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RAMSEY'S TEST, ADAMS' THESIS, AND LEFT-NESTED CONDITIONALS

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Abstract. Adams famously suggested that the acceptability of any indicative conditional whose antecedent and consequent are both factive sentences amounts to the subjective conditional probability of the consequent given the antecedent. The received view has it that this thesis offers an adequate partial explication of Ramsey's test, which characterizes graded acceptability for conditionals in terms of hypothetical updates on the antecedent. Some results in van Fraassen (1976) may raise hope that this explicatory approach to Ramsey's test is extendible to left-nested conditionals, that is, conditionals whose antecedent is itself conditional in form. We argue that this interpretation of van Fraassen's results is to be rejected. Specifically, we provide an argument from material inadequacy against a generalization of Adams' thesis for left-nested conditionals.

Ernest Adams' well-known thesis on the acceptability of conditionals says that the degree of acceptability for a given conditional amounts to the corresponding conditional probability of the consequent given the antecedent.[1] Notwithstanding the open question of whether ordinary speakers' normative assessments lend evidential support to the thesis, it seems to still be common ground in the philosophical community that it provides an adequate explication of Ramsey's test, which gives an informal characterization of acceptability for conditionals. Even that granted, it is to be stressed that Adams' thesis provides at best only a partial account of graded acceptability for conditionals. For it concerns only *simple* conditionals, that is, conditionals whose antecedent and consequent are factive sentences, that is, sentences which are atoms or obtainable from atoms by means of Boolean connectives. Some results in van Fraassen (1976) may raise hope that Adams' explicatory approach is extendible to instances of Ramsey's test for left-nested conditionals, that is, conditionals whose antecedent is itself conditional in form. We argue that this interpretation of van Fraassen's results is to be rejected. In particular, we show that, first, whatever type of probabilism for languages with a conditional that one may plausibly consider, conditional probability given a simple conditional is bound to meet certain constraints; and second, for some instances of Ramsey's test for left-nested conditionals, it is perfectly reasonable to violate these constraints. In effect, we provide an argument from material inadequacy against a generalization of Adams' thesis for left-nested conditionals. By the same token, we give a novel material inadequacy argument against Stalnaker's hypothesis,

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[1] For simplicity, in what follows we omit reference to utterances of sentences and just refer to sentences. Unless specified otherwise, we refer to indicative conditionals throughout as *conditionals.*

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which implies a generalization of Adams' thesis to left-nested conditionals. To begin with, we introduce some key notions and results which are relevant to the further discussion.

§1. Ramsey's test and Adams' thesis. In a famous footnote in his "General propositions and causality", Ramsey (1929) suggested a way of assessing the acceptability of conditionals by way of hypothetically updating a given epistemic state:[2]

> If two people are arguing "If p will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense "If p, q" and "If p, $\neg q$" are contradictories. We can say they are fixing their degrees of belief in q given p. (Ramsey (1929:155 n))

This statement seems to have two implications. For one, the degree of belief for a conditional $\ulcorner \varphi \rightarrow \psi \urcorner$ is given by the degree of belief regarding the consequent ψ given the antecedent φ.[3] For another, Ramsey's concluding qualification of the test suggests the following operational characterization of conditional degrees of belief: to fix the conditional degree of belief for ψ given φ, consider the degrees-of-belief distribution that is obtainable from your actual one by hypothetically adding φ to what you already know, and take the degree of belief for ψ relative to that hypothetical new distribution.[4] In what follows, we will refer to an interpretation of Ramsey's footnote that has these two implications as *Ramsey's test* (RT); the term *conditional degree of belief* will be used in the informal sense as characterized by RT. RT leaves the notion of hypothetical updating still fairly unconstrained. So, how to explicate RT in more exact terms? Before turning to something that may be prima facie deemed a natural explication strategy for RT, we make two preliminary remarks.

The first concerns the variety of instances RT may have. The simplest type of case of graded belief involving conditionals is graded belief regarding a *simple* conditional, that is, a conditional whose antecedent and consequent are both factive (i.e., atoms or Boolean combinations thereof). It seems incontrovertible that, in the same sense in which graded belief may pertain to simple conditionals, it may also pertain to certain types of embeddings of simple conditionals. One such type of embeddings are Boolean embeddings. Consider, for example, these sentences:

- It won't rain, but if it does rain, it won't pour.
- It is not the case that if it rains, it will pour.
- Either if it rains it will pour, or if it rains it won't pour.[5]

Another type of embeddings are iterated conditionals of both varieties, that is, right-nested conditionals—conditionals of the form $\varphi \rightarrow (\chi \rightarrow \psi)$—as well as left-nested

[2] Ramsey may have suggested this test not exclusively for indicative conditionals, but also for subjunctive conditionals. For the purposes of our discussion, this issue is irrelevant.

[3] When saying that a relevant epistemic subject *has a degree of belief of x regarding a sentence* $\ulcorner \varphi \urcorner$, we could equivalently say that *she believes to degree x that* φ.

[4] In this reading of Ramsey's footnote, we agree with Bennett (2001:53).

[5] The examples are from Adams (1998:268), who offers a partial account of embedded conditionals by showing that at least some such conditionals can be naturally reinterpreted in terms of unembedded ones.

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conditionals, that is, conditionals of the form $(\varphi \rightarrow \chi) \rightarrow \psi$. For example, consider these sentences:

- If you leave your umbrella behind, you'll get soaked if it rains.
- If you'll get soaked if it rains, then you'll get soaked.[6]

In view of such examples, it seems fair to say that a proper explication strategy for RT has to pass the test not only for simple but also for iterated conditionals.

The second preliminary remark concerns the syntax of the object-languages and the metalanguage that will be considered. Roman upper case letters "A," "B," "C," ... serve as metavariables for *factive* sentences in the relevant language; Greek lower case letters "φ," "ψ," "χ," ... serve as metavariables for any sort of sentence in the relevant language. The object languages that will be referred to are the following:

$\mathcal{L}_0$ a conditional-free language of propositional logic ($\neg$, $\wedge$, $\vee$), that is, a language that contains $\ulcorner \neg\varphi \urcorner$, $\ulcorner \varphi \wedge \psi \urcorner$, and $\ulcorner \varphi \vee \psi \urcorner$, whenever it contains φ and ψ, and that contains only factive sentences;

$\mathcal{L}_1$ a language of propositional logic that embeds $\mathcal{L}_0$ and that contains the sentence $\ulcorner A \rightarrow B \urcorner$ whenever it contains A and B;

$\mathcal{L}_{2_{\mathrm{SL}}}$ a language of propositional logic that embeds $\mathcal{L}_1$ and that contains the sentence $\ulcorner \varphi \rightarrow B \urcorner$ whenever it contains φ and B, with φ being either a factive sentence or a simple conditional;

$\mathcal{L}_{2_{\mathrm{SR}}}$ a language of propositional logic that embeds $\mathcal{L}_1$ and that contains the sentence $\ulcorner A \rightarrow \varphi \urcorner$ whenever it contains A and φ, with φ being either a factive sentence or a simple conditional;

$\mathcal{L}_{2_{\mathrm{R}}}$ a language of propositional logic that embeds $\mathcal{L}_1$ and that contains the sentence $\ulcorner A \rightarrow \varphi \urcorner$ whenever it contains A and φ.

We can now get back to our point of departure, to wit, the question of how to explicate RT.

