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A new bipolar map projection

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ABSTRACT. This paper discusses map projections suitable for representing the whole of the Earth's surface while drawing particular attention to the polar regions, and proposes a new projection. The projection is a latitudinally distorted variant of the transverse Mollweide projection, relative to which it roughly doubles the linear scale and trebles the areal coverage of the polar regions. It was adopted by the Scott Polar Research Institute in 2005.

Contents

Introduction 215
The new projection 216
Summary and comments 222
Annex: transverse Mollweide projection for a spherical Earth 222
Reference 222

Introduction

The major map projections can be classed as cylindrical, pseudocylindrical, conical, azimuthal, and polyconic (for example, Bugayevskiy and Snyder 1995: 43). Of these, only the cylindrical and pseudocylindrical classes lend themselves to the representation of the whole of the Earth's surface. Of course, it is a simple matter to represent the entirety of the Earth's surface in *two* maps, one for each hemisphere, and this greatly extends the choice of available projections. However, this paper concerns itself with single global projections.

In a cylindrical projection in its normal aspect, the x-coordinate is proportional to the longitude, while the y-coordinate is some monotonic function of the latitude, so that the whole of the Earth's surface is represented by a rectangle. The simplest example, and the prototype of the cylindrical projections, is the Plate Carrée projection (sometimes referred to as the geographical or latitude-longitude projection) in which the y-coordinate is proportional to the latitude (Fig. 1). In any normal-aspect cylindrical projection, the upper edge of the rectangle represents the North Pole and the lower edge the South Pole. Such a projection is clearly undesirable if the intention is to draw attention to the polar regions, since these are highly distorted.

This difficulty is obviated by choosing a transverse aspect of the projection, in which the hypothetical cylinder that is wrapped around the Earth touches it along a meridian rather than along the equator. The transverse aspect of the Plate Carrée projection is illustrated in Figure 2. The extremes of distortion in this case occur at longitudes 90 degrees either side of the central meridian, again represented by the upper and lower edges of the rectangle.

In the pseudocylindrical projections in their normal aspect, the y-coordinate is again some monotonic function of the latitude. The x-coordinate, however, is the product of a linear function of longitude and some function of latitude. The full extent of the projection of the Earth's surface need therefore no longer be rectangular, and this allows for the possibility of introducing other desirable properties, such as constant areal scale, into the projection. However, the problem of extreme distortion of the polar regions remains, and this can again be obviated by choosing a transverse aspect of the projection. Figures 3A–3C illustrate other cylindrical projections, and Figures 4A–4D pseudocylindrical projections, in their transverse aspects.

Choosing among the many projections — of which Figures 2, 3A–C, and 4A–D are illustrative — is a question of fitness for purpose and, to some extent, of aesthetics. The desirable characteristics of a map projection are equivalence (otherwise referred to as the equal-area property), in which a given area of the map represents the same area on the Earth's surface wherever it is located; equidistance, in which the scale in one particular direction is constant; and conformality, which preserves the shape of objects. No map projection can combine all three of these criteria. Of the examples given in Figures 2, 3A–C, and 4A–D, the Plate Carrée and Lambert cylindrical projections are equidistant, while all of the pseudocylindrical projections have the equal-area property. An argument in favour of the pseudocylindrical projections and against the cylindrical projections can perhaps be made on aesthetic grounds, and similarly perhaps an aesthetic judgement can be made in favour of those pseudocylindrical projections that have an elliptical outline (for example, Mollweide, Fournier II, Apianus II) rather than a parabolic (Craster) or sinusoidal (Sanson-Flamsteed) outline.

Since about 1980, the Scott Polar Research Institute has employed as part of its logogram the transverse Mollweide projection. This, as has been noted, is an equal-area projection (Bugayevskiy and Snyder 1995: 71), which is, in fact, rather undesirable from the point of view of its symbolic use by a polar research institute. Since the polar regions, defined somewhat arbitrarily as lying within 30° of the poles, occupy between them only

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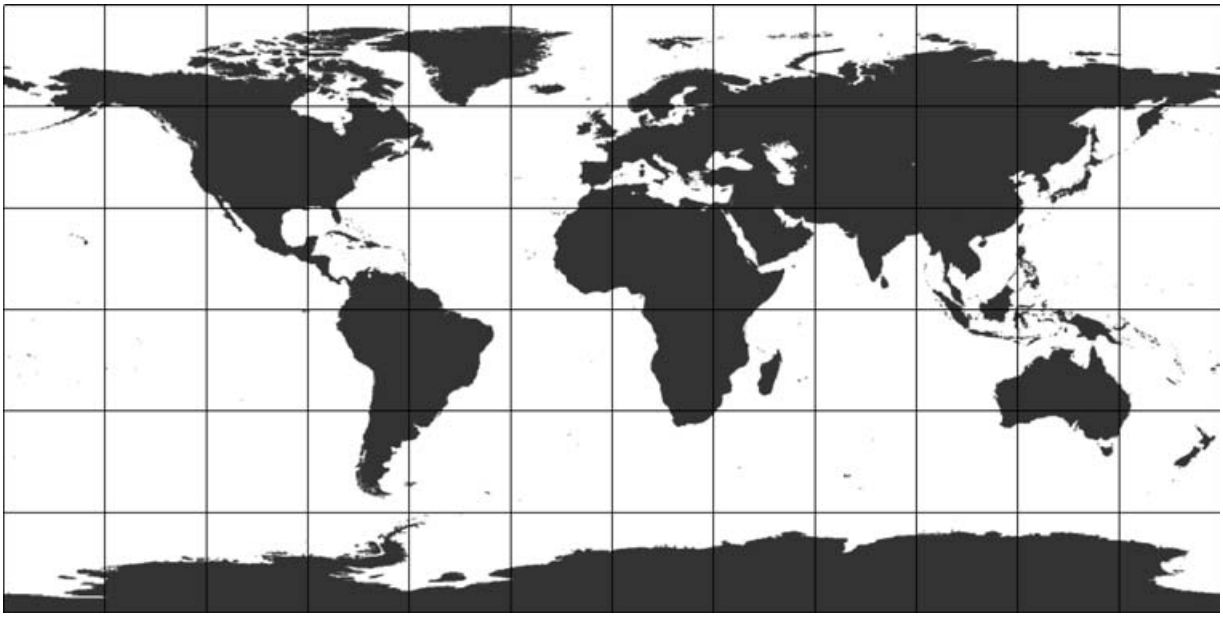


