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Computational study of three-dimensional equilibria with the bootstrap current

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The three-dimensional currentless equilibrium code, HINT is revised so that it can handle equilibria with a net toroidal current, for example the bootstrap current. The revised HINT code is applied to Large Helical Device-like equilibria with good nested flux surfaces.

1. Introduction

To study three-dimensional magnetohydrodynamic (MHD) equilibria with a net toroidal current, in this paper the HINT code (Hayashi 1989; Harafuji et al. 1989), which originally treated only currentless equilibria, is revised. The net toroidal current comprises such non-vanishing currents after the flux surface average as the bootstrap current, the Ohmic current, the Ohkawa current, etc. The calculation is based on the time-dependent relaxation technique using small values of resistivity and viscosity. The original HINT code was applied to finite-beta three-dimensional equilibria without a net toroidal current, because three-dimensional equilibria have the possibility of net current-free steady operation. The most important advantage of the HINT code is that it does not need to assume the existence of nested flux surfaces in equilibria. Thus the HINT code enables us to investigate quantitatively (i) the deterioration of magnetic surfaces due to the formation of magnetic islands, and (ii) the self-healing of magnetic islands in a finite-beta plasma in three-dimensional currentless equilibria (Hayashi et al. 1992, 1994, 1995). Although three-dimensional equilibria can exist under the currentless condition, net toroidal currents, namely the bootstrap current and/or the Ohkawa current, can flow according to the experimental conditions. The properties of Large Helical Device (LHD) (Iiyoshi et al. 1990) equilibria with a self-consistent bootstrap current have been examined (Nakajima and Okamoto 1992; Watanabe et al. 1992), and the effects of a net toroidal current on Mercier stability have also been investigated (Watanabe et al. 1995) under the assumption that nested good flux surfaces exist. The net toroidal current changes the rotational transform so drastically that the resonance condition of the magnetic island will be altered, which leads to the formation and/or self-healing of magnetic islands. To investigate such phenomena, the HINT code is revised. In this paper, we propose a revised HINT computational method and apply it to LHD-like equilibria with good nested flux surfaces.

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2. Modification of the HINT computation

In the HINT code, an MHD equilibrium is obtained starting from an arbitrary nonequilibrium initial plasma and vacuum field configuration by means of a time-dependent relaxation method with small values of resistivity η and viscosity ν. The calculations are performed in two steps, which were first developed by Park et al. (1986).

The first step (the A-step) is relaxation of the pressure along magnetic field lines. To speed up the relaxation of the pressure, we solve the artificial sound wave equations under a fixed magnetic field, until $\mathbf{B}\cdot\nabla p = 0$ is satisfied (Park et al. 1986; Hayashi 1989; Harafuji et al. 1989). This pressure relaxation scheme has been further developed using a method where field lines are traced (Hayashi et al. 1992). The newer scheme has turned out to be more efficient when island chains appear on magnetic surfaces.

The second step (the B-step) is relaxation of the magnetic field under a fixed pressure profile (Park et al. 1986; Hayashi 1989; Harafuji et al. 1989). To calculate the MHD equilibrium with a net toroidal current, we revise the Faraday equation in the B-step, according to the following idea. In the original HINT code, the Faraday equation is given as

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{j}). \tag{1}$$

In a steady state, $\mathbf{E}$ is expressed as the gradient of a scalar potential ϕ, i.e. $\mathbf{E} = -\nabla\phi$, in Ohm's law given as

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}. \tag{2}$$

If there exist closed flux surfaces, the calculation of a flux surface average is permitted on such surfaces. For some equilibria without a source of net current, the scalar potential becomes a single-valued function, so that

$$\langle \mathbf{E} \cdot \mathbf{B} \rangle = -\langle \nabla\phi \cdot \mathbf{B} \rangle = 0 = \eta \langle \mathbf{j} \cdot \mathbf{B} \rangle, \tag{3}$$

where $\langle\ \rangle$ means the flux surface average. Thus we obtain three-dimensional equilibria without a net current (Park et al. 1986).

