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Extreme-value distributions for some classes of non-uniformly partially hyperbolic dynamical systems

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Abstract. In this note, we obtain verifiable sufficient conditions for the extreme-value distribution for a certain class of skew-product extensions of non-uniformly hyperbolic base maps. We show that these conditions, formulated in terms of the decay of correlations on the product system and the measure of rapidly returning points on the base, lead to a distribution for the maximum of $\Phi(p) = -\log(d(p, p_0))$ that is of the first type. In particular, we establish the type I distribution for S^1 extensions of piecewise C^2 uniformly expanding maps of the interval, non-uniformly expanding maps of the interval modeled by a Young tower, and a skew-product extension of a uniformly expanding map with a curve of neutral points.

1. *Introduction*

Suppose that $\{X_i\}$ is a stochastic process and we define a stochastic process $\{M_n\}$ of successive maxima by $M_n = \max\{X_1, \ldots, X_n\}$. Extreme-value theory is concerned with the limiting distribution of $\{M_n\}$ under linear scalings $a_n(M_n - b_n)$ defined by constants $a_n > 0$, $b_n \in \mathbb{R}$. In the independent and identically distributed case there is a well-developed theory [**5, 10, 16**] and it is known that there are only three possible non-degenerate distributions under linear scaling, i.e. if $\{X_i\}$ is independent and identically distributed, $a_n > 0$, $b_n \in \mathbb{R}$ are scaling constants and $G(x)$ is a non-degenerate distribution defined by

$$\lim_{n\to\infty} P(a_n(M_n - b_n) \le x) = G(x)$$

then $G(x)$ has one of three possible forms (up to scale and location changes), which we call extreme type distributions.

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Collet [**11**] studied the return-time statistics to a point x_0 in the phase space of a one-dimensional non-uniformly expanding map modeled by a Young tower with exponential decay of correlations. He notes that his work can be interpreted in terms of extreme-value statistics for such systems. Collet showed that the function $F(x) = -\log d(x, x_0)$ on the systems he considered displays type I extreme-value statistics for μ-almost every x_0. Freitas and Freitas [**3**] showed the corresponding result for these maps when x_0 is taken to be the critical point c or critical value $f(c)$. Freitas *et al* [**4**] investigated the link between extreme-value statistics and return-time statistics, and showed that any multimodal map with an absolutely continuous invariant measure displays either type I, type II or type III extreme-value statistics. This result required no knowledge of the decay of correlations for these maps. They also proved that for these systems the excedance point process converges to a Poisson process. Dolgopyat [**2**, Theorem 8] has proved Poisson limit laws for the return-time statistics of visits to a scaled neighborhood of a measure-theoretically generic point in uniformly partially hyperbolic systems with exponential decay of correlations for C^k functions. He also gives distributional limits for periodic orbits, but again exponential decay is required and uniform partial hyperbolicity is assumed.

In this paper, using arguments based on Collet's work and recent work by Gouëzel [**12**] on the rate of decay of correlations for compact group extensions of non-uniformly expanding maps we establish, to our knowledge, the first extreme-value theory (or return-time statistics) for non-uniformly partially hyperbolic systems. Our main result is Theorem 2.1 which gives verifiable conditions on the base transformation and a sufficient (polynomial) rate of decay of correlations for a type I extreme-value distribution to hold for $\Phi(p) = -\log d(p, p_0)$ for $\mu \times \lambda_Y$-almost every $p_0 = (x_0, \theta_0) \in X \times Y$. This results in the extreme-value statistics for observations of a certain degree of regularity with maxima at such points p_0. The sufficient conditions of Theorem 2.1 are verifiable for a residual set of Hölder S^1-cocyles over certain classes of maps recorded in Corollary 2.2. The maps in this category include piecewise C^2 uniformly expanding maps and non-uniformly expanding maps with finite derivative which may be modeled by a Young tower with exponential return-time tails (such as logistic or unimodal maps, including the class studied by Collet). We also verify, in §5.2, that Gouëzel's map satisfies hypotheses of our theorem and hence our results also apply to this map. A key role in our verification is played by results, due to Gouëzel [**12**], on rates of decay of correlations for S^1 extensions of non-uniformly partially hyperbolic systems. We note that our type I law for $\Phi(p) = -\log d(p, p_0)$ also implies type II and type III laws for $\Phi(p) = d(p, p_0)^{-\alpha}$ and $\Phi(p) = \mathcal{C} - d(p, p_0)^{\alpha}$ (see [**9**, Lemma 1.3]).

Further, we verify the conditions on the base transformation for a class of intermittent like maps, including the Liverani–Saussol–Vaienti map. Unfortunately, the rate of decay of correlations of Hölder observations on compact group extensions of such systems is not known. Nevertheless we give a sufficient decay rate to ensure type I extreme-value statistics for $-\log d(p, p_0)$ for $\mu \times \lambda_Y$-almost every p_0. We believe it plausible that for sufficiently small $0 < \omega < 1$, where the germ of the indifferent fixed point is $x \to x + x^{1+\omega}$, this decay rate holds and will be proven to hold. We also verify all but one of the hypotheses of our theorem for the Viana Map. The hypothesis that fails concerns the density of the absolutely continuous invariant measure. It is not known whether the density belongs to $\mathbb{L}^{1+\delta}(\lambda)$ for any $\delta > 0$.

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2. *Framework of the problem*

Suppose that Y is a compact, connected, M-dimensional manifold with metric d_Y and X is a compact N-dimensional manifold with metric d_X. We let $D = M + N$ and define a metric on $X \times Y$ by

$$d((x_1, \theta_1), (x_2, \theta_2)) = \sqrt{d_X(x_1, x_2)^2 + d_Y(\theta_1, \theta_2)^2}. \tag{2.1}$$

We denote the Lebesgue measure on X by λ_X, the Lebesgue measure on Y by λ_Y and the product measure on $X \times Y$ by $\lambda = \lambda_X \times \lambda_Y$.