Fig. 1. Normal-aspect Plate Carrée projection with 30° graticule.

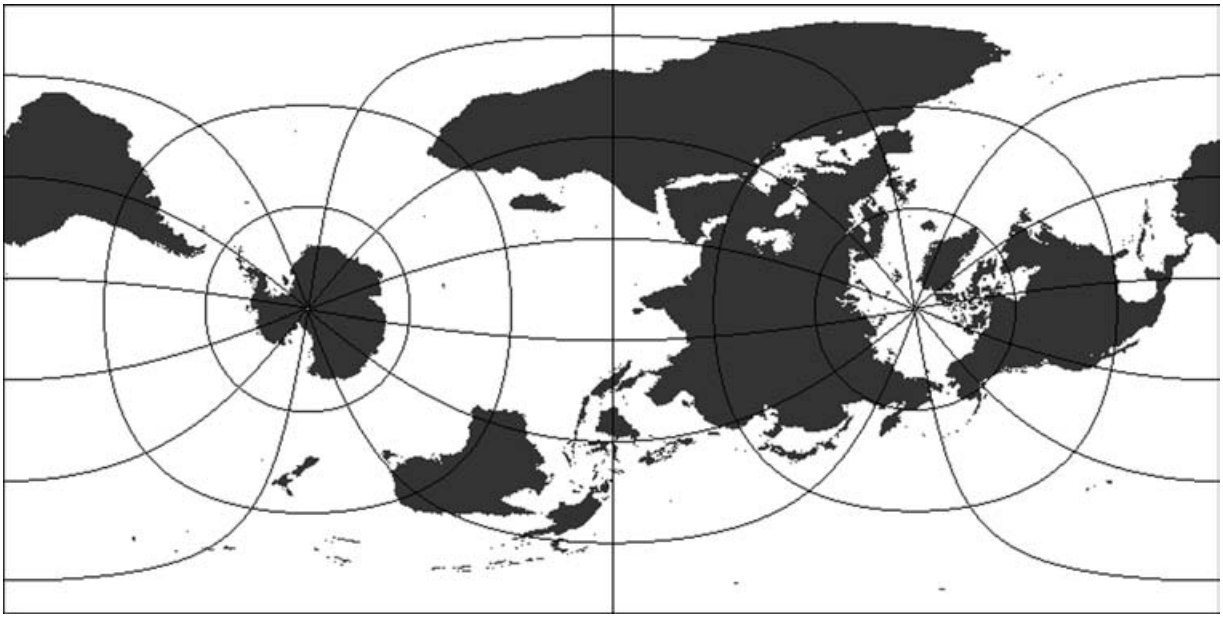


Fig. 2. Transverse-aspect Plate Carrée projection with 30° graticule. The central meridian is 80°E.

about 13% of the Earth's surface, it is clear that they are represented by less than one-sixth of the area of the figure. This characteristic is common to all of the transverse pseudocylindrical projections illustrated in Figures 4A–4D. The next section develops a new projection that deliberately rejects the equal-area principle in order to increase the prominence of the polar regions.

The new projection

The motivation for this research was specifically to develop a new logogram for the Scott Polar Research Institute. The requirements of the new projection were that it should have the same general form as the old one but exaggerate the area representing the polar regions without unduly further distorting them. In addition, the general outline of the world's continents should be reasonably recognisable. The choice was thus limited to projections based on the transverse pseudocylindrical projections, and, as Figures 4A–4D make clear, there is not a great deal to choose between these. It was therefore decided to base the new projection on the Mollweide projection, so that the old projection could be regarded as a special case of the new one. A very simple approach was adopted

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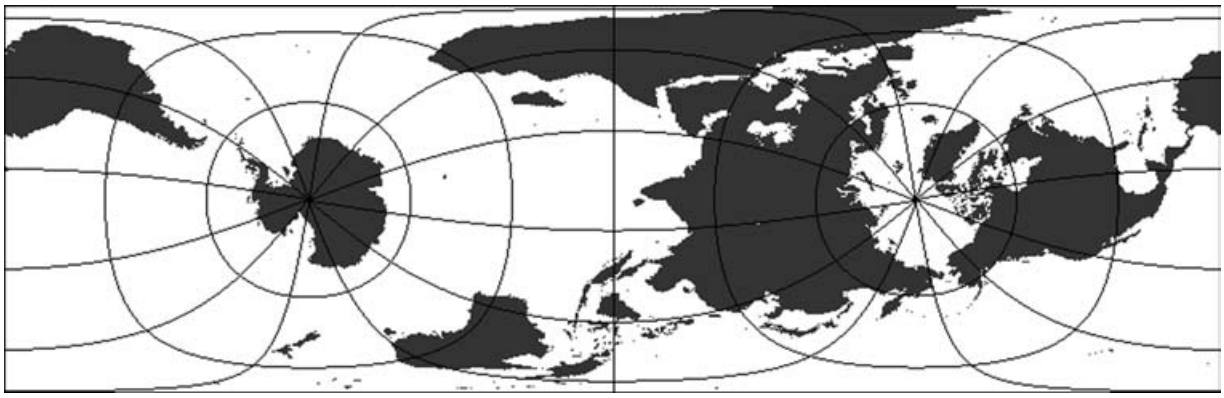


Fig. 3A. Lambert equal-area stereographic projection in transverse aspect, with central meridian of 80°E and 30° graticule.

