Wake vortex encounters in a flight simulator

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ABSTRACT

A real-time model of a wake vortex has been developed for an engineering flight simulator, which enables studies to be undertaken into the effects of wake vortex encounters with civil transport aircraft. A brief summary of the methods used to create a vortex model and integrate it with a real-time flight simulator is presented. Tools have been developed to determine the structural loads on an aircraft during a simulated wake vortex encounter and to visualise the airframe loads. Four wake vortex encounters were flown with different pilot recovery strategies and automatic flight controls. It is clear from analysis of these results that it is difficult for a pilot to appreciate the margin of tail loading during an encounter and a prototype training display is outlined to enable pilots to fully understand the effects of rudder inputs in recovery from these events.

NOMENCLATURE

BM_v	vertical tail bending moment
BM_{arm}	moment arm length
$C_{Lv\beta}$	vertical tail lift-curve slope
C_{Yv}	vertical tail sideforce coefficient
EAS	equivalent airspeed
K_g	gust alleviation factor
L_v	vertical tail shear load
q_v	dynamic pressure
S_v	vertical tail area
U_{de}	gust velocity
V_e	aircraft equivalent airspeed
β	aircraft side slip angle
β_v	side slip angle at vertical tail
δ_r	rudder deflection

1.0 BACKGROUND

Considerable interest was generated following the American Airlines Flight 587 Airbus A300-600 accident in November 2001 where the aircraft broke up following a wake vortex encounter. It had been assumed that aircraft had been designed to withstand the loads likely to occur during an interaction with the wake vortex of another aircraft and that there was sufficient margin to tolerate worst case pilot inputs during an encounter. That the tail broke up under structural loading implied that either the forces occurring in such events had been underestimated or that more consideration needed to be given to flight crew training to avoid pilot intervention which might result in over-stressing the aircraft.

The real-time wake vortex models used in flight simulation at that time were relatively simple, based on estimates of the vortex core radius and magnitude of airflows likely to occur in a wake vortex where the shape of the vortex was defined as a simple geometric volume emanating from the aircraft generating the vortex. However, studies of vortices have shown that a vortex is not a simple analytic shape and moreover, that there is considerable dynamic variation in the radial and longitudinal flows of a vortex[1], which may persist for several minutes. One concern is that pilots trained using a relatively benign vortex in a flight simulator may lack the skills to cope with an actual vortex. Additionally, an encounter with a single decaying wake vortex developing significant instabilities will not have the same characteristics as an encounter with an analytical parallel vortex model. In other words, if the simulated vortex does not have the appropriate characteristics, it may result in a negative transfer of training. Consequently, it would seem prudent to develop more realistic vortices and then study their effects in a flight simulator.

A similar situation occurred with wind shear accidents where, following a number of accidents, the cause of the problem was identified, flight simulators were used to establish strategies to cope

Paper No. 3501. Manuscript received 2 October 2009, revised 29 March 2010, accepted 16 April 2010.

https://doi.org/10.1017/S000192400000405X Published online by Cambridge University Press

with wind shear and a training solution (rather than an automatic avoidance system) was shown to be effective.

Certainly, the NSTB findings[2] following the AA587 accident show that inappropriate pilot response to the vortex encounter could lead to a catastrophic outcome; the corollary being that appropriate training in a simulator might avoid the repetition of a similar incident.

A vortex encounter is likely to be unexpected and, depending on the encounter, can result in the generation of large airframe accelerations within a few seconds. For example, the encounter may occur in the climb with automatic flight modes enabled. The disturbance can lead to large angles of incidence and sideslip inducing fast roll rates. It is also likely that the pilot will respond to such an upset with large inputs of aileron or rudder. However, the pilot will have no knowledge of the safety margins of the airframe during the encounter nor the severity of the wake vortex flows. In this situation, an automatic recovery system or specific pilot behaviour may provide an appropriate recovery action. The basis of this paper is that, with the availability of realistic real-time vortex data in a research flight simulator, some insight may be given into the most appropriate action.

2.0 WAKE VORTEX MODELLING

Detailed descriptions of the wake vortex modelling approach used in the studies described in this paper have been previously reported[3-5]. In this paper the authors only present an overview of the approach. Classic real-time models of aircraft wake vortices[6,7] range from representative bank-angle upsets to analytical models describing a two-dimensional plane containing a counter-rotating flow field. Although simple to implement, such wake models lack fidelity when considering the temporal evolution of a wake vortex. Many groups have applied computational fluid dynamics (CFD) to study the four-dimensional nature of wake vortex decay[8,9]. These studies have been able to model many of the instabilities exhibited during actual wake vortex decay, for example the Crow instability[1]. Although the computation of CFD models of wake vortex decay is expensive, the resulting data can be harnessed to provide a rich 4D flow-field and form the basis of a higher fidelity wake model. For this study, our wake model uses pre-computed time-dependent large eddy simulation calculations of the decay of parallel vortices from a B747-400. The model covers 200 seconds of the decay of the vortex pair after roll-up in a cylindrical volume measuring 400 meters in diameter and approximately the length of a complete long-wave instability.