For equilibria with an Ohmic current, Ohm's law as given by (2) can be represented as follows:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \left(\mathbf{j} - \sigma \mathbf{E}_{\mathrm{ex}} \right), \tag{4}$$

where σ is the conductivity and $\sigma\mathbf{E}_{\mathrm{ex}}$ is the Ohmic current. Since the Ohmic current can also be represented as $\mathbf{B} \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{Ohmic}} / \langle B^2 \rangle$, the revised version of (1) becomes

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times \left[\mathbf{v} \times \mathbf{B} - \eta \left(\mathbf{j} - \mathbf{B} \frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{Ohmic}}}{\langle B^2 \rangle} \right) \right], \tag{5}$$

where we assume that an equilibrium has closed flux surfaces. If there exist closed flux surfaces in the steady state, we have

$$\langle \mathbf{E} \cdot \mathbf{B} \rangle = 0 = \eta \left(\langle \mathbf{j} \cdot \mathbf{B} \rangle - \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{Ohmic}} \right). \tag{6}$$

Thus, in the steady state, $\langle \mathbf{j} \cdot \mathbf{B} \rangle$ becomes $\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{Ohmic}}$, and we obtain three-dimensional equilibria with a net Ohmic current. If a bootstrap current and an Ohkawa current are also present, (5) can be generalized as

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times \left[\mathbf{v} \times \mathbf{B} - \eta \left(\mathbf{j} - \mathbf{B} \frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{net}}}{\langle B^2 \rangle} \right) \right], \tag{7}$$

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where

$$\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{net}} = \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{Ohmic}} + \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{bootstrap}} + \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{Ohkawa}}. \tag{8}$$

Therefore we treat (7) in the revised B-step, instead of (1). Solving the relaxation equations numerically in the revised HINT computational method, we find a three-dimensional MHD equilibrium with a net current in a toroidal helical system.

The calculation of a net toroidal current (e.g. the bootstrap current) is done between the A- and B-steps under the conditions of a fixed pressure profile and a fixed magnetic field with closed flux surfaces. In the calculation of the bootstrap current, we employed the Boozer coordinates (ψ, θ, ζ), where ψ is the toroidal magnetic flux divided by 2π, and θ and ζ are the poloidal and toroidal angles respectively. If the collisionality of the electrons is the same as that of the ions, the bootstrap current is expressed in each collisionality limit as follows:

$$\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{bootstrap}} = -G_{\text{bs}} \left(L_1 \frac{dp}{d\psi} + L_2 n \frac{dT}{d\psi} \right), \tag{9}$$

where p is the plasma pressure, n is the density, T is the temperature ($p = 2nT$; $n = n_i = n_e$, $T = T_i = T_e$, and $n, T \propto p^{1/2}$ in this work). L_1 and L_2 are transport coefficients, which are composed of the viscosity and the friction coefficients, and G_{bs} is a geometrical factor given in Nakajima and Okamoto (1992), Watanabe et al. (1992, 1995) and Ichiguchi et al. (1997). To evaluate the bootstrap current through (9), only spectra of the magnetic field strength in the Boozer coordinates are needed, with some surface quantities (Nakajima and Okamoto 1992; Watanabe et al. 1992, 1995; Ichiguchi et al. 1997). To obtain the spectra of the magnetic field strength and surface quantities, we use the method of Boozer (1982) and Rome (1989) with the magnetic field line trace. To estimate the bootstrap current in various collisionality regimes, we use the connection formula derived in Watanabe et al. (1995), instead of (9).

3. Application of the revised HINT code

We apply the revised HINT code to an LHD-like equilibrium with the bootstrap current. We consider the equilibrium with $B_0 = 4$ T, $R_0 = 4$ m, $\beta_0 = 1.4\,\%$, $n_0 = 7.5 \times 10^{19}$ m^{-3}, and $T_0 = 4$ keV at the magnetic axis. In this calculation, the equilibrium has the structure of good nested flux surfaces. In this case, the bootstrap current I_{net} is about 50 kA, where I_{net} is defined as

$$I_{\text{net}} \equiv \int d\mathbf{S}_T \cdot \mathbf{j}_{\text{net}}, \tag{10}$$

$\mathbf{S}_T$ is the poloidal cross-section, and $\mathbf{j}_{\text{net}} = \mathbf{B}\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{net}} / \langle B^2 \rangle$. Poincaré plots of field lines, profiles of the pressure and the toroidal current, and the rotational transform are shown in Fig. 1. To evaluate the bootstrap current, Fourier spectra of the magnetic field strength in the Boozer coordinates are calculated as in Fig. 2, according to the method in Boozer (1982) and Rome (1989). As shown in Fig. 3, the profile of the bootstrap current is obtained by using the Fourier spectra. The profile of the bootstrap current can be explained by using the geometrical factor G_{bs}, the pressure gradient $\mathrm{d}p/d\psi$, and the temperature gradient $dT/d\psi$; see (9). In this calculation, the plasma is in the collisionless regime, except for the vicinity of the magnetic axis, as shown in Fig. 4, and the behaviour of the geometrical factor is

