14 Standard Model Effective Field Theory

14.1 Basic Framework

If there is a separation of scales between the electroweak scale v and the NP scale Λ, as is suggested by the absence of any new particles close to the electroweak scale in LHC searches so far, one can investigate the implications of the unbroken SM gauge symmetry on the correlations between various observables. To study these correlations in a model-independent manner, one can consider an OPE with dimension-5, dimension-6 or higher-dimension operators built out of SM fields that are invariant under the full SM gauge symmetry. This corresponds to an effective field theory where all the SM degrees of freedom are kept as dynamical degrees of freedom, and only the NP is integrated out.

The application of EFT approach [113, 114] here not only allows to resum large logarithms appearing if NP scales are much higher than the EW scale, but also serves as a convenient intermediate step between an ultraviolet completion formulated at a high NP scale and low-energy phenomenology. The type of EFT considered here is usually called SMEFT because at low-energies, it should reduce to the SM, provided no undiscovered but weakly coupled *light* particles exist, like axions or sterile neutrinos. Yet, as we will see later, the presence of new operators, in particular dimension-6 ones, whose Wilson coefficients can in principle be arbitrary, can introduce very significant modifications of SM predictions for flavor observables and also influence the values of SM parameters. Therefore the name SMEFT is in fact a misnomer because the Wilson coefficients of involved operators contain information about NP beyond the SM, and also there is a multitude of operators that are strongly suppressed in the SM but can be important in SMEFT. But we will use this name in what follows as it is used in the literature. A recent review of SMEFT with applications to Higgs physics can be found in [757]. Here we will concentrate on flavour physics and generally low-energy processes. In any case if no new particles will be found at the LHC in the coming years, the language of the SMEFT will dominate elementary particle physics in the coming decades, and it is crucial to learn it as soon as possible.

The effective Lagrangian of SMEFT at dimension-6 is given as follows

$$\mathcal{L}^{(6)} = \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)}, \tag{14.1}$$

with the contributing operators classified in full generality in [561, 562]. The second of these papers removed certain redundant operators present in the first one, and we will use the results of [562] here. The corresponding renormalization group analysis at LO of all these operators has been analyzed in [758–760].

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For three generations of fermions there are 2,499 independent operators $Q_k^{(6)}$ (151 irreducible flavour representations) that do not violate baryon and lepton number. This means that the RG analysis involves in full generality a $2,499 \times 2,499$ anomalous matrix with the evolution governed by the Higgs self-coupling λ [758], Yukawa couplings [759], and SM gauge interactions [760]. While several of the entries in this matrix have been known before, not only at LO and already appeared in previous sections, these papers provided a large number of new entries so that RG effects can be investigated, albeit only at LO, in full generality.

The Wilson coefficients $C_k^{(6)}$ contain 1,350 CP-even and 1,149 CP-odd parameters. They contain the information about NP at and above scales $O(\Lambda)$. In this totally model-independent approach, these coefficients should be determined from the pattern of deviations from SM expectations at the electroweak scale and low-energy scales considered in this book. Clearly, this goal cannot be realized in view of very many free parameters, but as we will see later, this classification of operators and the results for RG effects turn out to be very useful in concrete models.

It should also be emphasized that in addition to the 2,499 operators in question, there are 273 D = 6 $\Delta B = \Delta L = 1$ operators (7 irreducible flavor representations) and their Hermitian conjugates, which are the leading operators that permit proton decay in SMEFT [561, 562, 761–764].

The important point stressed already in the past and in particular emphasized in [760] is that RG evolution between the NP scale Λ and the electroweak scale involves operator mixing, and the pattern of deviations from SM expectations observed at the electroweak scale and low-energy scales can differ significantly from the pattern of Wilson coefficients at the NP scale Λ. Here we just quote one example known already for decades in the context of QCD RG evolution. Tiny effects of right-handed currents originating at a very high scale Λ, when accompanied by left-handed currents, also generated by NP or present in the SM, can in $\Delta F = 2$ observables cause very large effects in flavor observables. This is due to the presence of left-right operators with enhanced Wilson coefficients through RG evolution to scales $O(1\,\text{GeV})$ and chirally enhanced hadronic matrix elements, in particular in the K-meson system. See previous chapter.

However, the point made in [760] and in subsequent papers [484, 666, 719, 765–770] is not related to QCD but in particular to top Yukawa couplings. Even in the absence of left-handed currents generated by NP, the presence of right-handed currents in the presence of Yukawa couplings can generate in the process of RG evolution left-right operators at the electroweak scale, because left-handed currents are already present in the SM. Moreover, this effect is further enhanced through QCD RG evolution to low-energy scales as stated earlier. This underlines the importance of RG studies in the search for NP far beyond the LHC scales. As stressed in particular in [770], the bounds from $\Delta F = 2$ processes can affect in turn $\Delta F = 1$ and $\Delta F = 0$ processes.

I find the analyses in [562, 758–760] impressive, but I do not think that they will teach us much about NP at very high scales in the context of a bottom-up approach in which one wants to be totally model independent. Exceptions are observables in which only a very limited number of operators is allowed to contribute because of the special structure of given observables or additional assumptions beyond SM gauge invariance, in particular

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flavor symmetries and their breakdown. But these analyses form an important basis for a top-down approach in which some model assumptions at high scales are made and in particular are very useful in concrete NP models. While such a strategy towards the identification of NP beyond the LHC requires the study of many models, the number of models studied until now is by roughly two orders of magnitude lower than the numbers of free parameters involved in a general bottom-up approach.

In this section we will list all the operators collected in [562], and we will describe the results of the analyses in [758–760] in general terms. Subsequently, we will present several examples of RG equations in which only few operators will have nonvanishing coefficients at the high scale Λ, but at low-energy new operators will contribute to observables with Wilson coefficients generated through the RG evolution in question. In choosing these examples, we anticipated their application in concrete models to be presented partly in subsequent chapters and partly found in the rich literature. Of particular interest are the following scenarios:

Scenario I

At the high NP scale Λ only neutral flavor-violating right-handed (RH) quark currents are generated by NP with profound implications for $\Delta F = 2$ transitions as advertised earlier and their specific correlations with $\Delta F = 1$ observables. Concrete models with this structure are models with flavor-violating Z couplings considered in Section 15.5 and in particular models with vector-like quarks that we will analyze in Section 16.3.

Scenario II

At the high NP scale Λ only neutral flavor-violating left-handed (LH) quark currents are generated by NP with interesting but different implications for the correlation between $\Delta F = 2$ and $\Delta F = 1$ transitions than found in Scenario I. The corresponding models with vector-like quarks will be analyzed in Section 16.3. These results will also be important for Section 15.5 in which FCNC mediated by the Z boson will be considered.

Scenario III

This scenario is like Scenarios I and II but involving neutral flavor-violating RH and LH lepton currents instead of quark currents with implications through RG evolution on semileptonic operators. Concrete models of this scenario are the ones involving vector-like leptons. We will comment on them in Section 16.3.

Scenario IV

At high NP scale Λ only semileptonic operators are present as found in leptoquark models that will be analyzed in Section 16.4. RG evolution in the presence of Yukawa couplings and gauge interactions has then impact on lepton-flavor-violating lepton decays and implies modifications of SM couplings. This case is important in the context of the B physics anomalies that require enhanced contributions from semileptonic operators that in turn can have impact on lepton decays in question implying enhancements of the latter.

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The experimental bounds on lepton decays can in turn bound the required enhancements of semileptonic B decays as stressed in particular in [765, 767].

For instance, LFU breaking effects in $\tau \to \ell\bar{\nu}\nu$ (with $\ell_{1,2} = e, \mu$) are described by the observables

$$R_\tau^{\tau/\ell_{1,2}} = \frac{\mathcal{B}(\tau \to \ell_{2,1}\nu\bar{\nu})_{\rm exp}/\mathcal{B}(\tau \to \ell_{2,1}\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(\mu \to e\nu\bar{\nu})_{\rm exp}/\mathcal{B}(\mu \to e\nu\bar{\nu})_{\rm SM}} \tag{14.2}$$

and are experimentally tested at the few permille level [716]

$$R_\tau^{\tau/\mu} = 1.0022 \pm 0.0030, \quad R_\tau^{\tau/e} = 1.0060 \pm 0.0030. \tag{14.3}$$

Similar, modifications of the leptonic Z couplings are constrained by the LEP measurements of the Z decay widths, left-right, and forward-backward asymmetries. The bounds on lepton nonuniversal couplings are according to PDG

$$\frac{v_\tau}{v_e} = 0.959\ (29), \qquad \frac{a_\tau}{a_e} = 1.0019\ (15), \tag{14.4}$$

where v_ℓ and a_ℓ are the vector and axial-vector couplings, respectively, defined as $v_\ell = g_{\ell L}^{\ell\ell} + g_{\ell R}^{\ell\ell}$ and $a_\ell = g_{\ell L}^{\ell\ell} - g_{\ell R}^{\ell\ell}$. These few examples show how careful one has to be not to violate the existing bounds. Further examples can be found in [765, 767].

Scenario V

Again, as in Scenario IV, at the high scale only semileptonic operators are present, but this time we will study the impact of RG evolution on $\Delta F = 1$ nonleptonic decays like $K_L \to \pi\pi$ and the ratio ε'/ε. This case is again interesting in the context of leptoquark models and will be briefly discussed in Section 16.4.13, with more details found in [667].

Scenario VI

In this scenario we will consider only operators relevant for $b \to s\nu\bar{\nu}$ and $b \to s\ell^+\ell^-$ transitions with the goal to identify some correlations between them that follow from SM gauge invariance. Indeed the $b \to s\nu\bar{\nu}$ transition is closely related to the $b \to s\ell^+\ell^-$ transition because the neutrinos and left-handed charged leptons are related by $\mathrm{SU}(2)_L$ symmetry.

The same comments apply to the $d \to s\nu\bar{\nu}$ and $d \to s\ell^+\ell^-$ transitions. While, already at present, the correlations between these transitions imply stringent constraints on parameters of specific NP scenarios, significant LD uncertainties in $d \to s\ell^+\ell^-$ and no real plans for measuring $K_L \to \pi^0\ell^+\ell^-$ in the near future make these connections less interesting than the B physics case at present. Yet, there exist dedicated analyses of such correlations in the literature, and we will refer to them as we proceed.

Scenario VII

Of particular interest are scenarios that imply correlations between $b \to s\nu\bar{\nu}$ transitions studied at BELLE II and $d \to s\nu\bar{\nu}$ investigated at NA62 and KOTO. In MFV scenarios and simplified NP scenarios, these correlations are very stringent [216, 601]. Involving two meson systems, they test the flavor structure of different models.

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Definitely, these are just few simplified scenarios. In reality a given NP model may contain several of these scenarios that are correlated with each other and contain still other scenarios not listed here. The analysis of such models becomes then very involved, and one has to develop efficient computer codes to get useful results. We will list existing codes in the course of our presentation. Yet, we think that already these scenarios will give us some insight in the dynamics involved and even excluding some of them one day will teach us something.

Significant part of this chapter resulted from the collaboration with Christoph Bobeth and benefited from papers and discussions with other colleagues, whose names will appear later.

14.2 Full Set of Dimension-6 Operators

We will now list the operators $Q_k^{(6)}$ classified in [562] and listed in Tables 14.1 and 14.2. The sign conventions for covariant derivatives in that paper differ from ours and correspond to the conventions 2 in Section 1.8, and these are also the conventions used in [758–760]. But it is easy to translate these conventions to ours by simply reversing the signs of all couplings, and we will do it to be consistent with the rest of the book. The reader may check by comparing our expressions with those in the papers that indeed this is what has to be changed. We also warn the reader that our Yukawa matrices, like Y_u, are defined as the Hermitean conjugate w.r.t. [759]. But the most important warning is the following one:

The Wilson coefficients of the operators in the SMEFT are defined by the Lagrangian and not by the Hamiltonian as used in the rest of this book!

