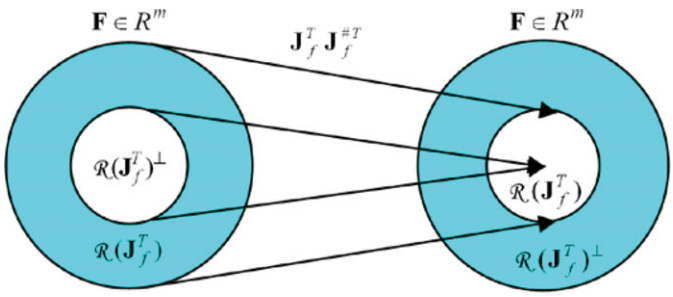


Fig. 5. (Colour online) Projection of mobile platform wrench onto the range space of $\mathbf{J}_f^T$.

Metric $\boldsymbol{W}_\tau$ is chosen such that $\boldsymbol{\tau}^T(\mathbf{W}_\tau^{-1}\boldsymbol{\tau})$ becomes physically consistent,[16] for instance, to minimize/maximize the strain energy of manipulator.

When the rank of $\mathbf{J}_f^T$ is less than m, then $\mathbf{J}_f^T$ will not have full row-rank, and in general, the lost wrench cannot be fully recovered. In this case if $\mathbf{J}_f^T$ has full column-rank and the lost wrench is not in the range space of $\mathbf{J}_f^T$, then the mobile platform wrench that best approximates the lost wrench in the least-square sense is calculated using the weighted left-generalized inverse of $\mathbf{J}_f^T$, for example, after removing the zero columns corresponding to the failed joints, as

$$\mathbf{J}_f^{\#T} = \left(\mathbf{J}_f \mathbf{W}_F \mathbf{J}_f^T\right)^{-1} \mathbf{J}_f \mathbf{W}_F. \tag{11}$$

The weighting metric $\boldsymbol{W}_F$ is chosen such that $\mathbf{F}^T(\mathbf{W}_F\mathbf{F})$ becomes physically consistent. When $\mathbf{J}_f^T$ is calculated after removing the zero columns, $\Delta\boldsymbol{\tau}_{\text{corr}}$ calculated using Eqs. (7) or (8) would correspond to healthy wires only.

3.3. *Criteria for full and partial recovery*

With the correctional input from healthy wires, when $\mathbf{F} \in \Re(\mathbf{J}_f^T)$, the deviation in the wrench of mobile platform will be zero using

$$\Delta\mathbf{F}_r = \mathbf{F} - \mathbf{F}_r = \left(\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\#T}\right) \mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f), \tag{12}$$

where $\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\#T}$ projects the lost wrench $\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f)$ due to failure of wire i (or k wires) to the orthogonal complement of the range space (i.e., to the null space) of $\mathbf{J}_f^T$, and hence, $\Delta\mathbf{F}_r = \mathbf{0}$ (refer to Fig. 4). Likewise, when $\mathbf{J}_f^T$ does not have full row-rank and the lost wrench belongs to the range space of $\mathbf{J}_f^T$, then $\Delta\mathbf{F}_r = \mathbf{0}$. Therefore, the condition for full recovery of the lost wrench after the failure of wire i (or k wires), i.e., when $\Delta\mathbf{F}_f \in \Re(\mathbf{J}_f^T)$, is

$$\mathbf{F}_{\Re^\perp} = \left(\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\#T}\right) \mathbf{F} = \mathbf{0}. \tag{13}$$

That is, the lost wrench will be fully recovered when all the components of the wrench $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$ projected onto the orthogonal complement of the range space of $\mathbf{J}_f^T$ are zero, provided that the overall wire forces $\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}}$ will not surpass the limit of the healthy wires and/or the corresponding overall actuator torques will not exceed their limit. Otherwise, the procedure could be applied for the wire corresponding to the entry of $\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}}$ that reaches/exceeds the wire/actuator limit.

When the lost wrench belongs to the orthogonal complement of the range space of $\mathbf{J}_f^T$, the correctional input of the remaining wires cannot set the deviation in the mobile platform wrench to zero, $\Delta\mathbf{F}_r \neq \mathbf{0}$ (refer to Fig. 5). The lost wrench that cannot be recovered could be characterized considering the range space of $\mathbf{I} - \mathbf{J}_f^T\mathbf{J}_f^{\#T}$, which is the same as the orthogonal complement of the range space of $\mathbf{J}_f^T$. Hence, the condition for partial recovery of the lost wrench after the failure of wire i (or k wires), i.e., when $\Delta\mathbf{F}_f \in \Re(\mathbf{J}_f^T)^\perp$, is

$$\mathbf{F}_{\Re^\perp} = \left(\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\#T}\right) \mathbf{F} \neq \mathbf{0}. \tag{14}$$

When one or more entries of $\mathbf{F}_{\Re^\perp}$ are zero, the corresponding components of the mobile platform force/moment could be completely recovered.

If the lost mobile platform wrench cannot be fully recovered, i.e., $\Delta\mathbf{F}_f \in \Re(\mathbf{J}_f^T)^\perp$, $\mathbf{F}_{\Re^\perp} = (\mathbf{I} - \mathbf{J}_f^T\mathbf{J}_f^{\#T})\mathbf{F} \neq \mathbf{0}$ and the recovered portion of the wrench will be calculated using Eq. (12) as

$$\mathbf{F}_r = \mathbf{F} - \left(\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\#T}\right) \sum \mathbf{J}_i^T(\tau_i - \tau_{ci}). \tag{15}$$

In case the force/torque of healthy wires/actuators cannot be changed after the failure, the error in the wrench that the platform could apply will be calculated as

$$\Delta\mathbf{F}_f = \mathbf{J}_f^T \mathbf{J}_f^{\#T} \sum \mathbf{J}_i^T(\tau_i - \tau_{ci}), \tag{16}$$

where $\mathbf{J}_f^T\mathbf{J}_f^{\#T} \sum \mathbf{J}_i^T(\tau_i - \tau_{ci})$ is the projection of the lost wrench to the range space of $\mathbf{J}_f^T$.

