Note on Losses in Cable Sheaths. By F. W. CARTER, Sc.D., St John's College.

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1. *Introduction.* In a recent communication to the Society*, the author referred to cable-sheath losses, and gave formulae for computing them in certain cases. These appertained to power cables in which were comprised a group of conductors, arranged symmetrically and encased in a single conducting sheath. In some distribution systems, however, the conductors for the several phases are encased in separate lead sheaths, which are either laid in proximity as separate cables, or grouped and comprehended in an outer sheath. The analysis previously given does not include such cases directly. Moreover, it is common practice either to lay the elementary cables with sheaths in contact, or to bond the sheaths together at the ends of suitable sections, in order to prevent differences of potential between them; and, when this is done, a circulating current flows in the circuit of the sheaths and bonds, sufficient to maintain equality of potential between the several sheaths. This current, to which reference was made in the former paper†, is additional to the eddy current discussed therein, the integral of which over the cross section of the sheath is zero. It is for convenience here referred to as the "circulating current," to distinguish it from the "eddy current," although there is no such distinction between them as the names imply.

Since the currents due to several independent causes are additive, the solution of the problem presented by multiple sheaths can be obtained by a process of successive approximation, in which the eddy or circulating current determined from a given fixed distribution of current is used as a fixed distribution to determine subsidiary eddy and circulating currents in neighbouring sheaths, and so on to any degree of approximation that may be desired. Whilst the complete solution of the physical problem is thus envisaged, that of the engineering problem is fortunately less exacting; for the degree of approximation needed is not in general such as to render the subsidiary currents of practical importance.

2. *Notation.* We use a notation corresponding to that employed in the former paper, viz.:

a = mean radius of sheath,

b = distance between centres of neighbouring sheaths,

k = product of thickness and specific conductivity of sheath,

* *Proceedings*, vol. 23 (1927), pp. 901–906.

† *L.c.*, note on § 3, p. 903.

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$p = 2\pi$ times the frequency of alternation,

$m = 2\pi kap$,

σ = product of current density in sheath by its thickness,

ϕ_1, ψ_1 = respectively potential function and flux function of eddy current field within sheath,

ϕ_0, ψ_0 = corresponding functions of eddy current field outside sheath,

ϕ', ψ' = field functions of disturbing field,

C = maximum value of current in conductor,

C' = maximum value of circulating current in sheath.

Where two or more quantities of the same nature are in question, the symbols are distinguished by dashes or subscripts. It is convenient in the interests of conciseness to perform the analysis in terms of complex quantities, but, for the sake of clearness, two symbols are used to distinguish the imaginary, viz.:

i designates the imaginary in space relations,

j designates the imaginary as an operator which controls phase.

3. *Primary Circulating Current.* We assume that there is no drop of potential in the bonds, or that the circulating current is determined by the mean E.M.F. per unit length of sheath. We consider first the single-phase case of Fig. 1*. The potential and flux functions outside the sheaths are given by†

$$\phi + i\psi = 2(C + C')\, i \log \left(\frac{r}{r'} e^{i(\theta - \theta')} \right) \quad \ldots\ldots\ldots(1).$$

The total flux between a point on the left hand sheath and a distant point is

$$-2(C + C') \log \frac{a}{(a^2 + b^2 - 2ab\cos\theta)^{\frac{1}{2}}}.$$

The E.M.F. per unit length in the sheath is, at this point,

$$2(C + C')\, jp \log \frac{a}{(a^2 + b^2 - 2ab\cos\theta)^{\frac{1}{2}}}$$

The mean value of this for the left hand sheath is

$$-2(C + C')\, jp \log \frac{b}{a},$$

* The outer sheath shown in this figure and the following is not now under consideration. It was discussed in the former paper, and is herein referred to only in section 6.

† The factor e^{jpt} is omitted throughout, for the sake of brevity.

