J. Plasma Physics (1999), *vol.* 62, *part* 3, *pp.* 345–350. Printed in the United Kingdom

© 1999 Cambridge University Press

Alfvén resonances and forced reconnection

CHANCHAL UBEROI* and ELLEN G. ZWEIBEL

[1] Bell Laboratories, Lucent Technologies, Murray Hill, NJ 07974, USA
[2] JILA, University of Colorado at Boulder, Campus Box 440, Boulder, CO 80309-0440, USA

(Received 1 December 1998 and in revised form 15 March 1999)

Abstract. The correspondence between the forced magnetic reconnection induced by perturbing the boundary of the simple Taylor model and the surface-wave-induced magnetic reconnection given by Alfvén resonance theory is pointed out explicitly by showing that the theory of forced magnetic reconnection is actually embedded in the Alfvén resonance theory. The advantages of viewing the forced reconnection as surface-wave-induced reconnection are briefly discussed in the context of the formation of small-scale structures at the magnetospheric boundary and solar coronal heating.

1. Introduction

Forced magnetic reconnection induced by perturbing the boundary of a simple Taylor model, a simple slab equilibrium of an incompressible plasma with a resonant surface inside, was investigated by Hahm and Kulsrud (1985) (hereinafter referred to as HK) by studying the time evolution of the MHD system near the neutral point for linear magnetic field variation. HK first consider a change in boundary that is rapid compared with any resistive time scale, and then follow the subsequent time evolution of the plasma. The entire process, as they note, seems to occur on four time stages. The first phase, A, corresponds to ideal MHD theory. The MHD equation governing the dynamics of linear perturbations of the boundary is solved as an initial-value problem near the resonant surface, with the assumption of long-wavelength perturbations. The similarity solutions of the equation show linear growth with time. In phase B, a small resistive correction is added, but it is seen that the dynamics of the plasma motion are unaffected by the resistivity. After a time of order $\tau_R^{1/3}\tau_A^{2/3}$, the reconnection affects the dynamics, and a full resistive theory must be employed. In phase C, the resistive MHD equations are treated by carrying out a tearing-mode analysis near the resonant surface similar to that given by Furth et al. (1973) (hereinafter referred to as FKR). In phase D, the reconnection process occurs and magnetic islands are formed at the tearing-mode time scale $\tau_A^{2/5}\tau_R^{3/5}$ defined by FKR.

The phenomenon of resonant absorption of Alfvén waves at a non-zero singular point is well known (see e.g. Hasegawa and Uberoi 1982, and references therein). Resonant absorption of Alfvén waves near a zero singular point was discussed by Uberoi (1994), who pointed out that the similarity solution as

* Permanent address: Department of Mathematics, Indian Institute of Science, Bangalore 560012, India.

https://doi.org/10.1017/S0022377899007813 Published online by Cambridge University Press

obtained by HK in phase A and the resistive time scales for which resistive effects become important in phases B and C can be obtained very easily from the general time-dependent solution of the Alfvén-wave equation governing the dynamics of ideal and resistive MHD in the presence of an inhomogeneous magnetic field given by Uberoi and Sedláček (1990) and Kappraff and Tataronis (1977) by assuming a linear magnetic field variation. Taking advantage of this mathematical similarity between the problems of resonant absorption of Alfvén waves at a neutral point and forced magnetic reconnection, Uberoi (1990, 1994) emphasized that zero-frequency Alfvén waves in a shear inhomogeneous MHD system are closely associated with time-dependent magnetic reconnection across the neutral layer. Subsequently, applications of these results to the understanding of tearing-mode instability and magnetic reconnection along the magnetospheric boundary were considered by Uberoi et al. (1996). It was argued that surface waves with wavelengths larger than the thickness of a neutral layer separating two different plasma regimes can induce tearing-mode instability. Magnetic reconnection was seen to occur on the tearing-mode time scale as defined by FKR. The size of the islands suggested that surface-wave-induced magnetic reconnection may be responsible for the smallest-scale FTEs that are observed at the Earths' magnetopause.