3.4. *Negative wire forces*

Using Eq. (1), the solution of $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$ for the vector of wire tensions is

$$\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} + \left(\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T\right)\mathbf{k} = \mathbf{J}^{\#T}\mathbf{F} + \mathbf{N}\lambda, \tag{17}$$

where $\mathbf{J}^{\#T}\mathbf{F}$ is the minimum norm or particular solution and $(\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T)\mathbf{k}$ and $\mathbf{N}\lambda$ represent the homogenous solution. $(\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T)\mathbf{k}$ is the projection of the arbitrary vector $\mathbf{k}$ onto the null space of $\mathbf{J}^T$, while the columns of $n \times (n - m)$ matrix $\mathbf{N}$ correspond to the orthonormal basis of the null space of $\mathbf{J}^T$.

The minimum norm solution could result in negative value for a wire tension, then the $n \times 1$ arbitrary vector $\mathbf{k}$, (or the

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$(n-m)\times 1$ arbitrary vector λ) is chosen such that positive tension is maintained in the wires. If $(\mathbf{I}-\mathbf{J}^{\#T}\mathbf{J}^T)\,\mathbf{k}$ (or $\mathbf{N}\,\lambda$) does not result in positive wire tension, for example, positive tension in at least m wires ($m \le n$) and zero tension in the remaining $n-m$ wires, when added to the minimum norm solution for any values of $\mathbf{k}$ (or λ), then the mobile platform is outside the wrench closure workspace and cannot be controlled. If the task requires this pose, then the manipulator will be considered as failed.

3.4.1. Zero external wrench. In the absence of external wrench (when there are no forces/moments applied by the mobile platform), $\mathbf{F}=\mathbf{0}$ and $\boldsymbol{\tau}=\mathbf{J}(\mathbf{I}-\mathbf{J}^{\#T}\mathbf{J}^T)\,\mathbf{k}=\mathbf{N}\,\lambda$, and the conditions for positive tension in wires are as follows. If the number of wires is $n=m+1$, then the $n\times 1$ matrix $\mathbf{N}$ that spans the null space basis of the transposed Jacobian matrix $\mathbf{J}^T$ (here referred to as the null space vector) should have entries with consistent sign. When $n>m+1$, the null space basis of $\mathbf{J}^T$ is defined by $n-m$ orthonormal vectors and at least one linear combination of these null space vectors should have entries with consistent sign (all non-negative or all non-positive).

Similarly, when one or more wires are failed, provided that the entries of the null space vector of $\mathbf{J}_f^T$ (or at least one linear combination of the null space vectors) have consistent signs, the tension of the remaining wires will be positive and the pose will be in the wrench closure workspace of the manipulator. If the entries of the null space vector do not have consistent signs, a zero (or a positive threshold) value for the wire with negative tension will result in the non-zero platform wrench with the least-square error.

3.4.2. Non-zero external wrench. In the presence of external wrench, $\mathbf{F}\neq\mathbf{0}$ and $\boldsymbol{\tau}=\mathbf{J}^{\#T}\mathbf{F}+(\mathbf{I}-\mathbf{J}^{\#T}\mathbf{J}^T)\,\mathbf{k}=\mathbf{J}^{\#T}\mathbf{F}+\mathbf{N}\,\lambda$. Then, if the minimum norm solution results in negative tension for wire i, as long as the entries of the null space vector of $\mathbf{J}^T$ (for $n=m+1$) or the entries of at least one linear combination of the null space vectors of $\mathbf{J}^T$ (for $n>m+1$) have consistent signs, then using the procedures presented in Sections 3.1 through 3.3 and treating wire i as the failed wire, the tension of wire i could be adjusted such that the negative value for τ_i, which results in the minimum norm solution for $\boldsymbol{\tau}$, is set to zero or a positive value (finite value or a threshold in order to avoid slackness).

When the entries of the null space vector of $\mathbf{J}^T$ (or all linear combinations of the null space vectors of $\mathbf{J}^T$) do not have consistent signs and the minimum norm solution results in negative tension for wire i, then it may be possible to have positive tension depending on the external wrench. This is because the external force could be modeled as an additional wire collinear with the axis of the external force. When l components of wrench $\mathbf{F}$ are non-zero, where $l\le m\le 6$, then the conditions for positive tension in wires are as follows. Rearranging the force/moment balance Eq. (1) as $\mathbf{J}^T\boldsymbol{\tau}-\mathbf{F}=\mathbf{0}$ and augmenting $\boldsymbol{\tau}$ with l ones, to form the $(n+l)\times 1$ vector $\boldsymbol{\tau}_{\text{aug}}$,

$$\mathbf{J}_{\text{aug}}^T\boldsymbol{\tau}_{\text{aug}}=\left[\mathbf{J}_1^T\mathbf{J}_2^T\cdots\mathbf{J}_{n-1}^T\mathbf{J}_n^T\mathbf{W}_1^T\ \cdots\ \mathbf{W}_l^T\right]\boldsymbol{\tau}_{\text{aug}}=\mathbf{0}, \quad (18)$$