The most apparent difference between classic wake vortex models used in flight simulation and the model used in this study is that this approach requires access to the pre-computed CFD dataset. In this case, the raw data set resulting from a single CFD simulation of wake vortex decay is over 20GBs. To ensure that the real-time wake model runs on a desktop PC, the data set must be compressed and structured so that access by a wake vortex interaction model is efficient. Preprocessing the CFD data is necessary, not only to reduce the volume of data, but to also ensure a sufficient level of data fidelity is maintained. Data compression using regular down-sampling of the volumetric data can potentially remove important flow features, especially when the underlying CFD calculations are computed on unstructured or hybrid meshes. The compression method aims to reduce information in regions of data that exhibit low gradients. The lossy compression technique applied in this study involves the process of repeatedly collapsing individual unstructured mesh elements (tetrahedra)[10]. Collapsing a single tetrahedron can remove many tetrahedra from the mesh, and the associated flow field data. To ensure that data fidelity is maintained, all tetrahedron collapse operations are firstly simulated and weighted depending on the error that the collapse will introduce. The weighted list containing the cost of each collapse operation is stored in a priority queue. Each collapse operation is then executed with the cheapest collapses occurring first. This process can be stopped at any time if the flow field errors become unacceptable. Using tetrahedral collapsing, a raw CFD data file containing 1,871,728 tetrahedra was reduced to 473,663 tetrahedra while constraining the flow field errors to $0{\cdot}05 ms^{-1}$. All 200 seconds of CFD data were processed as described, leaving a manageable volume of data to be used in a real-time flight simulator.

Stripping out any information concerning the underlying computational mesh reduces the size of data further. The resulting unstructured point cloud is then spatially structured to permit fast data access to flow vectors. A kd-tree[11] spatial data structure was used to provide hierarchical access to individual flow field vector data in log n time. Other spatial data structures would have been suitable (for example, binary space partitions or oct-trees[12]), but kd-trees require the least runtime overheads due to their simple axis-aligned spatial discrimination.

During a simulated wake vortex encounter, access to flow field data for many points over an aircraft geometrical strip model must be in less than 20ms for a frame rate of 50Hz. In a classic wake vortex model, extracting the flow field data consists of calculating velocity vectors from analytical equations as a function of a distinct point in the vortex. However, in a CFD based model, the flow field data must be queried using spatial search functions that navigate through the kd-tree. To query the spatial data, an aircraft strip model point in the aircraft body axis is chosen. This point is then transformed to the axis system of the wake vortex before the spatial query is executed. The resulting flow vector must then be transformed from the wake axis to the aircraft body axis to be used in the wake interaction computations. Unlike an analytical model that provides continuous data, the compressed and structured data provides the physically nearest data point to the requested location in the wake vortex. In practice, such an approach does not return erroneous values because data compression is only applied to regions with low gradients in the underlying data.

One important benefit of accessing time-dependent CFD data in a wake vortex model is that temporal features of wake decay can be encapsulated in addition to spatial features. Although the volume of data after compression and hierarchical structuring becomes manageable, the number of time-steps to be incorporated in the wake interaction model determines the size of primary memory needed for the wake model. For cases when the entire time-series cannot be preloaded into RAM, a time-step paging system has to be developed to avoid unacceptable losses of frame-rate if time-steps are loaded into RAM on demand. A multi-threaded paging system has been incorporated into the model developed at Sheffield[3].

3.0 STRUCTURAL TAIL LOADING MODEL

In order to provide pilot feedback during training a vertical tail loading model has been integrated with the wake vortex interaction model. Because the model has to run in real-time, a simple analytical tail loading model is used to compute instantaneous loads. The accident of AA587 highlighted that while the wake vortex encountered was not severe enough to compromise the structural integrity of the vertical stabiliser, subsequent reactions by the crew generated loads that exceeded the ultimate design load of the A300-600[2]. A goal in this study is to determine a real-time indication of the estimated loads on the vertical stabiliser, and further, to provide useful feedback for training purposes. The following paragraphs describe the computation of the tail shear load and bending moment used for pilot feedback and simple load analysis.

The vertical tail shear load, L_v, due to side slip β_v, and rudder deflection, δ_r, is computed from Equations 1 and 2.

$$C_{Yv} = \frac{\partial C_{Lv\beta}}{\partial \beta_v}\beta_v + \frac{\partial C_{Lv\beta}}{\partial \delta_r}\delta_r \quad \ldots (1)$$

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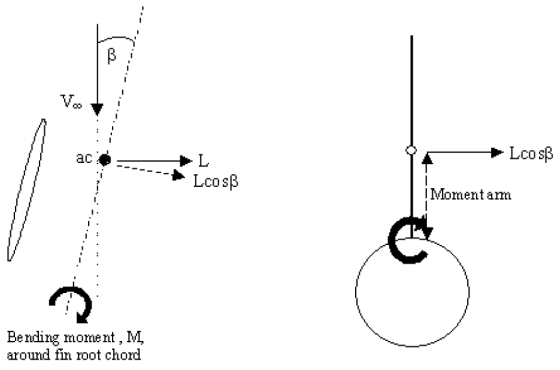


Figure 1. Direction of vertical tail shear force and bending moment.