We will call a function $\Upsilon : X \times Y \to \mathbb{R}$ Hölder continuous of exponent ζ if there exists some constant K such that

$$|\Upsilon(x) - \Upsilon(y)| \leq K d((x_1, \theta_1), (x_2, \theta_2))^{\zeta}$$

for all (x_1, θ_1) and (x_2, θ_2) in $X \times Y$. We define the C^{ζ} norm of Υ as

$$\|\Upsilon\|_{C^{\zeta}} = \sup_{(x,\theta)\in X\times Y} |\Upsilon(x, \theta)| + \sup_{\substack{(x,\theta),(y,\rho)\in X\times Y \\ (x,\theta)\neq(y,\rho)}} \frac{|\Upsilon(x, \theta) - \Upsilon(y, \rho)|}{d((x, \theta), (y, \rho))^{\zeta}}.$$

If $T : X \to X$ is a measurable transformation and $u : X \times Y \to Y$ a measurable function, then we may define f, the *Y-skew extension of T by u*, by

$$f : X \times Y \to X \times Y$$

$$f(x, \theta) = (Tx, u(x, \theta)). \tag{2.2}$$

We assume that $T : X \to X$ has an ergodic invariant measure μ_X with support X. We further assume that $f : X \times Y \to X \times Y$ preserves an invariant probability measure ν, which has density $H \in \mathbb{L}^1(\mu_X \times \lambda_Y)$ and H is locally $\mathbb{L}^p(\lambda)$ for some $p > 1$.

We are interested in the extreme-value statistics of observations which are maximized at a unique point (x_0, θ_0). For the given point (x_0, θ_0) we define a function Φ on $X \times Y$ by

$$\Phi(x, \theta) = -\log d((x, \theta), (x_0, \theta_0))$$

(here the dependence on (x_0, θ_0) is omitted for notational simplicity). For a given $v \in \mathbb{R}$ we define $u_n = v + (1/D) \log n$ and denote by Z_n (more precisely $Z_n^{(x_0,\theta_0)}$) the random variable

$$Z_n^{(x_0,\theta_0)} = \max(\Phi, \Phi \circ f, \ldots, \Phi \circ f^n).$$

We will prove the following result.

THEOREM 2.1. *Assume that the density $H \in \mathbb{L}^{1+\delta}(\lambda)$(locally) for some $\delta > 0$. Let $\kappa > 1$ be conjugate to $1 + \delta$. Further, assume that the following hold.*

(a) *There exist constants $C_1 > 0$, $\beta > 0$ and an increasing function $g(n) \approx n^{D\gamma'}$ (with $0 < \gamma' < \beta/D$) such that if*

$$E_n^X := \left\{ x \in X \;\middle|\; d_X(T^j x, x) < \frac{1}{n} \text{ for some } j \in \{1, 2, \ldots, g(n)\} \right\}$$

then $\mu_X(E_n^X) < C_1/n^{\beta}$.

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(b) *There exists* $0 < \hat{\alpha} \leq 1$ *such that, for all Hölder continuous functions* Υ *with Hölder exponent* $\hat{\alpha}$*, and* $\Psi \in \mathbb{L}^{\infty}(\nu)$*,*

$$\left| \int \Psi \circ f^j \Upsilon \, d\nu - \int \Upsilon \, d\nu \int \Psi \, d\nu \right| \leq C_2 \Theta(j) \|\Psi\|_{\infty} \|\Upsilon\|_{C^{\hat{\alpha}}} \tag{2.3}$$

where $\Theta(j) \leq j^{-\alpha}$ *and*

$$\alpha > \frac{(1 + D\kappa(3/2 - 1/\kappa))/D + 3/2}{\min\{\gamma', 1/2\}}.$$

Then for ν*-almost every* (x_0, θ_0) *and for every* $v \in \mathbb{R}$*,*

$$\lim_{n\to\infty} \nu(Z_n^{(x_0,\theta_0)} < u_n) = \exp\left(H(x_0, \theta_0) e^{-Dv}\right). \tag{2.4}$$

While we will prove Theorem 2.1 for an arbitrary fiber Y that is a compact connected M-dimensional manifold, our corollaries will involve the special cases $Y = S^1$ and $Y = [0, 1]$. This is because condition (b) of Theorem 2.1 requires a decay of correlations to hold and we only consider examples for which this decay is known to hold. Further, note that we require $0 < \hat{\alpha} \leq 1$. This is because, for the proof of Lemma 3.6, we need (b) of the above theorem to hold for Lipschitz continuous functions having compact support.

We will make the following definitions. A set will be called *residual* if its complement can be written as a countable union of nowhere dense sets. A C^r *cocycle* h on an interval I into a group Y will be defined as a C^r map, $h : I \to Y$. If h is a cocycle, the skew extension f will be defined as $f(x, \theta) = (Tx, \theta + h(x))$.

We now state the corollaries to the above theorem (see §5).

COROLLARY 2.2. *If* $Y = S^1$ *and* T *is one of the following transformations:*

(a) *a piecewise* C^2 *uniformly expanding map* $T : I \to I$ *of an interval* I*;*

(b) *a one-dimensional non-uniformly expanding map* $T : I \to I$ *of an interval* I *with bounded derivative and modeled by a Young tower with exponential decay of correlations,*

then for a residual set of Hölder cocycles $h : I \to S^1$*, for* $\mu_X \times \lambda_Y$*-almost every* (x_0, θ_0) *and for all* $v \in \mathbb{R}$*,*

$$\lim_{n\to\infty} \nu(Z_n^{(x_0,\theta_0)} < u_n) = \exp(H(x_0, \theta_0) e^{-Dv}). \tag{2.5}$$

In the next corollary, we take the skew extension over the map $F(\omega) = 4\omega$ with the map T_α defined in §5.2.