where for a 6-DOF manipulator, when $l=6$, $\mathbf{W}_1=[-F_x\ 0\ 0\ 0\ 0\ 0]$, $\mathbf{W}_2=[0\ -F_y\ 0\ 0\ 0\ 0]$, ..., $\mathbf{W}_6=[0\ 0\ 0\ 0\ 0\ -M_z]$ and $\boldsymbol{\tau}_{\text{aug}}=[\,\tau_1\cdots\tau_n\,1\ 1\ 1\ 1\ 1\ 1\,]^T$. For a 3-DOF planar manipulator, when $l=3$ and the plane of motion is defined by the X–Y plane, $\mathbf{W}_1=[-F_x\ 0\ 0]$, $\mathbf{W}_2=[0\ -F_y\ 0]$, $\mathbf{W}_3=[0\ 0\ -M_z]$ and $\boldsymbol{\tau}_{\text{aug}}=[\,\tau_1\cdots\tau_n\,1\ 1\ 1\,]^T$.

The null space basis of $\mathbf{J}_{\text{aug}}^T$ is defined by $n+l-m$ vectors. At least one linear combination of these $(n+l)\times 1$ null space vectors should have non-negative values for the first n entries (corresponding to wires) and positive values for the last l entries (corresponding to non-zero components of external wrench), for example, refer to ref [13] for the case that $\mathbf{F}$ is the gravitational force.

When the tension of wire i (k wires) is negative for the minimum norm solution, and the first n entries of the null space vector (or of the linear combination of the null space vectors) of $\mathbf{J}_{\text{aug}}^T$ are non-negative and the last l entries are positive, then the tension of wire i (k wires) could be adjusted such that the negative value for τ_i (tension of k wires), which results in the minimum norm solution for $\boldsymbol{\tau}$, is set to zero or a positive value.

Similarly, when wire i is failed and the required mobile platform wrench is in the range space of $\mathbf{J}_f^T$, provided that the entries of the null space vector of $\mathbf{J}_f^T$ (or at least one linear combination of the null space vectors) have consistent signs, the correctional tension from the remaining wires could fully recover the required mobile platform wrench if the minimum norm solution for remaining wires results in negative tension for one or more wires. When $\mathbf{F}\in\Re(\mathbf{J}_{\text{aug}\,f}^T)$ and the entries of the null space vector (or all linear combinations of the null space vectors) of $\mathbf{J}_f^T$ do not have consistent signs, provided that the entries of the null space vector of $\mathbf{J}_{\text{aug}\,f}^T$ fulfill the pertinent condition, the correctional tension from the remaining wires could fully recover the required mobile platform wrench if the minimum norm solution for remaining wires results in negative tension for one or more wires. If the entries of the null space vector (or all linear combinations of the null space vectors) of $\mathbf{J}_{\text{aug}\,f}^T$ do not satisfy the pertinent condition, then the platform wrench cannot be fully recovered. However, a zero (or a positive threshold) value for the wire with negative tension will result in the least-square error in the platform wrench.

3.4.3. Formulation of null space vector. The null space vector of $\mathbf{J}^T$ (or $\mathbf{J}_{\text{aug}}^T$, $\mathbf{J}_f^T$, $\mathbf{J}_{\text{aug}\,f}^T$) could be identified by different methods. Using Cramer's rule,[17] when $n=m+1$, vector $\mathbf{N}$, which spans the null space basis of $\mathbf{J}^T$, is calculated as

$$\mathbf{N}=[(-1)^{n-1}\det(\mathbf{M}_1)\quad(-1)^{n-2}\det(\mathbf{M}_2)\quad\cdots\quad\det(\mathbf{M}_n)\,]^T, \quad (19)$$

where $\boldsymbol{M}_i$ is the ith $m\times m$ submatrix (minor) of the transposed Jacobian matrix, formed by removing the ith column of $\mathbf{J}^T$. For wire-actuated parallel manipulators, the entries of vector $\mathbf{N}$ should have consistent sign, e.g., $(-1)^{n-i}\det(\mathbf{M}_i)\ge 0$ for $i=1,\ldots,n$.

4. Case Study

In order to model the wire-actuated parallel manipulators, a fixed reference frame Ψ(X,Y,Z) is assigned to the base, with

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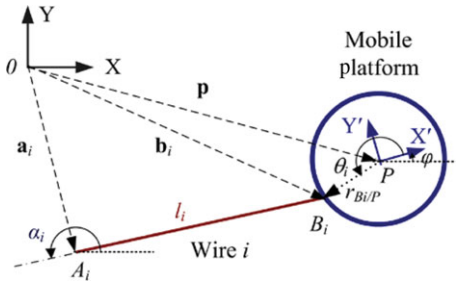


Fig. 6. (Colour online) Parameters of planar wire-actuated parallel manipulators.

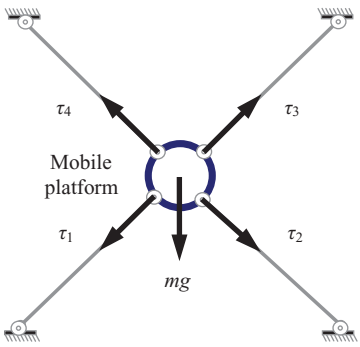


Fig. 7. (Colour online) Free-body diagram of mobile platform of Fig. 2(a).

origin at point 0, and a moving reference frame Γ(X′,Y′,Z′) is attached to the center of mass, point P, of the mobile platform. For the translational wire-actuated manipulators, the mobile platform shrinks to point P (a point mass). The position vector of point P in the base frame is $\boldsymbol{p} = [p_x\, p_y\, p_z]^T$. The orientation of the mobile platform with respect to the base frame Ψ(X,Y,Z) is given by the Euler angles.