However, the link between the theories of Alfvén resonances and forced magnetic reconnection was pointed out only implicitly (Uberoi 1994). The aim of the present paper is to point out explicitly the correspondence between forced magnetic reconnection and surface-wave-induced magnetic reconnection given by Alfvén resonance theory. We shall actually show that theory of forced magnetic reconnection is embedded in Alfvén resonance theory and that the resistive relaxation study of HK is a special case of this theory. This correspondence may not give rise to any new results or new physics, but is certainly important for basic studies in plasmas since it can give new perspective to the theories of tearing-mode instabilities, magnetic reconnection and the Alfvén-resonance mechanism. Further, by linking these two aspects of resistive-layer theory, we shall be able to carry over results from one problem to the other and to assess the realms of validity of various approximations. Some advantages of viewing forced magnetic reconnection by boundary perturbations as surface-wave-induced magnetic reconnection are finally discussed briefly.

2. Basic formulation and review

We choose for the equilibrium problem a sheared slab of magnetic field. $\mathbf{B} = \hat{\mathbf{y}}B_{0y}(x) + \hat{\mathbf{z}}B_{0z}(x)$. We satisfy force balance by assuming that the magnetic field strength $(B_{0y}^2 + B_{0z}^2)^{1/2}$ is constant. We also assume that $|B_{0y}/B_{0z}| \ll 1$ everywhere. This allows us to assume that waves and other perturbations that are independent of $\hat{\mathbf{z}}$ behave incompressibly. We take the plasma density ρ_0 to be uniform and constant.

We shall generally be interested in the case where B_{0y} has a null, which we choose to be at $x = 0$. We shall assume that in the neighbourhood of the null, B_{0y} has a linear profile:

$$B_{0y} = B_0 x/l_B$$

(which defines B_0/l_B).

https://doi.org/10.1017/S0022377899007813 Published online by Cambridge University Press

Finally, we assume that the plasma resistivity η is small, in a sense that we make precise below.

2.1. Forced reconnection

HK considered a thin strip of incompressible plasma initially confined within $|x| < l_B$, with $B_{0y} = B_0\, x/l_B$. If the boundaries are perturbed so as to satisfy the equations

$$x = \pm\,(l_B - \delta \cos ky), \quad \frac{\delta}{l_B} \ll 1,$$

then linear theory predicts that the plasma has two possible equilibrium states. One, EQI, has the same topology as the original state, but has a current sheet ($\hat{\mathbf{z}}J \to \infty$) at $x = 0$. The other, EQII, has no current sheet, but has magnetic islands centred on the original null surface $x = 0$.

HK showed that the relaxation of the plasma, considered as an initial-value problem, takes place in several stages, which are defined in terms of the resistive time τ_R, and the Alfvén time τ_A:

$$\tau_R \equiv \frac{4\pi l_B^2}{\eta}, \quad \tau_A \equiv \frac{(4\pi\rho_0)^{1/2}\, l_B}{B_0}. \tag{1}$$

At first, resistivity is negligible, and the plasma evolves toward EQI, with the current on the null line, $J_z(0)$, increasing linearly with time. After a time

$$\tau_H \approx \tau_R^{1/3} \tau_A^{2/3}, \tag{2}$$

resistive effects become important. The system then evolves according to the scaling originally found for tearing modes (FKR), with characteristic timescale

$$\tau_T \approx \tau_R^{3/5} \tau_A^{2/5}. \tag{3}$$

The final state is EQII.

2.2. Alfvén resonance and Alfvén surface waves

If a wave of frequency ω and wave vector $\mathbf{k} = \hat{\mathbf{y}}k$ is launched into a magnetized plasma with the properties summarized above then Alfvén resonance is said to occur at any location x_r at which the local Alfvén frequency

$$\omega_A \equiv \frac{kB_{0y}}{(4\pi\rho_0)^{1/2}}$$

is equal to $\pm\omega$ (Uberoi 1972; Hasegawa and Uberoi 1982). If the source of waves is monochromatic and steady then wave energy piles up at the singular layer. Initially, the current density $\hat{\mathbf{z}}J_z(x_r)$ grows linearly with time (Uberoi and Sedláček 1992). If one generalizes the definition of l_B to

$$l_B^{-1} \equiv \left.\frac{d \ln \omega_A(x)}{dx}\right|_{x_r}$$

and defines the resistive and Alfvén times τ_R and τ_A according to (1) then resistive effects become important within the layer after a time τ_H as given by (2) (Kappraff and Tataronis 1977). This is the time scale on which resistive heating at the Alfvén resonance reaches a steady state.