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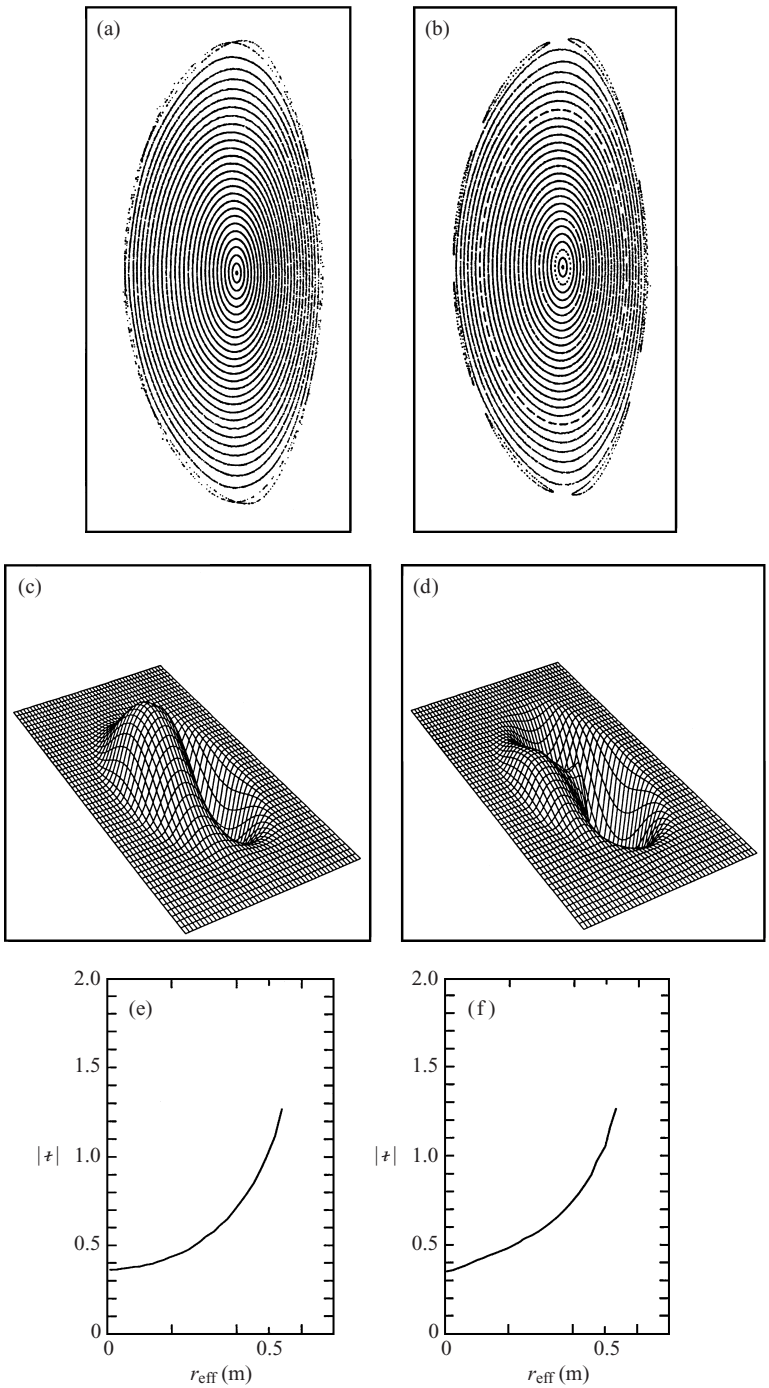


Figure 1. The LHD equilibrium for the case of $\beta_0 = 1.4\%$: (a,b) Poincaré plots of field lines at the vertically elongated poloidal cross-section, (c,d) the current profile at the vertically elongated poloidal cross-section, (e,f) the profile of the rotational transform $|\iota|$. (a,c,e) show results for the currentless LHD, and (b,d,f) are results for the LHD with the bootstrap current.