The methodology presented in the previous section is valid for both planar and spatial wire-actuated parallel manipulators. In this section, planar manipulators are used as the case study. The kinematics and static force analysis of planar manipulators are presented first, then the effect of failed wire(s) and the failure recovery are investigated, and the corresponding simulation results are reported.

4.1. Kinematic analysis

In wire-actuated parallel manipulators, the mobile platform is connected to the base by n wires (refer to Figs. 1 and 2), each wire with a length of l_i (Fig. 6). For planar manipulators, defining the plane of motion by X–Y plane, the position vector of the base attachment point of wire i (anchor), point A_i, in the fixed frame Ψ(X,Y,Z) is $\boldsymbol{a}_i = [a_{ix} a_{iy}]^T$, for $i = 1, \ldots, n$, and the position vector of wire attachment to the mobile platform, point B_i, in the platform frame Γ(X′,Y′,Z′) is ${}^{\Gamma}\boldsymbol{b}_i = [r_{Bi/P}\cos\theta_i \;\; r_{Bi/P}\sin\theta_i]^T$, where $r_{Bi/P}$ is the length of the line segment $\overline{PB_i}$. The orientation of lines $\overline{PB_i}$ (angular positions of points B_i on the mobile platform) with respect to Γ(X′,Y′,Z′) are denoted by angles θ_i. The position of B_i relative to the base frame, $\boldsymbol{b}_i = [b_{ix} b_{iy}]^T$, is calculated using the homogeneous transformation matrix relating Γ(X′,Y′,Z′) to Ψ(X,Y,Z),

$$\mathbf{A}_{\Psi,\Gamma} = \begin{bmatrix} \cos\varphi & -\sin\varphi & p_x \\ \sin\varphi & \cos\varphi & p_y \\ 0 & 0 & 1 \end{bmatrix}, \tag{20}$$

which is in terms of the mobile platform position $\boldsymbol{p} = [p_x\, p_y]^T$ and orientation φ. The orientation of wire i with respect to the base frame Ψ(X,Y,Z) is represented by angle α_i.

When only wire actuators are used, for non-redundant translational 2-DOF manipulators ($m = 2$; two translations on the plane), in the absence of gravity and external force, three wires are required ($n = 3$). For non-redundant 3-DOF planar manipulators ($m = 3$; two translations on the plane and a rotation about an axis normal to the plane), in the absence of gravity and external wrench, four wires are required ($n = 4$). For redundant actuation if solely wire actuators are utilized, then one or more wires should be added; whereas if hybrid actuation is considered, the anchors may be mounted on linear actuators.

For velocity analysis, the rate of change of wire lengths (and velocity of linear actuators) $\dot{\mathbf{l}}$ is related to the translational and rotational velocity (twist) of mobile platform, $\mathbf{V} = [v_x \;\; v_y \;\; \dot{\varphi}\,]^T$, with the Jacobian matrix $\mathbf{J}$,

$$\dot{\mathbf{l}} = -\mathbf{J}\mathbf{V} \tag{21}$$

To solve for the velocity of mobile platform $\mathbf{V}$, when matrix $\mathbf{J}$ is not square, $\mathbf{V} = -\mathbf{J}^{\#}\dot{\mathbf{l}}$. Considering the velocity relation $\dot{\mathbf{l}} = -\mathbf{J}\mathbf{V}$, vector $\dot{\mathbf{l}}$ is physically consistent (all entries have the same dimension of length/time). Hence, even though for the general motion the entries of the mobile platform twist are not unit consistent (include rotational and translational velocities), as long as $\mathbf{J}$ has full column-rank there is no need for a weighting metric on the task space velocity. Therefore, matrix $\mathbf{J}^{\#} = (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T$ is the un-weighted generalized inverse of matrix $\mathbf{J}$. For the 2-DOF translational manipulators, the mobile platform velocity vector is $\mathbf{V} = [\,v_x \;\; v_y\,]^T$.

4.2. Force analysis

The relationship between the vector of wire forces (and forces of linear actuators) $\boldsymbol{\tau} = [\tau_1 \cdots \tau_n]^T$ and the wrench applied by the mobile platform, $\mathbf{F}$, is $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau} = [\mathbf{J}_1^T \mathbf{J}_2^T \cdots \mathbf{J}_i^T \cdots \mathbf{J}_{n-1}^T \mathbf{J}_n^T]\boldsymbol{\tau} = \sum_{j=1}^{n} \mathbf{J}_j^T \tau_j$, where column i of matrix $\mathbf{J}^T$, i.e., $\mathbf{J}_i^T$, corresponds to the wrench that wire i (or linear actuator) applies on the mobile platform.

