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Survey or Review
Multi-level control of zero-moment point-based humanoid biped robots: a review

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SUMMARY
Researchers dream of developing autonomous humanoid robots which behave/walk like a human being. Biped robots, although complex, have the greatest potential for use in human-centred environments such as the home or office. Studying biped robots is also important for understanding human locomotion and improving control strategies for prosthetic and orthotic limbs. Control systems of humans walking in cluttered environments are complex, however, and may involve multiple local controllers and commands from the cerebellum. Although biped robots have been of interest over the last four decades, no unified stability/balance criterion adopted for stabilization of miscellaneous walking/running modes of biped robots has so far been available. The literature is scattered and it is difficult to construct a unified background for the balance strategies of biped motion. The zero-moment point (ZMP) criterion, however, is a conservative indicator of stabilized motion for a class of biped robots. Therefore, we offer a systematic presentation of multi-level balance controllers for stabilization and balance recovery of ZMP-based humanoid robots.

KEYWORDS: Biped robot; Stability; Zero-moment point; Balance; Multi-level control.

1. Introduction

Industrial robots are often rigidly attached to the ground. In contrast, mobile robots are able to move in specific environments depending on the tasks they are required to perform. Much attention has been paid to the design and control of mobile robots because they have significant characteristics such as versatile mobility and can sense and react to specific environments.[1] Mobile robots can be classified as legged robots, wheeled robots and tracks.

Legged robots offer significant advantages over wheeled robots and tracks in terms of workable environments, energy consumption, (e.g. biped robots could be exploited to consume less energy than wheeled robots when walking on sand environments) and adaptability.[2] Although wheeled robots are simple lightweight structures, they need regular terrains for motion and lack for instance the ability to climb stairs. Tracks help to overcome this drawback, but they consume a lot of energy because of the high amount of friction between the chains and the ground. In contrast, legged robots can move in regular and irregular terrains with versatile mobility. With configuration changes, they can easily adapt to irregular environments. The small contact areas of their feet mean legged robots can be efficiently operated.[2]

The biped robots, a type of two-feet legged robots, are designed to imitate human-like locomotion and perform certain tasks such as activities in danger environments, assistance to the elderly and

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entertainment.[3–11] The biped robot can consist of a trunk and legs with/without feet or even have an entirely human-like mechanism depending on the desired application.

Biped robots have advantages over multi-legged robots. They have considerably higher adaptability, enabling them to overcome obstacles like narrow paths or stairs with ease. This adaptability is particularly important when the robot is required to perform human-centred tasks. Because of the smaller ground contact area and fewer actuators used, their energy consumption can be lower than that of multi-legged robots. This is consistent with the assumption that two-legged animals have higher efficiency and adaptability than multi-legged animals.[12] For a historical review of legged machines, we refer readers to refs. [3, 13].

Some challenges encountered in the design of biped robots are as below:

- Biped robots have unstable structures because of the passive joint located at the unilateral foot-ground contact.[14–17]
- Because of the unilateral foot-ground contact and the varying configurations throughout the gait cycle, their mechanical description is highly nonlinear. During the single-support phase (SSP), the robot is under-actuated, turning into an over-actuated system during the double-support phase (DSP), however.[18] Consequently, the dynamic description and control laws change during transition from one phase to another.[15] It is notable that the biped robot can be fully actuated during the swing phase, with some limitations. In fact, the dynamics of the robot depend on which legs make contact with the ground. Moreover, exchange of leg support is accompanied by an impact disturbing the robot's motion.[15]
- Biped robots have many degrees of freedom (DOFs). A humanoid robot may have more than 30 DOFs, making its mechanical behaviour and control difficult.[19–21] To avoid this difficulty, most researchers have used simple models based on approximations and assumptions. A trade-off between simplicity and accuracy is necessary.
- Biped robots interact with different unknown environments. The surface could be elastic, sticky, soft or stiff. This requires robust algorithms for the generation of reference trajectories and control. These algorithms should be insensitive to possible disturbances and noises. Stabilization and online adaptive control schemes could solve this dilemma.

These challenges are associated with mechanics, control, electronics, artificial intelligence and human anatomy and so studying biped robots is an interdisciplinary exercise. Unified solutions are therefore difficult to develop. For further reading on difficulties encountered in biped design, we refer to ref. [22]. In our previous paper,[23] we concentrated on discussion of the different methods used for generation of biped walking patterns and the stability/balance criteria used for biped stabilization. In this paper, we have extended discussion of the stability/balance problem to include some other criteria used for balance recovery, and we have attempted to present systematically multi-level control systems of biped robots based on ZMP criterion.

The remainder of the paper is organized as follows. The gait cycle of biped locomotion is presented in Section 2. Section 3 discusses the stability/balance criteria of the biped mechanism, and Sections 4, 5 and 6 introduce multi-level control of biped locomotion based on ZMP criterion. Section 7 concludes.

Remark 1. Despite numerous 3D applications have been reported in the literature, this paper tries to capture the fundamental characteristics of biped walking by limiting our study to this super-abstractive structure.

2. Gait Cycle

The complete gait cycle of human walking consists largely of two successive phases: the DSP and the SSP with intermediate sub-phases.[24,25] The DSP arises when both feet contact the ground, resulting in a closed-chain mechanism, and the SSP starts when the rear foot is not supported by the ground and the front foot is flat on the ground. It should be noted that the DSP accounts for about 20% of the time taken for one stride of the gait cycle and the SSP about 80%.[16,25]

Because of the complexity of biped mechanisms, most researchers have simplified the gait cycle to explore their kinematics, biomechanics and control schemes. For details on different walking patterns of biped robot, see ref. [23] and the references therein.