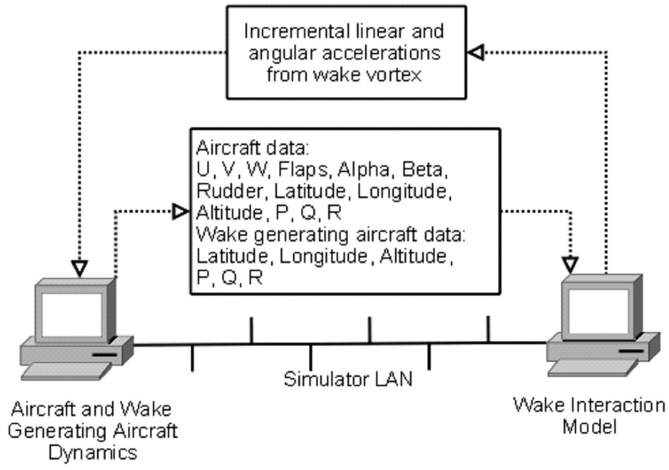


Figure 2. Wake model integration.

$$L_v = q_v S_v C_{Y_v} \quad \ldots (2)$$

where $dC_{Lv\beta}/d\delta_r$ is the rate of change of the vertical tail lift-curve slope with respect to the rudder deflection angle, δ_r. Aerodynamic data for rudder effectiveness was estimated using the technique described in ESDU13 Data Item No 87008. In this model, the angle-of-attack at the vertical tail, β_v is modified by the wake vortex flow-field, so the effect of the wake vortex influencing the aerodynamic loads is included by using a modified value of β_v in the loading computations. The vertical tail bending moment, BM_v, is the result of aerodynamic forces acting on the surface where the root of the surface is fixed to the aircraft fuselage, as defined in Equation (3).

$$BM_v = -\left(L_v BM_{arm}\right) \quad \ldots (3)$$

where BM_{arm} is the moment arm length. For the simplified model, it is assumed that the aerodynamic force produced by a particular vertical tail angle-of-attack acts through a single convenient point – the aerodynamic centre, perpendicular to the free stream. In other words, this represents an average force, assuming a constant lift distribution over the vertical tail. In reality, the vertical tail would be experiencing a continuous spanwise distribution of lift at an infinite number of locations. When calculating the root bending moment, the shear force direction is adjusted so that the force is directed normal to the surface face (Fig. 1).

Estimating the current load on the vertical tail allows comparisons to be made with values of the design limit and ultimate loads. A static gust approach[14] has been used to provide an estimate of the limit and ultimate loads for arbitrary vertical tails on civil transport aircraft. The lateral gust load formula for a rigid body is given in Equation (4).

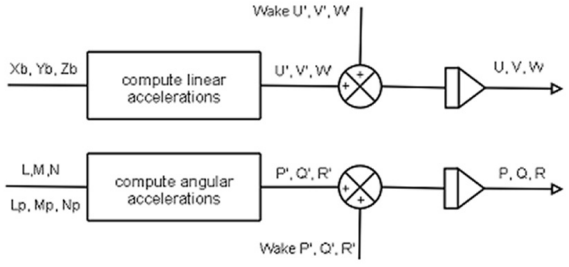


Figure 3. Wake component inputs into the equations of motion.

$$L_v = K_g U_{de} V_e C_{L\alpha v} S_v / 498 \quad \ldots (4)$$

where K_g is a gust alleviation factor, U_{de} is a gust velocity (feet per second, EAS), V_e is the aircraft equivalent airspeed, C_L is the lift-curve slope for the vertical tail, S_v is the vertical tail reference area (feet squared). Further details for determining values for K_g and U_{de} is given by Lomax[14]. Using Equation (4), the estimate for the B747-200 vertical tail design limit load and bending moment have been calculated as approximately 567kN and 2,613kNm, respectively. The load L_v is the design limit load, with the ultimate load determined by multiplying the limit load by 1·5.

4.0 FLIGHT SIMULATOR OVERVIEW AND WAKE INTEGRATION

The real-time engineering flight simulator in the Department of Automatic Control and Systems Engineering at the University of Sheffield is based on a distributed network of PCs connected by Ethernet. The functionality is distributed across eight PCs to provide the following simulation functions:

- acquisition of pilot inputs
- equations of motion, including modelling of the airframe and propulsion dynamics
- navigation and avionics
- an instructor station
- image generation (three IGs)
- wake vortex generation

The simulator includes a number of detailed non-linear flight models with a system-wide update rate of 50Hz. The PCs run under the Linux operating system and the software is written in C, which includes OpenGL for flight displays and OpenSceneGraph for scene rendering. A real-time UDP protocol is used for multi-cast network transmissions between computers at the 50Hz frame rate.

The distributed architecture enables additional PCs to be integrated with the flight simulator, where each PC is responsible for receiving packets and broadcasting a packet every 20ms. This architecture facilitated the integration of the wake vortex model described in Section 2. The wake model computer reads packets from the other computers to determine the aircraft position, velocities and accelerations (linear and rotational). During each frame, the resultant wake vortex flows are determined around the airframe and wake induced accelerations are computed and transmitted as a network packet to the flight model computer (Fig. 2), where the accelerations are added to the aircraft motion (Fig. 3). One further advantage of this architecture is that the wake vortex computer can also visualise the flows and also the airframe loading in real-time.