Considering the force relation and the $3 \times n$ matrix $\mathbf{J}^T$, where $n > 3$, the vector of wire forces (and forces of linear actuators) is physically consistent (all entries have the same dimension of force, i.e., mass times acceleration). Therefore, to formulate $\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} + (\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T)\,\mathbf{k} = \mathbf{J}^{\#T}\mathbf{F} + \mathbf{N}\,\lambda$, even though the entries of the mobile platform wrench vector, $\mathbf{F} = [F_{\text{ext}_x} \;\; F_{\text{ext}_y} \;\; M_{\text{ext}_z}]^T$, are not unit consistent, as long as $\mathbf{J}^T$ has full row-rank (i.e., $\mathbf{F}$ belongs to the range space of $\mathbf{J}^T$) there is no need for a weighting metric and $\mathbf{J}^{\#T}$ is calculated as

$$\mathbf{J}^{\#T} = \mathbf{J}(\mathbf{J}^T\mathbf{J})^{-1}. \tag{22}$$

When the actuation is provided by n wires, referring to Fig. 7, in the absence of gravity, for example, when the plane of motion is a horizontal plane, the static force and moment

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equations are

$$\sum_{1}^{n} \tau_i \cos\alpha_i = F_{\text{ext}_x}, \tag{23a}$$

$$\sum_{1}^{n} \tau_i \sin\alpha_i = F_{\text{ext}_y}, \tag{23b}$$

$$\sum_{1}^{n} \tau_i v_i = M_{\text{ext}_z}, \tag{23c}$$

or

$$\mathbf{F} = \mathbf{J}^T \boldsymbol{\tau} = \begin{bmatrix} \cos\alpha_1 & \cdots & \cos\alpha_n \\ \sin\alpha_1 & \cdots & \sin\alpha_n \\ v_1 & \cdots & v_n \end{bmatrix} \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix}, \tag{24}$$

where column i of the $3 \times n$ matrix $\mathbf{J}^T$, i.e., $\mathbf{J}_i^T$, represents the axis of wire i. The direction cosines corresponding to the axis of wire i are calculated as $\cos\alpha_i = l_{ix}/l_i$ and $\sin\alpha_i = l_{iy}/l_i$, and v_i is the moment of wire axis with respect to the origin of $\Gamma(X',Y',Z')$, point P, formulated in $\Psi(X, Y, Z)$ as

$$v_i = -\cos\alpha_i(b_{iy} - p_y) + \sin\alpha_i(b_{ix} - p_x). \tag{25}$$

When the plane of motion is a vertical plane and the combined weight of platform and its payload, mg, is not negligible, the applied wrench by the platform is $\mathbf{F} = [F_{\text{ext}_x}(F_{\text{ext}_y} - mg)M_{\text{ext}_z}]^T$. For 2-DOF translational manipulators, the vector of mobile platform forces is $F = [F_{\text{ext}_x}(F_{\text{ext}_y} - mg)]^T$ and

$$\mathbf{F} = \mathbf{J}^T \boldsymbol{\tau} = \begin{bmatrix} \cos\alpha_1 & \cdots & \cos\alpha_n \\ \sin\alpha_1 & \cdots & \sin\alpha_n \end{bmatrix} \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix}. \tag{26}$$

4.3. Recovering lost force/moment

In this section, three different planar wire-actuated parallel manipulators are investigated.

4.3.1. Translational four-wire manipulator. For the 2-DOF translational parallel manipulator of Fig. 2(b), the task space dimension is $m = 2$ and the number of wires is $n = 4$. The coordinates of the base attachment points $A_i, i = 1, \ldots, 4$, in the base frame are, respectively, $(-2, -1.5), (2, -1.5), (2, 1.5)$ and $(-2, 1.5)$. When the mobile platform pose is $\boldsymbol{p} = [1\ 0]^T$ m, the transpose of the Jacobian matrix is

$$\mathbf{J}^T = \begin{bmatrix} -0.894 & 0.555 & 0.555 & -0.894 \\ -0.447 & -0.832 & 0.832 & 0.447 \end{bmatrix}. \tag{27}$$

A null space vector of $\mathbf{J}^T$ is $\boldsymbol{N} = [0.874\ 0.591\ 1.000\ 0.113]^T$, which has positive entries. Therefore, this pose is in the wrench closure workspace of manipulator, i.e., for any external wrench, the wires could have positive tension. When $\mathbf{F} = \mathbf{0}$, a solution for $\lambda = 1$ is $\boldsymbol{\tau} = [0.874\ 0.591\ 1.000\ 0.113]^T$. In this case, when wire 1 is failed, the null space vector of $\mathbf{J}_f^T$ after removing the first column of $\mathbf{J}^T$ is $\boldsymbol{N}_f = [0.688\ 0.344\ 0.640]^T$; hence, with the remaining wires 2, 3 and 4 the manipulator is still in the wrench closure workspace. On the other hand, when wire 2 is failed the null space vector of $\mathbf{J}_f^T$ is $\boldsymbol{N}_f = [-0.726\ -0.585\ 0.363]^T$ with negative and positive entries; hence, with wires 1, 3 and 4 this pose is not in the wrench closure workspace of manipulator.

The proposed methodology is demonstrated for the non-zero wrench of $\mathbf{F} = [-16.986\ 21.737]^T$ N applied by the mobile platform. Using $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$, the minimum norm vector of wire forces is

$$\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} = [1.411 \quad -14.388 \quad 5.881 \quad 12.305]^T. \tag{28}$$

The minimum norm solution results in negative tension for wire 2. The tension of wire 2 could be adjusted to zero ($\tau_2 = 0$) by using the homogenous solution or employing the methodology presented in Section 3. Utilizing the proposed methodology and treating wire 2 as failed, when τ_2 of the minimum norm solution is set to zero, the wrench of mobile platform is calculated as $\mathbf{F}_f = \mathbf{J}^T\boldsymbol{\tau}_f = [-9.005\ 9.766]^T$.