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3. Stability

The biped mechanism is unstable during the SSP. One of the challenges in the design and control of biped robots is to maintain their balance while walking in different kinds of environment. The reason for the instability is under-actuation owed to the passive joint of the foot-ground contact. This means that control of the feet is dependent on control of the mechanism above the feet.[14,29] Common stability theories such as analysis of eigenvalues, gain and phase margins, and Lyapunov stability can be applied to particular modes of biped robot gait but cannot guarantee biped stability for all modes of motion.[30]

In general, there are two types of stability criteria that the trajectories of a biped mechanism depend on: static stability and dynamic stability. Static stability restricts the vertical projection of the centre of the mass of the biped to the inside of the support polygon. The support polygon is defined as the area represented by the stance foot during the SSP and the bounded area between the supported feet during the DSP.[18] This type of stability leads to slow gait and biped robots with large feet.[31] Thus, the position of the centre of mass (COM), $\boldsymbol{p}_{\text{com}}$, can be calculated as follows:

$$\boldsymbol{p}_{\text{com}} = \frac{\sum_{i=1}^{n_l} m_i \boldsymbol{p}_i}{\sum_{i=1}^{n_l} m_i}, \tag{1}$$

where n_l is the number of biped links, m_i is the mass of link (i) and $\boldsymbol{p}_i$ is the position of the COM of link (i). The ground projection of $\boldsymbol{p}_{\text{com}}$ can be found easily by determining its components.

Dynamic stability provides more freedom than the static stability since the projected COM of the biped may leave the support polygon and thus allow for faster gait.[31] There are four specific techniques for analysing dynamic stability:[30,32–34]

1. Zero-moment point (ZMP),
2. Centroidal angular momentum,
3. Footstep-based criteria,
4. Periodicity-based gait.

The first three criteria will be described in details due to their association with humanoid biped locomotion. The periodicity-based gait is not locally stable such that the biped cannot stop, turn etc.; therefore, it will not be considered in this paper.

3.1. Zero-moment point (ZMP)

The notion of the ZMP was proposed by Vukobratovic and colleagues,[35,207] who exploited the passive joint of the foot-ground contact. It is applied to the biped mechanism in the design of walking patterns and control schemes. The ZMP is the point on the ground at which the net moment vector of the inertial and gravitational forces of the entire body has zero components in the horizontal planes.[14,29] In brief, if ZMP is located inside the support polygon, then the system is stable and the centre of pressure (COP) of the foot coincides with the ZMP. If the ZMP is outside the support polygon, the system is unstable and the ZMP will be outside the stability margin comprising the fictitious ZMP (FZMP),[14] as shown in Fig. 1. The point P in the mentioned figure represents the location of the zero components of the net moments affecting the foot on the horizontal planes. It is clear that in the stable case one can determine the position of the ZMP by calculating the position of the COP. This is done by using force sensors at the sole plate of the foot. In effect, the theoretical calculation of the ZMP can be performed by the following two formulations:

1. Formulation 1 (more computational).
   By knowing the inertial and gravitational forces, one can find the ZMP coordinate as in Eqs. (2) and (3)[36]

$$x_{\text{ZMP}} = \frac{\sum_{i=1}^{n_l} m_i (\ddot{z}_i + g) x_i - \sum_{i=1}^{n_l} m_i \ddot{x}_i z_i - \sum_{i=1}^{n_l} I_{iy} \ddot{q}_{iy}}{\sum_{i=1}^{n_l} (\ddot{z}_i + g)}, \tag{2}$$

$$y_{\text{ZMP}} = \frac{\sum_{i=1}^{n_l} m_i (\ddot{z}_i + g) y_i - \sum_{i=1}^{n_l} m_i \ddot{y}_i z_i + \sum_{i=1}^{n_l} I_{ix} \ddot{q}_{ix}}{\sum_{i=1}^{n_l} (\ddot{z}_i + g)}, \tag{3}$$

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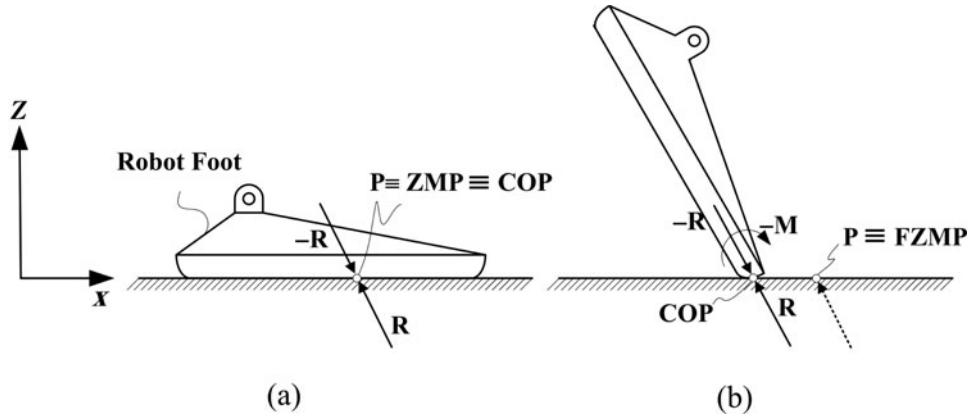


Fig. 1. Relationship between the ZMP, FZMP and COP. (a) Dynamically stable (b) Dynamically unstable.[14]

where x and y axes are parallel to the horizontal plane for the biped motion, z axis is pointing upwards, $(I_{ix}, I_{iy})^T$ is the inertial vector of link (i), $\ddot{q}_{ix}$ and $\ddot{q}_{iy}$ are the angular acceleration of link (i) about x and y respectively, g is the gravitational acceleration, $(x_{\text{ZMP}}, y_{\text{ZMP}}, 0)$ is the coordinate of the ZMP and (x_i, y_i, z_i) is the coordinate of the mass centre of link (i). The above equations can be used to investigate the stability of the biped mechanism during the SSP and DSP.

