9 Relativistic optical effects

Special relativity affects what an interstellar traveller observes. This includes the apparent direction, size, and brightness of a source, as well as the frequency of signals and the wavelength of light. We examine these phenomena – aberration and the Doppler effect – and see how they arise from the Lorentz transformations. General relativistic effects, due to light passing through gravitational fields, are not considered.

9.1 Source direction and size: aberration

Aberration is the effect by which the observed position of an object changes when its transverse velocity relative to the observer changes. This is an everyday experience. If rain is falling vertically to the ground, then when a person starts to run, they experience the rain arriving diagonally (figure 9.1). The rain appears to have acquired some horizontal velocity. Generally, if $\mathbf{u}_{rg}$ is the velocity of the rain relative to the ground, and $\mathbf{u}_{pg}$ is the velocity of the person relative to the ground, then $\mathbf{u}_{rp} = \mathbf{u}_{rg} - \mathbf{u}_{pg}$ is the velocity of the rain relative to the person.

The same thing occurs if we replace rain drops with photons. Consider a light source shining in the negative y' direction in its rest frame S' (figure 9.2). The velocity of the photons in this frame is $(u'_{x'}, u'_{y'}, u'_{z'}) = (0, -c, 0)$. This source moves at velocity v relative to the observer (at rest in frame S) in the x direction. Frame S' is like the ground in the previous example, but the person is now running to the left. Under a classical Galilean transformation, where special relativity is neglected, the observer measures velocities in their rest frame of $(u_x, u_y, u_z)_{\text{classical}} = (v, -c, 0)$. Thus the observer sees the photons arriving at angle

$$\theta_{\text{classical}} = \arctan\left(\frac{|u_x|}{|u_y|}\right) = \arctan\beta \tag{9.1}$$

where as usual $\beta = v/c$. I've taken the absolute values of the velocities because for now we only consider the magnitude of the aberration angle, not its sign. Using instead the relativistic Lorentz transformation (equation 7.13), the observer measures velocities in their rest frame of $(u_x, u_y, u_z)_{\text{relativistic}} = (v, -c/\gamma, 0)$. The observer then sees the photons arriving at an angle of

$$\theta_{\text{relativistic}} = \arctan\left(\frac{|u_x|}{|u_y|}\right) = \arctan\gamma\beta = \arcsin\beta. \tag{9.2}$$

For velocities small compared to the speed of light, the classical and relativistic results are very similar, as we can see from a Taylor expansion:

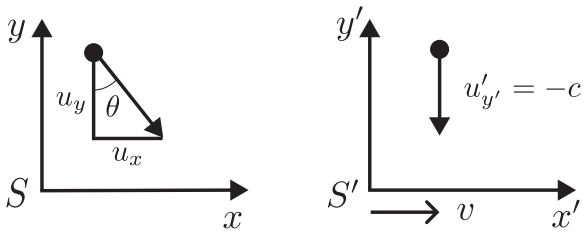
$$\theta_{\text{classical}} = \arctan\beta = \beta - \frac{\beta^3}{3} + \mathcal{O}(\beta^5) \tag{9.3a}$$

$$\theta_{\text{relativistic}} = \arcsin\beta = \beta + \frac{\beta^3}{2\cdot 3} + \mathcal{O}(\beta^5). \tag{9.3b}$$

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The concept of aberration. A person running experiences the rain as coming from a different direction ($\mathbf{u}_{rp}$, middle panel) compared to when standing still ($\mathbf{u}_{rg}$, left panel). Fig. 9.1



Aberration of light. In reference frame S', where the photon source is at rest, the photon moves vertically down. This frame moves at velocity v along the x-axis with respect to reference frame S. In frame S the photon appears to arrive at an angle θ from the vertical. Fig. 9.2

Figure 9.3 shows the size of the aberration as a function of the relative velocity. For a spacecraft with a velocity $\beta = 0.01$, the aberration is 0.6°, and the difference between the classical and relativistic predictions is $0.1''$, so becomes measurable.