https://doi.org/10.1017/S0022377899007813 Published online by Cambridge University Press

Uberoi (1994) has considered Alfvén resonance in the limit $\omega_A \to 0$. She shows that for a magnetic profile with linear shear, the onset of resistive behaviour still occurs at $t \approx \tau_H$. Moreover, if one replaces the continuously sheared magnetic profile by a discontinuous profile such that

$$\omega_A^2(x) = \omega_{A2}^2 - (\omega_{A2}^2 - \omega_{A1}^2)\,[H(x+a) - H(x-a)]$$

(where H is the Heaviside function, and ω_{A1} and ω_{A2} are constants), and considers waves with wavenumber $k \ll a^{-1}$, then the layer has two possible eigenmodes, which decay exponentially with distance outside it. One mode is antisymmetric, with the wave speed given by the Alfvén speed outside the layer, and the other mode is symmetric, with the wave speed equal to the Alfvén speed within the layer. In the limit that the Alfvén frequency within the layer, ω_{A1}, approaches zero, the eigenfunction for the symmetric mode is similar to the eigenfunction for the tearing mode. Uberoi asserts that this mode couples resonantly to the tearing mode if resistivity is included.

3. Unified picture

The overall similarity of the two problems discussed in Sec. 2, particularly the appearance of the time scale τ_H introduced in (2), leads us to claim that the resistive relaxation studied by HK is equivalent to resonant absorption in the limit of zero driving frequency. This provides an interpretation of the boundary perturbation, and makes the connection between the symmetric layer mode found by Uberoi (1994) and the tearing mode.

The Alfvén-wave equation (Uberoi 1972) for the geometry described in Sec. 2 can be written as

$$\frac{\partial}{\partial x}\left[\left(\frac{\partial^2}{\partial t^2} + \omega_A^2\right)\frac{\partial v_{1x}}{\partial x}\right] - k^2\left(\frac{\partial^2}{\partial t^2} + \omega_A^2\right)v_{1x} = 0, \tag{4}$$

where v_{1x} is the x component of the velocity perturbation. Resistivity is omitted in the derivation of (4). The x component of the fluid displacement, ξ_x, which is related to v_{1x} by $v_{1x} = \partial\xi_x/\partial t$, also satisfies (4). If we now assume the same linearly varying magnetic profile as HK then

$$\omega_A^2 = \frac{k^2 B_0^2 x^2}{4\pi\rho_0 l_B^2}.$$

Assuming that $k \ll \partial/\partial x$ and introducing again the Alfvén time scale τ_A from (1), (4) can be approximated as

$$\tau_A^2 \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 \xi_x}{\partial x^2}\right) + k^2 \frac{\partial}{\partial x}\left(x^2 \frac{\partial \xi_x}{\partial x}\right) = 0. \tag{5}$$

Equation (5) is identical to the starting equation (20) of HK, and appears in that paper as the equation governing the initial, non-resistive behaviour of the system. Hence equation (20) of HK has a singularity, and normal-mode analysis gives a continuum and singular solution near $x = x_0$ (where $x_0 = 0$).

HK found a similarity solution to (5). The similarity variable is xt, and the

https://doi.org/10.1017/S0022377899007813 Published online by Cambridge University Press

solution describes a layer with current-density amplitude increasing as t, and width decreasing as t^{-1}. The same power-law growth emerges from an integro-differential solution to (4), provided that one chooses the initial displacement to be non-zero and the initial velocity to be zero, as given by Uberoi and Sedláček (1992).

Resistivity becomes important after the time τ_H defined in (2). This can be seen directly by comparing the inductive and resistive electric fields implied by the solution to (5), or from asymptotic analysis in Laplace space.