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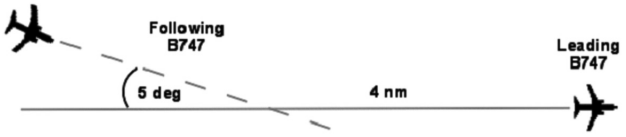


Figure 4. Wake vortex encounter geometry.

5.0 EXAMPLE WAKE ENCOUNTERS

In this section we describe three simulated wake vortex encounters and outline the effect of the disturbance on aircraft sideslip and vertical tail loading. The first simulation (Encounter 1) is manually piloted, with only aileron input in response to the disturbance (no rudder input is applied). The second simulation (Encounter 2) is also manually piloted, but in this case, the pilot makes an inappropriate response to the wake using the ailerons and the rudder. The third simulation (Encounter 3) uses an autopilot configured for altitude hold and heading hold. As in the first encounter, the autopilot applies no rudder input. The final simulation (Encounter 4) also uses an autopilot, but modified to provide lateral load alleviation and wing levelling using rudder and aileron input. Each simulated wake vortex upset in this study is based on a B747-200 encountering a wake vortex generated by a B747-400. The encounter geometry is shown in Fig. 4. The encountering aircraft has an initial airspeed of 220kt, with flaps and gear up, at an altitude of 3,000ft. These simulations have been configured so the following aircraft encounters the wake vortex in the horizontal plane of the vortex cores and as such can be considered worst-case scenarios.

In Fig. 5, although a sideslip angle of 13° is induced with the maximum tail loading occurring at 25s into the recorded encounter, the tail loading does not exceed the ultimate limit, although it does reach the design limit. In Fig. 6, the pilot detects the onset of the vortex at 29s applying maximum countering rudder (25°) followed by maximum opposite rudder at 32s. The effect of this second rudder input, when the angle of sideslip has increased to 16°, is evident from the tail loading which reaches the ultimate load value at 32·6s and again at 35s where it exceeds the ultimate load as the rudder is returned to the midpoint. In Fig. 7, the autopilot attempts to maintain altitude and heading during the upset, where it is significant that the induced sideslip is significantly less than the two previous cases. A lateral load alleviation system was used in the fourth encounter. The results of this simulation, given in Fig. 8, show that sideslip was well constrained and the roll disturbance was less than 20°. The automatic load alleviation scheme was initiated at 18·2s into the simulation when the sideslip rate reached $2°s^{-1}$. In comparison with the autopilot used in Encounter 3 (no rudder), the load alleviation system was able to constrain the load below the design limit and recovered the aircraft attitude in a shorter period. In other words, corrective rudder can be applied without overloading the tail. In these four cases, the yaw damper and the turn coordinator were disengaged so that their effect would not influence the recorded manoeuvres.

There is a likelihood that a wake encounter will occur in IFR conditions, where there is no visual contact with the wake. The pilot will respond to the disturbance, particularly in roll, with reference to the attitude indicator. In the recovery, the pilot has no knowledge of the load factor (G) or the side force (although there is a sideslip indicator). Of course, in the simulation of the wake vortex, the wing loading and the tail loading is known and could be displayed to the pilot. This concept is shown in Fig. 9, where the PFD is augmented with a dashed square and a load indicator symbol (a yellow, or light, circle). The vertical displacement of the symbol shows the instantaneous load factor, in this case where the top edge of the square corresponds to 2G. The 2G limit shown by the horizontal lines could easily be set to the specific load limits of a particular aircraft. The lateral displacement of the symbol shows the instantaneous tail loading, where the left and right edges of the square indicate the ultimate tail load.

The display serves two purposes. Firstly, the pilot can see the effect of a wake encounter in terms of the structural design limits and secondly, the pilot can appreciate the margin for rudder input during the encounter. It is straightforward to extend the display to colour code the symbol in terms of the severity and also to remove the display unless a nominal tail load limit is exceeded or alternatively, if a specific sideslip rate is exceeded. It is acknowledged that although this symbology is not proposed for aircraft displays, it may prove a useful training aid in debriefings to highlight potentially hazardous actions.

6.0 DISCUSSION

The vortex flow fields generated by CFD are accessed by the vortex interaction model to derive the vortex flows every frame. However, the shape and geometry of the vortex is also known and, with the use of billboard rendering techniques, enables the vortex to be seen in the simulator visual scene in daylight conditions. In other words, controlled vortex encounters can be set up where the pilot deliberately flies towards the vortex. This facility is not available on most training simulators, but in an engineering simulator the results of an encounter can be stored at the nominal frame rate, capturing the resultant aircraft motion and also the airframe loading. The wake vortex visualisation can be disabled at anytime. The authors are not aware of other simulation facilities providing these research opportunities.

However, one of the difficulties is the cost (in computing terms) of generating one vortex. The dataset is very large and moreover it is time consuming to generate. One vortex is fixed in terms of the altitude, airspeed, the configuration, mass, engines and geometry of the aircraft generating the vortex. In other words, a large number of vortices would be needed for a range of aircraft, airspeed and altitude initial conditions. In addition, the aircraft encountering the vortex is a specific type and the direction of the encounter in terms of both the vertical and lateral closure towards the centre of the vortex also affects the resultant disturbance. Again, many cases would need to be simulated and analysed to cover the full range of encounters. This study was restricted to two B747 aircraft, at an airspeed of 220kt, at a closure angle of 5°, level with the vortex. The following aircraft encountered the wake approximately 4nm behind the leading aircraft. Nevertheless, the vortex was seen to cause a significant disruption and the loads on the aircraft encountering the vortex were surprisingly close to the design limits assumed for the tail.