2. Formulation 2 (less computational).

Equations (2) and (3) are highly nonlinear; therefore, most researchers use Eqs. (4) and (5) alternately during the SSP, using the static equations of the fixed stance foot.[15]

$$x_{\text{ZMP}} = -\tau_y / F_z, \tag{4}$$

$$y_{\text{ZMP}} = \tau_x / F_z, \tag{5}$$

where τ_y and τ_x are the ankle joint torques about the referred axes and F_z is the normal component of the ground reaction force.

Because the ZMP coincides with the COP, the ZMP coordinate can be found during the DSP according to Eq. (6)[38]

$$\text{cop} = \text{cop}_f \frac{F_{zf}}{F_{zf} + F_{zr}} + \text{cop}_r \frac{F_{zr}}{F_{zf} + F_{zr}}, \tag{6}$$

where cop, cop_f and cop_r represent the centre of pressure for the biped robot during the DSP, the front foot COP and the rear foot COP respectively, and F_{zf} and F_{zr} are the normal components of the ground reaction forces for the front and rear foot, respectively.

In general, the ZMP-based biped mechanisms are characterized by the following.

- The stance foot of the biped should remain in full contact all the time.[39]
- All the joints of the biped mechanism are actuated and often rigidly controlled to track predetermined trajectories which can simplify the control task during the SSP.
- They do not exploit natural dynamics, so their locomotion usually tends to be less natural than human-like motion (often bent-knee walking) and with high energy consumption.
- Most conventional humanoid biped robots are driven by electric motors via gears and their walking patterns are based on the linear inverted pendulum mode (LIPM).

Several algorithms have been used to generate a stable reference trajectory for the biped mechanism which satisfies the ZMP constraint. This implies that the solution of Eqs. (2) and (3) to find a relationship between the COM of the biped and the ZMP trajectory requires a lot of computational effort. Therefore, these algorithms can be applied offline.[42,43] Alternatively, most researchers use simple models to generate the desired biped walking patterns such as the linear inverted pendulum for online implementation. Biped robot stability approaches based on the ZMP still lack efficiency, robustness, easy handling and natural motion.[39,44] For further reading, refer to refs. [27, 45–54].

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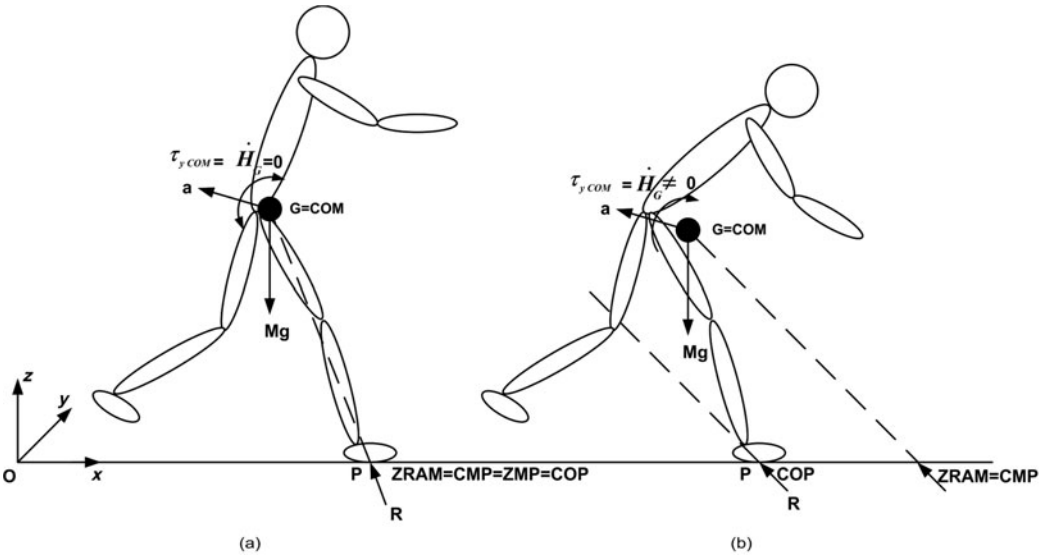


Fig. 2. The relationship between ZMP, CMP and ZRAM points. (a) Dynamically stable biped (b) Dynamically unstable biped.[55,54]

3.2. Centroidal angular momentum

According to the principle of dynamics, the rate of change of linear/angular momentum of a rigid body about its centre of gravity (COG) G is equal to the resultant external forces/moments. As a result the linear and angular momentums are zero for zero external disturbances (forces and moments). Exploiting this property, the angular momentum can be used as an index for rotational posture control of biped robots and even for legged systems.[55] Sano and Furusho[56,57] proposed using the angular momentum of the biped system as a controlled variable. It has been shown that the angular momentum during the SSP about the ankle joint of the stance foot is a function of the gravity effect and the ankle torque. Therefore, the ankle torque can be considered as an angular momentum control input. In their method, the torque control rather than the position control have been used for the ankle joints during the SSP and one of the hip joints during the DSP with specific selection of the desired angular momentum function. Goswami and Kallem[55] suggested an important criterion called the zero rate of change of angular momentum (ZRAM) point. They observe that the rate of change of centroidal angular momentum is a useful index/criterion for the analysis and control of posture control of biped robots in different interactive environments. Figure 2 shows the detail of their method.