COROLLARY 2.3. *Let* $T : S^1 \times [0, 1] \to S^1 \times [0, 1]$ *be the map* $T(\omega, x) = (4\omega, T_{\alpha(\omega)}(x))$ *where the maps* α *and* T_α*, an intermittent type map, are as defined in §5.2. Suppose that*

$$\alpha_{\max} < \frac{\min\{\gamma', 1/2\}}{\min\{\gamma', 1/2\} + (1 + D/2)/D + 3/2}.$$

Then for ν*-almost every* (ω_0, x_0) *and for each* $v \in \mathbb{R}$*,*

$$\lim_{n\to\infty} \nu(Z_n^{(\omega_0,x_0)} < u_n) = \exp(H(\omega_0, x_0) e^{-Dv}). \tag{2.6}$$

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There are other important classes of maps such as Y extensions of Manneville–Pommeau type maps (for a compact connected Lie group Y, for instance, $Y = S^1$) and the Viana type maps that satisfy most, but not all, of our hypotheses. It is not known for S^1 extensions of Manneville–Pommeau type maps whether a sufficiently high polynomial rate of decay satisfying our hypotheses holds. Similarly, for the Viana map, all of our hypotheses are satisfied except we do not know whether the density of the invariant measure is locally $\mathbb{L}^p$ for some $p > 1$. A further discussion of these maps may be found in §5.

3. *Preliminaries*

For the rest of the article, we will refer to the function f^0 as the identity function and χ_A as the characteristic function for A. Upper-case Greek letters, such as Φ and Ψ, will usually denote functions, while lower-case letters, such as ϕ, will usually denote scalar constants.

This section contains the statements of some lemmas from [**11**] and proofs of some other lemmas. Of note is Proposition 3.4 because it allows us to induce to the product system an important and desirable property of the base map T.

LEMMA 3.1. *For any $k > 0$ and any $u \in \mathbb{R}$,*

$$\sum_{j=1}^{k} \chi_{\{\Phi\circ f^j \geq u\}} \geq \chi_{\{Z_k \geq u\}} \geq \sum_{j=1}^{k} \chi_{\{\Phi\circ f^j \geq u\}} - \sum_{l\neq j} \chi_{\{\Phi\circ f^j \geq u\}}\chi_{\{\Phi\circ f^l \geq u\}}. \tag{3.1}$$

LEMMA 3.2. *For any integers r and $k \geq 0$,*

$$0 \leq \nu(Z_r < u) - \nu(Z_{r+k} < u) \leq k\nu(\Phi \circ f^0 \geq u). \tag{3.2}$$

LEMMA 3.3. *For any positive integers m, p and t,*

$$\left|\nu(Z_{m+p+t} < u) - \nu(Z_m < u) + \sum_{j=1}^{p} \int \chi_{\{\Phi\circ f^0 \geq u\}}\chi_{\{Z_m < u\}} \circ f^{p+t-j} d\nu\right|$$

$$\leq 2p \sum_{j=1}^{p} \int \chi_{\{\Phi\circ f^0 \geq u\}}\chi_{\{\Phi\circ f^0 \geq u\}} \circ f^j d\nu + t\nu(\Phi \circ f^0 \geq u). \tag{3.3}$$

The proofs for these lemmas can be found in [**11**].

PROPOSITION 3.4. *Let μ_X be the invariant, ergodic measure with respect to the map $T : X \to X$. Suppose*

$$E_n^X := \left\{x \in X \;\middle|\; d(T^j x, x) < \frac{1}{n} \text{ for some } j \leq g(n)\right\}$$

satisfies $\mu_X(E_n^X) \leq C/n^\beta$ for some constant $C > 0$ and some $\beta > 0$. Then, under the hypotheses of Theorem 2.1, $\nu(\tilde{E}_n) \leq C/n^\beta$ where

$$\tilde{E}_n = \left\{(x, \theta) \in X \times Y \;\middle|\; d(f^j(x, \theta), (x, \theta)) < \frac{1}{n} \text{ for some } j \leq g(n)\right\}.$$

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Proof. $(x, \theta) \in \tilde{E}_n$ implies $d(f^j(x, \theta), (x, \theta)) < 1/n$ for some $j \le g(n)$ and so

$$\sqrt{d_X(T^j x, x)^2 + d_Y(u^j(x, \theta), \theta)^2} < \frac{1}{n}$$

for such j and so $d_X(T^j x, x) < 1/n$. Thus, $x \in E_n^X$ and so $\tilde{E}_n \subset E_n^X \times Y$.

Define a new measure Δ on X as $\Delta(A) := \nu(A \times Y)$. If $\lambda_X(A) = 0$ then $\mu_X(A) = 0$ and so $\mu_X \times \lambda_Y(A \times Y) = 0$ and thus $\nu(A \times Y) = 0$. Therefore, Δ is absolutely continuous with respect to the Lebesgue measure on X. Further,

$$\begin{aligned} f^{-1}(A \times Y) &= \{(x, \theta) \mid (Tx, u(x, \theta)) \in A \times Y\} \\ &= \{x \in T^{-1}A, (x, \theta) \in u^{-1}Y\} \\ &= \{x \in (T^{-1}A \cap X), \theta \in Y\} \\ &= T^{-1}(A) \times Y \end{aligned}$$

and so $\nu(f^{-1}(A \times Y)) = \nu(T^{-1}(A) \times Y)$. Therefore

$$\Delta(T^{-1}(A)) = \nu(T^{-1}(A) \times Y) = \nu(f^{-1}(A \times Y)) = \nu(A \times Y) = \Delta(A).$$

To prove that Δ is ergodic for T, if $T^{-1}A = A$ then $\mu_X(A) = 0$ or 1 from which it follows that $\mu_X \times \lambda_Y(A \times Y) = 0$ or 1. Therefore by redefining H (recall that H is the density of ν) on a $\mu_X \times \lambda_Y$ measure zero set if necessary we have

$$\nu(A \times Y) = \int_{A \times Y} H \, d(\mu_X \times \lambda_Y) = 0 \text{ or } 1.$$

Therefore, $\Delta(A) = 0$ or 1.