Few pilots have experienced vortex encounters of this sort, making subjective comparison of simulator tests difficult. In addition, the flight simulator is a fixed-base simulator and it is possible that the large onset accelerations occurring in an encounter would influence the pilot's response in a manner that is different from an encounter that is detected by visual cues.

The most surprising aspect of the study was the ease with which the tail could be over-stressed by inappropriate pilot rudder input (Fig. 6). Perhaps of more concern is that pilots trained in aircraft where rudder inputs are necessary for lateral stability may react to a vortex with an instinctive rudder response. In such situations, the pilot has very little information available; the severity of the vortex flows can only be estimated from the un-commanded roll rates and pitch rates it induces and the margin of tail loading cannot be deduced. However, the studies at Sheffield have shown that typical vortex encounters do not produce tail loads close to the vertical tail load limits and that it is only inappropriate rudder action that can increase the resultant side force to overload the tail. Clearly the use of aileron to minimise the upset has very little effect on tail loading but can stabilise the aircraft.

It is the authors' opinion that automatic recovery is not appropriate for wake vortex encounters – the onset is very fast and the lateral modes of motion of an aircraft are relatively slow.

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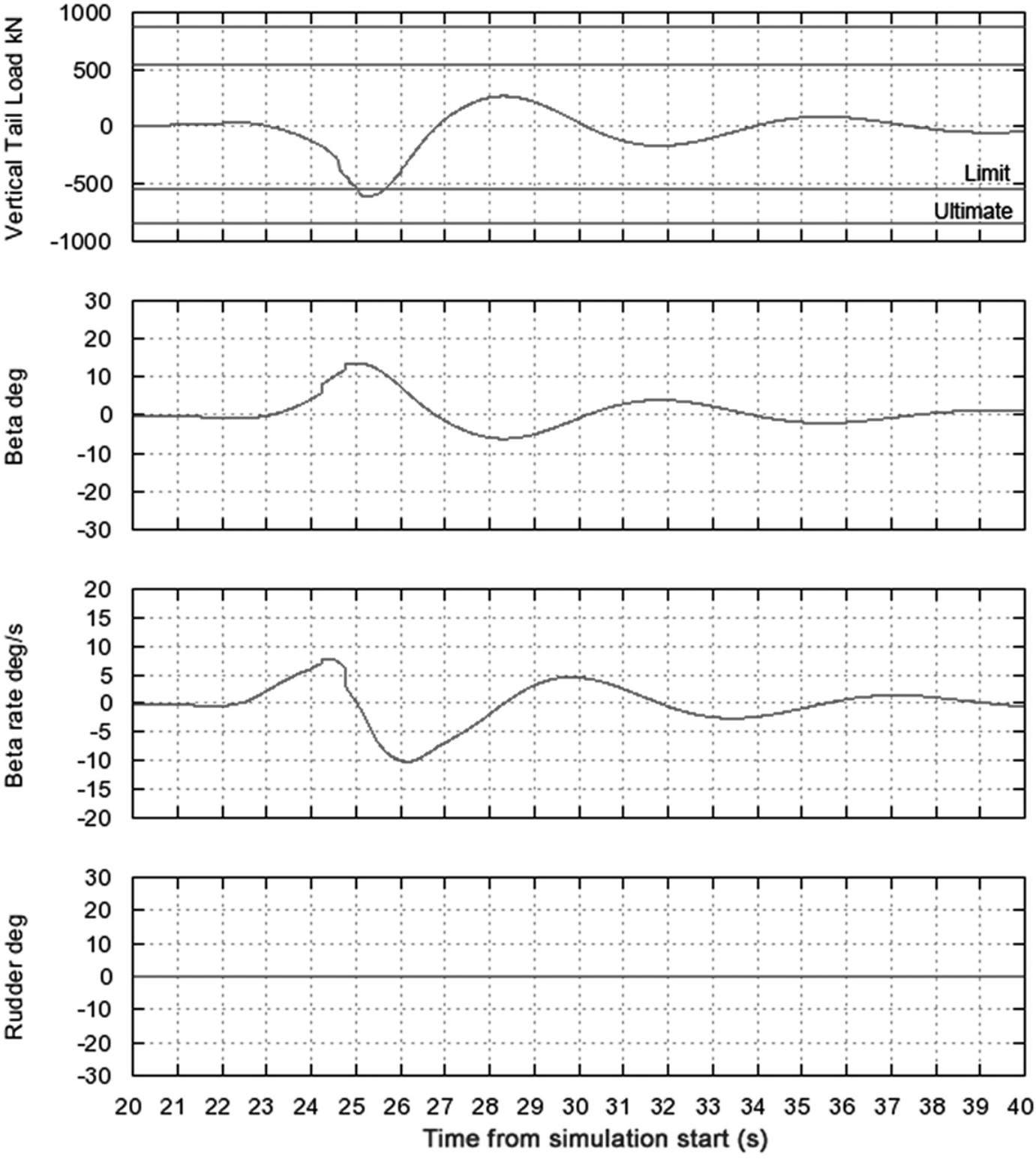


Figure 5. Manual piloted encounter with no rudder input.