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which causes current C' in resistance $1/(2\pi ak)$. Hence

$$C' + 2(C + C')jm \log \frac{b}{a} = 0 \quad \text{.....(2),}$$

or
$$C' = \frac{-2jm \log \frac{b}{a}}{1 + 2jm \log \frac{b}{a}} C \quad \text{.....(3),}$$

and
$$C + C' = \frac{C}{1 + 2jm \log \frac{b}{a}} \quad \text{.....(4).}$$

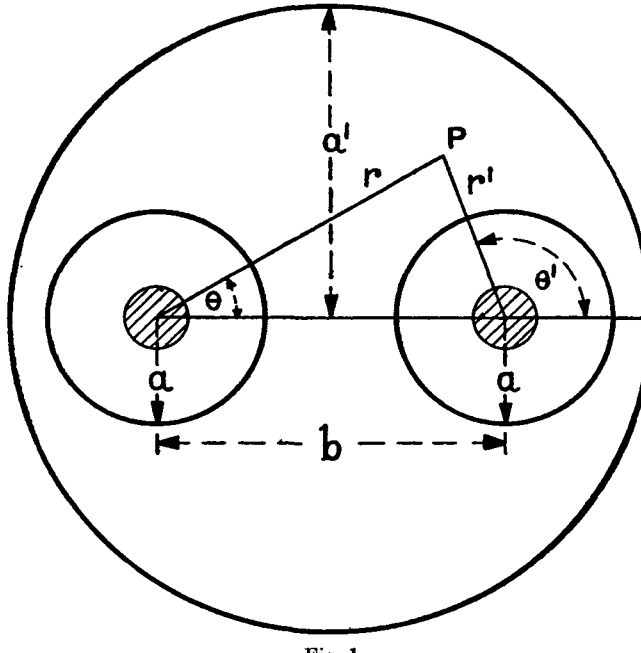


Fig. 1.

When the outward and return cable-sheaths differ in material, radius or thickness, the condition that the potential falls at the same rate in both is

$$\frac{C'}{m} + 2(C + C')j \log \frac{b}{a} = -\frac{C'}{m'} - 2(C + C')j \log \frac{b}{a'},$$

and accordingly

$$C' + 2(C + C')j \frac{mm'}{m + m'} \log \frac{b^2}{aa'} = 0 \quad \text{.....(5).}$$

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We consider next a symmetrical arrangement of conductors for three-phase transmission (Fig. 2). Since the sum of the currents is zero at all times, we may write $-(C_2+C_3)$ for C_1, and the problem reduces to that first considered. The solution is expressed by equations (3) or (4), in which C is now any phase current, and C' the corresponding circulating current. This is clearly true whether or not the load is a balanced one.

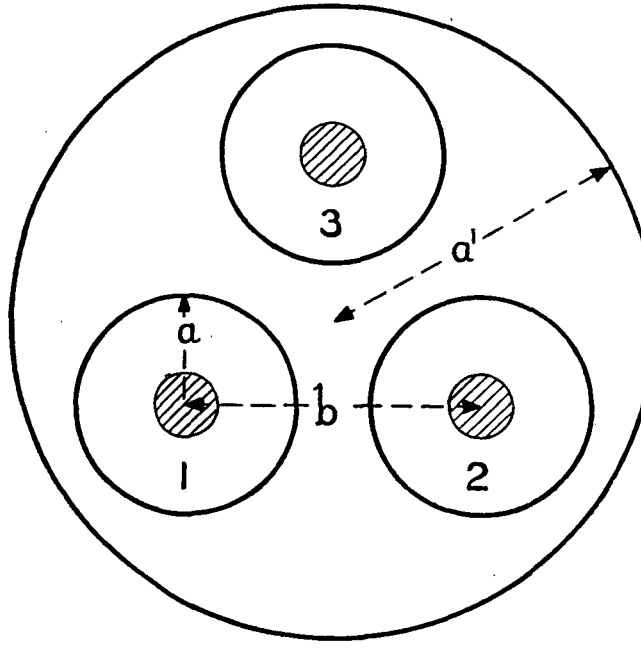


Fig. 2.