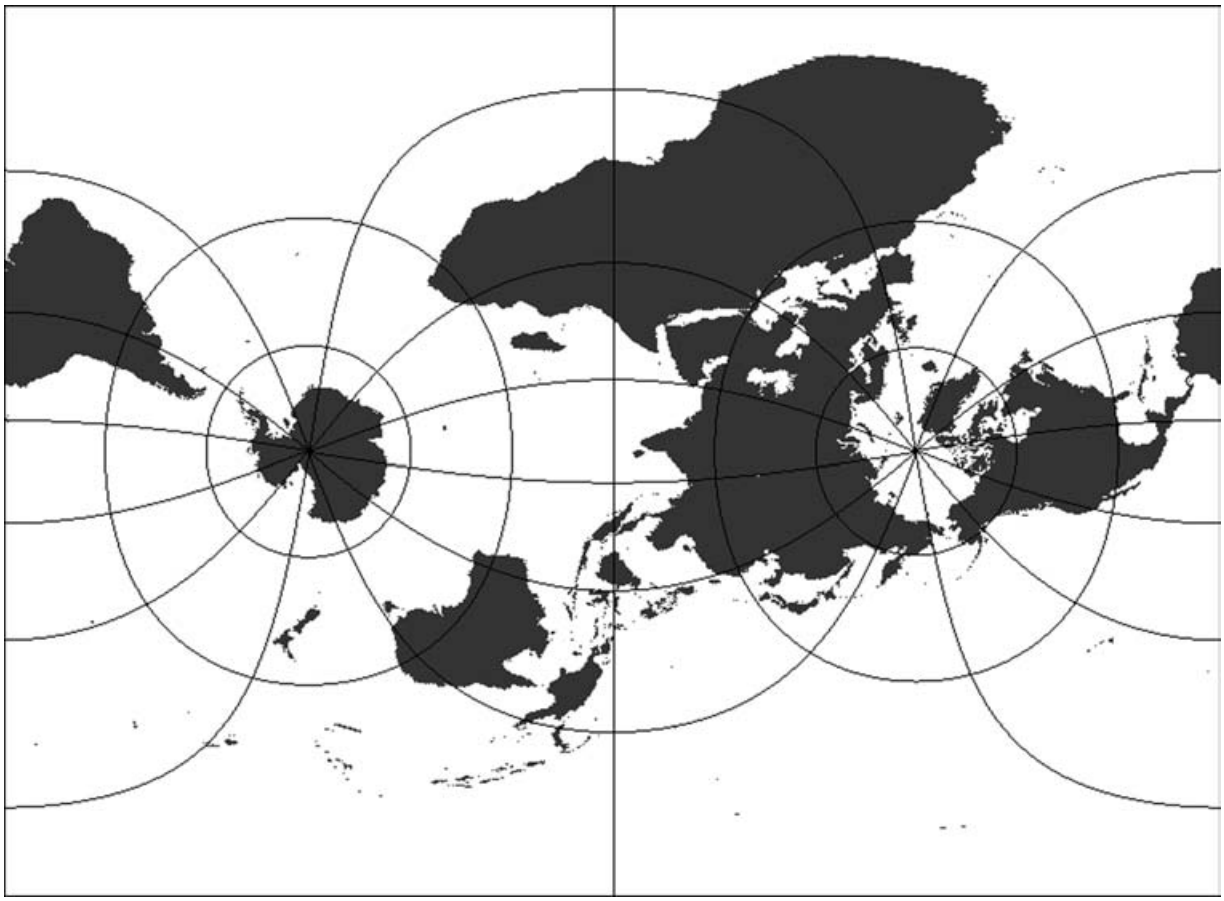


Fig. 3B. Miller modified Mercator projection in transverse aspect, with central meridian of 80°E and 30° graticule.

for the polar exaggeration. Writing $x_{t-Moll}(\phi, \lambda)$ and $y_{t-Moll}(\phi, \lambda)$ for the x- and y-coordinates of a point at latitude ϕ and longitude λ according to the transverse Mollweide projection (these functions are defined in the annex), the coordinates of the point in the new projection become

$$x_{new}(\phi, \lambda) = x_{t-Moll}(\phi^*, \lambda)$$
$$y_{new}(\phi, \lambda) = y_{t-Moll}(\phi^*, \lambda) \quad [1]$$

where

$$\sin\phi = (\sin\phi^*)^k \quad [2]$$

for ϕ and $\phi^* > 0$, and

$$-\sin\phi = (-\sin\phi^*)^k \quad [3]$$

for ϕ and $\phi^* < 0$. k is a constant that must be less than 1 in order to provide exaggeration of the polar regions,