They proposed three control strategies for balance control, manipulating the value of the rate of change of the angular momentum as follows:

$$OP \times \boldsymbol{R} + OG \times m_{\mathrm{COM}} \times g = \dot{\boldsymbol{H}}_{\boldsymbol{G}} + OG \times m_{\mathrm{COM}} \boldsymbol{a}, \tag{7}$$

with notations shown in Fig. 2.

These strategies can be applied through the following:

1. Enlarging the support polygon.
2. Moving G relative to point P; this coincides with strategy of COG position-based compensation discussed in Section 5.
3. Changing the direction of ground reaction forces by changing the centroidal (spin) acceleration of the biped mechanism; this coincides with the strategy of COG acceleration-based compensation discussed in Section 5.

Remark 2. Although some researchers have not directly considered the angular momentum as a balance criterion, the latter two strategies have been used successfully to compensate for modelling errors and external disturbances, as we will see later. In addition, sudden forward movements of the

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body and arms make use of the angular momentum to maintain balance.[32] Therefore, all the methods described in Section 5.2 try to regulate the centroidal angular momentum.

Popov and colleagues[54,58–61] proposed the centroidal moment pivot (CMP) as a stability criterion for measuring the unbalance of the biped mechanism. It is exactly the same criterion proposed by the authors of ref. [55]. It is the point where a line parallel to the ground reaction force, passing through the COM, intersects with the external contact surface,[54] as shown in Fig. 2.

It can be written in terms of the COM and ground reaction forces as in eqs. (8) and (9) and in terms of the ZMP location, the vertical ground reaction force, and the moment about the COM as in eqs. (10) and (11).

$$x_{\mathrm{CMP}} = x_{\mathrm{COM}} - \frac{F_x}{F_z + m_{\mathrm{COM}} g} z_{\mathrm{COM}}, \tag{8}$$

$$y_{\mathrm{CMP}} = y_{\mathrm{COM}} - \frac{F_y}{F_z + m_{\mathrm{COM}} g} z_{\mathrm{COM}}, \tag{9}$$

$$x_{\mathrm{CMP}} = x_{\mathrm{ZMP}} + \frac{\tau_{y\mathrm{COM}}}{F_z + m_{\mathrm{COM}} g}, \tag{10}$$

$$y_{\mathrm{CMP}} = y_{\mathrm{ZMP}} - \frac{\tau_{x\mathrm{COM}}}{F_z + m_{\mathrm{COM}} g}, \tag{11}$$

where $(x_{\mathrm{COM}},\ y_{\mathrm{COM}},\ z_{\mathrm{COM}})$ is the position of COM, and other notations are shown in Fig. 2

It is clear from eqs. (10) and (11) that if the centroidal moments of the biped are zero, the CMP will coincide with the ZMP and the biped is stable (balanced). This now means that the centroidal moments should be zero for a balanced biped; corrective moments for minimizing the non-zero spin angular momentum should be applied in the case of unbalanced bipeds. As a result, the ZMP may not coincide with the CMP in some cases. One of the important problems inherent in the notion of ZMP which motivates researchers to look for a general powerful stability index represented by angular momentum is that of the point-foot biped robot. In that case, the ZMP is constrained at the contact point and cannot be repositioned. The only way to stabilize such a system is to produce a non-zero moment about the COM of the biped. Kajita and colleagues[62,63] used the ZMP to generate reasonable walking patterns and integrated the notion of specific angular momentum as a high-level control to maintain the balance of the biped mechanism.

Remark 3. Most common humanoid robots depend on the ZMP in generating stable walking patterns. Motion planning is applied with the ZMP and the balance strategy can be performed by regulating the angular momentum and the ZMP stabilizer. In other words, as proved in ref. [63], the ZMP can be maintained inside the support polygon by selection of suitable values of the total linear and angular momentum and the position of the COM.

Remark 4. The foot rotation indicator (FRI)[29] and FZMP are equivalent terms which can be used to measure the unbalance of the biped mechanism without explaining their relationships with the linear/angular momentums, except in the work of the authors of ref. [63]. The ZRAM point and CMP are exactly equivalent in terms of regulating the angular momentum in relation to the COG. Even now there is no general criterion that can describe all stability cases of biped locomotion with different gaits.[55]

3.3. Footstep-based criteria

Pratt and Tedrake[32] proposed five subtasks that the biped should perform to generate balanced gait: (1) maintaining the body (trunk) orientation, (2) maintaining virtual leg length resulting from simplified linear inverted pendulum plus flywheel body, (3) swinging the swing leg, (4) transition of different phases, and (5) regulating the velocity of the COM. They called them velocity-based stability margins. Point 5 plays an important role in maintaining balance. It has been shown that the first three points are fully controllable and can be implemented by means of traditional high gain joint position control techniques, provided the requirements of joint torques are satisfied to avoid slipping. The fourth point can be dealt with by low impedance force control,[64] or a shift transition function that guarantees

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Table I. Comparison of different stability criteria[32,33].