Table 14.1 Dimension-6 operators other than the four-fermion ones.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
O_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	O_φ	$(\varphi^\dagger\varphi)^3$	$O_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$O_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$O_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$O_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
O_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$O_{\varphi D}$	$\left(\varphi^\dagger D^\mu \varphi\right)^\star \left(\varphi^\dagger D_\mu \varphi\right)$	$O_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$O_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$O_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	O_{eW}	$(\bar{\ell}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$O_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\ell}_p \gamma^\mu \ell_r)$
$O_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	O_{eB}	$(\bar{\ell}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$O_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{\ell}_p \tau^I \gamma^\mu \ell_r)$
$O_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	O_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$O_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$O_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	O_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{\varphi}\, W_{\mu\nu}^I$	$O_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$O_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	O_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$O_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$O_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	O_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$O_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$O_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	O_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$O_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$O_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	O_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$O_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

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Table 14.2 Four-fermion operators.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
O_{ll}	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{\ell}_s\gamma^\mu\ell_t)$	O_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	O_{le}	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{e}_s\gamma^\mu e_t)$
$O_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	O_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	O_{lu}	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{u}_s\gamma^\mu u_t)$
$O_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	O_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	O_{ld}	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{d}_s\gamma^\mu d_t)$
$O_{lq}^{(1)}$	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{q}_s\gamma^\mu q_t)$	O_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	O_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$O_{lq}^{(3)}$	$(\bar{\ell}_p\gamma_\mu\tau^I\ell_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	O_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$O_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$O_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$O_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$O_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$O_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$O_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$				B-violating	
O_{ledq}	$(\bar{\ell}_p^j e_r)(\bar{d}_s q_t^j)$	O_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C\ell_t^k\right]$		
$O_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	O_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$		
$O_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	O_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$O_{lequ}^{(1)}$	$(\bar{\ell}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	O_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$		
$O_{lequ}^{(3)}$	$(\bar{\ell}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$				

Yet, it is very straightforward to take this difference (just the sign) into account in the process of the matching of SMEFT Wilson coefficients onto the ones in the low-energy effective theory (LEFT) describing the physics below the electroweak scale. This will be evident from examples presented by us. To make it crystal clear, in contrast to conventions in covariant derivatives and Yukawa couplings, the signs of Wilson coefficients of the operators in the SMEFT presented by us are as in [758–760]. But when matching on LEFT, Wilson coefficients considered by us so far and defined through effective Hamiltonians, we will take this sign difference into account.

The well-known expression for the SM Lagrangian before electroweak spontaneous symmetry breakdown (EWSB) reads

$$\begin{aligned}\mathcal{L}_{\rm SM}^{(4)} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \left(D_\mu\varphi\right)^\dagger\left(D^\mu\varphi\right) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda\left(\varphi^\dagger\varphi\right)^2 \\ &+ i\left(\bar{l}\not{D}l + \bar{e}\not{D}e + \bar{q}\not{D}q + \bar{u}\not{D}u + \bar{d}\not{D}d\right) - \left(\bar{l}\,Y_e e\varphi + \bar{q}\,Y_u u\widetilde{\varphi} + \bar{q}\,Y_d d\varphi + \text{h.c.}\right),\end{aligned} \tag{14.5}$$

where the Yukawa couplings $Y_{e,u,d}$ are matrices in the generation space. As stated earlier, our sign convention for covariant derivatives differs from [562] as we use

$$\left(D_\mu q\right)^{\alpha j} = \left(\partial_\mu - ig_s t^A_{\alpha\beta}G^A_\mu - igS^I_{jk}W^I_\mu - ig'Y_q B_\mu\right)q^{\beta k}, \tag{14.6}$$

so that in front of gauge couplings in the vertices there are no minus signs. Here,

$$t^A = \frac{1}{2}\lambda^A, \qquad S^I = \frac{1}{2}\tau^I \tag{14.7}$$

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are the SU(3) and SU(2) generators, while λ^A and $\tau^I = \sigma^I$ are the Gell-Mann and Pauli matrices, respectively.[1] Complex conjugate of the Higgs field occurs either as $\varphi^\dagger$ or $\widetilde{\varphi}$, where $\widetilde{\varphi}^j = \varepsilon_{jk}(\varphi^k)^\star$, and ε_{jk} is totally antisymmetric with $\varepsilon_{12} = +1$.

It is useful to define Hermitian derivative terms that contain $\varphi^\dagger \overleftarrow{D}_\mu \varphi \equiv (D_\mu \varphi)^\dagger \varphi$ as follows:

$$\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi \equiv i\varphi^\dagger \left(D_\mu - \overleftarrow{D}_\mu \right) \varphi \qquad \text{and} \qquad \varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi \equiv i\varphi^\dagger \left(\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I \right) \varphi. \tag{14.8}$$

In the following formulas we will often set $\varphi = H$ as this notation is also used in the literature.

The gauge field-strength tensors and their covariant derivatives read

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_3 f^{ABC} G^B_\mu G^C_\nu, & \left(D_\rho G_{\mu\nu}\right)^A &= \partial_\rho G^A_{\mu\nu} + g_3 f^{ABC} G^B_\rho G^C_{\mu\nu}, \\ W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g_2 \varepsilon^{IJK} W^J_\mu W^K_\nu, & \left(D_\rho W_{\mu\nu}\right)^I &= \partial_\rho W^I_{\mu\nu} + g_2 \varepsilon^{IJK} W^J_\rho W^K_{\mu\nu}, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, & D_\rho B_{\mu\nu} &= \partial_\rho B_{\mu\nu}. \end{aligned} \tag{14.9}$$

Again the signs in front of gauge couplings differ from those in [562].

Dual tensors are defined by

$$\widetilde{X}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} X^{\rho\sigma}, \qquad \varepsilon_{0123} = +1, \tag{14.10}$$

where X stands for G^A, W^I, or B.

14.3 Rotations in the Flavor Space

14.3.1 SM Repetition

We have now specified the set of linearly independent operators. Yet this is not the full story. To proceed we have to specify the flavor basis in which we plan to perform calculations, including the RG evolution above the electroweak scale. To this end let us repeat what we already stated in Section 2.5 in the context of the SM.

The SM interaction Lagrangian of (2.3) is invariant under a $[\mathrm{U}(3)]^5$ flavor symmetry

$$Q_L \to V^u_L Q, \quad u_R \to V^u_R u_R, \quad d_R \to V^d_R d_R, \tag{14.11}$$

$$L \to V^e_L L, \quad e_R \to V^e_R e_R, \tag{14.12}$$

where $V^u_L, V^u_R, V^d_R, V^e_L$, and V^e_R are unitary 3×3 matrices. This is the consequence of the fact that there is the universality of the gauge couplings in a given family of fermions, left-handed or right-handed. In the SM the Yukawa sector (2.13) breaks flavor universality in a given family, and consequently $[\mathrm{U}(3)]^5$ symmetry explicitly simply because the Yukawa

[1] In the rest of the book we use σ^I for Pauli matrices but not to confuse the notation with $\sigma_{\mu\nu}$ present in the operators we use the symbol τ^I for them here.

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couplings to fermions must be flavor nonuniversal to reproduce the known mass spectrum of quarks and leptons. The preferred basis in which calculations are performed is the mass eigenstate basis in which the Yukawa and consequently mass matrices are diagonalized as explicitly given by

$$(V_L^d)^\dagger \Upsilon^D V_R^d = \hat{\Upsilon}^D, \quad (V_L^u)^\dagger \Upsilon^U V_R^u = \hat{\Upsilon}^U, \quad (V_L^e)^\dagger \Upsilon^E V_R^e = \hat{\Upsilon}^E, \tag{14.13}$$

with $\hat{\Upsilon}^i$ being diagonal.

Now because of the universality of gauge couplings and the unitarity of rotation matrices, FCNC are absent and appear only in the charged currents parametrized by CKM and PMNS matrices. It should be stressed that it is irrelevant whether we rotate the down-quarks from flavor to mass eigenstates as in (2.45) and assume flavor and mass eigenstates in the up-quark system to be equal, or rotate the up-quarks only. The physics remains unchanged. The same applies to the lepton sector.

14.3.2 Light Z' Boson with Flavor Nonuniversal Couplings

Let us next add to the SM spectrum a new neutral gauge boson Z' resulting from a U(1)′ gauge symmetry.[2] The Lagrangian describing the interaction of Z' with the SM fermions is given by [3]

$$\mathcal{L}(Z') = \sum_{i,j,\psi_L} \Delta_L^{ij}(Z')\, \bar{\psi}^i_L \gamma^\mu P_L \psi^j_L Z'_\mu + \sum_{i,j,\psi_R} \Delta_R^{ij}(Z')\, \bar{\psi}^i_R \gamma^\mu P_R \psi^j_R Z'_\mu, \tag{14.14}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. Here ψ represent classes of fermions with the same electric charge, i.e., u, d, e, ν, while i, j are generation indices: $1, 2, 3$. For instance, $u_3 = t$ and $e_2 = \mu$. We assume first for simplicity that Z' is not much heavier than the top quark so that it can be integrated out at the electroweak scale.

The couplings $\Delta^{ij}_{L,R}(Z')$ are given in the interaction basis as follows

$$\Delta_L^{ij}(Z') = g_{Z'} z_{\psi^i_L}\, \delta^{ij}, \qquad \Delta_R^{ij}(Z') = g_{Z'} z_{\psi^i_R}\, \delta^{ij}, \tag{14.15}$$

with $z_{\psi^i_L}$ and $z_{\psi^i_R}$ being fermion charges under U(1)′ corresponding to Z'.

Performing then rotations (14.11) and (14.12) to the mass eigenstate basis we find instead of (14.15) [771]

$$\Delta_L^{ij}(Z') = g_{Z'} B_L^{ij}(\psi_L), \qquad \Delta_R^{ij}(Z') = g_{Z'} B_R^{ij}(\psi_R), \tag{14.16}$$

where

$$B_L^{ij}(\psi_L) = [(V_L^\psi)^\dagger \hat{Z}_L^\psi V_L^\psi]^{ij}, \qquad B_R^{ij}(\psi_R) = [(V_R^\psi)^\dagger \hat{Z}_R^\psi V_R^\psi]^{ij}. \tag{14.17}$$

$\hat{Z}_{\psi,L}$ and $\hat{Z}_{\psi,R}$ are diagonal matrices

$$\hat{Z}_L^\psi = \mathrm{diag}[z_{\psi^1_L}, z_{\psi^2_L}, z_{\psi^3_L}], \qquad \hat{Z}_R^\psi = \mathrm{diag}[z_{\psi^1_R}, z_{\psi^2_R}, z_{\psi^3_R}], \tag{14.18}$$

with the diagonal elements composed of the U(1)′ charges of the SM fermions.

[2] Based on collaboration with Jason Aebischer, Maria Cerdá-Sevilla, and Fulvia De Fazio.
[3] The physics of Z' will be discussed in a general context in Section 15.4.

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Let us next assume that all these charges differ from each other so that the invariance under the $[U(3)]^5$ flavor symmetry is broken explicitly. To see the consequences of this breakdown, let us choose $V_L^u = \hat{\mathbf{1}}$ and $V_R^u = \hat{\mathbf{1}}$. This means that the Yukawa matrix or equivalently the mass matrix for up-quarks in this flavor basis is diagonal, and the same applies to the interactions of up-quarks with Z'. There is no flavor violation in the up-quark sector mediated by the Z' up to RG effects discussed later. But with $V_L^u = \hat{\mathbf{1}}$ we have $V_L^d = V_{\rm CKM}$. Therefore, inserted into (14.16) and (14.17) we find FCNC transitions in the down-quark sector with

$$B_L^{ij}(\psi_L) = [(V_{\rm CKM})^\dagger \hat{Z}_L^d (V_{\rm CKM})]^{ij}, \qquad B_R^{ij}(\psi_R) = [(V_R^d)^\dagger \hat{Z}_R^d V_R^d]^{ij}, \tag{14.19}$$

with $(i, j = d, s, b)$ and $\hat{Z}_{L,R}^d$ being diagonal matrices collecting U(1)′ charges of left-handed and right-handed down-quarks.

However, $V_L^u = \hat{1}$ and $V_R^u = \hat{1}$ is an assumption that specifies our model. It assumes that in the basis in which Yukawa matrices for up-quarks are diagonal also the interactions of Z' with the up-quarks are flavor diagonal. In other words, $\hat{Y}_u$ and Z' interactions with the up-quarks are aligned with each other. But we could just as well choose $V_L^d = \hat{\mathbf{1}}$ and $V_R^d = \hat{1}$, which would result in FCNC mediated by Z' in the up-quark sector and no FCNC in the down-quark sector up to RG effects.

These simple examples show that in the absence of $[U(3)]^5$ flavor symmetry we have more freedom, and the physics depends on how the Yukawa matrices and matrices describing interactions are oriented in the flavor space. We will soon generalize this discussion to the SMEFT, but let us first have a closer look at the unitary matrix V_R^d. It is present in the SM but does not appear in the SM interactions because of the absence of right-handed charged currents in the SM and flavor universality of right-handed Z^0 couplings. In the Z' case considered here this matrix is responsible for flavor violation in the right-handed down-quark sector. Let us find an expression for it.

Adopting the standard CKM phase convention, where the five relative phases of the quark fields are adjusted to remove five complex phases from the CKM matrix, we have no more freedom to remove the six complex phases from V_R^d. In the standard CKM basis V_R^d can then be parametrized as follows [610, 772]

$$V_R^d = D_U V_R^0 D_D^\dagger, \tag{14.20}$$

where V_R^0 is a "CKM-like" mixing matrix, containing only three real mixing angles and one nontrivial phase. The diagonal matrices $D_{U,D}$ contain the remaining CP-violating phases. Choosing the standard parametrization for V_R^0 we have

$$V_R^0 = \begin{pmatrix} \tilde{c}_{12}\tilde{c}_{13} & \tilde{s}_{12}\tilde{c}_{13} & \tilde{s}_{13}e^{-i\phi} \\ -\tilde{s}_{12}\tilde{c}_{23} - \tilde{c}_{12}\tilde{s}_{23}\tilde{s}_{13}e^{i\phi} & \tilde{c}_{12}\tilde{c}_{23} - \tilde{s}_{12}\tilde{s}_{23}\tilde{s}_{13}e^{i\phi} & \tilde{s}_{23}\tilde{c}_{13} \\ \tilde{s}_{12}\tilde{s}_{23} - \tilde{c}_{12}\tilde{c}_{23}\tilde{s}_{13}e^{i\phi} & -\tilde{s}_{23}\tilde{c}_{12} - \tilde{s}_{12}\tilde{c}_{23}\tilde{s}_{13}e^{i\phi} & \tilde{c}_{23}\tilde{c}_{13} \end{pmatrix}, \tag{14.21}$$

and

$$D_U = \text{diag}(1, e^{i\phi_2^u}, e^{i\phi_3^u}), \qquad D_D = \text{diag}(e^{i\phi_1^d}, e^{i\phi_2^d}, e^{i\phi_3^d}). \tag{14.22}$$

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A similar discussion can be made for leptons. We will continue with this simple model in Section 15.4.9. It illustrated rotations in the flavor space in simple settings, but to be general we return now to the SMEFT.