At low velocities, aberration is small, but is still significant in astronomical observations. The Earth orbits the Sun in a near-circular orbit with a velocity of $30\,\mathrm{km\,s^{-1}}$. The velocity of a star relative to an Earth-based observer therefore changes by $\pm\,30\,\mathrm{km\,s^{-1}}$ over the course of a year. This induces an observed change in position in an inertial reference frame of $\pm\,20.5''$. The difference between the classical and relativistic predictions here is just $0.1\,\mu$as. The rotation of the Earth about its axis induces another change in position of up to $\pm\,0.3''$ over the course of the day.

The magnitude of the velocity of the photon in S' is $(u_x^2 + u_y^2 + u_z^2)^{1/2}$. With the Lorentz transformation this is equal to c, as expected, but with the Galilean transformation it is greater than c.

Let us now consider the more general case in which a photon moves at an angle α' to the x'-axis in S', as shown in figure 9.4. S' is moving to the right relative to S as usual, but the origin of S' is now shown to the left of S to make the geometry a bit clearer. In S' the velocity of the photon along the x'-axis is $u'_{x'} = c\cos\alpha'$. In S the x-component of its velocity is $u_x = c\cos\alpha$. Substituting these into the Lorentz transformation for this velocity component (equation 7.13), we find that the photon is observed in S to be moving in the direction

$$\cos\alpha = \frac{\cos\alpha' + \beta}{1 + \beta\cos\alpha'}\,. \tag{9.4}$$

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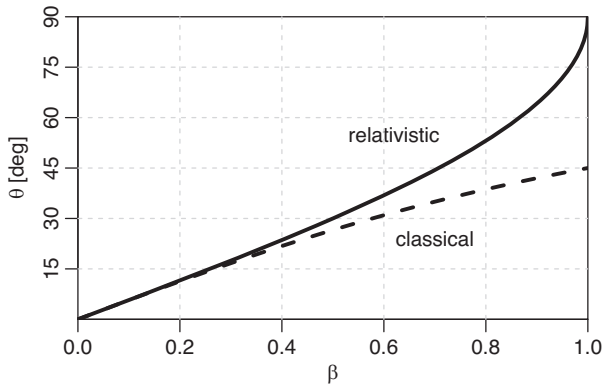


Fig. 9.3 The magnitude of the aberration (change in position) when a source moves with a transverse velocity relative to the observer of β, for the classical case (dashed line) and the relativistic case (solid line).

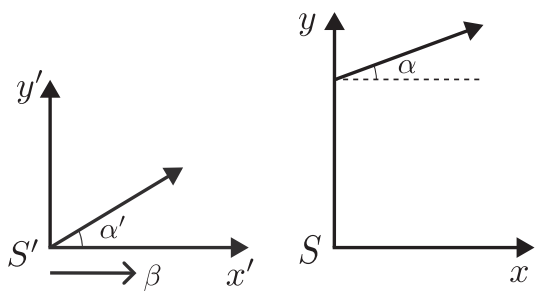


Fig. 9.4 Relativistic aberration. A light source at rest at the origin of frame S' emits photons in direction α' to the x'-axis. Frame S' moves with velocity β relative to frame S parallel to the x-axis. An observer at rest in S observes the photons to be moving in a direction α to the x-axis given by equation 9.4.

This function is shown in figure 9.5. Note that α is the angle at which the photons are travelling in S. The angle at which they appear to be coming from is therefore $\alpha + 180°$.[1] If a source is emitting photons only in direction α', then the observer in S will receive them only if the source is positioned at an angle $\alpha + 180°$ in S.

Consider a source emitting isotropically in its rest frame. For the spatial configuration shown in figure 9.4 with $\beta > 0$, the source is approaching the observer and the received angle α is smaller than the emitted angle α'. Once the source moves past the observer in S, $90° < \alpha < 180°$ and the source is receding; α is still less than α' (figure 9.5), but this corresponds to the received photons making a *larger* angle to the *negative* x-axis. Thus an approaching source always appears less inclined with respect to the line of sight than when it was at rest. For a receding source the opposite is true: it always appears more inclined with respect to the line of sight than when it was at rest. When observing from a spacecraft moving