Stability Criterion	Advantages	Disadvantages
ZMP	It is a sufficient and conservative stability criterion for biped robots which are fully actuated and position controlled to track predetermined trajectories.	1. The stance foot of the biped should remain in full contact at all times. 2. Unnatural motion with high energy consumption. 3. There are many situations in which the ZMP is violated, e.g. during toe-off human walking, biped with point feet, passive dynamic walkers. On the other hand, the ZMP could be satisfied while the biped robot is collapsed, e.g., setting zero torques of the biped mechanism and maintaining a flat foot while hitting the ground with its trunk.
Centroidal angular momentum	It is a good tool for recovering from a push or other disturbance, as there is a coupling between the angular momentum rate change and linear angular momentum and hence the COM speed.	Depends on minimizing spin angular momentum and can be violated through walking with a violently thrashing upper body, and falling over while the COM has zero spin angular momentum.
Footstep-based criteria	One of the successful strategies for recovering from push.	1. Computation of capture points may be difficult and obtaining a closed form solution might not be possible for a general humanoid robot. 2. Depends on an approximate model.

smooth transition of the ground reaction forces from the rear foot to the front one.[65] The most challenging problem is point 5 such that the COM velocity, beyond a certain point, is not controllable once the projection of the COM moves away from the support foot. The only possible strategy in this case is to take a step forward to maintain balance. Therefore, capture points have been proposed[33] to make a step for balance maintenance. A capture point is a point on the ground that can be stepped on by the biped robot in order to stop. Pratt and colleagues[33,66] extended the linear inverted pendulum to include a flywheel body at its COM to explicitly model angular momentum about the COM. A closed form solution of the capture region was devised. Their model is approximate, however, and its application to complex humanoid robots is questionable! Wight *et al.*[34] proposed a dynamic measure of balance to estimate the footstep to restore balance. The ZMP criteria can be augmented with this dynamic measure to broaden the stability conditions of the biped as it walks.

Comparison between the above mentioned stability criteria are described in Table I.

4. ZMP-Based Control

Human locomotion on level terrain with constant speed walking is controlled by a local pattern generator on the spinal cord without the use of brain commands. The control system of human walking in a cluttered environment is more complex because this may involve multiple local controllers and commands from the cerebellum.[3] Maintaining human balance while walking, standing, etc. is a complex process associated with different levels of the central nervous system.[67] In general, balance depends on the task characteristics and environment context as detailed in ref. [68]. The control system which is responsible for human walking can be described by hierarchical controls ranging from the highest level of action planning (motion planning) to the low-level control (reflex/local control).[3]

In robotics, multiple control loops have been used efficiently for generating feasible walking for biped mechanisms. As mentioned earlier, most ZMP-based humanoid robots need to use multi-level control to adapt to changes in the environment and internal perturbations. In contrast, the

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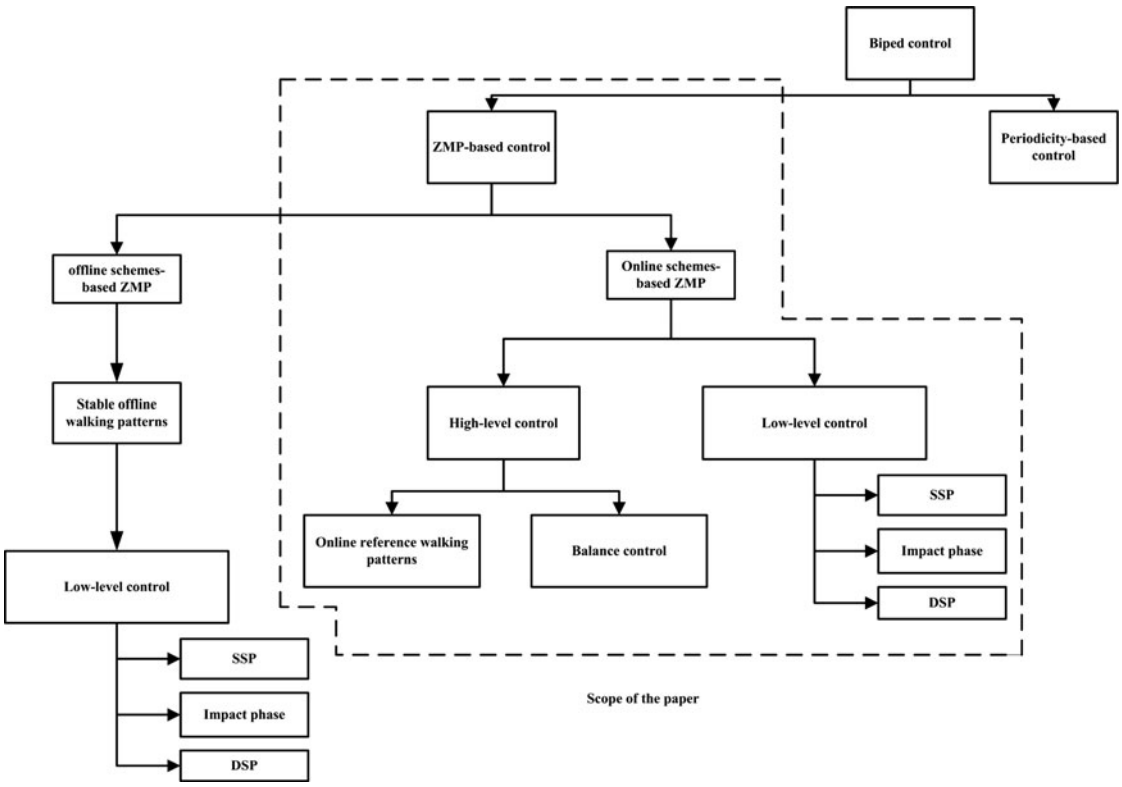


Fig. 3. General classification of multi-level control of biped mechanism according to its stability criterion.

periodicity-based biped does not afford robust walking and cannot stop or stand etc.; therefore, using a balance control is challenging. Figure 3 shows the classification of control levels of general biped mechanisms.

Remark 5. Offline walking patterns do not need multi-level controllers, because they have been prepared in advance to satisfy the dynamic/kinematic constraints.[65,69,70,102] It needs only low-level control to track the desired angular joint trajectories.

5. High-Level Control

Our paper suggests that the control systems of the biped mechanism consist of two levels: (1) high-level control which is responsible for motion planning and balance control, and (2) low-level control which can deal with the tracking of the desired angular joint trajectories.