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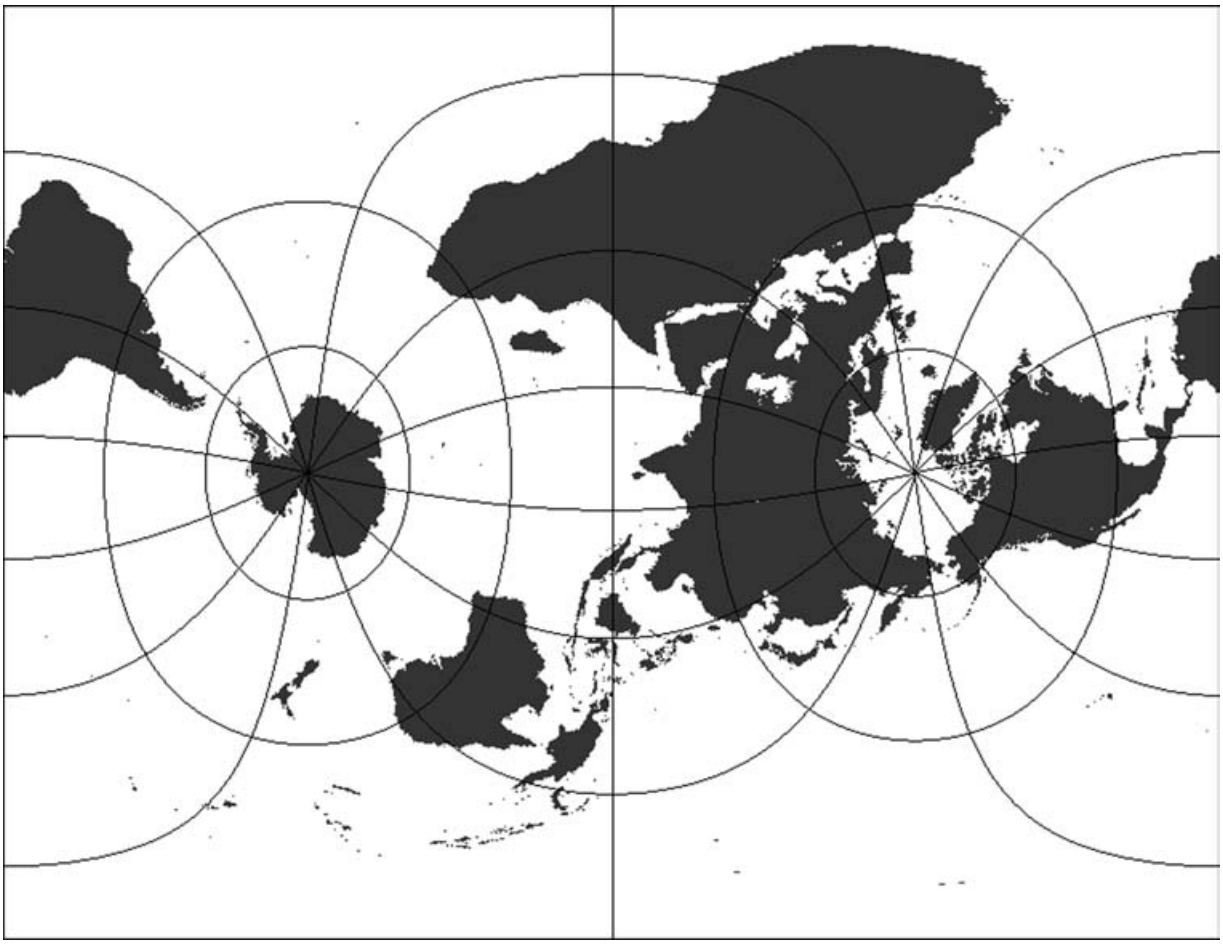


Fig. 3C. Gall stereographic projection in transverse aspect, with central meridian of 80°E and 30° graticule.

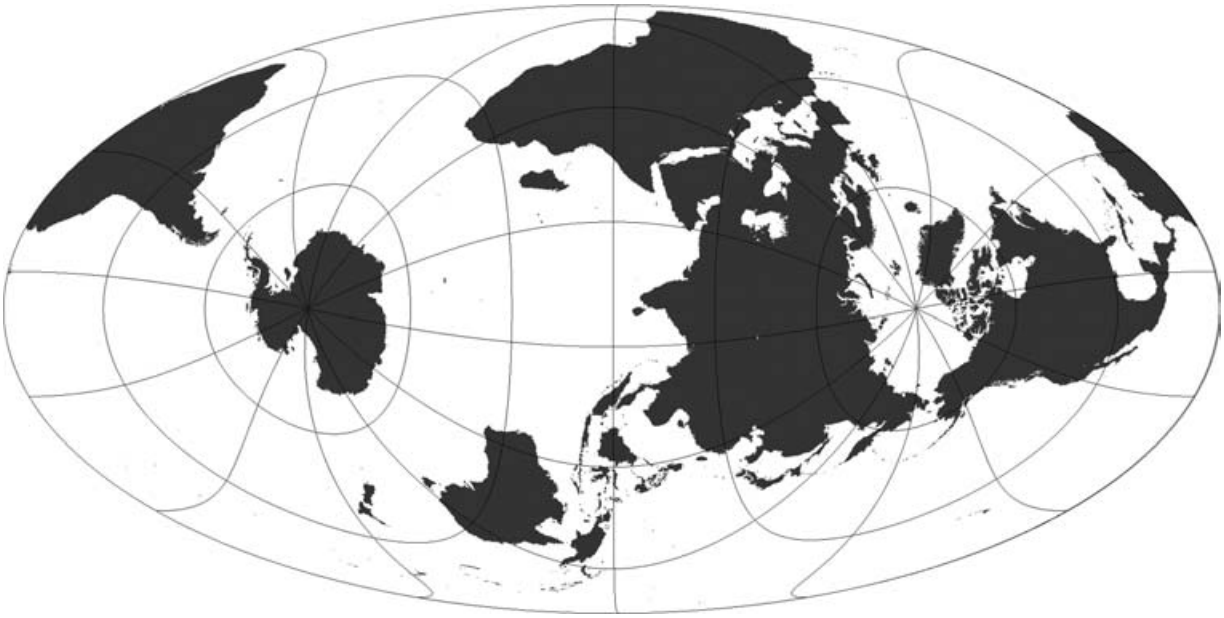


Fig. 4A. Mollweide projection in transverse aspect, with central meridian of 80°E and 30° graticule.