Since the measures on X are absolutely continuous with respect to Lebesgue, and hence unique, $\Delta(A) = \mu_X(A)$ from where it follows that

$$\nu(\tilde{E}_n) \le \nu(E_n^X \times Y) = \Delta(E_n^X) = \mu(E_n^X) \le \frac{C}{n^\beta}.$$

LEMMA 3.5. *Under the assumptions of Theorem 2.1, for ν a.e $(x_0, \theta_0) \in X \times Y$*

$$n \sum_{j=1}^{n^{\gamma'}} \nu(\Phi \circ f^0 > u_n, \Phi \circ f^j > u_n) \to 0 \quad \text{as } n \to \infty. \tag{3.4}$$

Proof. We begin by recalling that $H \in \mathbb{L}^{1+\delta}(\lambda) \subset \mathbb{L}^1(\lambda)$. Let

$$E_n = \left\{ (x, \theta) \;\middle|\; d(f^j(x, \theta), (x, \theta)) < \frac{1}{n} \text{ for some } j \le g(n) \right\}$$

where $g(n)$ is as in Theorem 2.1. Let $D\gamma' < \psi < \beta$ and $\delta > 0$. Define

$$L_n(x, \theta) := \sup_{r>0} \frac{1}{\lambda(B_r(x, \theta))} \int_{B_r(x,\theta)} H \chi_{E_n} \, d\lambda.$$

By the Hardy–Littlewood maximal principle, since $H\chi_{E_n} \in \mathbb{L}^1(\lambda)$,

$$\lambda(L_n(x, \theta) > \delta) \le \frac{C}{\delta} \|H\chi_{E_n}\|_1 \le \frac{C}{\delta} \nu(E_n) \le \frac{C}{\delta n^\beta}.$$

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Choose γ such that $\gamma(\beta-\psi)>1$. Replacing δ by $1/n^{\gamma\psi}$ and n by n^{γ} we get

$$\lambda\left(L_{n^\gamma} > \frac{1}{n^{\psi\gamma}}\right) \le \frac{C}{n^{\gamma(\beta-\psi)}}.$$

Therefore we have

$$\sum_n \lambda\left(L_{n^\gamma} > \frac{1}{n^{\gamma\psi}}\right) \le \sum_n \frac{C}{n^{\gamma(\beta-\psi)}}$$

which is summable. Hence, by the Borel–Cantelli lemma, for λ-almost every $(x_0,\theta_0)\in X\times Y$, we have $(x_0,\theta_0)\notin \limsup\{L_{n^\gamma} > 1/n^{\gamma\psi}\}$ and so there exists $N(x_0,\theta_0)$ such that

$$n \ge N(x_0,\theta_0) \implies L_{n^\gamma} \le \frac{1}{n^{\psi\gamma}},$$

that is,

$$\sup_{r>0} \frac{1}{\lambda(B_r(x_0,\theta_0))} \int_{B_r(x_0,\theta_0)} H\chi_{E_{n^\gamma}}\, d\lambda \le \frac{1}{n^{\gamma\psi}}.$$

Set $r=1/n^\gamma$ in the above to get

$$n^{\gamma D} \int_{B_{1/n^\gamma}(x_0,\theta_0)} H\chi_{E_{n^\gamma}}\, d\lambda \le \frac{1}{n^{\psi\gamma}}.$$

Therefore we have

$$\nu\left\{\left\{d((x,\theta),(x_0,\theta_0)) < \frac{1}{n^\gamma}\right\} \cap E_{n^\gamma}\right\} \le \frac{1}{n^{\psi\gamma+\gamma D}}. \tag{3.5}$$

Let $\tilde g(n)$ be a function that is increasing in n with the property $g(n/2)\le\tilde g(n)\le\tilde g(2n)\le g(n)$. Let $k=(n^{1/D}/2e^{-v})^{1/\gamma}$. Then we have

$$\begin{aligned}
&\left\{(x,\theta)\;\middle|\; d((x,\theta),(x_0,\theta_0)) \le \frac{e^{-v}}{n^{1/D}}, d(f^j(x,\theta),(x_0,\theta_0))\right.\\
&\qquad\left.\le \frac{e^{-v}}{n^{1/D}} \text{ for some } j\le \tilde g(n^{1/D}/e^{-v})\right\}\\
&\qquad\subset \left\{(x,\theta)\;\middle|\; d((x,\theta),(x_0,\theta_0)) \le \frac{e^{-v}}{n^{1/D}}, d(f^j(x,\theta),(x,\theta))\right.\\
&\qquad\qquad\left.< \frac{2e^{-v}}{n^{1/D}} \text{ for some } j\le \tilde g\left(\frac{n^{1/D}}{e^{-v}}\right)\right\}\\
&\qquad\subset \left\{(x,\theta)\;\middle|\; d((x,\theta),(x_0,\theta_0)) < \frac{2e^{-v}}{n^{1/D}}, d(f^j(x,\theta),(x,\theta))\right.\\
&\qquad\qquad\left.< \frac{2e^{-v}}{n^{1/D}} \text{ for some } j\le \tilde g\left(\frac{n^{1/D}}{e^{-v}}\right)\right\}\\
&\qquad\subset \left\{(x,\theta)\;\middle|\; d((x,\theta),(x_0,\theta_0)) < \frac{1}{k^\gamma}, d(f^j(x,\theta),(x,\theta))\right.\\
&\qquad\qquad\left.< \frac{1}{k^\gamma} \text{ for some } j\le \tilde g(2k^\gamma)\right\}\\
&\qquad\subset \left\{(x,\theta)\;\middle|\; d((x,\theta),(x_0,\theta_0)) < \frac{1}{k^\gamma}, d(f^j(x,\theta),(x,\theta))\right.\\
&\qquad\qquad\left.< \frac{1}{k^\gamma} \text{ for some } j\le g(k^\gamma)\right\}
\end{aligned} \tag{3.6}$$

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so that, by (3.5) and (3.6), for any $j \le g(n^{1/D}/2e^{-v})$

$$\nu\{\Phi \circ f^0 > u_n,\, \Phi \circ f^j > u_n\} \le \frac{(2e^{-v})^{\psi+D}}{n^{1+\psi/D}}.$$