[1] If we define the angles α and α' instead as the direction which the photons are coming from, then because $\cos(\alpha \pm \pi) = -\cos(\alpha)$, equation 9.4 becomes

$$\cos\alpha = \frac{\cos\alpha' - \beta}{1 - \beta\cos\alpha'} .$$

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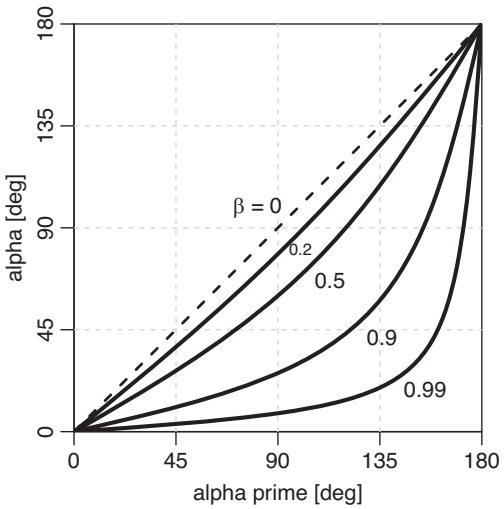


Fig. 9.5 The relation between the direction of travel of a photon α' in frame S' and the direction of travel α observed in frame S (configuration in figure 9.4) for various values of the velocity β of S' relative to S. This is given by equation 9.4. For negative values of β the curves are mirrored about the $\alpha = \alpha'$ ($\beta = 0$) line. Note that β gives the relative direction of motion of the two frames. Whether a specific source is observed to be approaching or receding depends also on its angular coordinate relative to the observer, that is, whether S is to the left or right of S' in figure 9.4.

at high velocity, stars in the forward direction will appear more bunched up in the direction of motion compared to when the spacecraft was at rest, because those stars are approaching the spacecraft. Conversely, stars behind the spacecraft will appear more spread out than they were when the spacecraft was at rest. This is illustrated in figure 9.6.

For small angles in both frames, we find that (see exercises)

$$\alpha \simeq \alpha' \sqrt{\frac{1-\beta}{1+\beta}} \quad (\alpha' \ll 1). \tag{9.5}$$

For an approaching source ($\beta > 0$), light is received at an angle closer to the direction of travel the faster it is moving.

Aberration is purely a result of the change in velocity of the observer relative to the source. It has nothing to do with their change in separation over time. Indeed, as we move away from a source of finite size at a finite distance, we would expect it to get smaller simply from the fact that we are getting further away from it. But our motion actually induces the opposite effect at any instant because of aberration (although in practice this is not noticeable).

Accounting for aberration is essential for navigation, as we shall see in chapter 14. Even for the low orbital velocity of the Earth mentioned above ($30\,\mathrm{km\,s^{-1}}$), the aberration of $\pm 20.5''$ for stars means that the pointing of telescopes has to be adjusted over the course of the year. Indeed, aberration was discovered and explained by James Bradley in 1727, in the course of trying to measure the parallax of nearby stars. Aberration is nearly 30 times as large as the largest stellar parallax, so it is not surprising that it was discovered nearly 100 years before the first stellar parallaxes were measured. Aberration, unlike parallax, does not depend on the distance to the stars. Furthermore, the displacement of a star due to aberration is 90° out of phase with respect to the displacement due to parallax.

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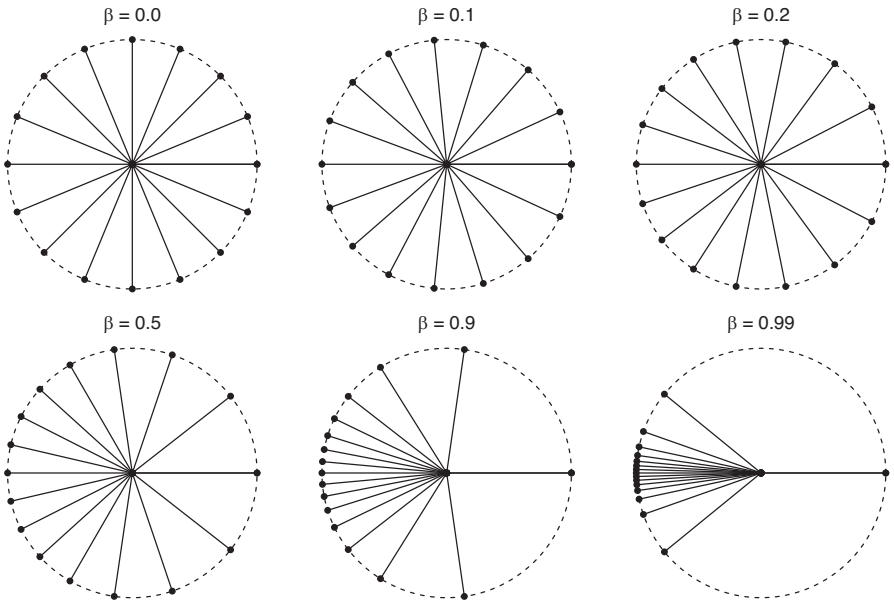