while a value of 1 clearly corresponds to the unmodified transverse Mollweide projection. The behaviour of the new projection can be simply understood as follows. It preserves the form of the transverse Mollweide projection, and preserves the positions of the meridians. However, it in effect relabels the lines of constant latitude, so that a

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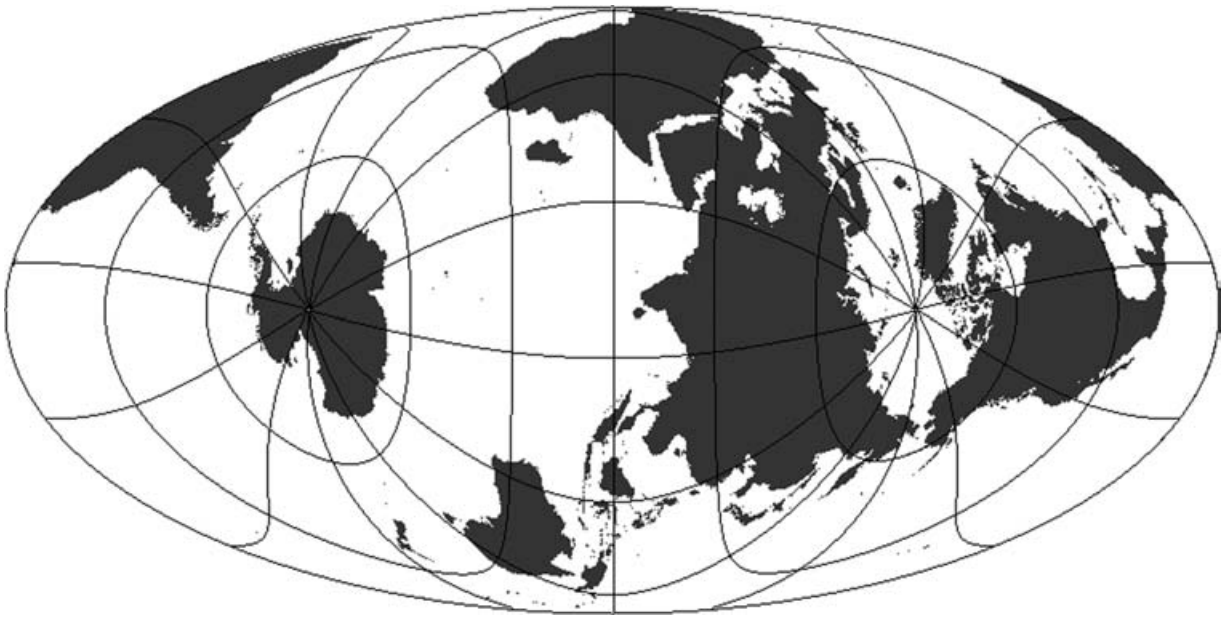


Fig. 4B. Fournier II projection in transverse aspect, with central meridian of 80°E and 30° graticule.

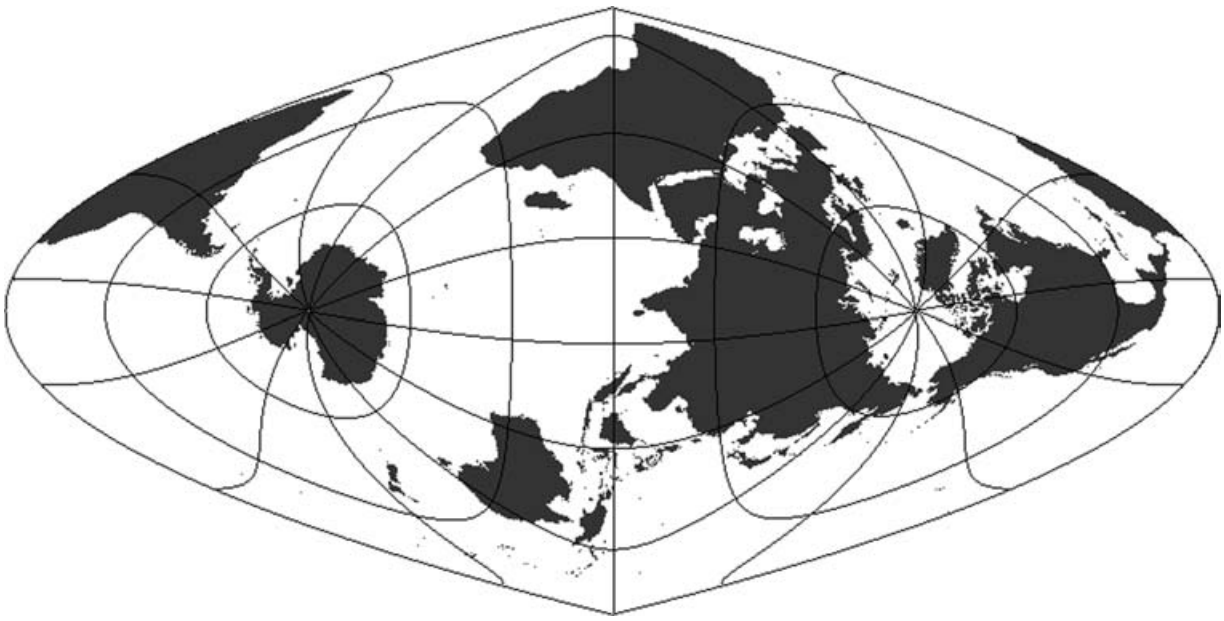


Fig. 4C. Craster projection in transverse aspect, with central meridian of 80°E and 30° graticule.