The wire forces are adjusted such that $\Delta\mathbf{F}_r = \mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) - \mathbf{J}^T\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_2^T\tau_2 - \mathbf{J}^T\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{0}$. Hence, the correctional force to be provided by wires 1, 3 and 4 is

$$\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_f^{\#T}\mathbf{J}_2^T\tau_2 = [0.470 \quad 0 \quad 7.572 \quad 13.149]^T. \tag{29}$$

Then the overall wire forces will be

$$\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}} = [1.881 \quad 0 \quad 13.453 \quad 25.454]^T, \tag{30}$$

which produces the original platform wrench.

4.3.2. 3-DOF four-wire manipulator. For the parallel manipulator of Figs. 2(a) and 7, the task space dimension is $m = 3$ and the number of wires is $n = 4$. The coordinates of the base attachment points $A_i, i = 1, \ldots, 4$, in the base frame are, respectively, $(-2, -1.5), (2, -1.5), (2, 1.5)$ and $(-2, 1.5)$. The position of connection points B_i on the mobile platform is set at a constant radius of $r_{Bi/P} = 0.25$ m. The angular coordinates, $\theta_i, i = 1, \ldots, 4$, of the wire connections to the mobile platform are, respectively, $180°, 0°, 0°$ and $180°$.

Negative wire tension. For the four-wire manipulator of Fig. 2(a), when the mobile platform pose is $\boldsymbol{p} = [0\ 0]^T$ m and $\varphi = 30°$, the transpose of the Jacobian matrix is

$$\mathbf{J}^T = \begin{bmatrix} -0.792 & 0.739 & 0.792 & -0.739 \\ -0.611 & -0.674 & 0.611 & 0.673 \\ 0.033 & -0.238 & 0.033 & -0.238 \end{bmatrix}. \tag{31}$$

The null space vector of $\mathbf{J}^T$ is $\boldsymbol{N} = [1.000\ 0.139\ 1.000\ 0.139]^T$. As all entries of $\mathbf{N}$ are positive, this pose is in the wrench closure workspace of the manipulator. Hence, any external wrench could be applied by the manipulator as long as the forces/torques of wires/actuators do not exceed the limit. When $\mathbf{F} = \mathbf{0}$, $\boldsymbol{\tau} = \mathbf{N}\lambda = \mathbf{N}/\|\mathbf{N}\| = [0.700\ 0.098\ 0.700\ 0.098]^T$ has the identity Euclidean norm.

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For $\boldsymbol{F} = [1.056\ 25.681\ -12.965]^T$ applied by the mobile platform, i.e., for a force of $[1.056\ 25.681]^T$ N and a moment of −12.965 N-m about the Z direction, using $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$, the minimum norm wire forces are

$$\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} = [-13.720 \quad 16.694 \quad 6.280 \quad 36.694]^T \quad (32)$$

with negative tension for wire 1, which is not allowed.

The augmented Jacobian matrix is

$$\mathbf{J}_{\text{aug}}^T = \begin{bmatrix} -0.792 & 0.739 & 0.792 & -0.739 & -1.056 & 0 & 0 \\ -0.611 & -0.674 & 0.611 & 0.673 & 0 & -25.681 & 0 \\ 0.033 & -0.238 & 0.033 & -0.238 & 0 & 0 & 12.965 \end{bmatrix}. \quad (33)$$

A null space vector of $\mathbf{J}_{\text{aug}}^T$ is $\boldsymbol{N}_{\text{aug}} = [1.000\ 0.452\ 0.486\ 0.511\ 0.324\ 0.013\ 0.016]^T$ and that of $\mathbf{J}_{\text{aug}f}^T$, after removing the first column of $\mathbf{J}_{\text{aug}}^T$, is $\boldsymbol{N}_{\text{aug}f} = [0.504\ 1.000\ 0.699\ 0.614\ 0.029\ 0.019]^T$, where the first four (and three) positive entries of $\boldsymbol{N}_{\text{aug}}$ (and $\boldsymbol{N}_{\text{aug}f}$) correspond to the wires and the last three positive entries correspond to the platform wrench. Therefore, as it was also indicated by $\mathbf{N}$, the negative tension for wire 1, which is calculated using the 2-norm solution, could be set to zero or a positive value.

Using the homogenous solution, the tension of wire 1 is zero when $\lambda = 13.720$ (and its tension is 2 N when $\lambda = 15.720$). Employing the methodology presented in Section 3, if the minimum allowable wire tension is 2 N, then treating wire 1 as a failed wire,

$$\boldsymbol{\tau}_f = [2 \quad 16.694 \quad 6.280 \quad 36.694]^T, \quad (34)$$

and the wrench of the mobile platform is calculated as $\mathbf{F}_f = \mathbf{J}^T\boldsymbol{\tau}_f = [-11.394\ 16.083\ -12.443]^T$.

The additional forces to be provided by wires 2, 3 and 4 should be

$$\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_f^{\#T}\mathbf{J}_1^T\tau_1 = [0 \quad 2.191 \quad 15.720 \quad 2.191]^T. \quad (35)$$

Then the overall wire forces will be

$$\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}} = [2 \quad 18.885 \quad 22.000 \quad 38.885]^T, \quad (36)$$

which results in $\Delta\mathbf{F}_r = (\mathbf{I} - \mathbf{J}_f^T\mathbf{J}_f^{\#T})\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) = \mathbf{0}$, and hence produces the original platform wrench of $\mathbf{F} = \mathbf{J}^T(\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}}) = [1.056 \quad 25.681 \quad -12.965]^T$.