In the general case of any number of dissimilar sheaths bonded together and unsymmetrically arranged, the circulating currents are given by

$$\left.\begin{aligned} -\frac{C_1'}{m_1} + 2(C_1+C_1')j\log a_1 + 2(C_2+C_2')j\log b_{12} + \ldots \\ = \text{symmetrically derived expressions for each of the other sheaths, together with} \\ C_1' + C_2' + \ldots = 0 \end{aligned}\right\} \ldots(6).$$

This includes the case in which the sheaths are bonded to parallel pipes, or to the sheaths of dead cables. The most important case of unsymmetrical arrangement is that in which three-phase power

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is conveyed by three similar cables laid in a plane with equal spacing (Fig. 3), so that

$$m_1 = m_2 = m_3, \quad a_1 = a_2 = a_3, \quad b_{12} = b_{23} = \tfrac{1}{2} b_{13}.$$

The equations then reduce to

$$\left.\begin{aligned} C_1' + 2jm\left[(C_1 + C_1')\log\frac{2b}{a} + \tfrac{2}{3}(C_2 + C_2')\log 2\right] &= 0 \\ C_2' + 2jm\left[(C_2 + C_2')\log\frac{b}{2a} + \tfrac{2}{3}(C_2 + C_2')\log 2\right] &= 0 \\ C_3' + 2jm\left[(C_3 + C_3')\log\frac{2b}{a} + \tfrac{2}{3}(C_2 + C_2')\log 2\right] &= 0 \end{aligned}\right\} \quad \ldots(7).$$

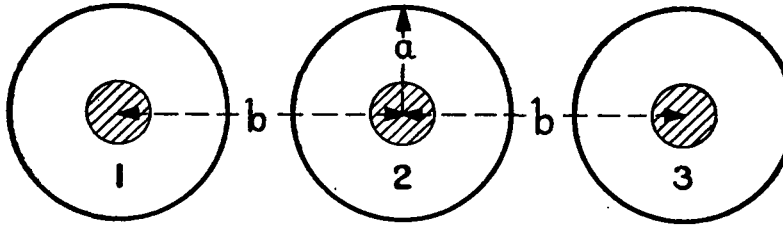


Fig. 3.

4. *Primary Eddy Current.* The primary eddy current is composed of elements due to line currents outside the sheath, and was substantially determined in the previous paper. It is dealt with here briefly for the sake of uniformity of treatment.

The equation satisfied by ϕ_1 at the surface of a sheath may be written thus:

$$\frac{\partial^2 \phi_1}{\partial \theta^2} = jma\frac{\partial(\phi_1 + \phi')}{\partial r} \quad \ldots\ldots\ldots\ldots\ldots\ldots(8).$$

In the problems with which we are concerned, the solution may be assumed as the real part (having regard to space only) of one or more expressions of the form

$$\Sigma A_n i\left(\frac{r}{a}\right)^n e^{in(\theta+\alpha)}.$$

In the case of Fig. 1, for the left hand sheath,

$$\begin{aligned} \phi' + i\psi' &= -2(C + C')\, i \log(r' e^{i\theta'}) \\ &= -2(C + C')\, i \log(re^{i\theta} - b) \\ &= -2(C + C')\, i\left[\log(-b) - \sum_1^\infty \frac{1}{n}\left(\frac{r}{b}\right)^n e^{ni\theta}\right]. \end{aligned}$$

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From equation 8,

$$\Sigma n(n+jm)A_n ie^{ni(\theta+\alpha)} + 2(C+C')jmi\sum_1^\infty \left(\frac{a}{b}\right)^n e^{ni\theta} = 0,$$

or
$$A_n = \frac{-2jm(C+C')}{n(n+jm)}\left(\frac{a}{b}\right)^n, \text{ and } \alpha = 0. \quad \ldots\ldots\ldots(9).$$

Hence

$$\phi_1 + i\psi_1 = -2jm(C+C')i\sum_1^\infty \frac{1}{n(n+jm)}\left(\frac{r}{b}\right)^n e^{ni\theta} \quad \ldots(10),$$

$$\phi_0 + i\psi_0 = 2jm(C+C')i\sum_1^\infty \frac{1}{n(n+jm)}\left(\frac{a^2}{br}\right)^n e^{ni\theta} \quad \ldots(11).$$

The current density

$$\sigma = \frac{2jm}{2\pi a}(C+C')\sum_1^\infty \frac{1}{n+jm}\left(\frac{a}{b}\right)^n \cos n\theta$$

$$= \frac{2jm}{2\pi a}(C+C')\left[\log \frac{b}{(a^2+b^2-2ab\cos\theta)^{\frac{1}{2}}} - \sum_1^\infty \frac{jm}{n(n+jm)}\left(\frac{a}{b}\right)^n \cos n\theta\right]$$