point with true latitude ϕ is plotted at the position given by the transverse Mollweide projection for a point with latitude ϕ^*. The choice of $k<1$ ensures that ϕ^* is farther away from the nearer pole than is ϕ.

The major implications of this model for the polar regions can be seen quite easily. The fraction of the total area occupied by latitudes above 60° becomes

$$1 - (\sin 60\text{þ})^{1/k}\,, \qquad [4]$$

which is greater than the value for the transverse Mollweide projection if $k<1$. There must inevitably be a decrease in the fractional area occupied by lower latitudes.

The linear scale at the poles becomes

$$\frac{2\sqrt{2}}{\pi\sqrt{k}} \qquad [5]$$

along the central meridian and

$$\frac{\pi\sqrt{2}}{4\sqrt{k}} \qquad [6]$$

along the perpendicular meridian, so these scales are increased by the same factor of $1/\sqrt{k}$ relative to the unmodified transverse Mollweide projection.

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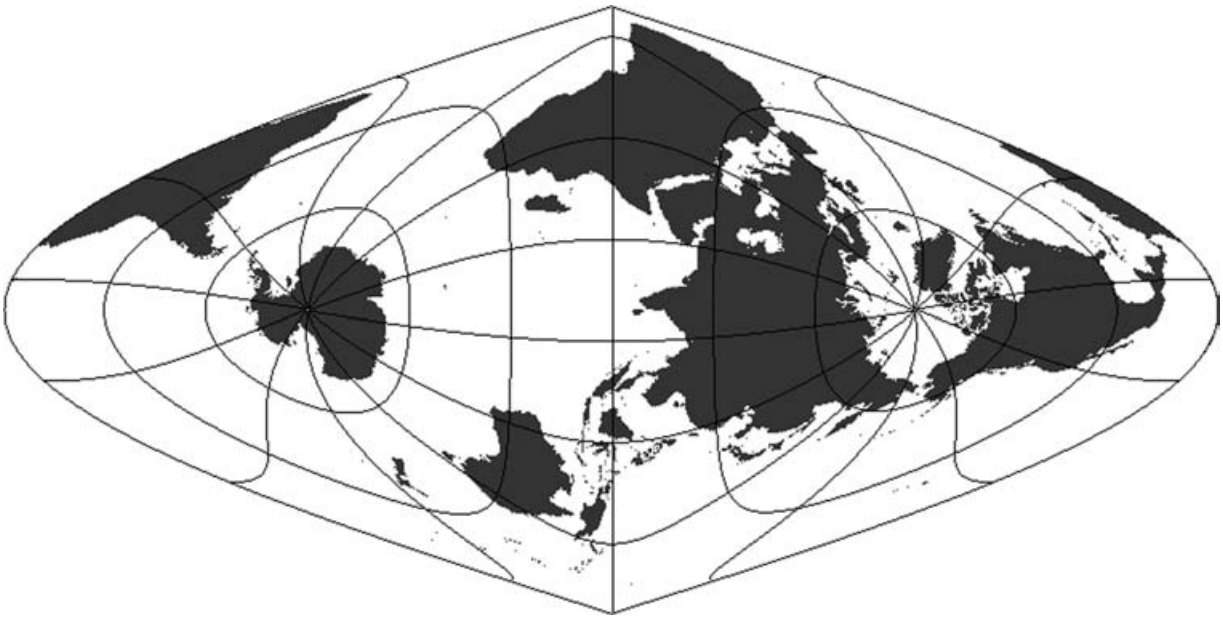


Fig. 4D. Sanson-Flamsteed projection in transverse aspect, with central meridian of 80°E and 30° graticule.