14.3.3 SMEFT Case

In a UV completion, having the complete Lagrangian including the Yukawa matrices along with the Wilson coefficients at the NP scale the flavor basis can be fixed. Eventually at the electroweak scale rotations to the mass eigenstate basis have to be performed. We will list some of such completions in Section 16.4.11.

A subtlety of choosing a basis for flavored SMEFT operators is the choice of the flavor basis in the space of the three fermion generations for each of the quark and lepton fields. Because all fermions are massless in the $SU(2)_L \times U(1)_Y$-symmetric phase, there is no a priori preferred basis, such as the mass basis below the electroweak scale. We will describe here very briefly what is done in numerical codes following [773]. The list of various codes will be made in Section 14.6.4.

The strategy of [773], which works in Warsaw operator basis, is to define a default weak basis in which the fermion mass matrices have a specific form. The running fermion mass matrices at the dimension-6 level, after a general $U(3)^5$ rotation, can be written as

$$M_d = \frac{v}{\sqrt{2}} U_q^\dagger \left(Y_d - \frac{v^2}{2} C_{d\phi} \right) U_d, \tag{14.23}$$

$$M_u = \frac{v}{\sqrt{2}} U_q^\dagger \left(Y_u - \frac{v^2}{2} C_{u\phi} \right) U_u, \tag{14.24}$$

$$M_e = \frac{v}{\sqrt{2}} U_l^\dagger \left(Y_e - \frac{v^2}{2} C_{e\phi} \right) U_e, \tag{14.25}$$

$$M_\nu = -v^2 U_l^T C_{ll\phi\phi} U_l. \tag{14.26}$$

Here $v \approx 246$ GeV is the electroweak vacuum expectation value (VEV) and Y_ψ, with $\psi = u, d, e$, are the SM Yukawa couplings. $C_{\psi\phi}$ is the Wilson coefficients of the SMEFT dimension-6 operator $Q_{\psi\phi} = \left(\phi^\dagger \phi\right) Q_\psi^Y$ with

$$\left(Y_e \bar{l} e \phi + Y_u \bar{q} u \widetilde{\phi} + Y_d \bar{q} d \phi \right) \equiv \sum_\psi Y_\psi Q_\psi^Y. \tag{14.27}$$

$C_{ll\phi\phi}$ is the lepton number-violating Weinberg operator $Q_{ll\phi\phi} = \left(\widetilde{\phi}^\dagger l\right)^T C \left(\widetilde{\phi}^\dagger l\right)$.

Next, using the freedom of $U(3)^5$ field rotations, a basis, called `Warsaw-down`, is chosen where

- M_d and M_e are diagonal with real positive ascending entries.
- M_u has the form $V^\dagger \hat{M}_u$, where $\hat{M}_u$ is diagonal with real positive ascending entries and V has the form of the CKM matrix in the standard phase convention.
- M_ν has the form $U^* \hat{M}_\nu U^\dagger$, where $\hat{M}_\nu$ is diagonal with real positive entries and U has the form of the PMNS neutrino mixing matrix in standard phase convention for the normally ordered neutrino mass spectrum. For inverted hierarchy, see [773].

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This basis choice is convenient as it removes all unphysical parameters present in the Yukawa couplings, and it allows to easily translate to the mass basis at the scale of electroweak symmetry breaking. However, one should stress that this diagonality is not invariant under SMEFT renormalization group evolution and only holds at a single scale. Therefore at the electroweak scale additional rotations to the mass eigenstate basis have to be performed. An additional subtlety is the fact that v is scale dependent itself and can even vanish at a high scale. While the overall factor does not affect the rotation matrices, the $O(v^2)$ terms in (14.23)–(14.25) are affected. To avoid this problem, the authors of [773] advocate using the on-shell definition of v in these terms.

As a variant of the default "Warsaw" basis, [773] also defines a basis denoted `Warsaw-up` that uses a flavor basis where the up-type quark mass matrix M_u rather than M_d is diagonalized, while M_d has the form $V\hat{M}_d$. These two choices are analogous to the ones made in the light Z' scenario discussed earlier, but now this procedure can also be used for a very heavy Z'.

Our presentation was rather brief. Much more details on rotations in flavor space with physical implications can be found in [484, 516, 563, 565, 719, 737, 767, 769, 770, 773, 774].

As we just mentioned, the diagonality of Yukawa couplings at one scale is not guaranteed to be preserved at another scale because of the RG effects. We will discuss them next.

14.4 Renormalization Group Equations

The scale dependence of Wilson coefficients is governed by the RG equations presented in [758–760] for quartic Higgs couplings, Yukawa couplings, and gauge couplings, respectively. They have the general structure

$$\boxed{\dot{C}_a \equiv (4\pi)^2 \mu \frac{\mathrm{d}C_a}{\mathrm{d}\mu} = \gamma_{ab}\, C_b,} \tag{14.28}$$

with γ_{ab} being the entries of a very big anomalous dimension matrix (ADM). The ADM is known for SMEFT at one-loop and depends on (1) the quartic Higgs coupling λ [758], (2) the fermion Yukawa couplings to the Higgs doublet [759], and (3) the three gauge couplings $g_{1,2,3}$ [760]. For small $\gamma_{ab}/(4\pi)^2 \ll 1$ the approximate solution retains only the first leading logarithm (1stLLA)

$$\boxed{C_a(\mu_{\rm ew}) = \left[\delta_{ab} - \frac{\gamma_{ab}}{(4\pi)^2} \ln \frac{\mu_\Lambda}{\mu_{\rm ew}}\right] C_b(\mu_\Lambda),} \tag{14.29}$$

which is sufficient as long as the logarithm is not too large, so that also $\gamma_{ab}/(4\pi)^2 \ln \frac{\mu_\Lambda}{\mu_{\rm ew}} \ll 1$ holds. Numerically one expects the largest enhancements when the ADM γ_{ab} is proportional to the strong coupling $4\pi\alpha_s \sim 1.4$ or the top-Yukawa coupling squared $y_t^2 \sim 1$. The QCD mixing is flavor-diagonal and hence cannot give rise to new genuine

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phenomenological effects in $\Delta F = 1, 2$ observables. On the other hand, top-Yukawa couplings are the main source of flavor-off-diagonal interactions responsible for the phenomenology discussed later. The $\mathrm{SU}(2)_L$ gauge interactions induce via $\gamma_{ab} \propto g_2^2$ and are parametrically suppressed compared to y_t-induced effects. The suppression is even stronger for the terms $\gamma_{ab} \propto g_1^2$. Yet, in certain situations these effects have to be included, in particular when QCD effects and top-Yukawa contributions are absent. This is, e.g., the case of Scenario V.

The ADMs presented in [758–760] look truly horrible. They imply in principle a large set of coupled differential equations of the RG equations of SM couplings (quartic Higgs, gauge, and Yukawa) and the ones of dim-6 Wilson coefficients. The solution of this system in full generality requires the application of numerical methods, and the imposition of boundary conditions might be highly nontrivial. The 1stLLA neglects all these "secondary mixing" effects that would be present in the general leading logarithmic approximation (LLA), which would also resum large logarithms to all orders in couplings. With "secondary mixing" we refer to the situation where an operator O_1 might not have an ADM entry with operator O_3 (no "direct mixing"), but still contribute to the Wilson coefficient $C_3(\mu_{\rm ew})$, via a direct mixing with some operator O_2 that in turn mixes directly into O_3. We refer to Appendix E for an analytic treatment of such cases.

Here we will first confine our presentation by keeping only contributions proportional to the up-type quark Yukawa coupling Y_u from [759], i.e., neglecting contributions from Yukawa matrices $Y_{d,e}$ of down-quarks and leptons unless the latter contributions are crucial. The same applies to terms involving gauge couplings. For instance, in Scenario V only gauge coupling contributions matter, and they have to be kept. In presenting the results here, we follow here [666, 667, 766] and list the equations only for operators that receive leading logarithmic contributions at the scale $\mu_{\rm EW}$ from the initial Wilson coefficients at the scale μ_Λ of the operators in the scenarios defined earlier.

Let us then have a closer look at simplified scenarios in question.

14.4.1 Scenario I

In this scenario only the coefficients $[C_{Hd}]_{ij}$ with $i, j = d, s, b$ are nonvanishing at μ_Λ. Keeping only Y_u couplings the RG evolution from μ_Λ down to $\mu_{\rm EW}$ generates $(\bar{L}L)(\bar{R}R)$ $\Delta F = 2$ operators through [766]

$$\boxed{[\dot{C}_{qd}^{(1)}]_{ijij} = [Y_u Y_u^\dagger]_{ij}[C_{Hd}]_{ij} + \cdots .} \tag{14.30}$$

The dots indicate other terms $\propto C_{\psi^2 H^2 D}$ that are not proportional to Y_u. We recall that our Y_u is defined as the Hermitean conjugate w.r.t. [759].

To proceed we have to specify Y_u and transform the quark fields from the flavor to the mass eigenbasis. As Y_u are scale dependent, it is useful to do it at the electroweak scale where EWSB takes place. The quark fields are then rotated by 3×3 unitary rotations in flavor space as done in Section 14.3 such that for $\psi = u, d$

$$(V_L^\psi)^\dagger M_\psi V_R^\psi = M_\psi^{\rm diag}, \qquad V \equiv (V_L^u)^\dagger V_L^d, \tag{14.31}$$

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with diagonal up- and down-quark mass matrices $M_\psi^{\rm diag}$. The nondiagonal mass matrices M_ψ include the contributions of dim-6 operators. The quark-mixing matrix V is unitary, similar to the CKM matrix of the SM; however, in the presence of dim-6 contributions the numerical values are different from those obtained in usual SM CKM-fits. We will choose the flavor basis such that down-quarks are already mass eigenstates, which fixes $V_{L,R}^d = \mathbb{1}$, and assume without loss of generality $V_R^u = \mathbb{1}$, yielding $q_L = (V^\dagger u_L, d_L)^T$. It should be noted that this treatment is different from the usual discussion of the CKM matrix, where the down-quarks are rotated. See (2.45).

Neglecting for simplicity dim-6 to M_ψ we have $V \approx V_{\rm CKM}$, and consequently, we find Y_u as a function of up-type quark masses m_k and the CKM parameters:

$$Y_u \overset{\text{dim}-4}{\approx} \frac{\sqrt{2}}{v} V_L^u M_U^{\rm diag} V_R^{u\dagger} = \frac{\sqrt{2}}{v} V_{\rm CKM}^\dagger M_U^{\rm diag}, \tag{14.32}$$

$$\boxed{[Y_u Y_u^\dagger]_{ij} = \frac{2}{v^2} \sum_{k=u,c,t} m_k^2 V_{ki}^* V_{kj} \approx \frac{2}{v^2} m_t^2 \lambda_t^{ij},} \tag{14.33}$$

with $v \approx (\sqrt{2} G_F)^{-1/2}$ being the Higgs vacuum expectation value. In the sum over k only the top-quark contribution is relevant ($m_{u,c} \ll m_t$), if one assumes that the unitary matrix V is equal to the CKM matrix up to dim-6 corrections. We expect only tiny contributions from $k = c$ in the case $ij = sd$, for $ij = bd, bs$ such contributions are entirely negligible. We recall that Y_d is diagonal and that all quark masses and CKM parameters are defined at the electroweak scale. When performing RG evolution to low-energy or high-energy scales the modification of their values due to QCD have to be taken into account, as they are significant. See Table 4.2.