Nevertheless, it is important to inhibit rudder inputs in the presence of large vortex flows and these conditions can be detected by sideslip angle and the rate of change of sideslip. Alternatively, in training, pilots need to be aware of the effect of inappropriate rudder commands during an encounter. In some ways, the training can be summarised as 'do nothing' in terms of rudder input but for some pilots (depending on their training) this may be counter-intuitive.

One aspect of the study at Sheffield is that the tail loading can be computed in real-time and the tail loading limits and instantaneous tail loading could be displayed (Fig. 9) to provide visualisation during training for vortex encounters or during replay of manoeuvres, review of training and in debriefing.

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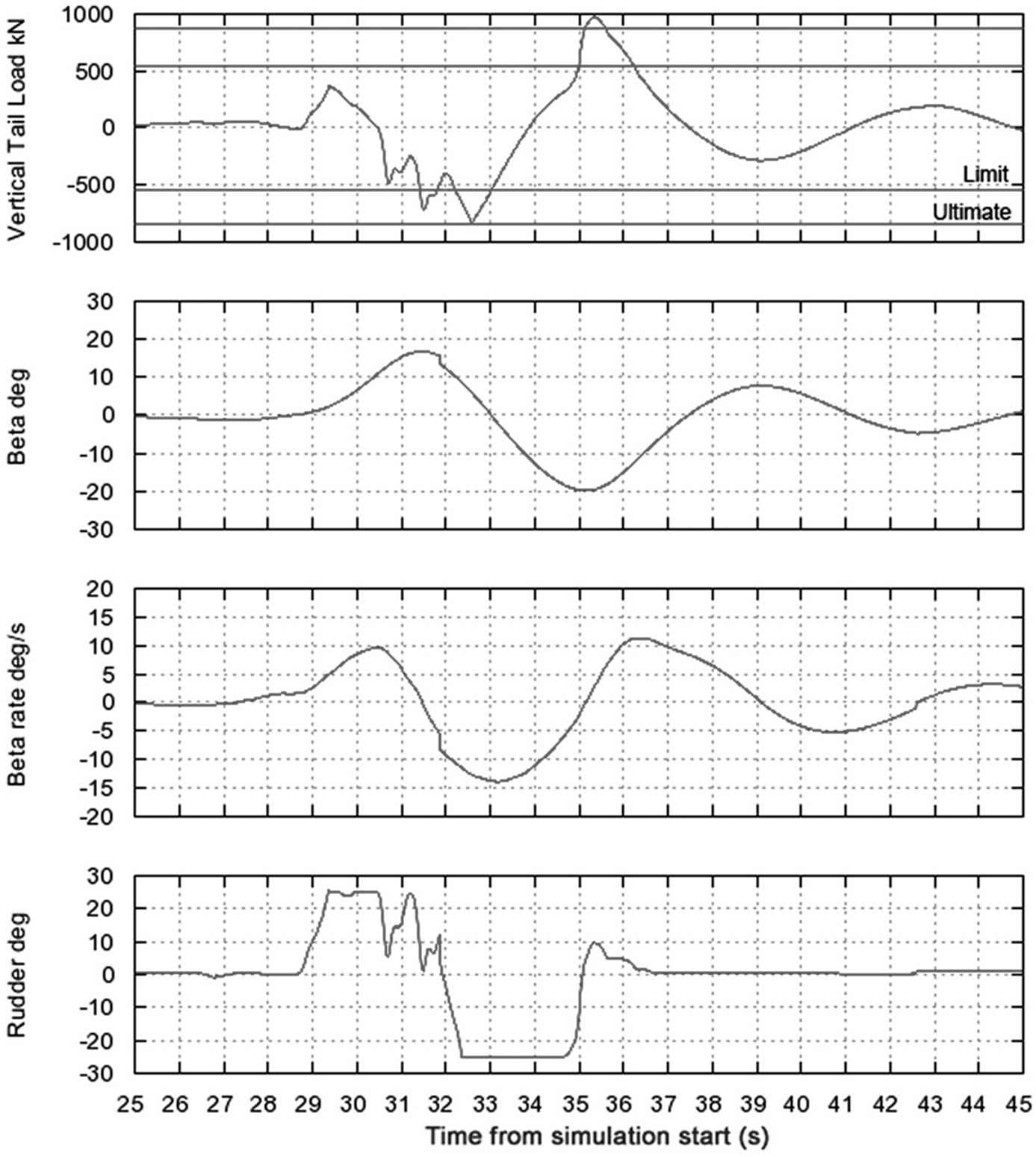


Figure 6. Manual piloted encounter with worst-case rudder input.