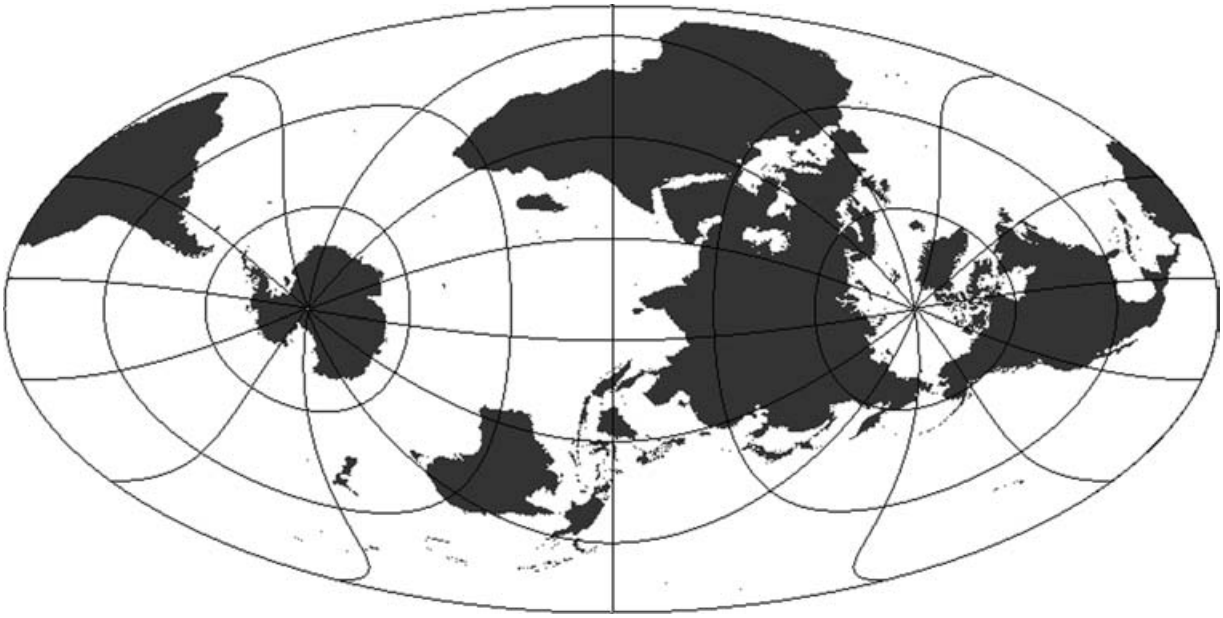


Fig. 4E. Apianus II projection in transverse aspect, with central meridian of 80°E and 30° graticule.

Figure 5 illustrates the effect of varying the exaggeration parameter k. It shows how the pattern of meridians is unchanged, while the lines of constant latitude move towards the Equator as k is decreased. Although decreasing the value of k increases the fractional area occupied by the polar regions as required, when it falls below about 0.25 the extreme compression of equatorial latitudes significantly reduces the recognisability of most continents. For this reason, an aesthetic judgement was made to choose a value of $k = 0.3$. This increases the fractional area occupied by the polar regions from 13% to 38%, that is, by a factor of about 2.8. The linear scales at the poles are increased by a factor of about 1.8.

Having selected a value for k, it remained to choose a suitable longitude for the central meridian. Since the most obvious distortion in the projection is in the region of the compressed equatorial region, the chosen criteria were to minimise the length along the y-axis represented by land, while at the same time attempting as far as possible to avoid splitting continents across the elliptical boundary of the projection. Again this involved an aesthetic judgement, and a central meridian of 150°W

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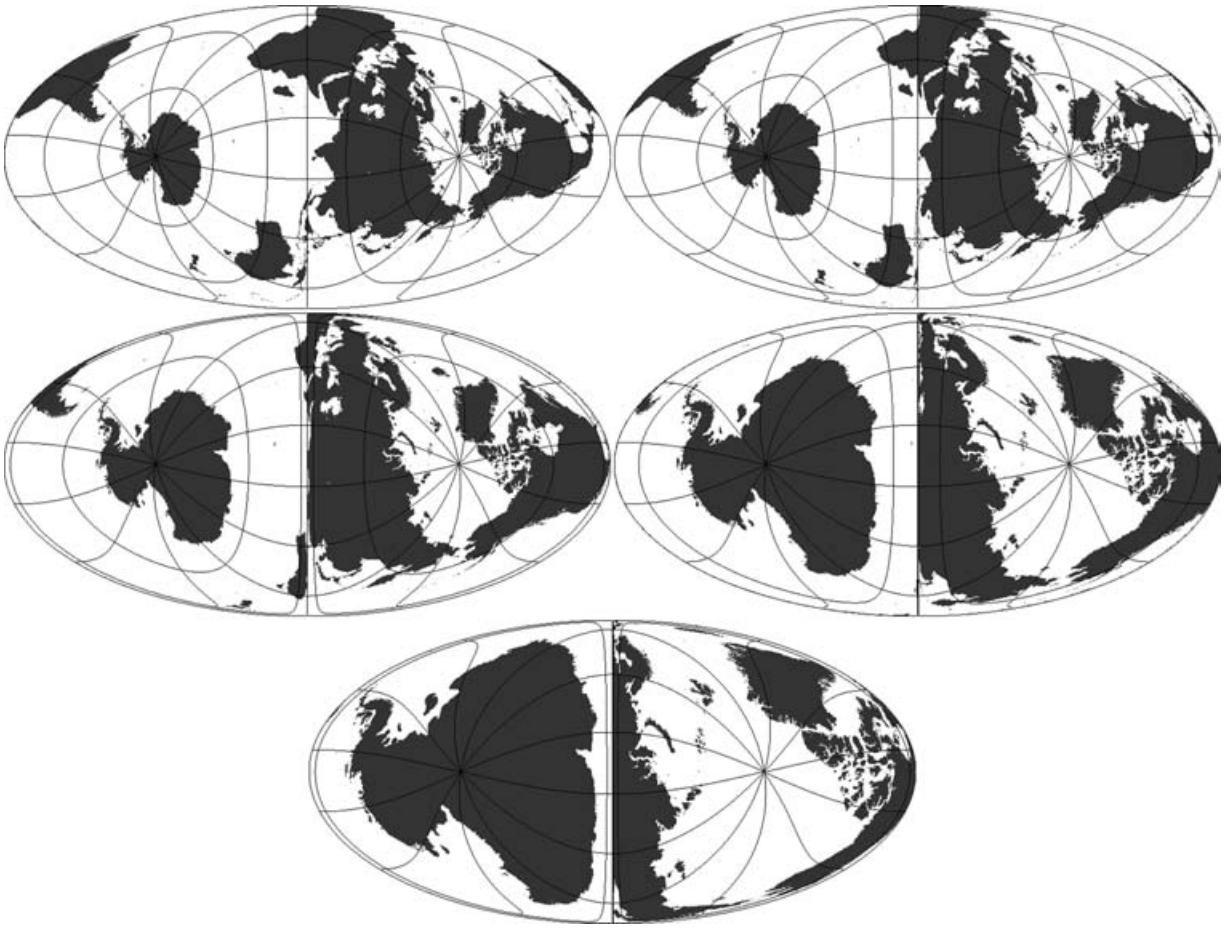