Next we want to relate the coefficients on both sides of (14.30) to objects that we know from other parts of this book. The decoupling of heavy SM degrees of freedom at $\mu_{\rm ew}$ gives rise to the $\Delta F = 2$ effective Hamiltonian [744, 745]

$$\boxed{\mathcal{H}_{\Delta F=2}^{ij} = \mathcal{N}_{ij} \sum_a C_a^{ij} O_a^{ij} + \text{h.c.},} \tag{14.34}$$

where the normalization factor and the CKM combinations are

$$\boxed{\mathcal{N}_{ij} = \frac{G_F^2}{4\pi^2} M_W^2 \left(\lambda_t^{ij}\right)^2, \qquad \lambda_t^{ij} = V_{ti}^* V_{tj},} \tag{14.35}$$

with $ij = sd$ for kaon mixing and $ij = bd, bs$ for B_d and B_s mixing, respectively. The important operators for Scenarios I and II are

$$O_{\rm VLL}^{ij} = [\bar{d}_i \gamma_\mu P_L d_j][\bar{d}_i \gamma^\mu P_L d_j], \qquad O_{\rm VRR}^{ij} = [\bar{d}_i \gamma_\mu P_R d_j][\bar{d}_i \gamma^\mu P_R d_j], \tag{14.36}$$

$$O_{\rm LR,1}^{ij} = [\bar{d}_i \gamma_\mu P_L d_j][\bar{d}_i \gamma^\mu P_R d_j], \qquad O_{\rm LR,2}^{ij} = [\bar{d}_i P_L d_j][\bar{d}_i P_R d_j], \tag{14.37}$$

which we met already in the previous chapter. Because of the addition of general flavor indices we had to change the notation of operators slightly.

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The SMEFT is then matched to the $\Delta F = 2$–LEFT at $\mu_{\rm ew}$ and at tree-level one finds the following modifications of the Wilson coefficients relevant for Scenario I [563]:

$$\Delta C^{ij}_{\rm LR,1} = -\mathcal{N}^{-1}_{ij}\left([C^{(1)}_{qd}]_{ijij} - \frac{[C^{(8)}_{qd}]_{ijij}}{2N_c}\right), \qquad \Delta C^{ij}_{\rm LR,2} = \mathcal{N}^{-1}_{ij}[C^{(8)}_{qd}]_{ijij}, \tag{14.38}$$

where $\mathcal{N}_{ij}$ is given in (14.35). The minus sign reflects the fact that $[C^{(1,8)}_{qd}]_{ijij}$ are the coefficients in the Lagrangian and $C^{ij}_{\rm LR,k}$ the coefficients in the Hamiltonian. Note that our SMEFT coefficients are $O(1/\Lambda^2)$ as we absorbed the $1/\Lambda^2$ factor in (14.1) into these coefficients. As can be seen, the $\Delta F = 2$ operator $O^{(8)}_{qd}$ does not receive direct leading logarithmic contributions. It is well known though that $C^{(8)}_{qd}$ would be generated via secondary QCD mixing from $C^{(1)}_{qd}$ [744].

From (14.38) and (14.30) we find that the presence of the RH operator O_{Hd} at a short-distance scale μ_Λ, i.e., $[C_{Hd}]_{ij}(\mu_\Lambda) \neq 0$, generates through Yukawa RG effects a leading-logarithmic contribution to the LR operator $O^{ij}_{\rm LR,1}$ at the electroweak scale $\mu_{\rm ew}$, given by [766]

$$\boxed{\Delta_{\rm 1stLLA} C^{ij}_{\rm LR,1}(\mu_{\rm ew}) = v^2 \frac{[C_{Hd}]_{ij}(\mu_\Lambda)}{\lambda^{ij}_t} x_t \ln\frac{\mu_\Lambda}{\mu_{\rm ew}}.} \tag{14.39}$$

The coefficient $[C_{Hd}]_{ij}(\mu_\Lambda)$ is related to the flavor-changing coupling of the Z to down-quarks. Although this relation and the phenomenological implications of (14.39) will be discussed in detail in Section 15.5, we would like to make the following important comment. The RG contribution just given is for μ_Λ sufficiently larger than $\mu_{\rm ew}$ not only the most important NP effect in this scenario but has also opposite sign to nonlogarithmic effects at the matching scale.

14.4.2 Scenario II

In this scenario only the coefficients $[C^{(1)}_{Hq}]_{ij}$ and $[C^{(3)}_{Hq}]_{ij}$ are nonvanishing. Keeping only Y_u couplings the RG evolution from μ_Λ down to $\mu_{\rm EW}$ generates $(\bar{L}L)(\bar{L}L)$ $\Delta F = 2$ operators through [766]

$$[\dot{C}^{(1)}_{qq}]_{ijij} = +[Y_u Y_u^\dagger]_{ij}[C^{(1)}_{Hq}]_{ij} + \cdots, \tag{14.40}$$

$$[\dot{C}^{(3)}_{qq}]_{ijij} = -[Y_u Y_u^\dagger]_{ij}[C^{(3)}_{Hq}]_{ij} + \cdots. \tag{14.41}$$

Matching in this case the SMEFT to the $\Delta F = 2$–LEFT at $\mu_{\rm ew}$ one finds the following modifications of the Wilson coefficients at tree-level [563]:

$$\Delta C^{ij}_{\rm VLL} = -\mathcal{N}^{-1}_{ij}\left([C^{(1)}_{qq}]_{ijij} + [C^{(3)}_{qq}]_{ijij}\right). \tag{14.42}$$

Again the minus sign reflects the fact that $[C^{(1,3)}_{qq}]_{ijij}$ are the coefficients in the Lagrangian and $C^{ij}_{\rm VLL}$ the coefficients in the Hamiltonian.

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Proceeding as in Scenario I, we then find that the presence of two operators $O_{Hq}^{(1)}$ and $O_{Hq}^{(3)}$ generates via (14.40), (14.41), and (14.42)

$$\boxed{\Delta_{\text{1stLLA}} C_{\text{VLL}}^{ij}(\mu_{\text{ew}}) = \frac{v^2}{\lambda_t^{ij}} \left([C_{Hq}^{(1)}]_{ij}(\mu_\Lambda) - [C_{Hq}^{(3)}]_{ij}(\mu_\Lambda)\right) x_t \ln \frac{\mu_\Lambda}{\mu_{\text{ew}}}.} \tag{14.43}$$

As we will see in Section 15.5 the coefficient $\sim (C_{Hq}^{(1)} - C_{Hq}^{(3)})$ is related to the flavor-changing coupling of the Z to up-quarks and not down-quarks considered here. The latter involve the linear combination $(C_{Hq}^{(1)} + C_{Hq}^{(3)})$. Important phenomenological consequences of this difference will be discussed in Section 15.5. It demonstrates again that neglecting RG effects driven by top-Yukawa couplings in the presence of $\text{SU}(2)_L$ invariance would miss important dynamics.

14.4.3 Self-Mixing in Scenarios I and II

The $\psi^2 H^2 D$-operators governing Scenarios I and II mix between themselves during the evolution from μ_Λ to μ_{ew}. This mixing turns out to be much smaller than the effects considered so far, at the level of a few percent for $\Lambda \leq 10\,\text{TeV}$. For this reason in writing the results in (14.39) and (14.43) this mixing has been neglected. For completeness we give the relevant formulas. They read [766]

$$\begin{aligned}\dot{C}_{Hq}^{(1)} &= 6\,\text{Tr}[Y_u Y_u^\dagger] C_{Hq}^{(1)} + 2\left(Y_u Y_u^\dagger C_{Hq}^{(1)} + C_{Hq}^{(1)} Y_u Y_u^\dagger\right) \\ &\quad - \frac{9}{2}\left(Y_u Y_u^\dagger C_{Hq}^{(3)} + C_{Hq}^{(3)} Y_u Y_u^\dagger\right) - Y_u C_{Hu} Y_u^\dagger,\end{aligned} \tag{14.44}$$

$$\dot{C}_{Hq}^{(3)} = 6\,\text{Tr}[Y_u Y_u^\dagger] C_{Hq}^{(3)} + Y_u Y_u^\dagger C_{Hq}^{(3)} + C_{Hq}^{(3)} Y_u Y_u^\dagger - \frac{3}{2}\left(Y_u Y_u^\dagger C_{Hq}^{(1)} + C_{Hq}^{(1)} Y_u Y_u^\dagger\right), \tag{14.45}$$

$$\dot{C}_{Hd} = 6\,\text{Tr}[Y_u Y_u^\dagger] C_{Hd}, \tag{14.46}$$

$$\dot{C}_{Hu} = -2 Y_u^\dagger C_{Hq}^{(1)} Y_u + 6\,\text{Tr}[Y_u Y_u^\dagger] C_{Hu} + 4\left(Y_u^\dagger Y_u C_{Hu} + C_{Hu} Y_u^\dagger Y_u\right), \tag{14.47}$$

$$\dot{C}_{Hud} = 6\,\text{Tr}[Y_u Y_u^\dagger] C_{Hud} + 3 Y_u^\dagger Y_u C_{Hud}. \tag{14.48}$$

We observe that in SMEFT the LH interactions $C_{Hq}^{(1,3)}$ do not generate RH interactions in the down-type sector (C_{Hd}) and vice versa. However, there is mixing of the LH interaction $C_{Hq}^{(1)}$ into the RH interactions of up-type sector (C_{Hu}) and vice versa. Moreover $C_{Hq}^{(1)}$ and $C_{Hq}^{(3)}$ mix with each other, while C_{Hd} evolves undisturbed by other operators.

14.4.4 Scenario III

Now the operators $O_{H\ell}^{(1,3)}$ and O_{He} are the driving force. The complex-valued coefficients of these operators are denoted by

$$\boxed{[C_{H\ell}^{(1)}]_{ab}, \qquad [C_{H\ell}^{(3)}]_{ab}, \qquad [C_{He}]_{ab},} \tag{14.49}$$

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where the indices $a, b = 1, 2, 3$ denote the different generations of up- and down-type leptons, that is neutrinos and charged leptons.

Through RG Yukawa interactions, these operators generate contributions to semileptonic four-fermion operators. Keeping only Y_u effects we find

ψ^4 $(\bar{L}L)(\bar{L}L)$:

$$[\dot{C}^{(1)}_{\ell q}]_{abij} = +[C^{(1)}_{H\ell}]_{ab}[Y_u Y_u^\dagger]_{ij}, \tag{14.50}$$

$$[\dot{C}^{(3)}_{\ell q}]_{abij} = -[C^{(3)}_{H\ell}]_{ab}[Y_u Y_u^\dagger]_{ij}, \tag{14.51}$$

ψ^4 $(\bar{R}R)(\bar{R}R)$:

$$[\dot{C}_{eu}]_{abij} = -2\,[C_{He}]_{ab}[Y_u^\dagger Y_u]_{ij}, \tag{14.52}$$

ψ^4 $(\bar{L}L)(\bar{R}R)$:

$$[\dot{C}_{\ell u}]_{abij} = -2\,[C^{(1)}_{H\ell}]_{ab}[Y_u^\dagger Y_u]_{ij}, \tag{14.53}$$

$$[\dot{C}_{qe}]_{ijab} = [Y_u Y_u^\dagger]_{ij}[C_{He}]_{ab}. \tag{14.54}$$

Here lepton-flavor indices are ab and quark-flavor indices ij.

This time the self-mixing of $O^{(1,3)}_{H\ell}$ and O_{He} is much simpler than in the first two scenarios. The three operators do not mix with each other ($N_c = 3$):

$$\dot{C}^{(1)}_{H\ell} = 2N_c\,\mathrm{Tr}[Y_u Y_u^\dagger]\,C^{(1)}_{H\ell}, \tag{14.55}$$

$$\dot{C}^{(3)}_{H\ell} = 2N_c\,\mathrm{Tr}[Y_u Y_u^\dagger]\,C^{(3)}_{H\ell}, \tag{14.56}$$

$$\dot{C}_{He} = 2N_c\,\mathrm{Tr}[Y_u Y_u^\dagger]\,C_{He}. \tag{14.57}$$

A detailed analysis of flavor physics in this scenario is in progress and should be available in the hep-arxiv when this book is published.

14.4.5 Scenario IV

We consider in this scenario $(\bar{L}L)(\bar{L}L)$ operators so that the operators $O^{(1,3)}_{\ell q}$ are the driving force. The complex-valued coefficients of these operators are denoted by

$$\boxed{[C^{(1)}_{\ell q}]_{abij}, \qquad [C^{(3)}_{\ell q}]_{abij},} \tag{14.58}$$

where lepton-flavor indices are ab and quark-flavor indices ij.