7.0 CONCLUSIONS

A realistic real-time wake vortex model has been implemented at the University of Sheffield as part of a three-year research grant. The vortex is derived from a non real-time CFD model and the characteristics accord well with the limited amount of data available to validate vortices. The vortex model provides three functions:

- It enables the vortex to interact with the real-time flight model at 50Hz as it is readily integrated with the real-time network.
- It enables the vortex to be visualised in the visual system, enabling a pilot to experience a wake vortex encounter where the encounter conditions are controlled and repeatable.
- It enables the tail and wing loading to be computed in real-time

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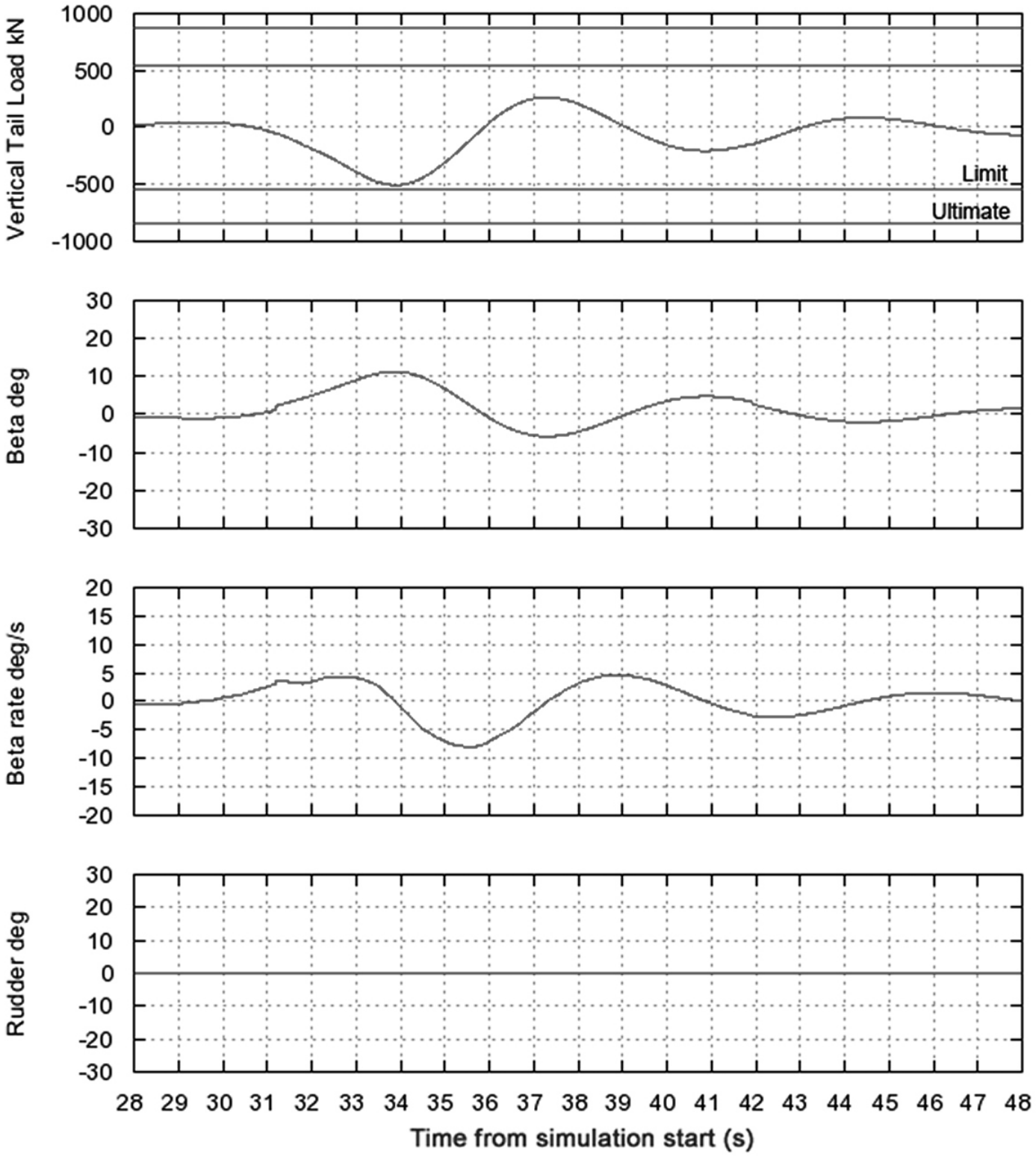


Figure 7. Autopilot encounter with no rudder input.

to provide visualisation and data recording of the tail loads occurring in a wake vortex encounter, including the effect of pilot inputs and automatic system inputs.

At speeds in excess of 200kt, depending on the trajectory of the intersection of the wake, the aircraft is subject to relatively large forces and moments for a short time (typically less than 10s). During this time, the aircraft may be subject to large rolling and yawing moments but the flight crew will have no knowledge of how close the aircraft tail loading is to its design limit. Inadvertent rudder input, even for a large transport aircraft can result in overloading the tail, with the likelihood of a catastrophic failure of the tail.

Clearly, the flight simulator offers a safe environment to practice for such events and to become aware of the effects.