Fig. 5. Modified transverse Mollweide projections with $k = 0.75$, 0.5 (top row), 0.25, 0.1 (middle row), 0.05 (bottom row). The graticule interval is 30°. The central meridian is at 80°E in all cases, so that the transverse Mollweide projection of Figure 4 corresponds to the case $k = 1$.

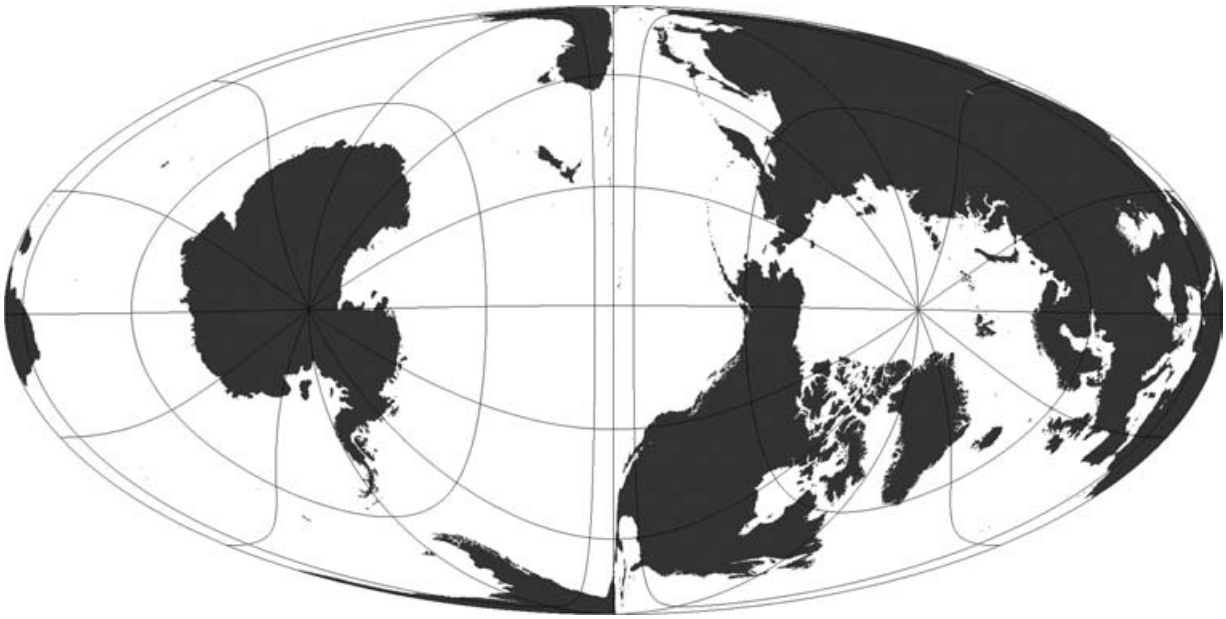


Fig. 6. Modified transverse Mollweide projection with $k = 0.3$ and central meridian of 150°W. The graticule interval is 30°.

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was selected. This final projection, which was adopted by the Scott Polar Research Institute in 2005, is shown in Figure 6.

Summary and comments

The new projection is a distorted variant of the transverse Mollweide projection, with a central meridian of 210°E and a distorted latitudinal scale that accentuates the polar regions at the expense of the equatorial regions. The chosen distortion ensures that about 38% of the area represents latitudes above 60°, compared with only about 13% for the standard transverse Mollweide projection. The linear scale at the poles is increased almost by a factor of 2 compared with the transverse Mollweide projection. Distortion is not excessive at latitudes above 60°, and the general form of the projection is recognisable down to latitudes of about 40°. Although the projection in principle shows the entirety of the Earth's surface, the compression of the equatorial regions means that in practice latitudes below about 20° are practically invisible. The projection has little cartographic value, since it lacks any of the properties (such as equidistance, conformality, or the equal-area property (Bugayevskiy and Snyder 1995: 20) that are usually regarded as desirable. Nevertheless, it meets its purpose of providing a recognisable representation of the Earth that gives particular prominence to the polar regions.

Annex: transverse Mollweide projection for a spherical Earth

The projection equations for the unmodified transverse Mollweide projection are as follows:

$$x_{t-Moll}(\phi, \lambda) = \frac{2\sqrt{2}\lambda' \cos\psi}{\pi}$$

$$y_{t-Moll}(\phi, \lambda) = \sqrt{2}\sin\psi$$

where the auxiliary angle [ ] is given by

$$\sin 2\psi + 2\psi = -\pi \sin\lambda \cos\phi$$

and the auxiliary angle λ' is given by

$$\tan\lambda' = \frac{\tan\phi}{\cos\lambda}.$$

This projects the Earth's surface onto an ellipse with a semimajor axis (along the x-axis) of $2\sqrt{2}$ and a semiminor (y-) axis of $\sqrt{2}$.

Reference

Bugayevskiy, L.M., and J.P. Snyder.1995.*Map projections: a reference manual*. London: Taylor & Francis.

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