Keeping only Y_u effects we find (summation over repeated indices ww is understood)

$$[\dot{C}^{(1)}_{\ell q}]_{abij} = +\frac{1}{2}[Y_u Y_u^\dagger]_{iw}[C^{(1)}_{\ell q}]_{abwj} + \frac{1}{2}[C^{(1)}_{\ell q}]_{abiw}[Y_u Y_u^\dagger]_{wj}, \tag{14.59}$$

$$[\dot{C}^{(3)}_{\ell q}]_{abij} = +\frac{1}{2}[Y_u Y_u^\dagger]_{iw}[C^{(3)}_{\ell q}]_{abwj} + \frac{1}{2}[C^{(3)}_{\ell q}]_{abiw}[Y_u Y_u^\dagger]_{wj}, \tag{14.60}$$

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which shows that these two Wilson coefficients in this approximation evolve independent of each other. Yet, when gauge couplings are included, they start to mix

$$[\dot{C}^{(1)}_{\ell q}]_{abij} = \frac{2}{9}g_1^2[C^{(1)}_{\ell q}]_{abww}\delta_{ij} + \frac{2}{3}g_1^2[C^{(1)}_{\ell q}]_{wwij}\delta_{ab} - g_1^2[C^{(1)}_{\ell q}]_{abij} + 9g_2^2[C^{(3)}_{\ell q}]_{abij}, \tag{14.61}$$

$$[\dot{C}^{(3)}_{\ell q}]_{abij} = 2g_2^2[C^{(3)}_{\ell q}]_{abww}\delta_{ij} + \frac{2}{3}[C^{(3)}_{\ell q}]_{wwij}\delta_{ab} + 3g_2^2[C^{(1)}_{\ell q}]_{abij} - 6\left(g_2^2 + \frac{1}{6}g_1^2\right)[C^{(3)}_{\ell q}]_{abij}. \tag{14.62}$$

But what is more important is the generation through electroweak interaction of purely leptonic operators:[4]

$$\begin{aligned}[\dot{C}_{\ell\ell}]_{abcd} = &-\frac{1}{3}g_1^2\left([C^{(1)}_{\ell q}]_{abww}\delta_{cd} + [C^{(1)}_{\ell q}]_{cdww}\delta_{ab}\right)\\ &- g_2^2\left([C^{(3)}_{\ell q}]_{abww}\delta_{cd} + [C^{(3)}_{\ell q}]_{cdww}\delta_{ab} - 2[C^{(3)}_{\ell q}]_{cbww}\delta_{ad} - 2[C^{(3)}_{\ell q}]_{adww}\delta_{bc}\right),\end{aligned} \tag{14.63}$$

which can contribute to pure leptonic lepton decays thereby putting significant bounds on the coefficients of semileptonic operators as stressed in particular in [765, 767] and mentioned already a few pages before. We refer to these papers for phenomenological implications of these equations.

14.4.6 Scenario V

In what follows we list RG equations, which govern the generation of nonleptonic (NL)-ψ^4 coefficients from semileptonic (SL)-ψ^4 ones. They can be derived from [760] and have been presented in detail in [667]. Only electroweak interactions are involved here. We find

NL-$\boldsymbol{\psi^4}$ $(\overline{L}L)(\overline{L}L)$:

$$[\dot{C}^{(1)}_{qq}]_{prst} = -\frac{1}{9}g_1^2\left([C^{(1)}_{\ell q}]_{wwst}\delta_{pr} + [C^{(1)}_{\ell q}]_{wwpr}\delta_{st} + [C_{qe}]_{stww}\delta_{pr} + [C_{qe}]_{prww}\delta_{st}\right), \tag{14.64}$$

$$[\dot{C}^{(3)}_{qq}]_{prst} = +\frac{1}{3}g_2^2\left([C^{(3)}_{\ell q}]_{wwst}\delta_{pr} + [C^{(3)}_{\ell q}]_{wwpr}\delta_{st}\right). \tag{14.65}$$

NL-$\boldsymbol{\psi^4}$ $(\overline{R}R)(\overline{R}R)$:

$$[\dot{C}_{uu}]_{prst} = -\frac{4}{9}g_1^2\Big([C_{eu}]_{wwst}\delta_{pr} + [C_{eu}]_{wwpr}\delta_{st} + [C_{\ell u}]_{wwst}\delta_{pr} + [C_{\ell u}]_{wwpr}\delta_{st}\Big), \tag{14.66}$$

$$[\dot{C}_{dd}]_{prst} = +\frac{2}{9}g_1^2\Big([C_{ed}]_{wwst}\delta_{pr} + [C_{ed}]_{wwpr}\delta_{st} + [C_{\ell d}]_{wwst}\delta_{pr} + [C_{\ell d}]_{wwpr}\delta_{st}\Big), \tag{14.67}$$

$$[\dot{C}^{(1)}_{ud}]_{prst} = +\frac{4}{9}g_1^2\Big([C_{\ell u}]_{wwpr}\delta_{st} + [C_{eu}]_{wwpr}\delta_{st} - 2[C_{\ell d}]_{wwst}\delta_{pr} - 2[C_{ed}]_{wwst}\delta_{pr}\Big). \tag{14.68}$$

[4] Note that now there are two new lepton indices cd, and summation is only over quark indices.

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NL-ψ^4 $(\bar{L}L)(\bar{R}R)$:

$$[\dot{C}_{qu}^{(1)}]_{prst} = -\frac{2}{9}g_1^2\left(4[C_{\ell q}^{(1)}]_{wwpr}\delta_{st} + 4[C_{qe}]_{prww}\delta_{st} + [C_{\ell u}]_{wwst}\delta_{pr} + [C_{eu}]_{wwst}\delta_{pr}\right), \tag{14.69}$$

$$[\dot{C}_{qd}^{(1)}]_{prst} = +\frac{2}{9}g_1^2\left(2[C_{\ell q}^{(1)}]_{wwpr}\delta_{st} + 2[C_{qe}]_{prww}\delta_{st} - [C_{\ell d}]_{wwst}\delta_{pr} - [C_{ed}]_{wwst}\delta_{pr}\right), \tag{14.70}$$

and finally for all other NL-ψ^4 operators

$$\begin{aligned}[\dot{C}_{ud}^{(8)}]_{prst} = 0, \quad [\dot{C}_{qu}^{(8)}]_{prst} = 0, \quad [\dot{C}_{quqd}^{(1)}]_{prst} = 0,\\ [\dot{C}_{qd}^{(8)}]_{prst} = 0, \quad [\dot{C}_{quqd}^{(8)}]_{prst} = 0.\end{aligned} \tag{14.71}$$

We observe that the SM gauge-mixing of SL-ψ^4 into NL-ψ^4 operators within SMEFT generates in 1stLLA only $(\bar{L}L)(\bar{L}L)$, $(\bar{L}L)(\bar{R}R)$, and $(\bar{R}R)(\bar{R}R)$ NL-ψ^4 operators from the corresponding semileptonic classes. The initial Wilson coefficients of the semileptonic operators at the scale μ_Λ enter only summed over the lepton-flavor-diagonal parts $[C_b^{(a)}]_{ww\ldots}$ (and $[C_{qe}]_{\ldots ww}$), summation over the index $w = 1,2,3$ is implied, because all leptons can run inside the loop. In consequence the underlying combination of the couplings is

$$\Sigma_{\chi,\mathrm{LQ}}^{ji} \equiv \sum_a (Y_{ja}^\chi)^* Y_{ia}^\chi. \tag{14.72}$$

Further details on this scenario can be found in [667], where the ratio ε'/ε was studied in leptoquark models. The results of this analysis are summarized in Section 16.4.13.

Scenario VI has been discussed by many authors, and we devote to it a separate subsection.

14.5 $SU(2)_L$ Correlations between $b \to s\nu\bar{\nu}$ and $b \to s\ell^+\ell^-$

The implication of SM gauge invariance for flavor observables has been discussed in many papers, in particular in [565, 607, 775–779]. Here, as an example, we will just write general formulas, which allow to study correlations on one hand between $b \to s\nu\bar{\nu}$ and $b \to s\ell^+\ell^-$ transitions and on the other hand between $d \to s\nu\bar{\nu}$ and $d \to s\ell^+\ell^-$ transitions.

To this end we recall general $\Delta F = 1$ Hamiltonian for the semileptonic FCNC transition of down-type quarks into leptons and neutrinos below the scale μ_{ew}

$$\boxed{\mathcal{H}_{d\to d(\ell\ell,\nu\nu)} = -\frac{4G_F}{\sqrt{2}}\lambda_t^{ji}\frac{\alpha_e}{4\pi}\sum_k C_k^{baji}Q_k^{baji} + \text{h.c.},} \tag{14.73}$$

with a, b being lepton indices, i, j down-quark indices and

$$\lambda_u^{ji} \equiv V_{ui}^* V_{uj}, \qquad u = \{u, c, t\}. \tag{14.74}$$

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There are eight semileptonic operators relevant for $d_i\ell_a \to d_j\ell_b$ when considering UV completions that give rise to SMEFT above the electroweak scale [565]

$$\begin{aligned} Q^{baji}_{9(9')} &= [\bar{d}_j\gamma_\mu P_{L(R)}d_i][\bar{\ell}_b\gamma^\mu\ell_a], & Q^{baji}_{10(10')} &= [\bar{d}_j\gamma_\mu P_{L(R)}d_i][\bar{\ell}_b\gamma^\mu\gamma_5\ell_a], \\ Q^{baji}_{S(S')} &= [\bar{d}_j P_{R(L)}d_i][\bar{\ell}_b\ell_a], & Q^{baji}_{P(P')} &= [\bar{d}_j P_{R(L)}d_i][\bar{\ell}_b\gamma_5\ell_a], \end{aligned} \tag{14.75}$$

and two for $d_i\nu_a \to d_j\nu_b$

$$Q^{baji}_{L(R)} = [\bar{d}_j\gamma_\mu P_{L(R)}d_i][\bar{\nu}_b\gamma^\mu(1-\gamma_5)\nu_a]. \tag{14.76}$$

The SM contribution to these Wilson coefficients is lepton-flavor diagonal

$$\boxed{C^{baji}_k = C_{k,\mathrm{SM}}\,\delta_{ba} + \frac{\pi}{\alpha_e}\frac{v^2}{\lambda_t^{ji}}\,C^{baji}_{k,\mathrm{NP}},} \tag{14.77}$$

where $v = 246\,\mathrm{GeV}$ and a normalization factor has been introduced for the NP contribution that proves convenient for matching with SMEFT in a given model. The nonvanishing SM contributions valid both in the B and in the K system,

$$C_{9,\mathrm{SM}} = \frac{Y_0(x_t)}{s_W^2} - 4Z_0(x_t), \qquad C_{10,\mathrm{SM}} = -\frac{Y_0(x_t)}{s_W^2}, \qquad C_{L,\mathrm{SM}} = -\frac{X_0(x_t)}{s_W^2}, \tag{14.78}$$

are given by the gauge-independent functions $X_0(x_t)$, $Y_0(x_t)$, and $Z_0(x_t)$ [133] that already appeared many times in this book.

The NP contribution to the Wilson coefficients of the $\Delta F = 1$ semileptonic operators in (14.75) and (14.76) at μ_{ew} in terms of the semileptonic SMEFT Wilson coefficients at μ_{ew} is given as follows [563, 565, 607]

$$\begin{aligned} C^{baji}_{9,\mathrm{NP}} &= [C_{qe} + C^{(1)}_{\ell q} + C^{(3)}_{\ell q}]_{baji}, & C^{baji}_{9',\mathrm{NP}} &= [C_{ed} + C_{\ell d}]_{baji}, \\ C^{baji}_{10,\mathrm{NP}} &= [C_{qe} - C^{(1)}_{\ell q} - C^{(3)}_{\ell q}]_{baji}, & C^{baji}_{10',\mathrm{NP}} &= [C_{ed} - C_{\ell d}]_{baji}, \\ C^{baji}_{L,\mathrm{NP}} &= [C^{(1)}_{\ell q} - C^{(3)}_{\ell q}]_{baji}, & C^{baji}_{R,\mathrm{NP}} &= [C_{\ell d}]_{baji}, \\ C^{baji}_{S,\mathrm{NP}} &= -C^{baji}_{P,\mathrm{NP}} = [C_{\ell edq}]^*_{abij}, & C^{baji}_{S',\mathrm{NP}} &= C^{baji}_{P',\mathrm{NP}} = [C_{\ell edq}]_{baji}. \end{aligned} \tag{14.79}$$

We caution the reader that C_{qe}, used in the literature, should here be written as C_{eq} so that the lepton indices come first as in the remaining WCs in these equations. But as we already stated after (14.73) a, b are lepton indices and i, j down-quark ones, so that no confusion should result from this notation. Here contributions from Z-mediating ψ^2H^2D–SMEFT operators $O^{(1,3)}_{Hq}$ to $C_{9,10,L}$ and O_{Hd} to $C_{9',10',R}$, respectively, have been omitted. In rare FCNC Kaon decays scalar and pseudoscalar Wilson coefficients are negligible but are relevant in $B_s \to \mu^+\mu^-$.

We observe that the number of operators in the SMEFT is in general larger than in the low-energy effective Hamiltonian, so on a completely model-independent basis, no general correlations can be derived. Indeed, from (14.79), it is clear that in complete generality, the size of NP effects in $b \to s\nu\bar{\nu}$ is not constrained by the $b \to s\ell^+\ell^-$ measurements.

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First, the decays with charged leptons are only sensitive to the combination $C_{\ell q}^{(1)} + C_{\ell q}^{(3)}$, while the decays with neutrinos in the final state probe $C_{\ell q}^{(1)} - C_{\ell q}^{(3)}$. Second, even if the Wilson coefficient $C_{\ell q}^{(3)}$ vanishes, cancellations between the operators with left- and right-handed charged leptons can lead to small deviations from the SM in $b \to s\ell^+\ell^-$ transitions even when large effects are present in $b \to s\nu\bar{\nu}$. However, in concrete NP models, often only a subset of the operators is relevant and in these cases correlations characteristic for these NP scenarios are obtained. We will see later on that the present bounds on $b \to s\nu\bar{\nu}$ transitions can in certain models be problematic for the explanation of the anomalies in $b \to s\ell^+\ell^-$ measurements discussed in Chapter 12.