A natural candidate for a more exact account of RT is suggested by Bayesianism. One core element of Bayesianism is *probabilism*, according to which degrees of belief should obey the standard laws of probability with respect to Boolean connectives. Another core element of Bayesianism is *Bayes' principle*, which says that updates of subjective probabilities on new information should go by conditioning: if Pr is the prior subjective probability distribution from which you start, φ your total new evidence, and Pr' the posterior subjective probability distribution you adopt on learning φ, then you have conditioned on φ iff, for every sentence ψ of your language, Pr' takes for ψ the prior conditional probability of ψ given φ, $\mathrm{Pr}(\psi \mid \varphi)$—where conditional probability is constrained by the ratio formula: $\mathrm{Pr}(\psi \mid \varphi) = \mathrm{Pr}(\varphi \wedge \psi)/\mathrm{Pr}(\varphi)$, provided $\mathrm{Pr}(\varphi) > 0$.[7]

Arguments for Bayesianism standardly refer to the formal language $\mathcal{L}_0$, thereby abstracting from logical notions other than truth functions available in natural languages, such as

[6] Again, the examples are from Adams (1998:268).

[7] Adams (1998:57) assumes, for convenience, that $\mathrm{Pr}(\psi \mid \varphi)$ takes the value 1 in case $\mathrm{Pr}(\varphi) = 0$. Alternatively, one may treat conditional probability to be undefined in this case, or to be definable by the ratio of infinitesimal probabilities. We can stay neutral here as to the treatment of conditional probability for the case in which the relevant condition takes probability zero.

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the indicative conditional.[8] This idealization seems legitimate, insofar as we are interested in questions unrelated to conditionals. As will be shown later, however, it may turn out to be futile if we want to give an account of partial belief pertaining to conditionals in particular. We adopt here, for the sake of argument, a Bayesian view about the statics and dynamics of partial belief for the language $\mathcal{L}_0$. The question is to what extent Bayesianism may provide (also) a viable position for languages that contain conditional sentences (assuming it is a viable position for conditional-free languages).

Consider a language of propositional logic containing a conditional. Assuming Bayesianism for the conditional-free fragment of the language, it is strongly suggested that RT as applied to simple conditionals is explicable by

ADAMS' THESIS (AT): If Pr is our subjective probability distribution on the conditional-free fragment of our language, then for any pair of factive sentences A and B where $\Pr(A) > 0$, the degree of belief for the associated conditional $A \rightarrow B$ amounts to $\Pr(B \mid A)$.

Is there a way of generalizing Adams' explication strategy to other types of instances of the test? The most natural way is simply to adopt probabilism also for any language of propositional logic that contains a conditional and to widen the domain of quantification for sentence variables in AT, from factive sentences to any type of sentences in the language, which may be themselves conditionals or sentences embedding conditionals. However, Lewis' (1976) triviality theorem shows that this approach is doomed to fail. Specifically, for a proof of this result we only need to consider a fragment of English that can be modelled by the formal language $\mathcal{L}_1$. For the argument, we just need two premises:

L1 Pr is a probability function that satisfies AT.

L2 For any A with $\Pr(A) > 0$, the probability function obtained by conditionalizing Pr on A in turn satisfies AT.

Starting from these premises, it follows that Pr is a trivial probability function; that is, for every pair of factive sentences A and C with $\Pr(A \wedge C) > 0$ and $\Pr(A \wedge \neg C) > 0$, $\Pr(C \mid A) = \Pr(C)$.[9] Obviously, such probability distributions can hardly represent our degrees of belief. Lewis' triviality result from L1 and L2 shows that we can embrace AT and a standard Bayesianism with respect to $\mathcal{L}_1$ only on pain of triviality. As $\mathcal{L}_1$ is the simplest type of language of propositional logic that contains any conditionals, this negative result says, in other words, that the joint adoption of Bayesianism and AT is not a viable option for any language with a conditional.[10]

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[8] We assume here that the indicative conditional is not definable as a material conditional, nor otherwise definable in terms of standard logical connectives. For a number of forceful arguments against the material conditional account, see Edgington (1995).

[9] More specifically, this is Lewis' first triviality result. His second triviality result says that the relevant class of probability functions is only defined for trivial languages, that is, languages which do not contain three pairwise incompatible formulas with each of them being possibly true. The third triviality result says that probability functions of the relevant class are at most four valued.

[10] It is to be granted that a language of propositional logic that does not contain an associated simple conditional for every pair of factive sentences is not susceptible to this triviality argument. But this way of ensuring safety from triviality would be of doubtful value because it would draw on an accidental limitation of the expressive means of the object language.

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Another argument, given in McGee (1989), shows that even if we keep to Bayes' principle only for conditional-free languages, AT gives rise to triviality if it is extended to right-nested conditionals—provided we adopt a certain probabilistic constraint regarding right-nested conditionals that seems to have intuitive force. Specifically, for generating the triviality result we just need to refer to $\mathcal{L}_{2_{SR}}$ and assume:

MG1 Pr is a probability function that satisfies the extension of AT to simple right-nested conditionals, that is, $\Pr(A \rightarrow \varphi) = \Pr(\varphi \mid A)$, for all A and φ in the domain of Pr such that $\Pr(A) > 0$ and φ is a factive sentence or a simple conditional;

MG2 Pr satisfies the import–export constraint, according to which $\Pr\big(B \rightarrow (A \rightarrow \varphi)\big) = \Pr\big((A \wedge B) \rightarrow \varphi\big)$, provided $\Pr(A \wedge B) > 0$.[11,12]

To our knowledge, there is no corresponding triviality result that shows that AT cannot be extended to left-nested conditionals. On the contrary, van Fraassen (1976) demonstrates that there are nontrivial probability functions which satisfy extensions of AT to left-nested conditionals and which respect Stalnaker's conditional logic C2, which is (like Lewis' system VC) one of the standard options of conditional logic.[13] This raises the question of whether a generalization of AT to left-nested conditionals may be considered as a viable option for modeling the conditional in natural languages. Our discussion will focus on:

ADAMS' THESIS EXTENDED (AT–EXT): For all factive sentences A, B, and C, it holds that $\Pr\big((A \rightarrow B) \rightarrow C\big) = \Pr(C \mid A \rightarrow B)$,

which extends AT to simple left-nested conditionals, for languages which are at least as strong as $\mathcal{L}_{2_{SL}}$.

To make our point against AT–EXT, it suffices to consider some plausible constraints on the fragment $\mathcal{L}_1$. To begin with, we show in the next section that in some RT instances for simple left-nested conditionals, it is perfectly reasonable to violate certain constraints for updates on conditionals. In Sections 3 and 4, we then argue that conditional probability given a simple conditional is bound to meet some of these constraints, whatever type of probabilism for $\mathcal{L}_1$ one may reasonably consider. The bottom line of our argument

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[11] McGee (1989:486–489) refers to the stronger language $\mathcal{L}_{2_R}$, where for each pair of a factive sentence A and an arbitrary sentence φ there is an associated conditional $A \rightarrow \varphi$. The proof goes through equally for the weaker language $\mathcal{L}_{2_{SR}}$. To wit, McGee's argument follows the same strategy as Lewis' argument, which essentially hinges on the factorization principle: $\Pr(A \rightarrow C \mid B) = \Pr(C \mid A \wedge B)$, if $\Pr(A \wedge B) > 0$. We can derive this principle from AT and import–export as follows: By AT, $\Pr(A \rightarrow C \mid B) = \Pr\big(B \rightarrow (A \rightarrow C)\big)$. By import–export, $\Pr(A \rightarrow C \mid B) = \Pr\big((A \wedge B) \rightarrow C\big)$ if $\Pr(A \wedge B) > 0$. Then, by AT, $\Pr(A \rightarrow C \mid B) = \Pr(C \mid A \wedge B)$ if $\Pr(A \wedge B) > 0$.