Therefore,

$$n \sum_{j=1}^{g(n^{1/D}/2e^{-v})} \nu\{\Phi \circ f^0 > u_n,\, \Phi \circ f^j > u_n\} \to 0 \iff \frac{g(n^{1/D}/(2e^{-v}))}{n^{\psi/D}} \to 0. \quad (3.7)$$

Since $\psi > D\gamma'$ we get the above result. □

LEMMA 3.6. *Let $B_r(x,\theta)$ be a ball of radius r and let $\epsilon > 0$ be arbitrary. Let κ be conjugate to $1+\delta$ (i.e, $1/(1+\delta) + (1/\kappa) = 1$) and let A be any measurable set. Then, under the assumptions of Theorem 2.1, there exist constants C_1 and C_2 so that*

$$|\nu(B_r \cap f^{-t}(A)) - \nu(B_r)\nu(A)| \le C_1 \|H\|_{1+\delta}^{\lambda,(x,\theta)} (\nu(A)+1) r^{(D+\epsilon)/\kappa} + \frac{C_2}{r^{1+\epsilon} t^{\alpha}}. \quad (3.8)$$

Proof. We construct a Hölder continuous approximation to the characteristic function for B_r. Let $r' = r - r^{1+\epsilon}$. Construct Φ_B by letting it be 1 on the inside of the ball of radius r' around (x,θ) and letting it decay to 0 at a linear rate between r and r'. The Lipschitz constant of this function may be chosen to be $(1/r^{1+\epsilon})$.

Next, we note that $\lambda(B_r \setminus B_r') = r^D - (r - r^{1+\epsilon})^D \le 2^D r^{D+\epsilon}$ and so we have

$$\begin{aligned} \|\Phi_B - \chi_{B_r}\|_1^{\nu} &= \int |\Phi_B - \chi_{B_r}|\, d\nu \le \nu(B_r \setminus B_{r'}) = \int H \chi_{B_r \setminus B_{r'}}\, d\lambda \\ &\le \|H\|_{1+\delta}^{\lambda,(x,\theta)} \|\chi_{B_r \setminus B_{r'}}\|_{\kappa}^{\lambda} \le C_1 \|H\|_{1+\delta}^{\lambda,(x,\theta)} r^{(D+\epsilon)/\kappa}. \end{aligned} \quad (3.9)$$

Finally,

$$\begin{aligned} &\left| \int \chi_B \chi_A \circ f^t\, d\nu - \int \chi_B\, d\nu \int \chi_A\, d\nu \right| \\ &\le \left| \int (\chi_B \chi_A \circ f^t - \Phi_B \chi_A \circ f^t)\, d\nu \right| \\ &\quad + \left| \int \Phi_B \chi_A \circ f^t\, d\nu - \int \Phi_B\, d\nu \int \chi_A\, d\nu \right| \\ &\quad + \left| \int \Phi_B\, d\nu \int \chi_A\, d\nu - \int \chi_A\, d\nu \int \chi_B\, d\nu \right| \\ &\le \|\chi_A \circ f^t\|_{\infty} \|\chi_B - \Phi_B\|_1^{\nu} + \frac{C_2 \|\chi_A\|_{\infty} \|\Phi_B\|_{\hat{\alpha}}}{t^{\alpha}} \\ &\quad + \nu(A) \|\chi_B - \Phi_B\|_1^{\nu}. \end{aligned} \quad (3.10)$$

A substitution of estimates from equation (3.9) completes the proof. □

4. *Proof of Theorem 2.1*

To prove Theorem 2.1, we begin by breaking n as a product of p and q with $p = \sqrt{n}$. We note that

$$\nu(Z_n < u_n) \approx \nu(Z_{n+qt} < u_n)$$

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where t is a monotonically increasing function chosen to satisfy $t/p \to 0$. The main estimate in the proof is

$$\nu(Z_{n+qt} < u_n) \approx (1 - p\nu(\Phi \circ f^0 \geq u_n))^q.$$

The function t needs to be chosen so that terms of the form

$$n \sum_{j=1}^{p} \nu(\Phi \circ f^0 \geq u_n, \Phi \circ f^j \geq u_n)$$

that appear in the error to the above approximation can be broken into sums over $1 \leq j \leq t$ and $t < j \leq p$ with t being small enough for growth of terms in the first sum to be killed by Lemma 3.5 while large enough for growth in the second sum to be killed by Lemma 3.6.

THEOREM 4.1. *Under the hypotheses of Theorem 2.1, for ν-almost every (x, θ) and for any $v \in \mathbb{R}$,*

$$\lim_{n\to\infty} \nu(Z_n^{(x,\theta)} < u_n) = \exp(H(x, \theta)e^{-Dv}). \tag{4.1}$$

Proof. Choose $(x, \theta) \notin \limsup_{n\to\infty} E_n$ such that

$$\lim_{a\to 0} \frac{1}{\lambda(B_a(x, \theta))} \nu(B_a(x, \theta)) = H(x, \theta).$$

Then from above

$$\lim_{n\to\infty} n\nu(B_{(e^{-v}/n^{1/D})}(x, \theta)) = e^{-Dv} H(x, \theta).$$

Choose

$$\epsilon > D\kappa\left(\frac{3}{2} - \frac{1}{\kappa}\right) \tag{4.2}$$

and $0 < \tau < \min\{\gamma', 1/2\}$ such that

$$\alpha > \frac{(1 + \epsilon)/D + 3/2}{\tau} > \frac{(1 + D\kappa(3/2 - 1/\kappa))/D + 3/2}{\min\{\gamma', 1/2\}}. \tag{4.3}$$

Define $t = n^\tau$, $p = \sqrt{n}$ and $q = \sqrt{n}$. Note that, by Lemma 3.2,

$$|\nu(Z_n < u_n) - \nu(Z_{q(p+t)} < u_n)| \leq qt\nu(\Phi \circ f^0 \geq u_n).$$