A solution to the latter problem is the equality $C_{\ell q}^{(1)} = C_{\ell q}^{(3)}$, which removes the contributions not only to $b \to s\nu\bar{\nu}$ transition but if valid for all down-quark flavors, also to $d \to s\nu\bar{\nu}$ transitions, that is $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. However, as emphasized in [765, 767], this equality can be valid only at one scale. RG evolution from NP scale to EW scale breaks this relation. See [667] for an explicit analysis of such RG effects in the context of leptoquark models that we will review in Section 16.4.

While such explicit models will be considered also in subsequent chapters, here we just illustrate general correlations on two examples presented in [607] and discussed in detail in Sections 15.4 and 15.5. Readers not familiar with these NP scenarios can skip these examples and return to them after reading these two sections.

We consider first the general case of Z' models in which a single Z' gauge boson with left-handed and right-handed couplings dominates the scene. In this case $C_{\ell q}^{(3)} = 0$ at the NP scale, and at this scale we have the relations

$$C_{L,\mathrm{NP}}^{baji} = \frac{C_{9,\mathrm{NP}}^{baji} - C_{10,\mathrm{NP}}^{baji}}{2}, \qquad C_{R,\mathrm{NP}}^{baji} = \frac{C_{9',\mathrm{NP}}^{baji} - C_{10',\mathrm{NP}}^{baji}}{2}. \tag{14.80}$$

But as we will see in Section 15.5, if NP contributions to the processes considered are fully dominated by induced FCNC couplings of the SM Z boson – this is the case, e.g., in the MSSM, in models with partial compositeness and models with vector-like quarks discussed in Section 16.3 – one finds

$$C_{L,\mathrm{NP}}^{baji} = C_{10,\mathrm{NP}}^{baji}, \quad C_{9,\mathrm{NP}}^{baji} = -\zeta C_{10}^{\mathrm{NP}}, \quad C_{R,\mathrm{NP}}^{baji} = C_{10',\mathrm{NP}}^{baji}, \quad C_{9}^{\mathrm{NP}} = -\zeta C_{10',\mathrm{NP}}^{baji}, \tag{14.81}$$

where $\zeta = 1 - 4s_W^2 = 0.08$ is the accidentally small vector coupling of the Z to charged leptons. Evidently the correlations in (14.80) and (14.81) differ from each other. In particular the sign in front of $C_{10,\mathrm{NP}}^{baji}$ in $C_{L,\mathrm{NP}}^{baji}$ is different and similar for right-handed coefficients. In this manner the Z' and Z scenarios can be distinguished from each other. The plots in [607] illustrate these differences.

Finally, if the presence of Z' induces FCNC couplings of Z through $Z - Z'$ mixing, the relations just given are modified and depend on the size of $Z - Z'$ mixing. This mixing is clearly model dependent, and the resulting correlations can vary from model to model. We will illustrate this case in Section 16.2 by using the 331 models studied in [615].

The preceding discussion does not include RG effects above the electroweak scale. While such an analysis is missing in the literature at the time of this writing, it should be available when this book is published.

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14.6 General Procedure and Useful Results

14.6.1 Preliminaries

We have seen on the examples just presented that specifying general properties about the dynamics at the NP scale, combined with the SMEFT technology, can give us some insight into various patterns of NP effects in low-energy flavor phenomenology. Quite generally, having a specific NP model or some simplified NP scenario with new heavy fields, gauge bosons, scalars, and fermions, the implications of this extension of the SM on low-energy observables can be found systematically by performing the following steps:

Step 1: Matching of NP Model onto the SMEFT
Assuming the NP scale $\Lambda_{\rm NP}$ to be much larger than the electroweak scale $\mu_{\rm EW}$ and integrating out new heavy particles with masses $O(\Lambda_{\rm NP})$ one finds the nonvanishing WCs of the SMEFT at $\mu = \Lambda_{\rm NP}$. Useful results related to this step present in the literature will be briefly described later. It should be emphasized that among the five steps listed here, this is the only one that is model dependent. Once this step has been completed, the remaining steps, even if sometimes tedious, are model independent in the sense that only SM fields and their interactions are involved.

Step 2: Renormalization Group Evolution in the SMEFT
This step has been already discussed earlier and is based on the one-loop ADMs collected in [758–760]. This step generates new operators with nonvanishing WC and modifies the ones we started with at $\mu = \Lambda_{\rm NP}$. If there is a full spectrum of new heavy particles with vastly different masses, this step consists of several steps and corresponding effective theories similar to what we have encountered in the QCD renormalization group evolution with effective theories characterized by the number of flavors $f = 5, 4, 3$. At every threshold between two effective theories matching between the two effective theories in question has to be made. Here, to simplify the presentation, we will assume that the first threshold on our route to low-energy scales is the electroweak scale.

Step 3: Matching of the SMEFT onto the LEFT
In this step the matching of WCs in SMEFT evaluated through Step 2 at $\mu = \Lambda_{\rm EW}$ onto those of the low-energy effective theory below the electroweak scale (LEFT) is performed. Here the issue of the relative signs of SMEFT WCs and LEFT WCs has to be taken into account as we already warned the readers several times and illustrated in a few examples. Also in this case useful results are present in the literature and will be briefly described later.

Step 4: Renormalization Group Evolution in the LEFT
The one-loop anomalous dimensions for the operators of LEFT have been collected in [780]. They include previously known results that are cited in that paper as well as new ones. Complete RG evolution below the electroweak scale down to scales relevant for B physics has been presented in [756]. This evolution involves QCD and QED.

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Step 5: Calculation of the Observables
Finally, having evaluated WCs at scales relevant for observables of interest, the latter have to be evaluated. This is often a difficult step, in particular in the case of nonleptonic decays of mesons but also in those semileptonic decays in which good knowledge of formfactors is required.

These five steps should be evident for any reader who studied the first three parts of this book, but we recalled them here as they constitute the generalization of what is usually done within the SM. In the context of electric dipole moments I find the presentation of the preceding steps in [781] very transparent.

During the last years a number of papers were published that contain results on SMEFT and LEFT that are useful for any analysis of NP effects in any model. The next three sections collect the results I found in the literature. I will describe them here very briefly, listing most important references. I hope this collection is useful, in particular for those readers who want to incorporate in their NP studies the full machinery of effective theories involved.

14.6.2 Useful Technology: SMEFT

- The complete matching of any NP model onto the SMEFT at tree-level has been calculated in [782]. Moreover, the classification of the new fields relevant for this work, presented before in a series of papers for new quarks [783], new leptons [784], new vectors [785], and new scalars [786], has been summarized and generalized to non-renormalizable UV-completions. Starting with a general Lagrangian involving scalars, fermions, and vector bosons, the authors have computed WCs of the SMEFT at the NP scale in terms of couplings and masses. In tables 1–3 of this paper one can find the list of new scalar bosons, new vector-like fermions, and new vector bosons contributing to the SMEFT at tree level. In the corresponding tables 7–9 in this paper a list of SMEFT operators generated by each of this field is given. Finally, in Appendix D of that paper the WCs of SMEFT operators as functions of masses and couplings of all new fields in tree-level approximations are given. Even if the formulas in this paper are very complicated, this paper is very useful both for the top-down approach and also for the bottom-up one. But as the authors stress, these results are insufficient for the study of NP that enters first at one-loop level.
- Feynman rules for the SMEFT in R_ξ-gauges can be found in [787, 788] and FEYNRULES implementation of SMEFT in [789]. In [790] generalization to a generic class of effective theories with operators of arbitrary dimension has been made.
- Complete one-loop matching for a real singlet heavy scalar on the SMEFT has been presented in [791].
- Formulas for Z penguin contributions in generic extensions of the SM that involve arbitrary charged fermions, scalars, and gauge bosons but do not generate FCNC mediated by Z at tree-level, have been presented in [136]. Because of the gauge invariance also the relevant box contributions are given there. This paper is particularly important for models with RH charged currents as in this case one-loop calculations require even more care than in the presence of only LH currents.

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- Another useful paper for one-loop calculations is [792], where a generic class of NP models featuring new heavy scalars and fermions that couple to the SM fermions via Yukawa-like interactions has been considered. The extension to a vector-like fourth generation with particular emphasis put on $b \to s\ell^+\ell^-$ has been presented in [793].
- One-loop matching of gauge-invariant dimension-6 operators for $b \to s$ and $b \to c$ transitions has been presented in [563].
- QCD improved matching for semileptonic B decays with leptoquarks has been presented in [794]. The enhancement of the Wilson coefficients by 8%(13%) for scalar (vector) leptoquarks softens the LHC bounds and increases the allowed parameter space of LQ models addressing the flavor anomalies.
- In [795] the complete one-loop contributions in the SMEFT to radiative lepton decays $\mu \to e\gamma$, $\tau \to \mu\gamma$, and $\tau \to e\gamma$ have been calculated as well as to the closely related anomalous magnetic moments and electric dipole moments of charged leptons. Also three body flavor-violating charged lepton decays, which we will consider in some detail in Section 17.2, have been considered, taking into account all tree-level contributions.
- In [796] a complete analysis of $\mu \to e\gamma$, $\mu \to 3e$, and $\mu \to e$ conversion in the framework of LEFT is performed with RG evolution of the relevant WCs between EW scale and low energies at the one-loop level.
- In [768] RG evolution of NP contributions to charged-current leptonic and semileptonic decays has been presented focusing on operators involving chirality flip at the quark level. It has been pointed out that the large mixing of the tensor operators into the (pseudo)scalar ones has important impact on the phenomenology.
- A master formula for neutrinoless double-beta decay rate that takes into account dimension-5, -7, and -9 operators within SMEFT and includes RG evolution down to chiral EFT for nucleons has been presented in [797].
- A master formula for ε'/ε, valid in any extension of the SM, has been presented in [798]. We will discuss it in detail in Section 14.7.
- Generation of $\Delta F = 2$ operators from $\Delta F = 1$ operators O_{Hd}, $O_{Hq}^{(1)}$, and $O_{Hq}^{(3)}$ through RG running in the SMEFT and one-loop matching on LEFT considered in Scenarios I and II (see Sections 14.4.1 and 14.4.2) [766] has been generalized to include all four-fermion Dim=6 $\Delta F = 1$ operators in [799, 800].
- Correlations of lepton-flavor-violating τ decays with $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ have been analyzed within the SMEFT in [801]. They will be of interest in the era of NA62, KOTO, and Belle II experiments.
- In the case of $(g-2)_\mu$ useful general formulas can be found in [802–804].

Several papers in addition to those already cited in other chapters should be mentioned in this context. These are [761, 781, 796, 805–810], where various results related to RG evolutions in the SMEFT and LEFT relevant for various processes are presented.

14.6.3 Matching of SMEFT on LEFT

The SMEFT allows to calculate the WCs of the SMEFT in the Warsaw basis at the electroweak scale. Below the electroweak scale flavor physics is described by an effective

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theory discussed within the SM in previous chapters of this book. However in the presence of NP there are new operators generated already at the NP scale and/or at the electroweak scale through RG running in the SMEFT and through the matching onto the low-energy theory so that the starting point for the RG evolution down to the low-energy scale differs from the one encountered within the SM. Not only the initial conditions for SM operators are modified by NP but also new WCs are present. The resulting effective theory below the electroweak scale, although as the SM based on the gauge symmetry of QCD × QED, differs then from the SM one, and we have termed it low-energy EFT (LEFT), although these days also the name Weak Effective Theory (WET) is used for it. It should be stressed that this theory does not involve W, Z gauge bosons, the Higgs boson, and the top-quark as dynamical degrees of freedom. Moreover, there are no elementary scalars in this theory.

Therefore the next task is to find all possible gauge-invariant operators of the LEFT. This task has been accomplished in [811]. There are 70 Hermitian D = 5 and 3,631 Hermitian D = 6 operators that conserve baryon and lepton number, as well as $\Delta B = \pm\Delta L = \pm 1$, $\Delta L = \pm 2$, and $\Delta L = \pm 4$ operators. Among the 3,631 operators in question, 1,933 are CP-even and 1,698 CP-odd. In that paper also the matching onto these operators from SMEFT up to order v^2/Λ^2 has been computed at tree-level. Because SMEFT has far fewer D = 6 operators than the LEFT, there are many relations among LEFT coefficients, which for B physics can be found in [563, 565] and more generally in [811].

The situation simplifies for a CP conserving $U(3)^5$-symmetric SMEFT. The full tree and one-loop matching of such a theory on LEFT has been performed in [812].

14.6.4 Numerical Codes

In view of the large number of operators, public computer codes have been generated to calculate many flavor observables and treat the RG equations in full generality both in the context of SMEFT and LEFT. A detailed presentation of these codes goes beyond the scope of this book and beyond author's skills. Their compact description can be found in [773], which defines an exchange format for BSM WCs, called `WCxf`. It consists of a standard data format for the numerical values of WCs and therefore facilitates the exchange of WCs between different computer codes. Furthermore in [773] a python package is provided, which allows to do the basis change between different bases in LEFT/SMEFT in an automated manner.