Finally, HK carried their solution into the fully resistive regime, and showed that magnetic islands form on the tearing-mode timescale $\tau_T \approx \tau_H^{3/5}\tau_A^{2/5}$. This suggests that reconnection induced by setting up a symmetric surface wave at a neutral layer would proceed at the same rate (Uberoi et al. 1996).

4. Conclusions

The correspondence between the theories of forced magnetic reconnection and Alfvén resonances indicates that the Taylor-model problem of inducing magnetic islands as discussed by HK can be viewed as surface-wave-induced magnetic islands in an inhomogeneous MHD system with sheared magnetic field. HK essentially studied the resonant problem with singularity $x_0 = 0$. The sheared MHD system can be seen as a series of different layers oscillating with the local Alfvén frequency $\omega_A(x)$. The neutral layer in the limiting sense oscillates with zero frequency. When $\omega_A(x) \neq 0$, we require a rapidly moving boundary to create waves that will give resonances. For $\omega_A(x) = 0$, the very slowly moving boundary will create the waves giving the resonances.

The advantage of viewing forced magnetic reconnection as surface-wave-induced reconnection (SWIMR) is that surface waves are observed in physical systems like the Earth's magnetopause, magnetotail and solar coronal loops. The applications of SWIMR have been discussed by Uberoi et al. (1996) to explain the observation of small-scale structures or FTEs at the magnetopause. Solar coronal heating has been discussed in the literature both in terms of forced current-sheet reconnection (Wang and Bhattacharjee 1992; Venkstein and Jain 1998) and resonant absorption of Alfvén surface waves in coronal loops (Ionson 1978; Bruhwiler and Zweibel 1992; Similon and Sudan 1989). The link between forced reconnection and Alfvén resonance theory indicates that the physical processes of Alfvén-resonant heating and magnetic reconnection in the coronal plasma can both be linked to excitation of a spectrum of surface waves at the footprints of the coronal loops. Higher-frequency surface waves are resonantly absorbed to heat the coronal loops, while lower-frequency surface waves can induce the tearing instability in the current sheets, leading to magnetic reconnection on the tearing-mode time scale.

Acknowledgements

We are happy to acknowledge the support of NASA through Space Physics Theory Program Grant NAG5-2256 and Astrophysical Theory Program Grant NAG5-4063, and the support of the NSF through Grant AST 9521179, all to the University of Colorado. C.U. is grateful for the hospitality of JILA, where portions of this work were carried out.

https://doi.org/10.1017/S0022377899007813 Published online by Cambridge University Press

References

Bruhwiler, D. L. and Zweibel, E. G. 1992 *J. Geophys. Res.* **97**, 10825.

Furth, H. P., Killeen, J. and Rosenbluth, M. N. 1963 *Phys. Fluids* **16**, 459.

Hahm, T. S. and Kulsrud, R. M. 1985 *Phys. Fluids* **28**, 2412.

Hasegawa, A. and Uberoi, C. 1982 *The Alfvén Wave*. Technical Information Center, US Department of Energy, Oak Ridge, TN.

Ionson, J. A. 1978 *Astrophys. J.* **226**, 650.

Kappraff, J. M. and Tataronis, J. A. 1977 *J. Plasma Phys.* **18**, 209.

Similon, P. L. and Sudan, R. N. 1989 *Astrophys. J.* **336**, 442.

Uberoi, C. 1972 *Phys. Fluids* **15**, 1673.

Uberoi, C. 1990 In: *Basic Plasma Processes on the Sun* (ed. E. R. Priest and V. Krishan), p. 275. Kluwer, Amsterdam.

Uberoi, C. 1994 *J. Plasma Phys.* **52**, 215.

Uberoi, C. and Sedláček, Z. 1992 *Phys. Fluids* **B4**, 6.

Uberoi, C., Lanzerotti, L. J. and Wolfe, A. 1996 *J. Geophys. Res.* **101**, 24979.

Venkstein, G. E. and Jain, R. 1998 *Phys. Plasmas* **5**, 1506.

Wang, X. and Bhattacharjee, A. 1992 *Phys. Fluids* **B4**, 1795.

https://doi.org/10.1017/S0022377899007813 Published online by Cambridge University Press