$\ldots\ldots\ldots\ldots(12).$

When θ is zero and m is small*, σ approximates to

$$\sigma = \frac{2jm}{2\pi a}(C+C')\log\frac{b}{b-a}.$$

Adding the circulating current density from equation 2 gives as resultant †,

$$\frac{2jm}{2\pi a}(C+C')\log\frac{a}{b-a},$$

which is zero when the sheaths are in contact ($b=2a$). Since it is a physical necessity that there should be no discontinuity in current in passing through the point of contact, it may be inferred that the proposed method of successive approximation does not tend to become inoperative as the sheaths are brought together.

The primary eddy current in the case of Fig. 2 can be deduced immediately as the sum of components. Thus for sheath (1) the potential and flux functions are given by

$$\phi_1 + i\psi_1 = 2jmi\sum_1^\infty \frac{1}{n(n+jm)}\left(\frac{r}{b}\right)^n e^{ni\theta}\left[(C_2+C_2') + (C_3+C_3')e^{-ni\frac{\pi}{3}}\right]$$

$\ldots\ldots\ldots\ldots(13),$

* For lead sheaths, and for a frequency of 50 cycles, m is generally between 0·04 and 0·1.

† This formula was given, substantially, by Prof. W. Cramp and Miss N. I. Calderwood; but was wrongly assumed by them to apply to the whole sheath, instead of the point nearest to the return sheath only ($\theta=0$). See "The use of single-core sheathed cables for alternating currents," *Journal I.E.E.*, vol. 61 (1923), p. 477. See also "Losses in grounded sheaths of single conductor cables," by H. B. Dwight, *The Electric Journal*, 1924, vol. 21, p. 62, and "Investigation of conflicting cable-sheath loss formulae," by L. C. Hutchison, *Journal I.E.E.*, vol. 65 (1927), p. 903.

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with a corresponding expression for $\phi_0 + i\psi_0$; and similarly for the other sheaths. For the case of Fig. 3 the formula becomes

$$\phi_1 + i\psi_1 = 2jmi \sum_1^\infty \frac{1}{n(n+jm)} \left(\frac{r}{b}\right)^n e^{ni\theta} \left[(C_2 + C_2') + \frac{1}{2^n}(C_3 + C_3')\right] \quad \ldots\ldots\ldots\ldots(14),$$

for cable (1), with a similar expression for cable (3); whilst for the middle cable

$$\phi_1 + i\psi_1 = 2jmi \sum_1^\infty \frac{1}{n(n+jm)} \left(\frac{r}{b}\right)^n e^{ni\theta} \left[(C_3 + C_3') + (C_1 + C_1')\, e^{ni\pi}\right] \quad \ldots\ldots\ldots\ldots(15).$$

5. *Secondary Currents.* The potential and flux functions of the eddy current in any sheath having been determined, the corresponding functions for the secondary currents produced by them in other sheaths are determinable. Let r', θ' be the coordinates of a point referred to a pole at $be^{i\alpha}$, and let the field functions of an eddy current in a sheath centred at this pole be, at outside points,

$$\Sigma A_{q,n} i \left(\frac{a}{r'}\right)^n e^{ni(\theta' + \beta)}.$$

Changing the sign of i this becomes

$$\Sigma A_{q,n} i \left(\frac{a}{r'}\right)^n e^{-ni(\theta' + \beta)}$$

Now

$$r' e^{i(\theta' + \beta)} = r e^{i(\theta + \beta)} - b e^{i(\alpha + \beta)}.$$

Hence the field function becomes, near $r = a$,

$$\Sigma A_{q,n} i a^n \left[r e^{i(\theta+\beta)} - b e^{i(\alpha+\beta)}\right]^{-n}$$
$$= \Sigma A_{q,n} i \left(\frac{a}{b}\right)^n e^{ni(\pi - \alpha - \beta)} \left[1 + \sum_1^\infty \frac{(n+s-1)!}{(n-1)!\, s!} \left(\frac{r}{b}\right)^s e^{si(\theta - \alpha)}\right] \quad \ldots\ldots\ldots\ldots(16).$$