Now, for $1 \leq l \leq q$

$$\begin{aligned}
&|\nu(Z_{l(p+t)} < u_n) - (1 - p\nu(\Phi \circ f^0 \geq u_n))\nu(Z_{(l-1)(p+t)} < u_n)| \\
&\quad = |p\nu(\Phi \circ f^0 \geq u_n)\nu(Z_{(l-1)(p+t)} < u_n) + \nu(Z_{l(p+t)} < u_n) - \nu(Z_{(l-1)(p+t)} < u_n)| \\
&\quad \leq \Bigg| p\nu(\Phi \circ f^0 \geq u_n)\nu(Z_{(l-1)(p+t)} < u_n) \\
&\qquad - \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^j \geq u_n\}} \chi_{\{Z_{(l-1)(p+t)} < u_n\}} \circ f^{p+t}\, d\nu \Bigg| \\
&\qquad + \Big| \nu(Z_{l(p+t)} < u_n) - \nu(Z_{(l-1)(p+t)} < u_n) \\
&\qquad + \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^j \geq u_n\}} \chi_{\{Z_{(l-1)(p+t)} < u_n\}} \circ f^{p+t}\, d\nu \Big|
\end{aligned}$$

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$$= \Big| p\nu(\Phi \circ f^0 \geq u_n)\nu(Z_{(l-1)(p+t)} < u_n)$$
$$- \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^j \geq u_n\}} \chi_{\{Z_{(l-1)(p+t)} < u_n\}} \circ f^{p+t}\, d\nu \Big|$$
$$+ \Big| \nu(Z_{lp+lt} < u_n) - \nu(Z_{lp+lt-(p+t)} < u_n)$$
$$+ \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^j \geq u\}} \chi_{\{Z_{lp+lt-(p+t)} < u_n\}} \circ f^{p+t}\, d\nu \Big|. \tag{4.4}$$

By Lemma 3.3 we have

$$\Big| \nu(Z_{lp+lt} < u_n) - \nu(Z_{(l-1)(p+t)} < u_n) + \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^0 \geq u_n\}} \circ f^j \chi_{\{Z_{(l-1)(p+t)}\}} \circ f^{p+t}\, d\nu \Big|$$
$$\leq 2p \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^0 \geq u_n\}} \chi_{\{\Phi \circ f^0 \geq u_n\}} \circ f^j\, d\nu + t\nu(\Phi \circ f^0 \geq u_n). \tag{4.5}$$

For the remaining part,

$$\Big| p\nu(\Phi \circ f^0 \geq u_n)\nu(Z_{(l-1)(p+t)} < u_n) - \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^j \geq u_n\}} \chi_{\{Z_{(l-1)(p+t)} < u_n\}} \circ f^{p+t}\, d\nu \Big|$$
$$\leq \sum_{j=1}^{p} \Big| \nu(\Phi \circ f^0 \geq u_n)\nu(Z_{(l-1)(p+t)} < u_n) - \int \chi_{\{\Phi \circ f^j \geq u_n\}} \chi_{\{Z_{(l-1)(p+t)} < u_n\}} \circ f^{p+t}\, d\nu \Big|$$
$$\leq pC_1 \frac{e^{-\nu(D+\epsilon)/\kappa}}{n^{(D+\epsilon)/(D\kappa)}} + p\frac{C_2 n^{(1+\epsilon)/D}}{e^{-\nu(1+\epsilon)} t^{\alpha}} \tag{4.6}$$

for large n by Lemma 3.6.

Define

$$\Gamma_n := t\nu(\Phi \circ f^0 \geq u_n) + 2p \sum_{j=1}^{p} \int \chi_{\{\Phi \circ f^0 \geq u_n\}} \chi_{\{\Phi \circ f^0 \geq u_n\}} \circ f^j\, d\nu$$
$$+ pC_1 \frac{e^{-\nu(D+\epsilon)/\kappa}}{n^{(D+\epsilon)/(D\kappa)}} + p\frac{C_2 n^{(1+\epsilon)/D}}{e^{-\nu(1+\epsilon)} t^{\alpha}}.$$

Therefore we have, for $1 \leq l \leq q$

$$|\nu(Z_{l(p+t)} < u_n) - (1 - p\nu(\Phi \circ f^0 \geq u_n))\nu(Z_{(l-1)(p+t)} < u_n)| \leq \Gamma_n.$$

Since $n\nu(\Phi \circ f^0 \geq u_n) \to e^{-D\nu} H(x, \theta)$, for n large enough, $p\nu(\Phi \circ f^0 \geq u_n) < 1$, and so on applying the above formula inductively we get

$$|\nu(Z_{q(p+t)} < u_n) - (1 - p\nu(\Phi \circ f^0 \geq u_n))^q| \leq q\Gamma_n + \frac{C_3 \|H\|_{1+\delta}(1 - p\nu(\Phi \geq u_n))^q}{n^{1/\kappa}}.$$

We now show that $q\Gamma_n \to 0$ as $n \to \infty$ and this will complete the proof because

$$\left(1 - \frac{pq\nu(\Phi \circ f^0 \geq u_n)}{q}\right)^q \to \exp(e^{-D\nu} H(x, \theta)).$$

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By Lebesgue's differentiation theorem, for ν-almost every (x, θ)

$$n\nu(\Phi \circ f^0 \geq u_n) \to e^{-D\upsilon} H(x, \theta)$$

and so since

$$\frac{t}{p} \to 0 \quad \text{as } n \to \infty$$

we have

$$\lim_{n\to\infty} qt\nu(\Phi \circ f^0 \geq u_n) = 0.$$

Also,

$$nC_1 \frac{e^{-\upsilon(D+\epsilon)/\kappa}}{n^{(D+\epsilon)/(D\kappa)}} \to 0$$

because $\epsilon > (3D\kappa/2) - D$. Further,

$$n\frac{C_2 n^{(1+\epsilon)/D}}{e^{-\upsilon(1+\epsilon)}t^\alpha} \to 0 \quad \text{because } \alpha > \frac{3/2 + (1+\epsilon)/D}{\tau}$$

by equation (4.3).