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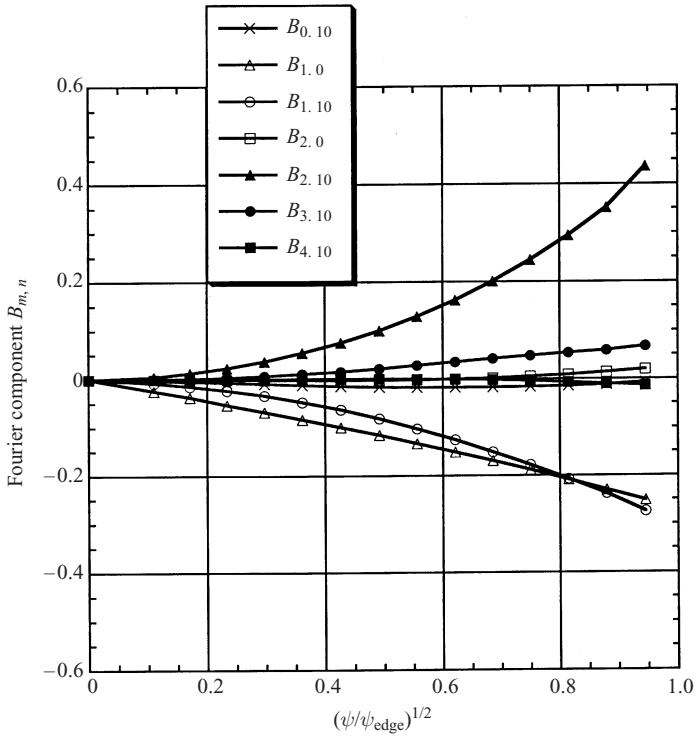


Figure 2. Fourier components of the magnetic field strength as a function of $(\psi/\psi_{\mathrm{edge}})^{1/2}$. We show the seven largest components used in the calculation of the bootstrap current, except for $B_{0,0}$.

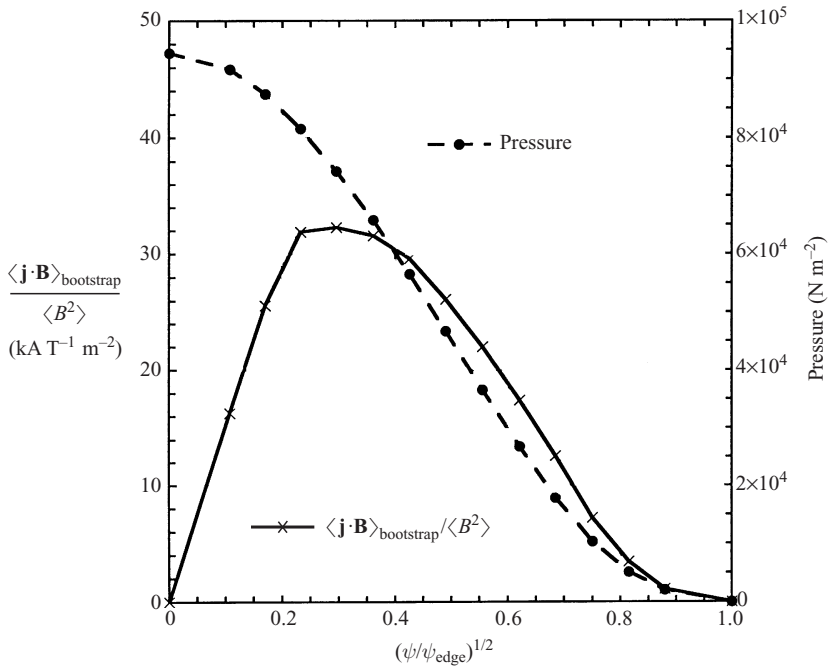


Figure 3. Profiles of the bootstrap current $\langle \mathbf{j}\cdot\mathbf{B}\rangle_{\mathrm{bootstrap}}/\langle B^2\rangle$ and the pressure as functions of $(\psi/\psi_{\mathrm{edge}})^{1/2}$.

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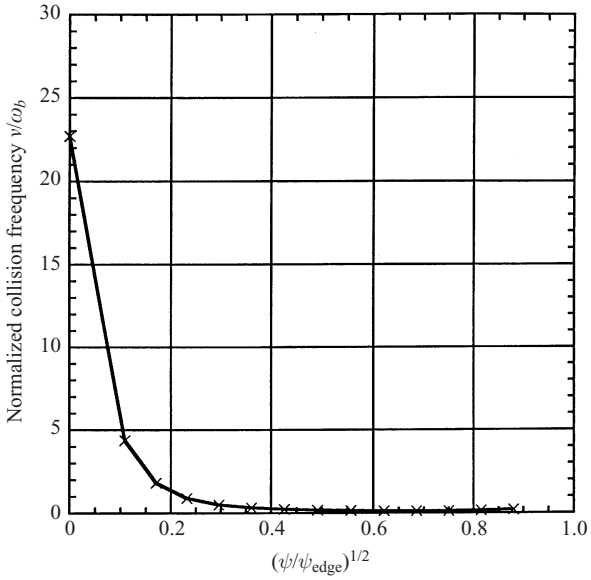


Figure 4. Collision frequency normalized by the bounce frequency ω_b.