Fig. 9.6 The effect of aberration on the observed positions of objects, such as stars. The stars (points on the circle) are at rest with respect to each other. When the spacecraft (at the centre of the circle) is at rest with respect to the stars, the stars are regularly spaced around the spacecraft (the top-left panel with $\beta = 0$). The faster the spacecraft moves along the horizontal axis to the left, the closer to the direction of travel the stars appear to lie. In the context of figure 9.4, the stars (rest frame S') are moving to the right with respect to the spacecraft (frame S), so β is positive.

By changing the angular position of point sources, it follows that aberration also changes the angular size of extended sources. The solid angle element is $d\Omega = d\phi\, d\alpha \sin\alpha$, where ϕ is the azimuthal angle in spherical coordinates. Because ϕ is defined in terms of the lengths perpendicular to the direction of motion, that is, in terms of y and z in equation 7.10, and because these lengths are invariant between S and S', the angle ϕ is also invariant. Thus

$$\frac{d\Omega}{d\Omega'} = \frac{\sin\alpha\, d\alpha}{\sin\alpha'\, d\alpha'} = \frac{d(\cos\alpha)}{d(\cos\alpha')} = \gamma^2(1-\beta\cos\alpha)^2. \tag{9.6}$$

In the last step I evaluated $d(\cos\alpha')/d(\cos\alpha)$ using the inverse version of equation 9.4 – with primes swapped and the sign of β changed – to obtain a result in terms of α rather than α'. As the quantity on the right of the above equation appears in many places in relativistic optics, I denote it with

$$\psi = \gamma(1-\beta\cos\alpha). \tag{9.7}$$

(We will examine this function below in figure 9.9 when we discuss the Doppler effect.) For a moving observer, the solid angular size of an object is multiplied by ψ^2 compared to its size when the observer was at rest. When directly approaching a source, $\alpha = 0$ and $\beta > 0$ for the spatial configuration[2] shown in figure 9.4, so this factor is $\gamma^2(1-\beta)^2 = (1-\beta)/(1+\beta)$ (equation 7.5), which is less then unity. Thus the approaching source appears

[2] The observer would likewise be approaching the source if the source S' lay to the right of the observer S, with $\alpha = 180°$ and $\beta < 0$.

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smaller. Receding sources, conversely, appear larger. This is consistent with figure 9.6. We will use this result again in section 9.3 when we consider the brightness of extended sources.

There are many other interesting optical effects induced by relativistic motion. One goes by the name of the Terrell–Penrose rotation. If we take a photograph of an extended object, we observe the photons that arrive at the observer (camera) at the same time, but which may have left different parts of the object at different times due to the finite speed of light. This makes small moving objects appear rotated, and large moving objects will appear distorted. In practice this is usually dominated by other effects, so is hard to observe.

9.2 Source frequency and wavelength: the Doppler effect

When a light source emits regular pulses at frequency ν' in its rest frame, then the frequency at which those pulses are received by an observer is affected by their relative motion via two phenomena. First, the finite speed of light means that pulses arrive less frequently when the source is receding from the observer, because the source is further away at each successive pulse and so has to cover a larger distance at the same velocity. Second, there is a time dilation between the source's and the observer's rest frames. The first effect is a classical phenomenon, and is familiar as the lowering of pitch (frequency) of acoustic waves as a vehicle with a siren drives past and recedes into the distance. The second effect is purely relativistic.