5.1. Online reference walking patterns

Generating walking patterns for biped robots demands finding a relationship between COG and ZMP trajectories. Depending on the reference trajectories of the COG and the feet, all the motions of the joints can be deduced.[15] Numerous approaches have been used to generate the motion of biped robots but only three methods have been used successfully: optimization-based gait, COG-based gait and interpolation-based gait. The optimization tool can deal successfully with minimum energy or input control effort, optimal design and miscellaneous dynamic and kinematic constraints.[65,69,70,102] ZMP criteria can be seen as a constraint or an objective function depending on the designer's purpose. Because of the computational complexity of the optimization tool it can often be used offline. The COG-based gait adopts the inverted pendulum model as a simplified model for biped robots, assuming all masses are concentrated in the COG and there is a pushing force at the ankle joint of the support foot without applied torque.[71] This model has been suggested for online realization of walking patterns for the biped robot. To compensate for the error produced by this approximation and other unknown disturbances, modification of the walking patterns is necessary via a balance control strategy

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which will be described later. The so-called interpolation-based gait requires suitable polynomial or spline functions for the COG and the foot trajectory such that it can track the desired ZMP or just satisfy it[21,26,27,72–76] with offline/online compensated ZMP. The method of Takanishi *et al.*[76] can be categorized within interpolation-based gait because the authors used fast Fourier series to find a suitable relationship between the trunk motion and ZMP trajectory. Because our paper focuses on online stable biped walking, we will examine the COG-based gait in detail. Real-time motion planning is difficult to perform for the following reasons: (1) the proposed algorithm for the walking generator should be solved in one iteration, otherwise offline algorithms should be used, and (2) it is difficult to connect the predefined walking pattern smoothly online because of the discontinuous COG velocity in the transition instances.[77] The online walking pattern generators can be satisfied by tuning input parameters simultaneously as desired. These parameters could be sway amplitude, step time, stride, etc., which are prepared before the step starts and updated at the start of each step.[78] For more details about walking pattern generation, refer to refs. [23, 79–89].

Because of the complex nonlinear dynamics of the biped robot, it is difficult to obtain a closed form solution. Therefore, many researchers have focused their attention on simplifying the complex dynamics of biped robots in order to use simple algorithms for generating walking patterns and control. As mentioned earlier, the inverted pendulum model is used as an approximate model for the biped mechanism because of the similarity between the two mechanical systems in terms of both the static and the periodic stability. This approximation can be connected with the periodic stability of the inverted pendulum, however. Kajita and Toni[71,90] proposed the LIPM, exploiting previous related work.[13,91–97] The LIPM includes finding a simple mathematical relationship between the trajectory of the COG of the biped robot and the ZMP to ensure stable biped motion. It is assumed that all masses of the biped model are concentrated in its COG and there is a reaction force between the ground and the foot without ankle torque.[43,98,99] Thus, the relationship between the COG and ZMP during the SSP can be computed as follows:[15,100–102]

$$x_{\mathrm{ZMP}} = x_{\mathrm{COG}} - \frac{H}{g}\ddot{x}_{\mathrm{COG}}, \tag{12}$$

$$y_{\mathrm{ZMP}} = y_{\mathrm{COG}} - \frac{H}{g}\ddot{y}_{\mathrm{COG}}, \tag{13}$$

where $(x_{\mathrm{COG}}, y_{\mathrm{COG}}, z_{\mathrm{COG}})$ is the position of the COG, and H is the height of the COG which is assumed to be fixed.

Remark 6. Modelling of the biped robot during the DSP can be performed using different strategies: (1) the linear inverted pendulum model,[101] (2) the linear pendulum model,[100] and (3) suitable transition function connecting the end of the first SSP to the beginning of the second SSP; it could be linear function or the same equation of the SSP but with opposite sign as performed in ref. [15]. Consequently, the global behaviour of the bipeds could depend on pendulum model strategy during the complete gait cycle. Equivalence of these methods has been proved in ref. [102].

Remark 7. Certainly there are miscellaneous methods used for generating walking patterns of the biped as detailed in our previous paper;[23] however, we focus our study on COG-based walking pattern to capture the most important issues of balance control associated with biped locomotion.

Remark 8. In order to decouple sagittal and frontal motion, hip height needs to be constant and avoid singularity at the knee joint. But then the robot walks with bent knees. For further reading on stretch-legged walking, refer to refs. [28, 103–105].

The LIPM concept could be accepted for quadrupeds because of their slender legs. One-third of the mass of the human being is contained in the legs, however. Therefore, the use of the LIPM in bipeds is questionable.[16] Consequently, Park and Kim[43] suggested using the gravity-compensated inverted pendulum mode with two proposed masses to reduce the errors accompanied by the LIPM. One mass denotes the dynamics of the swing leg and the other represents the stance foot and the trunk. Other modifications to the LIPM have been proposed to avoid the deviation of the desired trajectories of ZMP. These modified approaches include the two masses inverted pendulum mode,[106]

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Table II. Detailed description of the methods and comments on online reference walking patterns.