[12] Endorsing import–export, McGee abandons the extension of AT to right-nested conditionals. For an alternative way of blocking the triviality result, by extending AT to right-nested conditionals and abandoning import–export, see Jeffrey and Stalnaker (1994).

[13] This result is implied by van Fraassen's tenability result for C2 logic regarding a weak interpretation of Stalnaker's hypothesis, according to which for each probability function Pr, there is some connective "$\rightarrow$" such that Pr satisfies Stalnaker's Hypothesis Schema (SHS): $\Pr(\varphi \rightarrow \psi) = \Pr(\psi \mid \varphi)$, for all φ, ψ in the domain of Pr with $\Pr(\varphi) > 0$. In intuitive terms, van Fraassen's result for C2 shows that any probability space can be embedded within a model that obeys a restriction of SHS with respect to (i) simple conditionals, (ii) simple right-nested conditionals, and (iii) simple left-nested conditionals. Van Fraassen's further tenability result for the conditional logic CE shows the same even for SHS unrestrictedly. For a thorough discussion of van Fraassen's tenability results against the background of various triviality results, see Hájek & Hall (1994, Sect. 5.5).

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is that degree of belief in a simple left-nested conditional, $\Pr((A \rightarrow B) \rightarrow C)$, is not representable as the conditional probability of C given $A \rightarrow B$, that is, $\Pr(C \mid A \rightarrow B)$. As a consequence, it is suggested that AT–EXT does not provide an adequate explication of RT instances for simple left-nested conditionals. A fortiori, AT extended to left-nested conditionals in general is to be rejected as materially inadequate as well.

§2. The Judy Benjamin problem revisited. Van Fraassen's Judy Benjamin problem concerns updates on conditionals. He explains the problem as follows:[14] A soldier (Judy Benjamin) is dropped in an area that is divided into two halves, Red territory (R) and Blue territory ($\neg R$), each of which in turn is divided in equal parts, Second Company area (S) and Headquarters Company area ($\neg S$). As Judy Benjamin has no information about her location, her degree of confidence that she is in a particular quarter is the same for each quarter. That is, representing her epistemic state as a probability function $\Pr_0$, we have: $\Pr_0(R \wedge S) = \Pr_0(R \wedge \neg S) = \Pr_0(\neg R \wedge S) = \Pr_0(\neg R \wedge \neg S) = 1/4$. She then receives a radio message:

> I can't be sure where you are. If you are in Red territory, the odds are 3 : 1 that you are in Headquarters Company area.

After this, the radio gives out. How should Judy adjust her subjective probabilities on getting this new information? Van Fraassen discusses the following prima facie plausible constraints for the result $\Pr_1$ of updating $\Pr_0$ on the given new information: (i) $\Pr_1(\neg S \mid R) = 3/4$ and $\Pr_1(S \mid R) = 1/4$, that is, the probability of $R \wedge \neg S$ should be three times higher than the probability of $R \wedge S$; (ii) $\Pr_1(\neg R) = \Pr_0(\neg R)$, that is, the probability for $\neg R$ should be unaffected; and (iii) $\Pr_1(C \mid X) = \Pr_0(C \mid X)$, for all C, with $X \in \{\neg R, R \wedge S, R \wedge \neg S\}$, that is, none of her conditional degrees of belief given any sentence in $\{\neg R, R \wedge S, R \wedge \neg S\}$ should change either.

Generalizing from the particular example and formulating it in terms of probability functions on a language $\mathcal{L}_0$ of propositional logic with factive atoms, we can say:

DEFINITION 2.1. *Let* $\Pr_0$ *be our probability function on* $\mathcal{L}_0$. *Let* $\{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$ *be a set of sentences in* $\mathcal{L}_0$ *of pairwise truth-functionally inconsistent sentences whose disjunction is a tautology such that, for all* i, $\Pr_0(B_i \mid A) > 0$. *Let* E *be the new evidence of the form* ⌜*If A, then the odds for* $B_1, \ldots, B_n$ *are* $c_1 : \cdots : c_n$⌝. *Any posterior probability function* $\Pr_1$ *is said to meet the Judy Benjamin requirements for* $\Pr_0$ *and* E *iff for all* C *in* $\mathcal{L}_0$:

(JB1) $\Pr_1(B_i \mid A) = \frac{c_i}{\sum_{1 \leqslant j \leqslant n} c_j}$;

(JB2) $\Pr_1(A) = \Pr_0(A)$;

(JB3) $\Pr_1(C \mid X) = \Pr_0(C \mid X)$, *for all* $X \in \{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$.

DEFINITION 2.2. *For the above provisos on a given prior* $\Pr_0$ *and a set* $\{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$ *of sentences in* $\mathcal{L}_0$, *the Judy Benjamin problem with regard to new evidence* E *of the form* ⌜*If A, then the odds for* $B_1, \ldots, B_n$ *are* $c_1 : \cdots : c_n$⌝ *is to give a posterior probability function* $\Pr_1$ *that meets the Judy Benjamin requirements for* $\Pr_0$ *and* E.

The Judy Benjamin requirements JB1–JB3 are formulated with respect to scenarios where the new information has the form of a conditional with the consequent being about

[14] van Fraassen (1981); see also van Fraassen et al. (1986).

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odds. RT for simple left-nested conditionals, on the other hand, may also involve simple conditionals, where the consequent does not have that form. In the next section, we will come back to the systematic connection between updates on new information involving conditional odds and updates on associated simple conditionals in general. The purpose of this section is to show that the Judy Benjamin requirements as transferred to updates on simple conditionals are not correct in general.

To begin with a positive point, it is to be noted that insofar as for the Judy Benjamin example the respective instances of JB1–JB3 have intuitive force, the same holds for variants of the example where the respective conditional odds we come to learn are different. In particular, it holds for the scenario we obtain from the Judy Benjamin example by replacing "If you are in Red territory, the odds are 3 : 1 that you are in Headquarters Company area" with "If you are in Red territory, the odds are 1 : 0 that you are in Headquarters Company area"—for which it seems correct to impose the corresponding instances of the Judy Benjamin requirements on a posterior probability distribution, namely, (JB1′) $\mathrm{Pr}_1(B \mid A) = 1$, (JB2′) $\mathrm{Pr}_1(A) = \mathrm{Pr}_0(A)$, and (JB3′) $\mathrm{Pr}_1(C \mid X) = \mathrm{Pr}_0(C \mid X)$, for $X \in \{\neg A, A \wedge B\}$. Note that it makes no difference for the correctness of JB1′–JB3′ whether the new evidence has the form "If you are in Red territory, the odds are 1 : 0 that you are in Headquarters Company area" or the form "If you are in Red territory, then you are in Headquarters Company area." In view of this point, one may be tempted to adopt the Judy Benjamin–type constraints JB1′–JB3′ for *every* case of updating on a simple conditional. We contend, however, that the said constraints do fail in some instances of updating on simple conditionals; to wit, there are instances of RT, that is, hypothetical updates on simple conditionals, where it is perfectly reasonable to make requirements that conflict with JB1′–JB3′. As a first case in point, we give an example where JB2′ fails.