For the remaining part

$$qp\sum_{j=t}^{p} \nu(\{\Phi \circ f^0 \geq u_n\} \cap f^{-j}\{\Phi \circ f^0 \geq u_n\})$$

$$\leq qp^2\nu(\Phi \circ f^0 \geq u_n)^2 + qp^2 C_1 \frac{e^{-\upsilon(D+\epsilon)/\kappa}}{n^{(D+\epsilon)/(D\kappa)}} + qp^2 \frac{C_2 n^{(1+\epsilon)/D}}{e^{-\upsilon(1+\epsilon)}t^\alpha}. \tag{4.7}$$

We show that the terms on the right hand side converge to 0 as $n \to \infty$. Since

$$qp\nu(\Phi \circ f^0 \geq u_n) \to e^{-D\upsilon} H(x, \theta),$$

$$qp^2\nu(\Phi \circ f^0 \geq u_n)^2 \sim \frac{e^{-2D\upsilon} H(x, \theta)^2}{q} \to 0 \quad \text{as } q \to \infty.$$

Next, by (4.2),

$$qp^2 C_1 \frac{e^{-\upsilon(D+\epsilon)/\kappa}}{n^{(D+\epsilon)/(D\kappa)}} \sim \frac{1}{n^{3/2-(D+\epsilon)/(D\kappa)}} \to 0.$$

And, further,

$$qp^2 \frac{C_2 n^{(1+\epsilon)/D}}{e^{-\upsilon(1+\epsilon)}t^\alpha} \sim \frac{1}{n^{\tau\alpha-3/2-(1+\epsilon)/D}} \to 0.$$

Also, from Lemma 3.5,

$$qp\sum_{j=1}^{t} \nu(\Phi \circ f^0 > u_n, \Phi \circ f^j > u_n) \to 0 \quad \text{because } t = n^\tau \text{ and } \tau \leq \gamma'. \tag{4.8}$$

This completes the proof. □

5. *Applications and examples*

We now verify the conditions of Theorem 2.1 and hence establish Corollaries 2.2 and 2.3. We will also discuss briefly two other important classes of maps: extensions to the Manneville–Pommeau type maps; and the Viana type maps. In the course of the discussion we will sketch why these maps satisfy all but one of the hypotheses of Theorem 2.1.

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5.1. *Uniformly and non-uniformly expanding maps of an interval modeled by Young towers.*

5.1.1. *Piecewise C^2 uniformly expanding maps of the interval.* We suppose that $T : I \to I$ is a piecewise C^2 map of an interval I onto itself in the sense that there is a finite partition $\{I_i\}$ of the interval I, T is C^2 on the interior of each I_j, $T : I_j \to I$ is onto and monotone, and $|T'(x)| > 1 + \delta$ for all x lying in the interior of each I_j. It is known from [**1**] that such maps possess an absolutely continuous mixing invariant measure μ and there exists a C such that $1/C \le (d\mu/dm) \le C$. Let $x, y \in \{z \mid d(z, T^j z) < 1/n\} \cap I_i$. We can see that, by the mean-value theorem,

$$\begin{aligned}(1+\delta)d(x, y) &< (1+\delta)^j d(x, y) < |(T^j)'|d(x, y)\\ &= d(T^j x, T^j y) \le d(T^j x, x) + d(T^j y, y) + d(x, y)\end{aligned}$$

and so $d(x, y) < ((2/\delta)/n)$. Thus on summing over the contribution of each I_i we get an estimate of the form $m\{x \mid d(x, T^j x) < 1/n\} \le C_2/n$. Thus, for any $1 > \gamma' > 0$,

$$m\left\{x \;\middle|\; d(x, T^j x) < \frac{1}{n} \text{ for some } j \in \{1, \ldots, n^{2\gamma'}\}\right\} \le \frac{1}{n^{1-2\gamma'}}.$$

In particular, choosing $\gamma' < 1/4$, we see that hypothesis (a) of Theorem 2.1 holds.

Such maps possess a Young tower with exponential return-time tails [**8**], hence, as shown in [**12**] for a residual set of S^1 cocycles $h : I \to S^1$, the skew extension f of the base map T has exponential decay of correlations. Thus this class of maps satisfies the conditions of Theorem 2.1.

5.1.2. *Non-uniformly expanding maps modeled by a Young tower.* Suppose $T : X \to X$ is a non-uniformly expanding map of an interval with bounded derivative, i.e. $\sup_{x\in X} |T'(x)| < C$, modeled by a Young tower with exponential return-time tails. Collet [**11**] has shown that there exists a $\beta > 0$ for which $\mu(E_n^X) < C/n^\beta$ and so by Proposition 3.4 we may conclude that the system $f : X \times S^1 \to X \times S^1$ defined by $f(x, \theta) = (Tx, \theta + h(x))$ for any measurable cocycle h satisfies this property. Further, Gouëzel shows in [**12**] that for a residual set of Hölder cocycles, such systems satisfy the second hypothesis of Theorem 2.1 for an arbitrary α (by showing that decay is in fact exponential). Since the map f along the group S^1 is an isometry, its density with respect to the Lebesgue measure is 1 and hence the density of the invariant measure is just the density for T. Collet [**11**] shows that this density lies in $\mathbb{L}^p$ for some p larger than 1, and so all the hypotheses of Theorem 2.1 are satisfied.

5.2. *Skew product with a curve of neutral points.* We consider Gouëzel's map studied, for instance, in [**14**]. Define $F : S^1 \to S^1$ by $F(\omega) = 4\omega$ and $T_\alpha : [0, 1] \to [0, 1]$ as

$$T_\alpha(x) = \begin{cases} x(1 + 2^\alpha x^\alpha) & \text{if } 0 \le x \le \frac{1}{2},\\ 2x - 1 & \text{if } \frac{1}{2} < x \le 1, \end{cases} \tag{5.1}$$

where $\alpha : S^1 \to (0, 1)$ is a map with minimum $\alpha_{\min}$ and a maximum $\alpha_{\max}$ and satisfies the following:

- α is C^2;
- $0 < \alpha_{\min} < \alpha_{\max} < 1$;

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- α takes the value $\alpha_{\min}$ at a unique point x_0 with $\alpha''(x_0) > 0$;
- $\alpha_{\max} < 3\alpha_{\min}/2$.