As the codes in question are useful for various phenomenological applications, we want to list them here and describe some of them very briefly so that the readers will know what to expect from them. More details can be found in [1332]. A list of different aspects and the corresponding codes is then given as follows:

- Flavor observables: `flavio` [813], `EOS` [814], `FlavorKit` [815], `SPheno` [816], `FormFlavor`[817], `SuperIso` [818], `HEPfit` [819], `FlavBit` [820],
- Running and/or Matching: `DsixTools` [821], `wilson` [822], `MatchingTools` [823],

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- SMEFT related: `SMEFTsim` [789], `SMEFT Feynman Rules` [787],
- Dark matter: `DirectDM` [824].

Let us describe a few of them in more detail and mention other tools that are very useful.

- `DsixTools` [821] is a Mathematica package for handling the dimension-6 SMEFT. It allows to perform the full one-loop RG evolution of the Wilson coefficients from the NP scale down to the electroweak scale in the Warsaw basis numerically. It also contains modules devoted to the matching of SMEFT to $\Delta B = \Delta S = 1, 2$ and $\Delta B = \Delta C = 1, 2$ operators of the LEFT and their QCD and QED RG evolution below the electroweak scale. The application of this code to B physics anomalies by these authors can be found in [727]. References to the literature in which the relevant calculations have been done can be found in [821]. Readers interested in them should definitely look at these two papers.
- `flavio` [813] is an open-source tool for phenomenological analyses in flavor physics and other precision observables in the SM and beyond. It allows to compute predictions for a plethora of observables in quark and lepton-flavor physics and electroweak precision tests. It includes also the neutron electric dipole moment and anomalous magnetic moments of leptons. The list of presently included observables can be found in tables 1 and 2 of that paper. NP effects are parametrized as WCs of dimension-6 operators in the SMEFT or below the electroweak scale, that is in the LEFT. It contains the database of experimental measurements of these observables, a statistical package that allows to construct Bayesian and frequentists likelihoods, and convenient plotting and visualization routines.
- `FlavBit` [820] is a GAMBIT module for computing flavor observables and likelihoods. See also [825].
- `CoDEx` [826] is a Mathematica package that allows to calculate WCs of all effective operators at the electroweak scale that are generated from a given BSM theory, including the matching at tree and one-loop level on to the SMEFT and subsequent SMEFT renormalization group evolution down to the electroweak scale.
- `MatchingTools` [823] is a code that performs the tree-level matching of a given BSM model onto SMEFT in an automated way.
- `BasisGen` [827] allows for an automatic generation of an operator basis, given the fields and symmetries of any QFT.
- A further match-runner code is `wilson` [822]. In `wilson` the complete 1-loop running in the SMEFT, the complete tree-level matching from SMEFT onto LEFT at the EW scale, and the complete 1-loop running below the EW scale is implemented for all possible operators in SMEFT and LEFT. It is based on `WCxf` [773] mentioned earlier.
- The culmination of all these efforts can be found in [484], where the technology of [773, 813, 822] is combined resulting in a very efficient code, called *SMEFT likelihood (smelli)* that includes 257 observables, among them *all* our main players. Wilson coefficients in any BSM model can be specified at any scale, with one-loop renormalization

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group running above and below the electroweak scale automatically taken care of. The smelli package allows to compute the global likelihood as a function of the SMEFT WCs.

- In the standard determinations of the CKM parameters, it is usually assumed that the values of these parameters obtained from tree-level decays are not polluted by the presence of NP. To test this assumption and eventually take into account possible shifts in CKM parameters caused by the presence of NP, a strategy has been developed in [828]. It amounts to the redefinition of four Wolfestein parameters to four effective parameters by absorbing in them the effects of dim = 6 operators to four chosen observables. Subsequently, all other observables are expressed in terms of them. Once these four effective parameters are determined, one can insert their values in other observables. In my view it is not obvious whether one gains anything in this manner in the context of the SMEFT because the observables outside the chosen set will still contain additional contributions from dim = 6 operators that are unknown without a concrete model. Moreover, the values of the effective parameters depend on the chosen set of observables. Yet, one cannot exclude at present that one day, when the data improve, one will be able to find four precisely measured observables that will allow to find four optimal effective parameters that will facilitate the usual CKM fits in the presence of NP.

The necessity for efficient computer codes is illustrated by the following problematic, which is also present in the UV completions that do not predict the values of CKM parameters or generally SM parameters. These are usually determined at the electroweak scale or lower scales like m_b. But we would like to know them at the NP scale. If only QCD running is involved, this is straightforward as we have demonstrated in various chapters before. But in the SMEFT one has to take all effects into account. These are EW interactions, Yukawa interactions, and also the fact that the evolution of the SM parameters from EW scale to NP scales depends on the SMEFT Wilson coefficients themselves. This means that starting with a set of WCs at the NP scale, we have to make sure that the values of the SM parameters at this scale for the numerical values of WCs in question gives after RG to low energies the correct values of SM parameters determined in low-energy experiments. An iterative procedure explained in [822] accomplishes this task.

14.6.5 General Comments on the Future

Certainly this impressive technology will be improved in the coming years.

- The complete matching of any tree level model onto SMEFT at $\mu = \Lambda_{\rm NP}$ from [782] should be generalized to one-loop matching.
- The one-loop RG evolution in SMEFT from Λ down to EW scale from [758–760] should be generalized to NLO level to remove scale and renormalization scheme uncertainties present in one-loop matching.
- The tree-level matching of SMEFT operators onto LEFT operators at EW scale from [563, 811] should be generalized to one-loop level.
- The one-loop RG evolution in LEFT from EW scale down to a low-energy scale from [756, 780] should be extended to NLO. In fact, to a large extent this could be done

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already now using existing NLO results in the literature, see previous chapter. But one should still investigate whether some elements are missing.

- The numerical codes listed earlier should be extended to include more observables and the NLO contributions listed earlier as soon as they will be available.

Still the existing technology, which is dominantly incorporated in computer codes, allows one already now to compute not only the low-energy Lagrangian relevant for the phenomenology of weak decays of mesons and leptons but also with the help of flavio [813] a multitude of observables outside the flavor physics. In this context the paper [484] is very useful.

14.6.6 Nonlinear Realizations

It should be emphasized that SMEFT assumes the EW symmetry to be broken by a single fundamental Higgs doublet, which is known under the name of a linearly realized effective theory. This assumption can be in principle tested by measuring LEFT parameters from low-energy observables, SMEFT parameters from observables above the electroweak scale, and having the matching between these two theories [566, 811, 829]. I stress that in practice such a test will be very difficult in view of the presence of many operators.

When the breakdown of EW symmetry is realized nonlinearly, one deals with the EW chiral Lagrangians including a light Higgs. In this case the Higgs is a composite Nambu–Goldstone boson as considered in models in which SM gauge symmetry is broken dynamically. As far as Higgs physics and collider physics is concerned, these studies go back to the early 1990s [830–832]. More recent analyses can be found in [833–841]. In particular the complete electroweak chiral Lagrangian with a light Higgs at NLO has been presented in [838, 839] and renormalization group evolution of Higgs effective field theory in [840, 841].

14.7 ε'/ε beyond the SM

14.7.1 Master Formula for ε'/ε beyond the SM

In Chapter 10 we have presented various analytical formulas for ε'/ε in the SM. Writing ε'/ε as a sum of the SM and BSM contributions,

$$\frac{\varepsilon'}{\varepsilon} = \left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} + \left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm BSM}, \tag{14.82}$$

we will now present a master formula for the BSM part of this expression, which is valid in *any* theory beyond the SM that is free from nonstandard light degrees of freedom below the electroweak scale. This is in fact possible as we know all BSM $K \to \pi\pi$ hadronic matrix elements calculated in [246] at the low-energy scale $\mu_{\rm low}$ and also the RG evolution from $\mu_{\rm low}$ to $\mu_{\rm ew}$ at which the WCs are calculated.

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Such a master formula for $(\varepsilon'/\varepsilon)_{\rm BSM}$ has been presented in [798]. It exhibits the dependence on each WC at the scale $\mu_{\rm ew}$. Technical details that led to this formula can be found in [798] and in particular in [701].

Before presenting this formula a few comments are in order:

- The WCs are defined by

$$\boxed{\mathcal{H}_{\Delta S=1} \equiv -\sum_i \left[C_i(\mu_{\rm ew})O_i + C_i'(\mu_{\rm ew})O_i'\right],} \tag{14.83}$$

  with O_i' obtained from O_i by interchanging $P_L \leftrightarrow P_R$.
- As we have seen in the previous chapter, there are many operators in this sum when all five quarks below the electroweak scale are taken into account. Relating them to the SM basis is very complicated, and the idea in [701, 798] is to avoid this step by using a simpler basis, not involving sums over quarks. This basis is discussed in detail in [701]. As we will see soon, this allows in no time to calculate the BSM part of ε'/ε using numerical tables in both papers.
- For this reason the minus sign in (14.83) is not an issue. In calculating the BSM part, we simply forget about the definitions of SM operators Q_6, Q_8, etc. that we used so far and calculate the WCs defined by (14.83).
- While [798] is very short but allows already to calculate $(\varepsilon'/\varepsilon)_{\rm BSM}$, we advise motivated readers to study [701] as it contains a lot of information about ε'/ε in the SMEFT, which we can only briefly present here. The complexity of the analysis illustrated in Figure 14.1 will hopefully not stop the readers to look at this paper. The classes A–E of operators indicated in this sketch are explained there.

The master formula of [798] for the BSM part in (14.82) then reads

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm BSM} = \sum_i P_i(\mu_{\rm ew})\ {\rm Im}\left[C_i(\mu_{\rm ew}) - C_i'(\mu_{\rm ew})\right] \times (1\ {\rm TeV})^2, \tag{14.84}$$

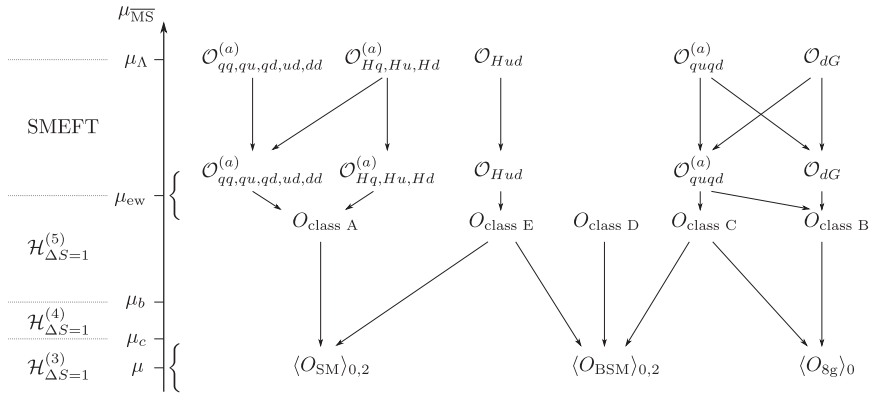


Figure 14.1 Sketch of the different contributions to ε'/ε discussed in section 2 of [701].

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where

$$\boxed{P_i(\mu_{\rm ew}) = \sum_j \sum_{I=0,2} p_{ij}^{(I)}(\mu_{\rm ew},\mu_{\rm low}) \left[\frac{\langle O_j(\mu_{\rm low})\rangle_I}{\rm GeV^3}\right],} \tag{14.85}$$

with the sum over i extending over the Wilson coefficients C_i of all operators and their chirality-flipped counterparts, that is $36 + 36'$ linearly independent four-quark operators and $1 + 1'$ chromo-magnetic dipole operators discussed in previous chapter. The C_i' are the WCs of the corresponding chirality-flipped operators as mentioned earlier. The relative minus sign accounts for the fact that their $K \to \pi\pi$ matrix elements differ by a sign. Among the contributing operators are also operators present already in the SM, but their WCs in (14.84) include only BSM contributions. The list of all contributing operators has been given in the previous chapter, but it is of course also given in [701, 798].

As the coefficients defined by (14.83) carry dimension, to make the coefficients $P_i(\mu_{\rm ew})$ and $p_{ij}^{(I)}(\mu_{\rm ew},\mu_{\rm low})$ dimensionless, we have included the factor $(1\,\text{TeV})^2$. The coefficients $p_{ij}^{(I)}(\mu_{\rm ew},\mu_{\rm low})$ include the QCD and QED RG evolution from $\mu_{\rm ew}$ to $\mu_{\rm low}$ for each WC as well as the relative suppression of the contributions to the $I = 0$ amplitude due to $\text{Re}A_2/\text{Re}A_0 \ll 1$ for the matrix elements $\langle O_j(\mu_{\rm low})\rangle_I$ of all the operators O_j present at the low-energy scale. The index j includes also i so that the effect of self-mixing is included. The $P_i(\mu_{\rm ew})$ do not depend on $\mu_{\rm low}$ to the considered order because the $\mu_{\rm low}$-dependence cancels between matrix elements and the RG evolution operator. Moreover, it should be emphasized that their values are *model-independent* and depend only on the SM dynamics below the electroweak scale, which includes short-distance contributions down to $\mu_{\rm low}$ and the long-distance contributions represented by the hadronic matrix elements. The BSM dependence enters our master formula in (14.84) *only* through $C_i(\mu_{\rm ew})$ and $C_i'(\mu_{\rm ew})$. That is, even if a given P_i is nonzero, the fate of its contribution depends on the difference of these two coefficients. In particular, in models with exact left-right symmetry this contribution vanishes as first pointed out in [842].