EXAMPLE 2.1. *There was a burglary. A watch has been stolen, and the homeowner has been murdered, but it is not clear at this point that there is a connection between these two events; perhaps someone murdered the homeowner, and then someone else saw an opportunity to steal the watch. There is a chance that Bob was the thief. On the other hand, while Bob may be capable of theft, it seems less likely that he committed the murder. For in view of his personality, the possibility that he was the murderer seems so remote that we would not consider it seriously—although it cannot be ruled out. Importantly, even supposing that if Bob stole the watch, he also murdered the victim, the probability that he is the murderer is still smaller than our prior probability for his being the thief.*

Now consider:

Thief:	*Bob stole the watch.*
Murderer:	*Bob murdered the victim.*
Evidence1:	*If Bob stole the watch, then he also murdered the victim.*

The provisos given above impose the following constraints on an application of RT to our prior probability distribution and Evidence1:

- $\mathrm{Pr}_1(\mathsf{Murderer} \mid \mathsf{Thief}) = 1$ *(by* $\mathrm{Pr}_1(\mathsf{Evidence1}) = 1$ *and AT);*
- $\mathrm{Pr}_1(\mathsf{Murderer}) < \mathrm{Pr}_0(\mathsf{Thief})$*;*
- $\mathrm{Pr}_1(\mathsf{Murderer}) =$
  $\mathrm{Pr}_1(\mathsf{Murderer} \mid \mathsf{Thief})\,\mathrm{Pr}_1(\mathsf{Thief}) + \mathrm{Pr}_1(\mathsf{Murderer} \mid \neg\mathsf{Thief})\,\mathrm{Pr}_1(\neg\mathsf{Thief})$.

Hence, JB2′ fails, for $\mathrm{Pr}_1(\mathsf{Thief}) \leqslant \mathrm{Pr}_1(\mathsf{Murderer}) < \mathrm{Pr}_0(\mathsf{Thief})$. ♦

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As a case in point against JB3′, consider the following example:[15]

EXAMPLE 2.2. *We live in a house in neighborhood N. Nearby lives a werewolf who assumes her wolf form every full-moon night. We know that before going out to search for prey, the werewolf flips a fair coin. If the coin lands heads, she goes to neighborhood N, and if it lands tails, she goes to neighborhood M. We also know that the werewolf always kills anyone who ventures outside in the neighborhood she is stalking. Conversely, she never kills beyond the neighborhood in which she is stalking. Tonight there is a full moon. The odds for the werewolf being in our neighborhood as against her being in the neighborhood M are 1:1. We suddenly learn that Jones, who was still in our house a few minutes ago, has left the house. There are two doors to the house, the front and the back; either way out leads into neighborhood N.*
Now consider:

Killing: *Jones was killed by the werewolf.*
Front: *Jones went out the front door.*
Evidence2: *If Jones went out the front door, then he was killed by the werewolf.*

What we know strongly suggests that the question of the outcome of the werewolf's coin toss is independent of the question of what exit was taken by Jones. It is important to note that mere supposition of Evidence2 *should not make a difference regarding the independence of these two questions of each other. These provisos impose the following constraints on an application of RT to our prior probability distribution and* Evidence2:

- $\mathrm{Pr}_1(\mathsf{Killing}) = \mathrm{Pr}_1(\mathsf{Killing} \mid \mathsf{Front})$*;*
- $\mathrm{Pr}_1(\mathsf{Killing} \mid \mathsf{Front}) = \mathrm{Pr}_1(\mathsf{Evidence2})$ *(by AT);*
- $\mathrm{Pr}_1(\mathsf{Evidence2}) = 1$*;*
- *hence,* $\mathrm{Pr}_1(\mathsf{Killing}) = 1$.

As a result, JB3′ fails, for $\mathrm{Pr}_1(\mathsf{Killing} \mid \neg\mathsf{Front}) = 1 \neq .5 = \mathrm{Pr}_0(\mathsf{Killing} \mid \neg\mathsf{Front})$. ♦

To sum up, insofar as our reasoning was sound, we could establish two facts:

FACT 2.1. *There are instances of RT for simple left-nested conditionals where it is perfectly reasonable to violate JB2′.*

FACT 2.2. *There are instances of RT for simple left-nested conditionals where it is perfectly reasonable to violate JB3′.*

In the next section, we turn to Vann McGee's probabilism for languages with a conditional, as given in McGee (1989). To put our point in the positive, we will show that starting from McGee's probabilism, any update on a simple conditional will meet the Judy Benjamin–type constraints JB1′–JB3′. To put our point in the negative, starting from McGee's probabilism, we are going to show that AT–EXT does not provide an adequate account of RT for simple left-nested conditionals.

§3. An argument against AT–EXT from McGee's probabilism. McGee (1989) offers an account of probability for the language $\mathcal{L}_{2_R}$. For our purposes, it suffices to focus on his probabilism for a certain type of fragment of the language $\mathcal{L}_1$. McGee gives three

[15] The example is from Lance (1991:270 f). In that paper, however, the example is used as a case for an entirely different point. Compare Section 3.

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constraints on probability functions for the full language $\mathcal{L}_1$ which are strong enough to fix uniquely for any probability function Pr on the base language $\mathcal{L}_0$, an extension of Pr to $\mathcal{L}_1$. The first two constraints are:[16]

PROBABILISTIC CENTERING (PC): For any conjunction of the form $A \wedge (A \rightarrow B)$, $\Pr\big(A \wedge (A \rightarrow B) \equiv A \wedge B\big) = 1$.

INDEPENDENCE (IND): For any conjunction of the form $C \wedge (A_1 \rightarrow B_1) \wedge \cdots \wedge (A_n \rightarrow B_n)$, if C is truth-functionally incompatible with each of $A_1, \ldots, A_n$ and, for all i, $\Pr(A_i) > 0$, then $\Pr\big(C \wedge (A_1 \rightarrow B_1) \wedge \cdots \wedge (A_n \rightarrow B_n)\big) = \Pr(C) \times \Pr\big((A_1 \rightarrow B_1) \wedge \cdots \wedge (A_n \rightarrow B_n)\big)$.

Starting from any given probability distribution on the base language $\mathcal{L}_0$, PC and IND uniquely fix a probability distribution on the fragment $\mathcal{L}_1^*$ of $\mathcal{L}_1$ that contains all and only all Boolean combinations of factive sentences and simple conditionals whose antecedent has a positive value relative to Pr—call this henceforth the *positive antecedent fragment* of $\mathcal{L}_1^*$ (for a given relevant probability distribution on $\mathcal{L}_0$). The said constraints furthermore uniquely fix a probability distribution on the full language $\mathcal{L}_1$, given a further constraint that fixes the value for simple conditionals whose antecedent has zero probability. McGee lets simple conditionals take the probability value one, if their antecedent takes the value zero—call this constraint *Zero Antecedent* (or ZA, for short).[17] This constraint seems too strong, insofar as we do not want *every* simple conditional to be fully believable, only on the ground that we fully disbelieve its antecedent; and indeed, McGee himself grants that he employs ZA only for technical convenience and for lack of a better account.[18] Fortunately, for our argument, we need to assume only McGee's probabilism for respective positive antecedent fragments of $\mathcal{L}_1$—that is, we only assume standard probability theory combined with PC and IND. Before turning to the implications of these constraints, let us have a closer look at them.