The map $T : S^1 \times [0, 1] \to S^1 \times [0, 1]$ is defined as $T(\omega, x) = (F(\omega), T_{\alpha(\omega)}(x))$. From [**14**, Theorem 2.10], the density H of the map T is $\mathbb{L}^1$ with respect to the product $\mu \times$ Leb where μ is the invariant measure on S^1 for F (and is the same as the Lebesgue measure). Since F is uniformly expanding, by exactly the same argument as in §5.1.1 we see that hypothesis (a) of Theorem 2.1 is satisfied. Further, from [**15**],

$$\left| \int \Upsilon \Psi \circ T^n - \int \Upsilon \int \Psi \right| \leq C n^{1-1/\alpha_{\max}} \|\Upsilon\|_{\hat{\alpha}} \|\Psi\|_\infty \tag{5.2}$$

and so hypothesis (b) is also satisfied. Further, by [**14**, Theorem 2.10], the density H is Lipschitz on every compact subset of $S^1 \times (0, 1]$. The only places in the proof of Theorem 2.1 that we require the density to be in $\mathbb{L}^{1+\delta}$ is to estimate the volume of balls, and this requirement can be replaced by the Lipschitz requirement on every compact subset. Recall that B_r is a ball about a fixed point (ω, x) of radius r and that

$$\|\Phi_B - \chi_B\|_1^\nu \leq \nu(B_r \setminus B_{r'}) = \int H \chi_{B_r \setminus B_{r'}} \, d\lambda.$$

Fix a closed ball Γ with center (ω, x). For r sufficiently small, $B_r \subset \Gamma$ and so $\|H|_\Gamma\|_\infty < \infty$. Therefore

$$\int H \chi_{B_r \setminus B_{r'}} \, d\lambda \leq \|H|_\Gamma\|_\infty \lambda(B_r \setminus B_{r'})$$

and so bounds of the type of Lemma 3.6 may be obtained with κ set equal to 1. Now, if we choose $\epsilon > D/2$, in equation (4.7) we have

$$qp^2 \frac{e^{-v(D+\epsilon)}}{n^{1+\epsilon/D}} \to 0.$$

The last term in equation (4.7) will converge to 0 if the function α is chosen so that $\alpha_{\max}$ satisfies

$$\alpha_{\max} < \frac{\min\{\gamma', 1/2\}}{\min\{\gamma', 1/2\} + (1 + D/2)/D + 3/2}.$$

5.3. *Some other extensions.*

5.3.1. *The Viana maps.* Let T be a uniformly expanding map of the circle S^1 given by $T(\theta) = d\theta \bmod 1$ for $d \geq 16$. Suppose $b : S^1 \to S^1$ is a Morse function, that $u_\alpha(\theta, x) = a_0 + \alpha b(\theta) - x^2$ and that a_0 is chosen so that $x = 0$ is pre-periodic for $a_0 - x^2$. Let $\Phi_\alpha(\theta, x) = (T(\theta), u_\alpha(\theta, x))$. From [**7**], for small enough α, there is an interval $I \subset (-2, 2)$ for which $\Phi_\alpha(S^1 \times I) \subset \text{int}\,(S^1 \times I)$.

Along the base, this map exhibits a uniformly expanding behavior, and thus, from Proposition 3.4, we can conclude that the first hypothesis of Theorem 2.1 is satisfied. Also, it has been shown in [**13**] that such a system displays a decay of correlations at the rate of $O(e^{-c\sqrt{n}})$ which is faster than any polynomial. From [**7**], we know that the density of the absolutely continuous invariant measure lies in $\mathbb{L}^1(\lambda)$. If we knew that this density was

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in $\mathbb{L}^{1+\delta}(\lambda)$ for small $\delta > 0$, then all the hypotheses of Theorem 2.1 would be satisfied and in that case the limiting distribution obtained would be

$$\lim_{n\to\infty} \nu(Z_n^{(x,\theta)} < u_n) = \exp(H(x,\theta)e^{-2v}).$$

5.3.2. *Manneville–Pommeau type maps.* We will consider the Liverani–Saussol–Vaienti map $T : [0, 1] \to [0, 1]$ defined as

$$T(x) = \begin{cases} x(1 + 2^{\omega}x^{\omega}) & x \in [0, \frac{1}{2}), \\ 2x - 1 & x \in [\frac{1}{2}, 1]. \end{cases}$$

Near the origin, this map is $x \mapsto x + 2^{\omega}x^{1+\omega}$ and the density near the origin is seen to be $h(x) \approx x^{-\omega}$ so $h \in \mathbb{L}^{1/\omega-\epsilon}$ for any $\epsilon > 0$. It is a result from [**9**] that

$$\mu_X \left\{ x \;\middle|\; d(T^j x, x) < \frac{1}{n} \text{ for some } 0 \le j \le g(n) \right\} \le \left(\frac{g(n)}{\sqrt{n}} \right)^{1-\omega}$$

so if we choose u to be a cocycle, $g(n) = n^{(1-\omega)/24}$ and $\beta = (1 - \omega)/8$, we see that for $Y = S^1$ we have $D = 2$, $\gamma' = (1 - \omega)/24 < \beta/D$ and $\mu_X(E_n^X) < C/n^{\beta}$. Further, since we have an isometry along the fiber, the density H for ν will lie in $\mathbb{L}^{1/\omega-\epsilon}$ and so all the hypotheses of Theorem 2.1 are met except that the rate of decay of correlations for such an extension $f = (T, u)$ is not known. If a rate satisfying condition (b) can be established, we will be able to establish the extreme-value law.

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