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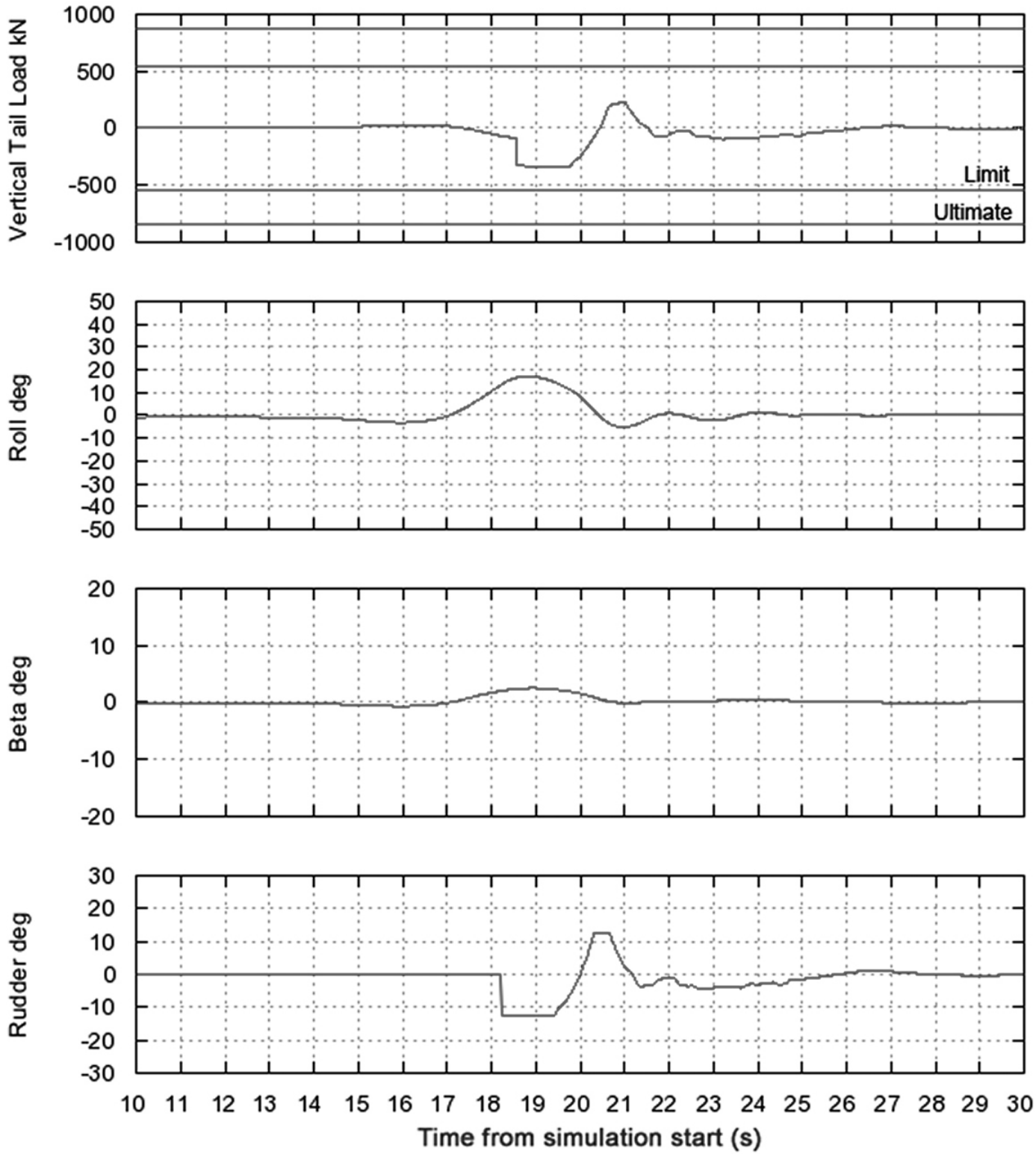


Figure 8. Autopilot encounter with lateral load alleviation.

With minimal modification to the aircraft displays, it is possible to provide visual feedback on the severity of specific pilot response as a training aid. In addition, a stand-alone visualisation tool can provide a useful training aid for off-line analysis and debriefing.

ACKNOWLEDGEMENTS

This work was funded by the UK Engineering and Physical Sciences Research Council under Grant GR/R84047/01 and their support is gratefully acknowledged.

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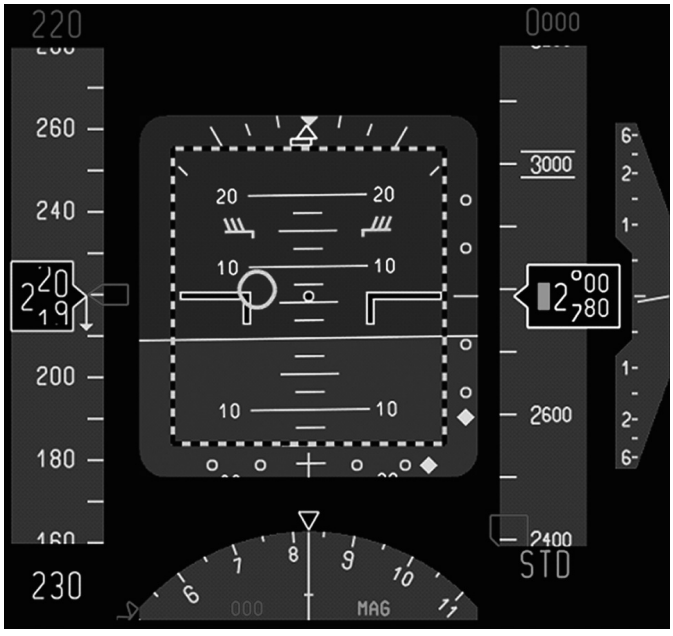


Figure 9. PFD augmented with loading indication.

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