PC seems incontrovertible, considering that it follows from two weak provisos: for one, that degrees of belief respect logic; for another, that conditional logic includes the standard principle $A \wedge (A \rightarrow B) \equiv A \wedge B$. The status of IND, however, is more controversial. Notably, in his (1991), Mark Lance argues that there are clear counterexamples against McGee's constraint, henceforth referred to as *Conjunction* (CON),

$$\Pr\big((A \rightarrow B) \wedge (C \rightarrow D)\big) = \frac{1}{\Pr(A \vee C)} \times \big[\Pr(A \wedge B \wedge C \wedge D) + \Pr(\neg A \wedge C \wedge D) \times \Pr(A \rightarrow B) + \Pr(A \wedge B \wedge \neg C) \times \Pr(C \rightarrow D)\big],$$

which is derivable from standard probability theory in combination with PC and IND.[19] Assuming standard probability theory and PC, Lance's argument suggests that there are clear counterexamples against IND.[20] One may make his point more directly by way of certain instances of IND. For example, take the scenario of Example 2.2 and consider for the sentences

[16] McGee (1989:501).
[17] McGee (1989:504–506).
[18] McGee (1989:506). For further criticism of ZA, see also McGee (1994).
[19] The proof is easy, but tediously long. For brevity, we omit it.
[20] See Lance (1991:275 n).

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P: Jones went out the front door.

Q: Jones was killed by the werewolf.

R: Jones went out the back door.

the associated conjunction

S: $(P \wedge Q) \wedge (R \rightarrow \neg Q)$.

Since, by assumption, P and R are truth-functionally incompatible, as an instance of IND, it follows that $\Pr(S) = \Pr(P \wedge Q) \times \Pr(R \rightarrow \neg Q)$. As a consequence, if the probability for either conjunct of S is positive, so is the probability for the conjunction S. One may argue that this result is inadequate, as follows: Given that Jones went out the front door and that he was killed by the werewolf, it is certain that he was killed upon going out either door. This suggests that S should receive probability zero. Importantly, this reasoning does not hinge on any assumption about the probabilities for the conjuncts of S. Hence, if it holds, it holds also in case the probability for either conjunct of S is positive. Or so one may argue.[21]

In reply to this, we grant that the *conditional degree of belief* for the simple conditional $R \rightarrow \neg Q$ given $P \wedge Q$ should be zero. However, from this alone, it does not follow that S is to receive probability zero. Indeed, the inference is valid, assuming that conditional degree of belief with respect to simple conditionals given a factive condition can be modeled as a *conditional probability*—more generally, that updates of degrees of belief for a positive antecedent fragment on hypothetical new factive evidence go by conditioning. On this assumption, the conditional degree of belief of $R \rightarrow \neg Q$ given $P \wedge Q$ amounts to the conditional probability of $R \rightarrow \neg Q$ given $P \wedge Q$. Hence, by the ratio formula, the probability of S must be zero. However, as already noted (see Section §1.), we can combine Bayes' principle with AT for languages with an indicative conditional only on pain of triviality. In fact, as the argument strategy for Lewis' (1976) triviality theorem for a language $\mathcal{L}_1$ does not essentially involve conditionals whose antecedents have probability 0, it works out as well for the restriction of any prior probability to the associated positive antecedent fragment of $\mathcal{L}_1$.[22] Thus, assuming that the above objection against IND essentially draws on Bayes' principle for updates of a probability distribution on a positive antecedent fragment on hypothetical factive evidence, the objection is to be rejected as unsound; and insofar as Lance's other considerations against IND are no more forceful, they can be put aside here as well.[23] This said, we do not mean to subscribe

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[21] This example was suggested to us by Richard Bradley (in personal communication), for which we thank him.

[22] As mentioned in note 11, Lewis' triviality theorem essentially rests on the factorization hypothesis: $\Pr(A \rightarrow C \mid B) = \Pr(C \mid A \wedge B)$, if $\Pr(A \wedge B) > 0$. To derive this constraint, we only need to consider the respective positive antecedent fragment of $\mathcal{L}_1$ for the probability function we pick out. To wit: (i) $\Pr(A \rightarrow C \mid B) = \Pr_B(A \rightarrow C)$ (by closure under conditioning); (ii) $\Pr_B(A \rightarrow C) = \Pr_B(C \mid A)$ (by AT for $\Pr_B$); (iii) $\Pr_B(C \mid A) = \Pr_B(C \wedge A)/\Pr_B(A)$ (by the ratio formula), from which, it follows that $\Pr(A \rightarrow C \mid B) = \Pr(C \wedge A \wedge B)/\Pr(A \wedge B) = \Pr(C \mid A \wedge B)$.

[23] That Lance's two challenges against IND are no more forceful, should become clear in view of the following: (1) Consider instances of the form $(A \rightarrow B) \wedge (C \rightarrow B)$, where A and C are truth-functionally incompatible. For instance, consider "Jones went out the front door," "Jones was killed by the werewolf," and "Jones went out the back door" as values for A, B, and C, respectively. Suppose that the probability that Jones left is .5 and that otherwise, the circumstances are as described in Example 2.2. It follows then, by CON, that $\Pr\big((A \rightarrow B) \wedge (C \rightarrow B)\big) =$

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here to McGee's case for IND either.[24] Rather, we treat IND as still pending in status. In the absence of a sound, nonquestion-begging argument against IND, it is fair to consider the constraint as a viable option. We can now turn to some implications of McGee's probabilism that will be relevant to the further discussion:

EXTENDABILITY (EX): For every probability function Pr on $\mathcal{L}_0$, there is a probability function Pr′ on the associated positive antecedent fragment of $\mathcal{L}_1$ such that Pr′ extends Pr and meets PC and IND.[25]

(1/.5) × (0 + .125 × .5 + .125 × .5) = .25. However, according to Lance, it should be .5. He argues as follows (p. 271): "[E]ach conditional comes down to the question of where the werewolf went tonight. If she went to our neighborhood she killed anyone outside. That is, if the werewolf is in our neighborhood, then if Jones went out the front, it is *certain* that he was killed. Similarly, if the werewolf is somewhere else, it is certain that Jones was not killed upon going out either door. So if the werewolf is in our neighborhood, both conditionals are true (or highly assertible). If not, both are false." Lance's reasoning seems to come down to the following: By standard probability theory and the ratio formula, $\Pr\big((A \to B) \wedge (C \to B)\big) = \Pr\big((A \to B) \wedge (C \to B) \,|\, D\big) \times \Pr(D) + \Pr\big((A \to B) \wedge (C \to B) \,|\, \neg D\big) \times \Pr(\neg D)$, where D stands for "The werewolf is in our neighborhood." The first summand amounts to .5, for, by assumption, $\Pr(D) = .5$, and $\Pr\big((A \to B) \wedge (C \to B) \,|\, D\big) = 1$. The second summand amounts to 0, for $\Pr\big((A \to B) \wedge (C \to B) \,|\, \neg D\big) = 0$. In reply to this, we indeed grant that the *conditional degree of belief* for $(A \to B) \wedge (C \to B)$ given D should equal 1, and that the conditional degree of belief for $(A \to B) \wedge (C \to B)$ given $\neg D$ should equal 0 (for given D, it is certain that $(A \to B) \wedge (C \to B)$—analogously for $\neg D$). However, from this it does not follow that the same holds for the associated *conditional probabilities*. It would follow, if we were free to assume AT–EXT. But this very question is at issue here. To accept the above reasoning against IND as sound would hence beg the question with respect to the status of AT–EXT. (2) Suppose that C is probabilistically independent of both A and B, and also of $A \wedge B$. Then by AT, $\Pr\big((A \wedge C) \to B\big) = \Pr(A \to B)$. Lance (p. 272) furthermore submits, without arguing, that in this case also the following holds: (i) $\Pr\big[(A \to B) \wedge \big((A \wedge C) \to B\big)\big] = \Pr\big((A \to B) \wedge (A \to B)\big)$, and (ii) $\Pr\big((A \to B) \wedge (A \to B)\big) = \Pr(A \to B)$. But whereas (ii) holds on CON, (i) does not. For example, suppose Jones threw a fair coin, d, twice, while Smith threw another fair coin, e, once, and let "Jones threw heads on his first throw," "Jones threw heads twice," and "Smith threw tails on his throw" be the values for A, B, and C, respectively. Then $\Pr(A \to B) = .5$. By application of CON, however, we obtain: $\Pr\big[(A \to B) \wedge \big((A \wedge C) \to B\big)\big] = \big[1/\Pr\big(A \vee (A \wedge C)\big)\big] \times \big[\Pr(A \wedge B \wedge C) + \Pr(\neg A \wedge A \wedge C \wedge B) \times \Pr(A \to B) + \Pr\big(A \wedge B \wedge \neg(A \wedge C)\big) \times \Pr\big((A \wedge C) \to B\big)\big] = [1/\Pr(A)] \times [.125 + 0 + .125 \times .5] = 2 \times .1875 = .375$. Again, Lance's objection seems to draw on informal assessments of conditional degree of belief. For given $A \to B$, it is indeed certain that $(A \wedge C) \to B$ (in other words, the associated conditional degree of belief should be one). Assuming that conditional degree of belief can be modelled as conditional probability, Lance's suggested assessment follows straightforwardly. After all, by standard probability theory, we have: $\Pr\big((A \to B) \wedge ((A \wedge C) \to B)\big) = \Pr\big[\big((A \wedge C) \to B\big) \,|\, (A \to B)\big] \times \Pr(A \to B)$. Since, by assumption, the first multiplicand amounts to one, from this, it follows that $\Pr\big[(A \to B) \wedge \big((A \wedge C) \to B\big)\big] = \Pr(A \to B)$. Again, as in (1), an extended version of AT is used for an account of RT. This time, the relevant conditional is not only left-nested (the antecedent has the form $A \to B$), but also right-nested (the consequent has the form $(A \wedge C) \to B$). But insofar as the status of AT–EXT is pending, a fortiori, we are not free to assume the extension of AT to left-and-right-nested conditionals either. This completes our reply to Lance's two objections against IND.