Single wire failure. When the mobile platform pose is $\boldsymbol{p} = [0\ -2]^T$ m (outside the rectangle defined by the four anchors) and $\varphi = 0^\circ$, the transpose of the Jacobian matrix is

$$\mathbf{J}^T = \begin{bmatrix} -0.962 & 0.962 & 0.447 & -0.447 \\ 0.275 & 0.275 & 0.894 & 0.894 \\ -0.069 & 0.069 & 0.224 & -0.224 \end{bmatrix}, \quad (37)$$

and the null space vector of $\mathbf{J}^T$ is $\boldsymbol{N} = [1.000\ 1.000\ -0.307\ \ -0.307]^T$. The entries of $\mathbf{N}$ have inconsistent signs, hence for $\mathbf{F} = \mathbf{0}$ this pose is outside the wrench closure workspace.

When the wrench $\boldsymbol{F} = [0\ 28.877\ 0]^T$ is applied by the mobile platform, i.e., for a pure force of 28.877 N in the Y direction (equivalent to a force of −28.877 N applied on the platform), the augmented Jacobian matrix is

$$\mathbf{J}_{\text{aug}}^T = \begin{bmatrix} -0.962 & 0.962 & 0.447 & -0.447 & 0 \\ 0.275 & 0.275 & 0.894 & 0.894 & -28.877 \\ -0.069 & 0.069 & 0.224 & -0.224 & 0 \end{bmatrix}. \quad (38)$$

A null space vector of $\mathbf{J}_{\text{aug}}^T$ is $\boldsymbol{N}_{\text{aug}} = [1.000\ 1.000\ 0.118\ 0.118\ 0.026]^T$. Using $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$, the minimum norm wire forces result in positive tension for all four wires,

$$\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} = [4.531 \quad 4.531 \quad 14.751 \quad 14.751]^T. \quad (39)$$

When wire 1 is slack, $\tau_1 = 0$, the wrench of the mobile platform is calculated as $\mathbf{F}_f = \mathbf{J}^T\boldsymbol{\tau}_f = [4.357\ 27.633\ 0.311]^T$.

The null space vector of $\mathbf{J}_{\text{aug}f}^T$ after removing the first column of $\mathbf{J}_{\text{aug}}^T$ is $\boldsymbol{N}_{\text{aug}f} = [0\ 1.000\ 1.000\ 0.062]^T$, where the first three entries correspond to the healthy wires (wires 2, 3 and 4) and the fourth entry corresponds to the non-zero component of $\mathbf{F}$. The zero entry of $\boldsymbol{N}_{\text{aug}f}$ indicates that there will be no contribution from wire 2 in order to provide $\boldsymbol{F} = [0\ 28.877\ 0]^T$. For the given platform wrench, considering $\boldsymbol{N}_{\text{aug}}$ (and $\boldsymbol{N}_{\text{aug}f}$), this pose is in the wrench closure workspace. Likewise, the projection of platform wrench $\mathbf{F}$ onto the range space of $\mathbf{J}_f^T$ is a zero vector, which indicates that the failure of wire 1 could be fully recovered by the remaining wires assuming that the correctional force will not result in a wire/actuator force/torque exceeding the limit.

In order to fully recover from the failure of wire 1, the wire forces are adjusted such that $\Delta\mathbf{F}_r = \mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) - \mathbf{J}^T\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_1^T\tau_1 - \mathbf{J}^T\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{0}$. Hence, the correctional force to be provided by wires 2, 3 and 4 (to be added to the minimum norm solution) should be

$$\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_f^{\#T}\mathbf{J}_1^T\tau_1 = [0 \quad -4.531 \quad 1.392 \quad 1.392]^T. \quad (40)$$

Then, the overall wire forces will be

$$\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}} = [0 \quad 0 \quad 16.143 \quad 16.143]^T, \quad (41)$$

which results in $\Delta\mathbf{F}_r = (\mathbf{I} - \mathbf{J}_f^T\mathbf{J}_f^{\#T})\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) = \mathbf{0}$, and hence produces the original platform wrench of $\mathbf{F} = \mathbf{J}^T(\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}}) = [0.000\ 28.877\ 0]^T$.

4.3.3. Six-wire manipulator. For the six-wire parallel manipulator of Fig. 8, the coordinates of the base attachment points A_i, $i = 1, \ldots, 6$, in the base frame are, respectively, $(-2, -1.5)$, $(2, -1.5)$, $(2, -1.5)$, $(2, 1.5)$, $(-2, 1.5)$ and $(-2, 1.5)$. The position of connection point B_i on the mobile platform is set at a constant radius of $r_{Bi/P} = 0.25$ m. The

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Table I. Example failures for the six-wire parallel manipulator.

	$\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} = [2.457 \quad -1.917 \quad 23.424 \quad 68.455 \quad 32.820 \quad 0.112]^T$	
	Case 1: $\tau_{c2} = 0, \tau_{c3} = 0, \tau_{c6} = 0$	Case 2: $\tau_{c1} = 0, \tau_{c2} = 0, \tau_{c6} = 0$
$\mathbf{F}_f = \mathbf{J}^T\boldsymbol{\tau}_f$	$[24.758 \quad 78.041 \quad 21.009]^T$	$[50.081 \quad 75.321 \quad 22.568]^T$
$\Delta\boldsymbol{\tau}_{\text{corr}} = \mathbf{J}_f^{\#T}\sum \mathbf{J}_i^T\tau_i$	$[0.607 \quad 0 \quad 0 \quad 17.201 \quad -19.093 \quad 0]^T$	$[0 \quad 0 \quad -2.065 \quad -1.227 \quad -2.454 \quad 0]^T$
$\boldsymbol{\tau}_f + \Delta\boldsymbol{\tau}_{\text{corr}}$	$[3.064 \quad 0 \quad 0 \quad 85.656 \quad 13.727 \quad 0]^T$	$[0 \quad 0 \quad 21.359 \quad 67.228 \quad 30.366 \quad 0]^T$
$\Delta\mathbf{F}_r = (\mathbf{I} - \mathbf{J}_f^T\mathbf{J}_f^{\#T})\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f)$	$[0 \quad 0 \quad 0]^T$	$[-2.630 \quad 2.097 \quad 1.395]^T$
$\mathbf{F}_{\Re^\perp} = (\mathbf{I} - \mathbf{J}_f^T\mathbf{J}_f^{\#T})\mathbf{J}^T\boldsymbol{\tau}$	$[0 \quad 0 \quad 0]^T$	$[45.969 \quad -47.232 \quad -3.210]^T$