Method	Description	Comments
Indirect ZMP control	• It uses the inverted pendulum to approximate the dynamics of the multi-body biped robot (see Fig. 4(a)). • The ZMP appears as the input of the system's plant. • The controller attempts to track the desired COG position /velocity. • Example of this controller can be seen in Fig. 5.	• The problem of the non-minimum phase property is inherent in the one-mass inverted pendulum (or the cart-table model).[111] It can be solved either using two-mass inverted pendulum[111] or pole-zero cancellation.[112] • It could be possible to use any linear control technique (PID controller, H_∞ control etc.) for control of inverted pendulum model (or the cart-table model) to track the desired ZMP or COG states.[109] However, preview control and its extensions[113,115–117] and linear predictive control[118–121] have been used extensively. • The ordinal control system cannot make good tracking for ZMP trajectory as shown in ref. [15]. Moreover, the disadvantages of the preview control mentioned above are: (1) it has difficulties in specifying the desired ZMP precisely at the heel or toe of the foot to realize human-like walking,[114] (2) it does not consider the constraints of the ZMP, and (3) it is sensitive to disturbance and noise. Therefore, linear predictive control has been proposed to deal with strong perturbations.[118–121] • Arbulu *et al.*[122] have suggested, without proof, that using the cart-table model can guarantee high walking speed with continuous dynamic response. • Large computational effort is often accompanied by most ZMP/COG tracking control methods.[15]
Direct ZMP control	• It uses the cart-table model to approximate the dynamics of the multi-body biped robot (see Fig.4 (b)). • The ZMP is the output of the system. • Example of this controller can be seen in Fig. 6.	
Analytical method	• Apart from the control theory, the researchers have solved the Eqs. (12) and (13) analytically using two successful techniques: (1) the two-point boundary value problem (BVP), and (2) the convergent and divergent components of the motion.[114] • The two-point boundary value problem can be solved by expressing the desired ZMP trajectory as a function (polynomial, Fourier series functions, etc.) and then determining the COG position and velocity to achieve the desired ZMP trajectory (please see refs. [77, 78, 124–129]). • Instead of the BVP, Takenake *et al.*[114] have proposed to decompose motions of the inverted pendulum into convergent and divergent components. Modification of the ZMP can be decreased by only controlling the divergent components.	• In general, the analytical method can be easy and less computational than other control methods mentioned above. • For detailed discussion of disadvantages of the BVP, see ref. [114].

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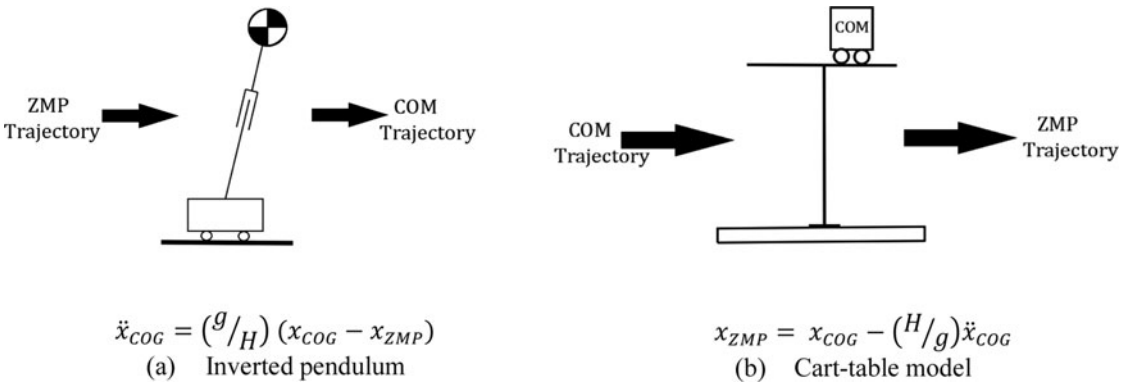


Fig. 4. The inverted pendulum model vs. the cart-table system.[122]

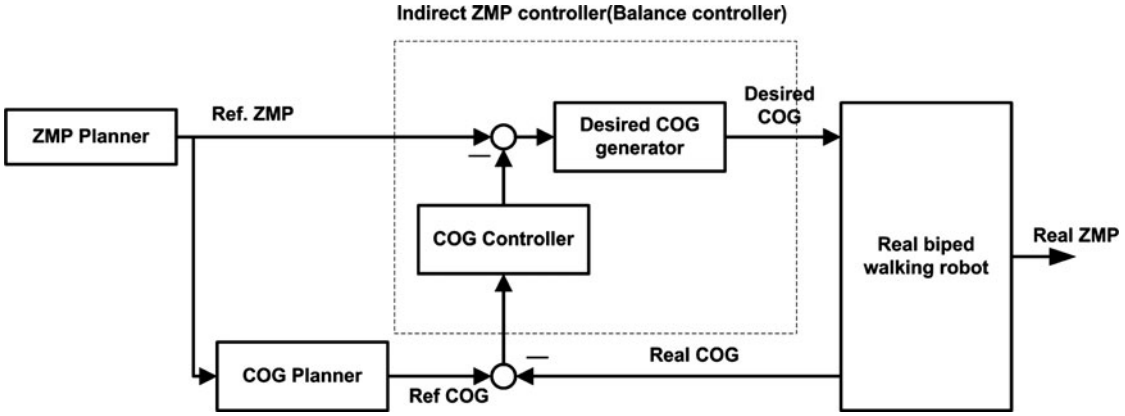


Fig. 5. Indirect control of ZMP-based biped robot for tracking of desired COG position.[110]

the multiple masses inverted pendulum mode,[107] and the virtual height inverted pendulum mode.[108] For a comprehensive comparison of the mentioned methods, refer to ref. [108]. Table II shows in details most of the methods used for online trajectory generation of the biped gait.

Remark 9. Assuming the ZMP trajectory during the SSP is fixed at some point of contact could have undesirable effects on biped motion. It can limit the performance of the humanoid robot; it cannot vary step length or walking period, etc. in the same way a human does. Therefore, linear transition of the ZMP is preferred for achieving some level of control of COM motions.[130,131] Vanderborght *et al.*[15] showed that the acceleration of variable ZMP is smaller than the fixed ZMP; however, the ZMP comes closer to the boundary of the support area. In effect, more work is needed to determine the optimal trajectory of ZMP with satisfied constraints.