[24] In support of IND, McGee (1989:490–493) gives two types of argument, for one an argument from probability updating, for another, an argument from Dutch Books (1989:493–501). An appraisal of these arguments would go beyond the scope of this paper.

[25] This follows from Lemma A.1, in Appendix.

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UNIQUENESS (UN): For every probability function Pr on $\mathcal{L}_0$, if there is a probability function Pr′ on the associated positive antecedent fragment of $\mathcal{L}_1$ such that Pr′ extends Pr and meets PC and IND, then there is only one such extension.[26]

ADAMS' THESIS GENERALIZED: Assuming probability theory, PC and IND together imply AT.[27]

We now introduce *simple conditional updating* (SCU), a special update function for updates on simple conditionals, which will be instrumental in establishing the material inadequacy of AT–EXT:

DEFINITION 3.1. *Let* $\{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$ *be a set of pairwise truth-functionally inconsistent sentences whose disjunction is a tautology such that for the prior probability function* Pr_0, $\mathrm{Pr}_0(B_i \mid A) > 0$ *for all* i, *and suppose one is caused to change, for all* i, *one's degrees of belief from* $\mathrm{Pr}_0(A \rightarrow B_i)$ *to* $\mathrm{Pr}_1(A \rightarrow B_i)$, *where* $\sum_{i=1}^{n} \mathrm{Pr}_1(A \rightarrow B_i) = 1$. *Then one updates by SCU on this change iff for all sentences* C *of* $\mathcal{L}_0$,

$$\mathrm{Pr}_1(C) \;=\; \sum_{i=1}^{n} \mathrm{Pr}_0(C \mid A \rightarrow B_i) \times \mathrm{Pr}_1(A \rightarrow B_i).$$

SCU itself only fixes an update of probability for $\mathcal{L}_0$ with respect to simple conditionals. However, assuming McGee's probabilism, SCU provides in effect a two-stage model of updating probability for the associated positive antecedent fragment of $\mathcal{L}_1$. For, by EX and UN, every instance of SCU uniquely fixes an update for this type of fragment.

SCU is of special interest here, for assuming that RT instances for simple left-nested conditionals accord with AT–EXT, it follows that they are instances of SCU—which can be shown as follows: Suppose that for the prior distribution Pr_0, $\mathrm{Pr}_0(A \rightarrow B)$ and $\mathrm{Pr}_0(A \rightarrow \neg B)$ are both positive, and that one is caused to raise one's degree of belief for $A \rightarrow B$ to 1 and to lower one's degree of belief for $A \rightarrow \neg B$ to 0. Then one updates by SCU on this change iff for all sentences C of $\mathcal{L}_0$, $\mathrm{Pr}_1(C) = \mathrm{Pr}_0(C \mid A \rightarrow B)$. Assuming that instances of RT for simple left-nested conditionals are correctly modeled by AT–EXT, the posterior probability distribution Pr_1^* that one obtains by hypothetically raising one's probability for $A \rightarrow B$ to 1 and hypothetically lowering one's degree of belief for $A \rightarrow \neg B$ to 0 is to meet, then, the same constraint; that is, for any C, $\mathrm{Pr}_1^*(C) = \mathrm{Pr}_0(C \mid A \rightarrow B)$. In other words, Pr_1^* is obtainable by SCU.

Having seen that, assuming McGee's probabilism together with AT–EXT, instances of RT for simple left-nested conditionals can be modeled as applications of SCU, let us have a closer look at the behavior of SCU. Here is a provable result about SCU:[28]

THEOREM 3.1. *Let* $\{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$ *be a set of pairwise truth-functionally inconsistent sentences of* $\mathcal{L}_0$ *whose disjunction is a tautology such that for the prior probability function* Pr_0 *on* $\mathcal{L}_0$, $\mathrm{Pr}_0(B_i \mid A) > 0$ *for all* i. *Let a person's new degrees-of-belief function* Pr_1 *come from her old degrees-of-belief function* Pr_0 *by SCU on a change in her*

[26] Again, this follows from Lemma A.1.

[27] See Lemma A.2, in Appendix. As McGee (1989, Sect. 3) notes, modulo standard probability theory, one can think of PC and IND as a way of generalizing AT to a language with Boolean combinations of factive sentences and simple conditionals.

[28] For a proof, see Appendix.

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degrees of belief in $A \rightarrow B_i$*, for all* i*, where* $\sum_{i=1}^{n} \Pr_1(A \rightarrow B_i) = 1$*. We then have for all* $C \in \mathcal{L}_0$*:*

(i) $\Pr_1(A) = \Pr_0(A)$*;*

(ii) $\Pr_1(C \mid X) = \Pr_0(C \mid X)$*, for all* $X \in \{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$*;*

(iii) the restriction of $\Pr_1$ *to* $\mathcal{L}_0$ *is a probability function.*

By this theorem, it follows that SCU results in a posterior probability function $\Pr_1$ on $\mathcal{L}_0$ that meets the Judy Benjamin–type constraints JB1′–JB3′.[29] With these results in place, the argument against AT–EXT from McGee's probabilism runs as follows:

1. Assuming McGee's probabilism and that AT–EXT provides an adequate model of RT for left-nested conditionals, instances of RT for simple left-nested conditionals are instances of SCU.
2. According to SCU, on updating on a simple conditional, we should meet both JB2′ and JB3′.
3. But by Facts 2.1 and 2.2, we can make perfect sense of updates on simple conditionals that violate JB2′ or JB3′.
4. Hence, assuming McGee's probabilism, AT–EXT does not provide an adequate model of RT for left-nested conditionals.