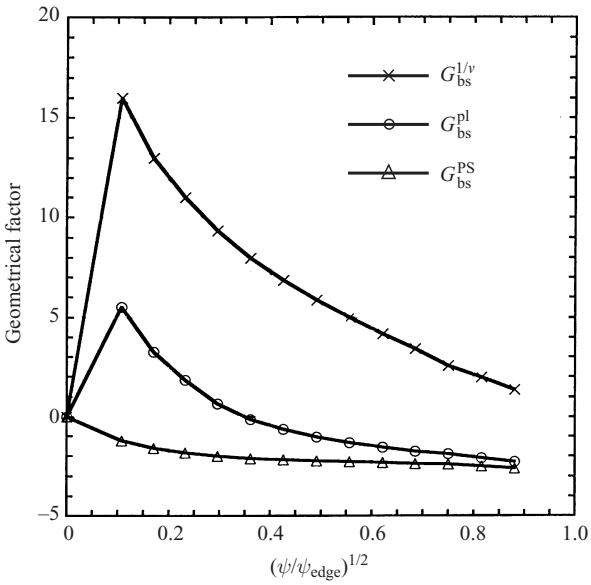


Figure 5. Geometrical factors for the $1/\nu$, plateau (pl), and Pfirsch–Schlüter (PS) regimes.

dominated by that for the $1/\nu$ regime (see Fig. 5). Both the pressure gradient and density become zero at the outermost surface. These behaviours are reflected in the profile of the bootstrap current. Adding the bootstrap current shown in Fig. 3 to the Pfirsch–Schlüter current, the current profile is modified from that shown in Fig. 1(c) to that shown in Fig. 1(d), and the rotational transform ι is increased as shown in Fig. 1(f) (compare with Fig. 1e).

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The error of force balance R_f in the above calculation is less than 10^{-5}, thus the new computational method is correctly working, here R_f is defined as

$$R_f = \frac{\int d\tau \, (\nabla p - \mathbf{j} \times \mathbf{B})^2}{\int d\tau [(\nabla p)^2 + (\mathbf{j} \times \mathbf{B})^2]}. \tag{11}$$

4. Conclusions and discussion

We have revised the HINT code to study three-dimensional MHD equilibria with the bootstrap current. To develop the original HINT code, we have proposed a modification of the Faraday equation in the scheme in the original HINT computation. The bootstrap current is calculated using the method of Boozer (1982) and Rome (1989). The revised HINT code has been applied to the LHD-like equilibrium, and its validity of the code has been ascertained.

The effects of a net current, especially the bootstrap current, on magnetic islands are under investigation. We are going to develop the revised HINT code to calculate equilibria with island chains.

The calculation scheme of the bootstrap current in equilibria with islands can be briefly described as follows. According to the following ideas, we are developing a new version of the revised HINT code. Even if an equilibrium contains islands or stochastic regions, we can construct magnetic coordinates (ψ, θ, ζ) by using the method of quasimagnetic coordinates (Reiman and Greenside 1986). The pressure profile in the island becomes flat when the numerical equilibrium has achieved a sufficiently relaxed state. Thus the gradients of pressure and temperature, $dp/d\psi$ and $dT/d\psi$, become zero in the islands, and the value of the bootstrap current is zero, on assuming that the formulation (9) of the bootstrap current is also appropriate in the quasimagnetic coordinates. Note that the bootstrap current also becomes zero in the stochastic region outside the outermost surface, because $p = 0$ in this region. To numerically distinguish sufficiently large islands from negligibly small islands (or rational surfaces), we trace a field line started from a fixed point (an O-point) and calculate the residue $\mathscr{R}$ of Cary and Hanson (1986). If the residue satisfies the condition $1 > \mathscr{R} > \epsilon_R$, the island is declared to be numerically visible, where ϵ_R is a small parameter, $1 \gg \epsilon_R \geqslant 0$, and defines the size of the negligibly small islands. In order to decide the boundaries of islands, contour lines of the pressure can be used. We define the island region as being given by the condition $p^{(+)}_{\text{island}} > p > p^{(-)}_{\text{island}}$, where $p^{(\pm)}_{\text{island}} = p_{\text{island}} \pm \epsilon_p$, p_{island} is the pressure at an O-point of the island, and ϵ_p is a small parameter with $1 \gg \epsilon_p / p_{\text{island}} > 0$ and gives the width of the island region. Note that, during the relaxation process, the pressure profile in islands is not exactly flat. The bootstrap current is given as zero in the island region with the condition $p^{(+)}_{\text{island}} > p > p^{(-)}_{\text{island}}$, while the current in all plasma regions except islands is calculated by (9).

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