Consider the source to be emitting photons isotropically in its rest frame S'. The source is moving at velocity β relative to an observer O, who in their rest frame S observes the photons moving in a direction α, as shown in figure 9.7. Suppose the period between pulses in S' is T'. In frame S, the source emits a pulse when it is at position x_1 at time t_1, and the next pulse when it has moved to position x_2 at time t_2. Due to time dilation, the time in frame S between the pulses is $t_2 - t_1 = \gamma T'$ (equation 7.11). In order to be received by the observer, the first pulse must travel a distance r_1 and the second pulse a distance r_2. The time that elapses between the arrival of the two pulses at the observer, which is the period in S, is

$$T = \left(t_2 + \frac{r_2}{c}\right) - \left(t_1 + \frac{r_1}{c}\right) = (t_2 - t_1) + \frac{r_2 - r_1}{c} = \gamma T' + \frac{r_2 - r_1}{c} . \tag{9.8}$$

From the geometry in figure 9.7, if the distance travelled by the source between the pulses is small compared to the distance to the observer, then $(r_2 - r_1) \simeq -\Delta r = -(x_2 - x_1)\cos\alpha$. The distance travelled by the source is $x_2 - x_1 = (t_2 - t_1)\beta c = \gamma T' \beta c$. Substituting this into equation 9.8 gives

$$T = T'\gamma(1 - \beta\cos\alpha) . \tag{9.9}$$

When using this equation, recall that the direction to the source from the observer is $\alpha + 180°$.[3]

As the frequency is the inverse of the period, we can also write

$$\nu = \frac{\nu'}{\gamma(1 - \beta\cos\alpha)} = \frac{\nu'}{\psi}, \tag{9.10}$$

[3] If we instead define α as the direction to the source, then the minus sign in equation 9.9 becomes a positive sign.

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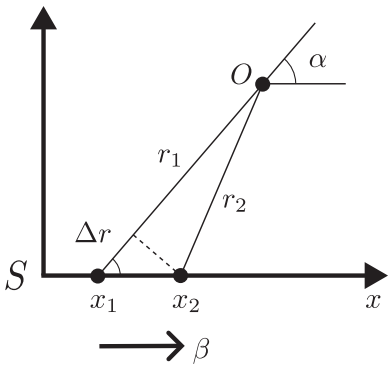


Fig. 9.7 The Doppler effect. A source emits light pulses at frequency ν' in its rest frame. It moves at velocity β relative to an observer *O* at rest in frame *S*. When the observer receives light pulses moving in direction α relative to the direction of travel, the observed frequency is given by equation 9.10. The points x_1 and x_2 are the positions of the source in *S* when it emits its pulses. This is the same configuration as in figure 9.4.

where we see again the ψ term appearing (equation 9.7). The above equation applies for any periodic signal, so could refer to the frequency of the photons, and therefore also to their energies, as $E = h\nu$. For photons specifically $c = \lambda\nu = \lambda'\nu'$, so the wavelength dependence is as for the period:

$$\lambda = \lambda'\gamma(1 - \beta\cos\alpha) = \lambda'\psi. \tag{9.11}$$

A source moving radially away from the observer emitting photons back to the source has $\alpha = 180°$, $\beta > 0$ (source to the right of the observer in figure 9.7). Using $\gamma(1+\beta) = \sqrt{(1+\beta)/(1-\beta)}$ (equation 7.6) in equation 9.11 with $\alpha = 180°$ we get

$$\lambda = \lambda'\sqrt{\frac{1+\beta}{1-\beta}} \quad \text{(radial)}. \tag{9.12}$$

This is plotted in figure 9.8 as the line marked 'radial (receding)'. For a source receding at $\beta = 0.1$, $\lambda/\lambda' = 1.11$: the wavelength is increased, or redshifted. The quantity $\lambda/\lambda' - 1$ is called the redshift in astronomy. For an approaching source we could either have $\alpha = 0°$ and $\beta > 0$, or $\alpha = 180°$ and $\beta < 0$. Equation 9.12 uses $\alpha = 180°$, and with $\beta = -0.1$ we get $\lambda/\lambda' = 0.90$, meaning the wavelength is decreased, or blueshifted.