§4. An argument against AT–EXT from weak centering. For the above argument against AT–EXT, two points were essential: first, the observation that the Judy Benjamin–type constraints JB2′ and JB3′ should be violated in certain instances of RT on learning a conditional $A \rightarrow B$, it can be perfectly reasonable to alter, for some factive C, the conditional probability of C given X, where $X \in \{\neg A, A \wedge B, A \wedge \neg B\}$; second, the proviso of McGee's probabilism for positive antecedent fragments of $\mathcal{L}_1$. Opponents to IND may suggest that our first argument highlights a problem with McGee's particular probabilism, rather than with AT–EXT itself. Even so, our second argument suggests that the prospects for AT–EXT are rather dim in general.

The only proviso we need in our second argument is that for any probabilism for $\mathcal{L}_1$ to be considered as a serious option, it is to respect a certain principle of standard conditional logic. Both on Stalnaker's system C2 and Lewis' system VC, we have:

LOGICAL CENTERING: $A \wedge (A \rightarrow B)$ and $A \wedge B$ are logically equivalent.[30]

This, together with the standard probabilistic constraint that probability respect logical entailment, gives

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[29] This result bears also on the Judy Benjamin problem. To wit, assuming AT, it is hard to give counterexamples to the following principle:

> We learn ⌜If A, then the odds for $B_1, \ldots, B_n$ are $c_1 : \cdots : c_n$⌝, where $\{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$ is a set of pairwise truth-functionally inconsistent sentences whose joint disjunction is a tautology, just in case for all i, our degrees of belief for $A \rightarrow B_i$ change to $\frac{c_i}{\sum_{0<j\leqslant n} c_j}$.

If this is sound, we can immediately obtain from SCU an update function that provides a solution to the Judy Benjamin problem. For a different solution to the Judy Benjamin problem which makes use of Adams conditioning (as presented in Bradley (2005)), see Douven & Romeijn (2009).

[30] In fact, on both mentioned systems, the schema $A \wedge (A \rightarrow B) \equiv A \wedge B$ is a logical principle. That is, both systems support even McGee's stronger constraint PC. For an overview of C2, VC, and some other systems of conditional logic, see Nute & Cross (2001).

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WEAK PROBABILISTIC CENTERING (WPC): $\Pr\big(A \wedge (A \rightarrow B)\big) = \Pr(A \wedge B)$.

In fact, to our knowledge, WPC is valid on all given serious accounts that supply means of defining probability distributions on $\mathcal{L}_1$ that satisfy AT.[31] As various authors have already noted, assuming probability theory and WPC, AT is equivalent to

PROBABILISTIC INDEPENDENCE (PI): $\Pr(A \rightarrow B \mid A) = \Pr(A \rightarrow B)$.[32]

Assuming that AT–EXT is a model of RT for left-nested conditionals, PI implies that any hypothetical update on a simple conditional should leave our degree of belief in the antecedent unaffected, which is one of the Judy Benjamin–type constraints, namely JB2′.

With these results in place, the argument against AT–EXT from weak centering runs as follows:

1. Assuming WPC, AT is equivalent to PI.
2. Assuming that AT–EXT provides an adequate model of RT for left-nested conditionals, PI implies the constraint JB2′.
3. But by Fact 2.1, we can make perfect sense of updates on simple conditionals that violate JB2′.
4. Hence, assuming WPC, AT–EXT does not provide an adequate model of RT for left-nested conditionals.

In defense of AT–EXT, one may object also that our second argument is only conditional in that it relies on the provisos of standard probability theory for the language $\mathcal{L}_1$, and on the constraints AT and WPC. Several points are to be made in reply to this. First, the first proviso should be incontrovertible, at least in the context of the present discussion. Second, any counterinstance to AT would by the same token be a counterinstance to AT–EXT, by the following highly plausible bridge principle that connects probability for simple conditionals and probability for simple left-nested conditionals: $\Pr(A \rightarrow B) = \Pr\big((\top \rightarrow A) \rightarrow B\big)$. Third, even notwithstanding that there may be room for also arguing against the adequacy of AT, our examples were deliberately chosen to make sure that the relevant instances of AT are unproblematic. That is, the revealed inadequacy problem cannot be simply reduced to an inadequacy problem for AT, if there is any. Lastly, abandoning WPC would be a costly way of making AT–EXT sustainable in that it would require us to give up an elementary principle of standard conditional logics. In view of these points, it seems fair to contend that the dismissal of any premise of our argument would in effect undermine the case for AT–EXT rather than offer a way of salvaging it.

A final note: van Fraassen (1976:278) surmises that PI may provide "a new fulcrum for the application of a philosophical critique or defence" of Stalnaker's hypothesis, which implies AT–EXT. On its weakest plausible interpretation, the hypothesis says that for each probability function Pr that could represent a rational agent's system of beliefs, there is some connective "$\rightarrow$" such that $\Pr(X \rightarrow Y) = \Pr(Y \mid X)$ for all X, Y in the domain of Pr with $\Pr(X) > 0$.[33] Our argument against AT–EXT, by the same token, provides a novel material inadequacy argument against Stalnaker's hypothesis.

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[31] We refer to van Fraassen (1976), McGee (1989), Jeffrey & Stalnaker (1994), and Milne (1997). For a different view, see Ellis (1979:79 f).

[32] See, e.g., van Fraassen [1976:278], Skyrms [1980:91], and Hájek & Hall [1994:81 f].

[33] For arguable precisifications of Stalnaker's hypothesis, see Hájek & Hall [1994].

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§5. Conclusion. We can make perfect sense of updates on simple conditionals that violate the Judy Benjamin–type constraints JB2′ or JB3′. On the other hand, whatever probabilism for conditionals one may *plausibly* adopt, the assumption that AT–EXT provides an adequate model of RT for left-nested conditionals implies that we are to embrace at least the rigidity constraint JB2′. We conclude that AT–EXT is not a viable option in that it narrows the space of admissible hypothetical updates in a way that is, in the absence of any theoretical underpinning, ill-motivated. In effect, our result poses also a novel kind of threat to Stalnaker's hypothesis, from material inadequacy.

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Appendix: Proof of Theorem 3.1.

LEMMA A.1. *Every probability function* Pr *on* $\mathcal{L}_0$ *can be extended in a unique way to a function* Pr′ *on the associated positive antecedent fragment of* $\mathcal{L}_1$ *for* Pr *that satisfies PC and IND.*[34]

LEMMA A.2. *Let for any given probability function* $\Pr_0$ *on the base language* $\mathcal{L}_0$, Pr *be an extension of* $\Pr_0$ *to the associated positive antecedent fragment of* $\mathcal{L}_1$ *where* Pr *satisfies PC and IND.* Pr *then also satisfies AT.*[35]

THEOREM 3.1. *Let* $\{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$ *be a set of pairwise truth-functionally inconsistent sentences of* $\mathcal{L}_0$ *whose disjunction is a tautology such that for the prior probability function* $\Pr_0$ *on* $\mathcal{L}_0$, $\Pr_0(B_i \mid A) > 0$ *for all* i. *Let a person's new degrees-of-belief function* $\Pr_1$ *come from her old degrees-of-belief function* $\Pr_0$ *by SCU on a change in her degrees of belief in* $A \rightarrow B_i$, *for all* i, *where* $\sum_{i=1}^{n} \Pr_1(A \rightarrow B_i) = 1$. *We then have for all* $C \in \mathcal{L}_0$:

(i) $\Pr_1(A) = \Pr_0(A)$;

(ii) $\Pr_1(C \mid X) = \Pr_0(C \mid X)$ *for all* $X \in \{\neg A, A \wedge B_1, \ldots, A \wedge B_n\}$;

(iii) the restriction of $\Pr_1$ *to* $\mathcal{L}_0$ *is a probability function.*

Proof. Assume the conditions of the theorem to be met. Then (i)–(iii) can be seen to hold, as follows:

(i) By the definition of $\Pr_1$ we have:

$$\Pr_1(A) \;=\; \sum_{i=1}^{n} \Pr_0(A \mid A \rightarrow B_i) \Pr_1(A \rightarrow B_i).$$

The right-hand side of this rewrites as

$$\sum_{i=1}^{n} \frac{\Pr_0\big(A \wedge (A \rightarrow B_i)\big)}{\Pr_0(A \rightarrow B_i)} \Pr_1(A \rightarrow B_i),$$

[34] McGee [1989:527–531] shows that for every probability function on $\mathcal{L}_0$, there is a unique probability function on $\mathcal{L}_1$ that meets PC, IND, and ZA. Every probability function on $\mathcal{L}_1$ that satisfies PC, IND, and ZA is representable by its restriction to the associated positive antecedent fragment of $\mathcal{L}_1$, which is a probability function that satisfies PC and IND. Thus, Lemma A.1 holds.

[35] McGee (1989:493) shows that every probability function on $\mathcal{L}_1$ that satisfies PC, IND, and ZA also satisfies AT. As every probability function on the positive antecedent fragment of $\mathcal{L}_1$ that satisfies PC and IND is extendible to a probability on $\mathcal{L}_1$ that satisfies PC, IND, and ZA, it follows that Lemma A.2 holds.

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which, by Lemma A.2, amounts to

$$\sum_{i=1}^{n} \frac{\mathrm{Pr}_0\big(A \wedge (A \rightarrow B_i)\big)}{\mathrm{Pr}_0(A \wedge B_i)} \mathrm{Pr}_0(A)\,\mathrm{Pr}_1(A \rightarrow B_i).$$

By PC, this in turn amounts to

$$\sum_{i=1}^{n} \frac{\mathrm{Pr}_0(A \wedge B_i)}{\mathrm{Pr}_0(A \wedge B_i)} \mathrm{Pr}_0(A)\,\mathrm{Pr}_1(A \rightarrow B_i),$$

which can be rewritten as

$$\mathrm{Pr}_0(A) \sum_{i=1}^{n} \mathrm{Pr}_1(A \rightarrow B_i).$$

As the second factor amounts to 1, it follows that $\mathrm{Pr}_1(A) = \mathrm{Pr}_0(A)$.
(ii.a) On conditional probability given $\neg A$: By the definition of Pr_1,

$$\mathrm{Pr}_1(C \wedge \neg A) \;=\; \sum_{i=1}^{n} \mathrm{Pr}_0(C \wedge \neg A \mid A \rightarrow B_i)\,\mathrm{Pr}_1(A \rightarrow B_i).$$

By IND, this term rewrites as

$$\sum_{i=1}^{n} \frac{\mathrm{Pr}_0(C \wedge \neg A)\,\mathrm{Pr}_0(A \rightarrow B_i)}{\mathrm{Pr}_0(A \rightarrow B_i)} \mathrm{Pr}_1(A \rightarrow B_i).$$

As $\sum_{i=1}^{n} \mathrm{Pr}_1(A \rightarrow B_i) = 1$, it follows that $\mathrm{Pr}_1(C \wedge \neg A) = \mathrm{Pr}_0(C \wedge \neg A)$. Thus, by (i), $\mathrm{Pr}_1(C \mid \neg A) = \mathrm{Pr}_0(C \mid \neg A)$.
(ii.b) On conditional probability given $A \wedge B_i$ ($i \leqslant n$): By the definition of Pr_1, for any sentence C of $\mathcal{L}_0$ and any $i \leqslant n$,

$$\begin{aligned}\mathrm{Pr}_1(C \wedge A \wedge B_i) \;=\; & \mathrm{Pr}_0(C \wedge A \wedge B_1 \mid A \rightarrow B_1)\,\mathrm{Pr}_1(A \rightarrow B_1) \\ & + \cdots + \mathrm{Pr}_0(C \wedge A \wedge B_i \mid A \rightarrow B_i)\,\mathrm{Pr}_1(A \rightarrow B_i) \\ & + \cdots + \mathrm{Pr}_0(C \wedge A \wedge B_n \mid A \rightarrow B_n)\,\mathrm{Pr}_1(A \rightarrow B_n).\end{aligned}$$

By PC, the summands related to j's distinct from i amount to zero, and the i-related summand amounts to

$$\frac{\mathrm{Pr}_0(C \wedge A \wedge B_i)}{\mathrm{Pr}_0(A \rightarrow B_i)} \mathrm{Pr}_1(A \rightarrow B_i),$$

which, by Lemma A.2, amounts to

$$\frac{\mathrm{Pr}_0(C \wedge A \wedge B_i)}{\mathrm{Pr}_0(A \wedge B_i)} \mathrm{Pr}_0(A) \frac{\mathrm{Pr}_1(A \wedge B_i)}{\mathrm{Pr}_1(A)}.$$

By (i), this rewrites as

$$\frac{\mathrm{Pr}_0(C \wedge A \wedge B_i)}{\mathrm{Pr}_0(A \wedge B_i)} \mathrm{Pr}_1(A \wedge B_i).$$

From this, it follows that

$$\mathrm{Pr}_1(C \mid A \wedge B_i) \;=\; \frac{\mathrm{Pr}_0(C \wedge A \wedge B_i)}{\mathrm{Pr}_0(A \wedge B_i)} \;=\; \mathrm{Pr}_0(C \mid A \wedge B_i).$$

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(iii.a) That Pr_1 ranges over [0, 1] follows by the fact that $\sum_{i=1}^{n} \mathrm{Pr}_1(A \rightarrow B_i) = 1$ and the fact that Pr_0 takes values in [0, 1]. (iii.b) That Pr_1 takes the value 1 for tautologies follows by the first of the aforementioned facts and the fact that $\mathrm{Pr}_0(C \mid A \rightarrow B_i) = 1$ for all i. (iii.c) That $\mathrm{Pr}_1(C \vee D) = \mathrm{Pr}_1(C) + \mathrm{Pr}_1(D)$ if C and D are truth-functionally inconsistent can be seen as follows. By the definition of Pr_1,

$$\mathrm{Pr}_1(C \vee D) \;=\; \sum_{i=1}^{n} \mathrm{Pr}_0(C \vee D \mid A \rightarrow B_i)\, \mathrm{Pr}_1(A \rightarrow B_i).$$

As Pr_0 is a probability function and C and D are truth-functionally inconsistent, the right-hand side amounts to

$$\sum_{i=1}^{n} \big(\mathrm{Pr}_0(C \mid A \rightarrow B_i) + \mathrm{Pr}_0(D \mid A \rightarrow B_i)\big)\, \mathrm{Pr}_1(A \rightarrow B_i),$$

which, by the definition of Pr_1 and the fact that $\sum_{i=1}^{n} \mathrm{Pr}_1(A \rightarrow B_i) = 1$, amounts to $\mathrm{Pr}_1(C) + \mathrm{Pr}_1(D)$. □

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