The numerical values of the $P_i(\mu_{\rm ew})$ are collected in the tables presented in [701, 798]. As seen in (14.85), the P_i depend on the hadronic matrix elements $\langle O_j(\mu_{\rm low})\rangle_I$ and the RG evolution factors $p_{ij}^{(I)}(\mu_{\rm ew},\mu_{\rm low})$. The numerical values of the hadronic matrix elements rely on LQCD in the case of SM operators and DQCD in the case of BSM operators. Once lattice QCD will provide precise values for the latter, an update for P_i will be possible.

Inspecting the results in the tables of [701] the following observations can be made, here given in a simplified form in terms of three islands of hadronic matrix elements: SM-like island governed by SM operator Q_8, chromo-magnetic one governed by the chromo-magnetic penguin operator O_{8g} and the BSM-island conquered in [246]:

- The largest P_i values on the SM-like island can be traced back to the large values of the matrix element $\langle Q_8\rangle_2$, the dominant electroweak penguin operator in the SM, and the enhancement by 22 of the $I = 2$ contributions relative to $I = 0$ ones.
- The small P_i values on the chromo-magnetic island are the consequence of the fact that each one is proportional to $\langle O_{8g}\rangle_0$, which has recently been found to be much smaller than previously expected [843, 844]. Moreover, as $\langle O_{8g}\rangle_2 = 0$, all contributions in

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this class are suppressed by the factor 22 relative to contributions from other classes. Nevertheless, there are NP scenarios where they play an important role (see, e.g., [845, 846]).

- The large P_i values on the BSM-island can be traced back to the large hadronic matrix elements of scalar and tensor operators calculated in [246]. In particular the largest P_i are found for

$$\boxed{(\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\alpha)(\bar{u}^\beta \sigma^{\mu\nu} P_L u^\beta), \qquad (\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\beta)(\bar{u}^\beta \sigma^{\mu\nu} P_L u^\alpha),} \tag{14.86}$$

$$\boxed{(\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\alpha)(\bar{d}^\beta \sigma^{\mu\nu} P_L d^\beta), \qquad (\bar{s}^\alpha P_L d^\alpha)(\bar{u}^\beta P_L u^\beta),} \tag{14.87}$$

  where α, β are color indices. The operators in (14.86) are related by Fierz identities to scalar-scalar ones and are allowed by the full gauge symmetry of the SM. This is not the case for the operators in (14.87), which violate hypercharge conservation and are only consistent with $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$.

The question then arises of which creatures are living on these islands. To this end the analysis in [782], mentioned previously, turns out to be very useful. Here we provide only a short answer for the SM- and BSM-islands:

- SM-island: Z', W', G', vector-like quarks
- BSM-island: heavy scalars

We will encounter the ones on the SM-island in several concrete models in later chapters, but we will also look briefly at BSM-island as well. This will allow us to illustrate on examples how the master formula in question is used. The detailed phenomenology of ε'/ε in concrete models on the BSM-island is left for the future.

The master formula given in (14.84) tells us what happens below the electroweak scale $\mu_{\rm ew}$ with the information collected in the coefficients $P_i(\mu_{\rm ew})$. While the values of the coefficients C_i and C_i' are certainly model dependent, some general lessons from the extensive SMEFT analysis in [701] can be drawn.

14.7.2 Lessons from the SMEFT Analysis of ε'/ε

In [701] the first model-independent anatomy of the ratio ε'/ε in the context of the SMEFT has been presented. This was only possible thanks to the 2018 calculations of the $K \to \pi\pi$ matrix elements of BSM operators, namely of the chromo-magnetic dipole operators by lattice QCD [843] and DQCD [844] and in particular through the calculation of matrix elements of all four-quark BSM operators, including scalar and tensor operators, by DQCD [246]. Even if the latter calculation has been performed in the chiral limit, it offers for the first time a look into the world of BSM operators contributing to ε'/ε.

The main goal in [701] was to identify those NP scenarios, which are probed by ε'/ε and which could help to explain the emerging anomaly in ε'/ε discussed in Chapter 10.

Since 2015 a number of analyses, addressing the ε'/ε anomaly in concrete models, appeared in the literature. They are collected in Section 14.7.3. We will discuss some of

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them in the next chapters. But all these analyses concentrated on models in which NP entered exclusively through modifications of the WCs of SM operators, in particular of the WC of the dominant electroweak penguin operator Q_8. Thus all these analyses have been performed on the SM-like island corresponding to the first term in (13.1).

This is a significant limitation if one wants to have a general view of possible BSM scenarios responsible for the ε'/ε anomaly. In particular, in the absence of even approximate values of hadronic matrix elements of BSM operators contributing to the second term in (13.1), no complete model-independent analysis was possible. The 2018 calculations of BSM $K \to \pi\pi$ matrix elements, in particular of those of scalar and tensor operators in [246], allowed to perform the analysis also on the BSM-island. Combined with the LEFT and in particular SMEFT technology in [701] this study widens significantly our view on BSM contributions to ε'/ε. The analysis in [701] has two main virtues:

- It opens the road to the analyses of ε'/ε in any theory beyond the SM and allows with the help of the master formula in (14.84) to search very efficiently for BSM scenarios behind the ε'/ε anomaly. In particular, the values of P_i collected in this paper indicate which routes could be more successful than others both in the context of the LEFT and the SMEFT. By implementing our results in the open source code `flavio` [813], testing specific BSM theories becomes particularly simple.
- Through this SMEFT analysis it was possible to identify correlations between ε'/ε and various observables that depend sensitively on the operators involved. Here $\Delta S = 2$, $\Delta C = 2$, and electric dipole moments (EDM) play a prominent role but also correlations with $\Delta S = 1$ and $\Delta C = 1$ provide valuable informations. We ask the interested readers to look at [701] for more details. We will return to some of these correlations in the following chapters.

The main messages from [701] are as follows:

- Tree-level vector exchanges, like Z', W', and G' contributions (SM-like island), discussed already by various authors and vector-like quarks [666] can be responsible for the observed anomaly, but generally one has to face important constraints from $\Delta S = 2$ and $\Delta C = 2$ transitions as well as direct searches, and often some fine-tuning is required. Here the main role is played by the electroweak operator Q_8 with its WC significantly modified by NP.
- Models with tree-level exchanges of heavy colorless or colored scalars (BSM-island) are a new avenue, opened with the results for BSM operators from DQCD in [246]. In particular scalar and tensor operators, having chirally enhanced matrix elements and consequently large coefficients P_i, are candidates for the explanation of the anomaly in question. Moreover, some of these models, in contrast to models with tree-level Z' and G' exchanges, are free from both $\Delta S = 2$ and $\Delta C = 2$ constraints. The EDM of the neutron is an important constraint for these models, depending on the couplings, but does not preclude sizable NP effects in ε'/ε.
- Models with modified $W^\pm$ or Z^0 couplings can induce sizable effects in ε'/ε without appreciable constraints from semileptonic decays such as $K^+ \to \pi^+\nu\bar{\nu}$ or $K_L \to \pi^0\ell\bar{\ell}$. In the case of an SM singlet Z' mixing with the Z, sizable Z-mediated contributions are

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disfavored by electroweak precision tests. Yet, as discussed in [847] and in references given there, also such models could contribute to our understanding of the role of NP in ε'/ε. This is in particular the case of models with vector-like quarks [666].

Of particular interest in the results of [246] is large mixing of the scalar-scalar operators into tensor-tensor operators that is responsible for the enhancement of the matrix elements of tensor-tensor operators in the process of $O(1/N)$ meson evolution. As pointed out in the latter paper, this feature has some analogy to the observation made in [756, 768], where the QED short-distance RG evolution of NP contributions to charged-current induced leptonic and semileptonic meson decays has been presented, focusing on chirality-flipped operators at the quark level. It has been pointed out that the large mixing of the tensor-tensor operators into the scalar-scalar ones has an important impact on the phenomenology. Recently this aspect has also been discussed in the context of $R(D^{(*)})$ anomalies in [848, 849].

In fact, the one-loop QED diagrams responsible for the latter mixing are the same as the one-loop QCD diagrams with gluon replaced by photon, QCD coupling replaced by the QED one, and color matrices replaced by charge ones. Even if the RG evolution of the QED coupling constant is different from the QCD one, the pattern of mixing analyzed in [756, 768] is very similar to the one found in [246].

The reason why in [756, 768] tensor operators have the impact on the scalar ones, as opposed to the case discussed here, is simply related to the known fact, encountered already in Chapter 5 and Section 7.2, that while the evolution of the matrix elements of operators is governed by the ADM of operators, the evolution of their Wilson coefficients, analyzed in [756, 768], is governed by the corresponding transposed matrix.

We expect then that while in [756, 768, 848, 849] the large mixing in question had impact on the phenomenology of B-meson decays, in the case at hand, it will have significant impact on ε'/ε. But a detailed phenomenological analysis has still to be done.

The future of ε'/ε in the SM and in the context of searches for NP will depend on how accurately it can be calculated. This requires improved lattice calculations not only of the matrix elements of SM operators but also of the BSM ones, which are known presently only from the DQCD approach in the chiral limit. Moreover, the impact of FSI on the values of P_i have to be investigated. As these values are dominated by $I = 2$ matrix elements, we expect that these effects amount to at most 10–20% of the present values. It is also hoped that lattice QCD will be able to take into account isospin breaking corrections and that other lattice collaborations will attempt to calculate hadronic matrix elements of all relevant operators. In this context we hope that the new analysis of the RBC-UKQCD collaboration with improved matrix elements to be expected this year will shed new light on the hinted anomaly. Such future updates can be easily accounted for by the supplementary details on the master formula in the appendices of [701].

On the short-distance side the NNLO results for QCD penguins should be available soon [649]. The dominant NNLO corrections to electroweak penguins were calculated almost 20 years ago [647]. As pointed out in [701] all the estimates of ε'/ε at NLO prior to this paper suffer from short-distance renormalization scheme uncertainties in the electroweak penguin contributions and also scale uncertainties in $m_t(\mu)$ that are removed only in the NNLO matching at the electroweak scale [647]. We have stressed it in Section 10.4.

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Table 14.3 Papers studying implications of the ε'/ε anomaly.

NP Scenario	References	Correlations with
LHT	[850]	$K_L \to \pi^0\nu\bar{\nu}$
Z-FCNC	[766, 799, 851]	$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$
Z'	[851]	$K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$ and ΔM_K
Simplified models	[601]	$K_L \to \pi^0\nu\bar{\nu}$
331 models	[852, 853]	$b \to s\ell^+\ell^-$
Vector-like quarks	[666]	$K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$ and ΔM_K
Supersymmetry	[854–858]	$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$
2-Higgs doublet model	[846, 859, 860]	$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$
Right-handed currents	[861, 862]	EDMs
Left-right symmetry	[863, 864]	EDMs
Leptoquarks	[667]	all rare Kaon decays
SMEFT	[701]	several processes
SU(8)	[865]	$b \to s\ell^+\ell^-$, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$
Diquarks	[866, 867]	ε_K, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$
3HDM+ν_R	[868]	$R(K^{(*)})$, $R(D^{(*)})$
Vectorlike compositeness	[869]	$R(K^{(*)})$, $R(D^{(*)})$, ε_K, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$

The fact that these NNLO contributions imply a *negative* shift of about -1.1×10^{-4} in ε'/ε gives another motivation for the search for new physics responsible for it, and thus for the analyses just described and those listed soon.

As far as BSM operators are concerned, an NLO analysis of their Wilson coefficients should be completed in 2020, but its importance is not as high as of hadronic matrix elements due to significant additional parametric uncertainties residing in any NP model. In any case, in the coming years the ratio ε'/ε is expected to play a significant role in the search for NP. In this respect, the results presented in [701, 798] will be helpful in disentangling potential models of new CP violating sources beyond the SM as well as constraining the magnitude of their effects.

14.7.3 ε'/ε in Specific Models

A number of authors investigated what kind of NP could give sufficient upward shift in ε'/ε and what would then be the implications for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. There is no space to discuss them all here but some of them will be discussed in later chapters. The list of papers known to the author in May 2019 is collected in Table 14.3. In these models ε'/ε can be enhanced significantly without violating existing constraints. The exceptions are leptoquark models, which we will discuss in Section 16.4.13.

It should be mentioned that in most models listed in Table 14.3, only modifications of the WCs of SM operators by NP contributions have been considered. The exceptions are the SMEFT analyses discussed earlier and the inclusion of chromomagnetic penguins in [846]. Finally, I can recommend an interesting paper in [1326] that demonstrates the correlation of ε'/ε with hadronic B decays via $U(2)^3$ flavor symmetry. This symmetry will be discussed in the following chapter.

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