Assuming the field of the eddy current set up in a sheath centred at the origin to be given by

$$\phi_1 - i\psi_1 = \Sigma A_{q+1,n} i \left(\frac{r}{a}\right)^n e^{ni(\theta + \gamma)},$$

equation 8 gives

$$n(n+jm) A_{q+1,n} e^{ni\gamma} + jm \left(\frac{a}{b}\right)^n e^{-ni\alpha} \Sigma A_{q,s} \left(\frac{a}{b}\right)^s \frac{(n+s-1)!}{(n-1)!\,(s-1)!} e^{si(\pi - \alpha - \beta)} = 0 \quad \ldots\ldots\ldots\ldots(17).$$

This connects eddies of successive orders.

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The coefficient of $-i$ in equation (16) is the flux function, and the mean value of the E.M.F. at the sheath, due to the flux, is

$$-jp\Sigma A_{q,n}\left(\frac{a}{b}\right)^n \cos n(\pi-\alpha-\beta).$$

Hence, in a symmetrical system, a circulating current is produced, the value of which is given by

$$C'_{q+1} = -jm\Sigma A_{q,n}\left(\frac{a}{b}\right)^n \cos n(\pi-\alpha-\beta) \quad \ldots\ldots(18).$$

It will be seen that the lowest power of m in the expression for an eddy or circulating current is given by the order (q) of the current.

6. *First Approximations.* It has been remarked that the problem of cable-sheath losses does not call for the labour of meticulous calculation; and a first approximation to the correct value is usually sufficient. Formulae of such approximation are given below for the more usual arrangements, working under conditions of balanced loading. It is usual and convenient to express the result as a fraction of the resistance loss in the conductors, the value of which, for each conductor, will be written $\frac{1}{2}RC^2$ per unit length. Absolute units and hyperbolic logarithms are used throughout.

i. For separate single-phase cables (Fig. 1),

$$\text{Sheath-loss} = \frac{2mp}{R}\left\{2\left(\log\frac{b}{a}\right)^2 + \sum_1^\infty \frac{1}{n^2}\left(\frac{a}{b}\right)^{2n}\right\} \quad \ldots(19)^*.$$

ii. If the sheathed conductors are encased in an outer sheath of radius a', and constant m', add

$$\frac{4m'p}{R}\sum_{1(n\text{ odd})}^\infty \frac{1}{n^2}\left(\frac{b}{2a'}\right)^{2n} \quad \ldots\ldots(20).$$

iii. For separate three-phase cables arranged symmetrically (Fig. 2),

$$\text{Sheath-loss} = \frac{2mp}{R}\left\{2\left(\log\frac{b}{a}\right)^2 + \sum_1^\infty \frac{1+2\sin^2\frac{n\pi}{6}}{n^2}\left(\frac{a}{b}\right)^{2n}\right\}$$
$$\ldots\ldots(21).$$

* In equations (19), (21), (23) and (24), the first term within the brackets, which arises from the circulating current, assumes the sheaths in contact or bonded together. In the alternative the term should be omitted, or suitably modified.

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iv. If the sheathed conductors are encased in an outer sheath of radius a', and constant m', add

$$\frac{4m'p}{R}\sum_{1}^{\infty}\frac{\sin^2\frac{n\pi}{3}}{n^2}\left(\frac{b}{\sqrt{3}a'}\right)^{2n} \quad \text{.....(22).}$$

v. For separate three-phase cables, arranged with equal spacing in a plane (Fig. 3),

Loss in centre cable-sheath

$$=\frac{2mp}{R}\left\{2\left(\log\frac{b}{a}-0{\cdot}231\right)^2+\sum_{1}^{\infty}\frac{1+2\sin^2\frac{n\pi}{2}}{n^2}\left(\frac{a}{b}\right)^{2n}\right\} \quad \text{.....(23).}$$

Loss in outer cable-sheaths

$$=\frac{2mp}{R}\left\{2\left[\left(\log\frac{b}{a}+0{\cdot}462\right)^2+0{\cdot}16\right]+\sum_{1}^{\infty}\frac{1}{n^2}\left(1-\frac{1}{2^n}+\frac{1}{2^{2n}}\right)\left(\frac{a}{b}\right)^{2n}\right\}$$

.....(24).

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