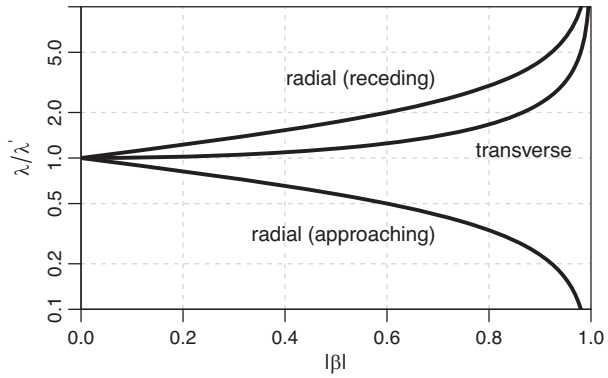
For a source moving tangentially with respect to the observer, $\alpha = 90°$, so

$$\lambda = \lambda'\gamma \quad \text{(transverse)} \tag{9.13}$$

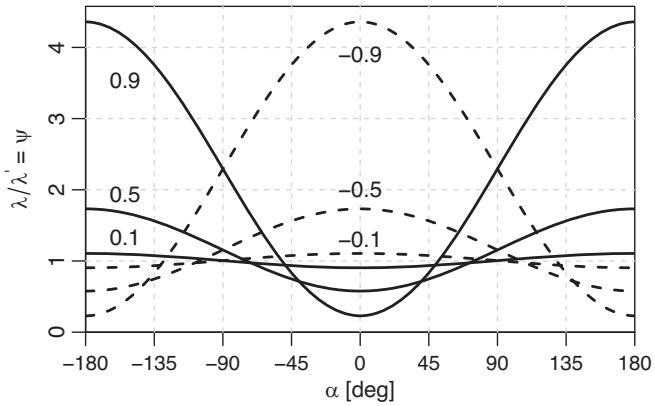
shown in figure 9.8 as the line marked 'transverse'. Successive pulses travel the same distance, so there is now no Doppler effect arising from the different light travel times. Here the Doppler effect is caused only by the time dilation, so is a purely relativistic phenomenon. As $\gamma \geq 1$, this transverse Doppler effect always produces a redshift for the observer. Given that $\lambda \propto 1/\nu$, wavelength behaves like time under time dilation, and indeed equation 9.13 has the same form as equation 7.11.

The variation of the general Doppler shift (equation 9.11) with angle for various velocities is shown in figure 9.9. To interpret this, consider a large spherical surface covered with stars that are at rest with respect to one another (in frame S'), and a spacecraft at the centre of the sphere moving in some direction. Figure 9.9 shows how the Doppler shift, λ/λ', varies for

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The radial and transverse Doppler effects. Note the logarithmic vertical scale. Fig. 9.8



The variation of the Doppler effect (equation 9.11) with angle of travel of the photons for various velocities (β, labelled on the curves) for the configuration in figure 9.4. The curves for negative β are just phase shifted by 180° relative to the curves for positive β. A source moving radially away from the observer emitting photons back to the source has $\beta > 0$ and $\alpha = \pm 180°$ (or $\beta < 0$ and $\alpha = 0°$). The term $\psi = \gamma(1 - \beta \cos\alpha)$ appears in many expressions in relativistic optics. Fig. 9.9

photons moving in direction α, which corresponds to light received from a star that lies at an angle $\alpha + 180°$ from the positive x-axis. Recall that β is the velocity of the stars relative to the spacecraft (figure 9.4), so a positive β corresponds to the spacecraft moving to the left in the rest frame of the stars.

Consider the line with $\beta = 0.5$ in figure 9.9. A value of $\alpha = 0°$ corresponds to light received from stars lying further off to the left directly in front of the spacecraft. These stars are approaching the spacecraft and their light is blueshifted ($\lambda/\lambda' < 1$). Increasing α, that is, examining the light that is increasingly off-axis in the direction of travel, we see that λ/λ' increases to the point that at sufficiently large angles the light becomes redshifted ($\lambda/\lambda' > 1$). For larger β the curve has a similar shape, but the variation of the Doppler shift is larger.