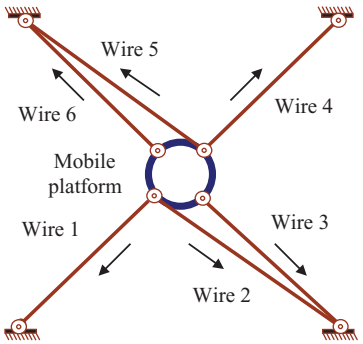


Fig. 8. (Colour online) Redundant six-wire parallel manipulator.

angular coordinates, θ_i, $i = 1, \ldots, 6$, of the wire connections to the platform are, respectively, 180°, 180°, 0°, 0°, 0° and 180°.

When the mobile platform pose is $\boldsymbol{p} = [-0.5 \; -1]^T$ m and $\varphi = -30°$, the transpose of the Jacobian matrix is

$$\mathbf{J}^T = \begin{bmatrix} -0.899 & 0.974 & 0.987 & 0.656 & -0.547 & -0.475 \\ -0.438 & -0.224 & -0.162 & 0.754 & 0.837 & 0.880 \\ 0.207 & -0.073 & 0.088 & 0.245 & 0.113 & -0.131 \end{bmatrix}. \tag{42}$$

For the wrench of $\boldsymbol{F} = [45.951 \; 74.774 \; 23.202]^T$ applied by the mobile platform, using $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$, the minimum norm wire forces are reported in Table I. Different failure cases have been investigated for this manipulator. Due to space limitation, two cases are discussed here.

In case 1, wires 2, 3 and 6 fail with zero tension. The required platform wrench is in the range space of $\mathbf{J}_f^T$, as indicated by $\mathbf{F}_{\Re^\perp} = [0 \; 0 \; 0]^T$, and hence the manipulator fully recovers the lost wrench by adjusting the tension of the remaining healthy wires (wires 1, 4 and 5) as reported in Table I.

In case 2, when wires 1, 2 and 6 fail (with zero tension), because the three healthy wires (wires 3, 4 and 5) are connected to the same attachment point on the mobile platform, they cannot provide/resist rotation of platform about that point. In this case $\mathbf{F}_{\Re^\perp} = [45.969 \; -47.232 \; -3.210]^T$ indicates that the required platform wrench is not in the range space of $\mathbf{J}_f^T$, and hence cannot be fully recovered. To reduce the error in the platform wrench, the correctional forces from wires 3, 4 and 5, which result in the minimum norm solution and approximate the lost wrench in the least-square sense, are $\Delta\boldsymbol{\tau}_{\text{corr}} = [0 \; 0 \; -2.065 \; -1.227 \; -2.454 \; 0]^T$, using an identify matrix as the weighting metric. The deviation (least square error) in the platform wrench after applying the correctional forces, i.e., the error of recovered wrench, is $\Delta\mathbf{F}_r = (\mathbf{I} - \mathbf{J}_f^T\mathbf{J}_f^{\#T})\mathbf{J}^T(\boldsymbol{\tau} - \boldsymbol{\tau}_f) = [-2.630 \; 2.097 \; 1.395]^T$, and the recovered wrench of platform is $\boldsymbol{F}_r = [48.581 \; 72.676 \; 21.808]^T$.

5. Conclusion

Failure analysis of wire-actuated parallel manipulators was discussed in this paper considering their failure modes. Specifically, the effect of zero or different/limited wire forces on the force/moment capabilities of these manipulators was examined. A methodology, based on the projection of the lost wire force onto the orthogonal complement of the null space of the Jacobian matrix of the failed manipulator, was presented. The correctional force to be provided by the healthy wires was formulated and the conditions for the full recovery of the lost wrench of the mobile platform, while maintaining positive tension in wires, were proposed.

It was presented that when the result of projecting the required mobile platform wrench onto the orthogonal complement of the range space of the reduced Jacobian matrix is a null vector, the lost wrench would be entirely recovered provided the entries of the null space vector of the Jacobian matrix after failure have consistent sign. Otherwise, the condition on the entries of the null space vector (or entries of at least one linear combination of the null space vectors) of the augmented Jacobian matrix after failure should be satisfied for full recovery, i.e., the entries corresponding to the wires should be non-negative and the ones corresponding to the external wrench should be positive. In addition, conditions on the lost platform wrench that could not be recovered were established. For both full and partial recovery cases, the correctional input from the healthy wires should maintain the overall force of the wires and torque of the actuators within their limits.

The minimum norm solution for wire tension vector could result in the negative value for wire tension. By treating the wire with negative tension as failed, the methodology could also be utilized to set the tension to a positive value if the considered pose is in the wrench closure workspace of manipulator.

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