For any velocity there is an angle α at which the signal crosses from blueshift to redshift. This is found by setting $\lambda/\lambda' = 1$ in equation 9.11; the result is plotted in figure 9.10. For small positive velocities, stars lying in the forward direction of the spacecraft (photons moving in directions $-90° < \alpha < 90°$) are blueshifted, and all those in the backward direction are redshifted. As the velocity increases (β more positive), the opening angle of the

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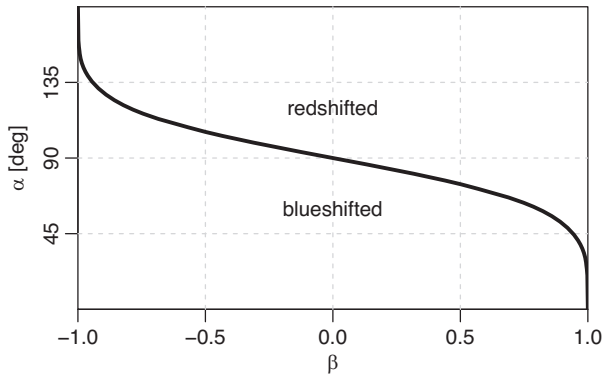


Fig. 9.10 The variation of the angle at which there is no Doppler shift ($\lambda = \lambda'$ in equation 9.11) as a function of β, for the configuration in figure 9.4.

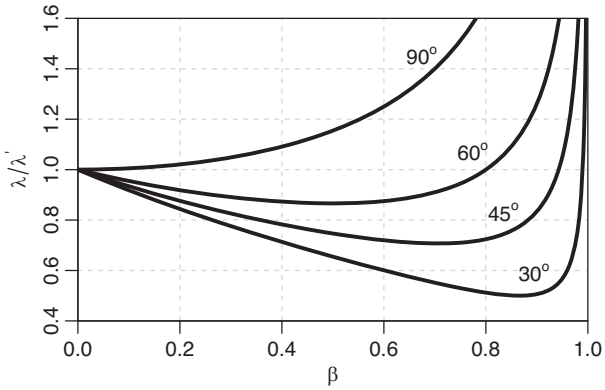


Fig. 9.11 The variation of the Doppler shift with velocity for various angles (α, labelled on the curves) for the configuration in figure 9.4 as given by equation 9.11.

cone over which we see stars blueshifted gets smaller. At $\beta = 0.5$, for example, only stars lying within 74.5° of the direction of travel (photons travelling in directions with $|\alpha| < 74.5°$) will be redshifted.

What is less evident in figure 9.9 is that the Doppler shift does not vary monotonically with velocity at a given angle. This variation is shown in figure 9.11. Take for example sources lying 60° from the direction of travel, and so have photons travelling at $\alpha = 60°$. As the speed increases from zero to more positive β, these sources initially become more blueshifted, reach a maximum blueshift (minimum λ/λ'), and then become increasingly redshifted.

9.3 Source brightness

The brightness of a source is also affected by relative motion. The spectral radiance is the power per unit area per unit solid angle per unit frequency, with units $\mathrm{W\,m^{-2}\,str^{-1}\,Hz^{-1}}$. (See section 1.5 for a summary of radiation terms.) The 'unit area' in this definition is the

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area at the receiver at some arbitrary distance from the source. In general, the spectral radiance can vary with direction and with the frequency of the radiation. Let J_ν be the spectral radiance received by an observer from a source that is moving at a velocity β in the usual configuration (figure 9.4). Let $J'_{\nu'}$ be the spectral radiance the observer would receive from the same position when at rest relative to the source. Using the expressions for time dilation, aberration, and Doppler shift (which applies to the energies of the photons), we find that the spectral radiance transforms as

$$J_\nu = J'_{\nu'} \frac{1}{\psi^3}. \tag{9.14}$$

Recall that ψ (equation 9.7) is independent of wavelength, so the spectral radiance transforms at all wavelengths in the same way. For an approaching source, $\psi < 1$ (see figure 9.9), and so the source will appear brighter than when it was at rest. Receding sources, in contrast, appear fainter. It may be useful to observe that, because J_ν and ν^3 both transform from S to S' with a factor ψ^3, the quantity J_ν/ν^3 is invariant between reference frames.

Because frequency and wavelength are not linearly related, the spectral radiance of a source defined per unit frequency (J_ν) is not the same as its spectral radiance defined per unit wavelength (J_λ). From $|J_\nu d\nu| = |J_\lambda d\lambda|$ and $d\lambda = -(\lambda^2/c)\, d\nu$ we get $J_\nu = J_\lambda \lambda^2/c$ and likewise for the primed quantities. Using this and the expression for the Doppler shift, $\lambda/\lambda' = \psi$, in equation 9.14, we see that the transformation for J_λ is

$$J_\lambda = J'_{\lambda'} \frac{1}{\psi^5}. \tag{9.15}$$

This expression (and the one for frequency) refers to monochromatic light. By integrating spectral radiance over a range of wavelengths (or frequencies) we get the radiance (units $\mathrm{W\,m^{-2}\,str^{-1}}$). Using $d\lambda = \psi\, d\lambda'$ we see that the relativistic transformation of the radiance is

$$J = \int J_\lambda d\lambda = \frac{1}{\psi^4} \int J'_{\lambda'}\, d\lambda' = \frac{1}{\psi^4} J'. \tag{9.16}$$

We of course get the same result if we replace wavelength with frequency. For the case of a source moving radially with respect to the observer, the radiance at the receiver is, using equation 7.6,

$$J = J' \left(\frac{1-\beta}{1+\beta} \right)^2 \tag{9.17}$$

where $\beta > 0$ means the source is moving radially away from the observer. An extended source receding at 10% the speed of light appears just two-thirds as bright (0.67) compared to when at rest.

In going through the derivation leading to the ψ^3 term in equation 9.14, we find that a factor of ψ^2 arises from the transformation of the solid angles, and another factor of ψ arises through a factor of γ from the time dilation (rate of reception of the photons) and a factor of $(1 - \beta\cos\alpha)$ from the aberration (of the projected area). The transformation for the photon frequencies cancels, because $\delta\nu/\nu$ is invariant. If we had a point source, as opposed to an extended source, the factor from the solid angle transformation would not appear, and so the spectral intensity (power per unit area per unit wavelength) would transform as $I_\lambda = I'_{\lambda'}/\psi^3$. The wavelength integral of this, which is just the intensity (power per unit area), would transform as $I = I'/\psi^2$.

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9.4 Summary

Special relativity affects the appearance of objects that are moving at large velocities relative to the observer.

Aberration describes the change in apparent direction to a source on account of a change in its velocity relative to the observer. It is a result of vectorial velocity addition, and so has a classical equivalent (e.g. a person running through the rain). One consequence of aberration is that for a spacecraft moving at relativistic velocities, stars appear to bunch towards the direction of travel, whereas the backward direction appears sparser (figure 9.6). An extended object in the forward direction appears smaller and also brighter (larger spectral radiance). Aberration is easy to measure astronomically: the directions to stars seen from the Earth vary by $\pm 20.5''$ due to the Earth's orbit around the Sun ($\pm 30\,\mathrm{km\,s^{-1}}$).

The Doppler effect describes the change in frequency of a pulsating signal, such as the frequency (and therefore wavelength) of light. Pulses from an object moving towards an observer are received at higher frequency. Thus for a spacecraft moving at relativistic velocities, stars in the forward direction are blueshifted. Stars in the backward direction are receding and are redshifted. Even an object moving transversely relative to an observer appears redshifted, as a consequence of time dilation.

These optical effects are relevant to interstellar travel in a number of ways. In communicating back to the Earth, the spacecraft's signal will be redshifted, so a receiver will have to be correctly tuned. To point instruments in the right direction during a fast flyby, aberration must be taken into account. In chapter 14, we will see how we can exploit aberration and the Doppler effect for navigating by the stars.

9.5 Exercises

1. Prove equation 9.5 for the aberration. Hint: Consider the Lorentz transformation for the y-component of the velocity.
2. The cosmic microwave background is the highly redshifted radiation from the hot early universe described by the Planck function (equation 15.9), with a temperature observed at our current epoch of about 2.7 K. Explain why its spectral radiance should be a Planck function for an observer at any other redshift.
3. How fast would a spacecraft have to move so that the peak of the cosmic microwave background radiation in the direction of travel appears blue (400 nm)? This will produce a concentric rainbow pattern around the direction of travel. At what angle to the direction of travel is the red circle (700 nm)?
4. How does figure 9.11 look if you extend the range to larger angles, and to negative velocities?

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