5.2. Balance control

All legged creatures have amazing balance strategies to avoid falling suddenly and to maintain their overall rotational stability, dynamic stability and postural stability.[55] Humans can use a variety of complex strategies in order to maintain balance and avoid potential disturbances. The balance system proactively monitors the external environment and predicts the effect of forces generated by voluntary movements on the body, making the adjustments necessary to maintain posture and equilibrium. It is only when these adjustments fail or unexpected destabilization occurs that reactive balance response emerges.[68] Thus, two familiar mechanisms are used to achieve balance conditions, the reactive mechanism (feedback control) and the proactive mechanism (feedforward control).[67] Table III describes in details the four known strategies for balance control of biped locomotion.

Remark 10. Reactive response occurs after unexpected disturbances are applied to the human.[67] In such cases, the human may use three essential strategies to maintain balance according to the amount

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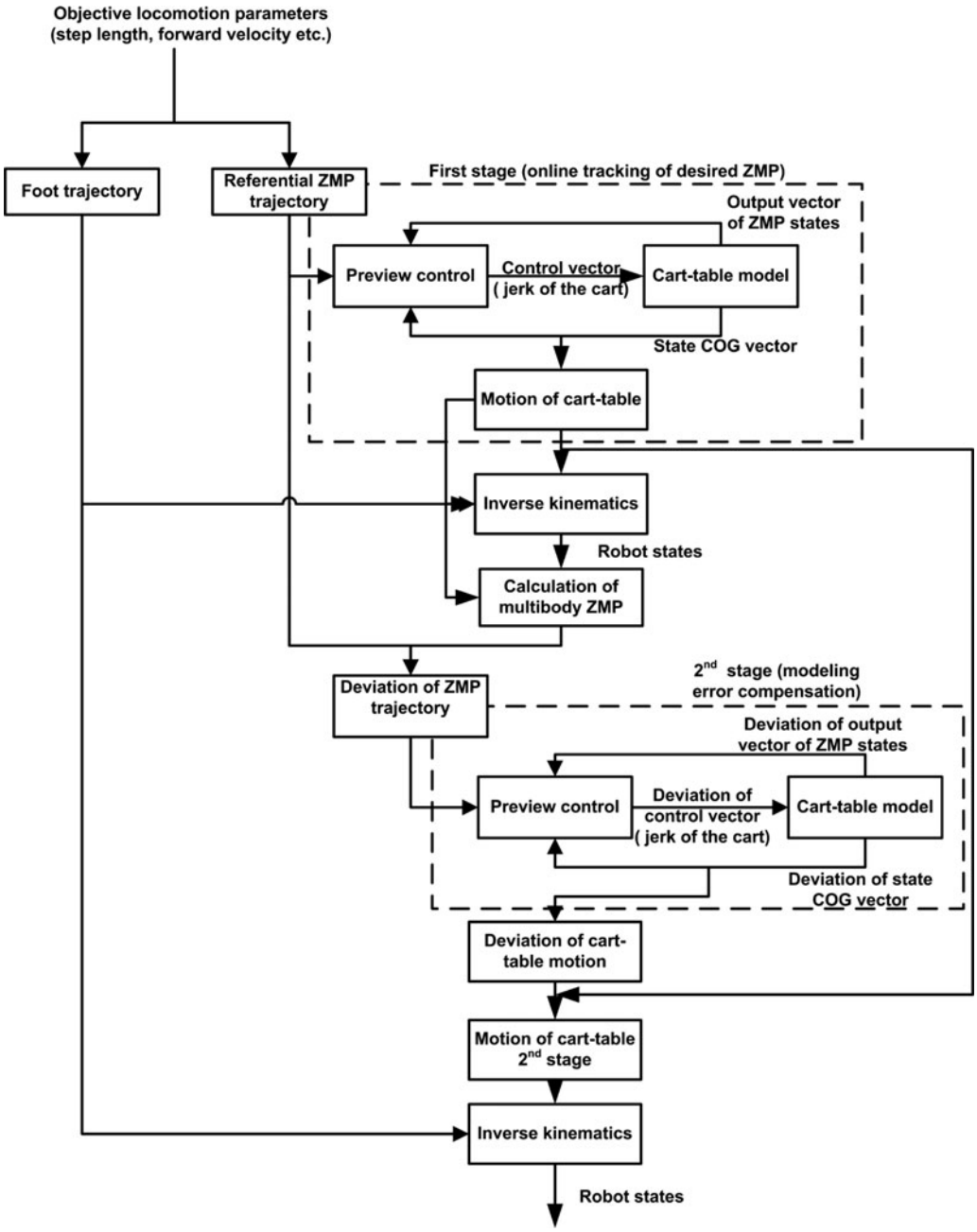


Fig. 6. The two stages of the preview controller for Lucy biped.[15] Stage one can track the desired predicted ZMP trajectory and the second stage compensates for the ZMP errors produced by the differences between the simplified model and the actual multi-body model.

of the disturbed forces applied to the body. For small disturbances, the impact is absorbed by the ankle joint; moving the COM of the human body forward or backward as a flexible inverted pendulum about the ankle joint.[67,130] This has motivated the use of an inverted pendulum with virtual spring and damper elements to excite the balance control using ankle strategy. When the impact is larger, the hip and knee joints are used such that the body is stiffened as a two-segment inverted pendulum. Whereas for severe and fast perturbations or for maintaining vertical trunk orientation, stepping the foot could be the best solution (see Fig. 7). For further details about these strategies refer to refs. [131–134].

Remark 11. As we see from Table III, the robotics community has discussed extensively the problem of the balance control of the biped robot, but in most cases the balance control strategies

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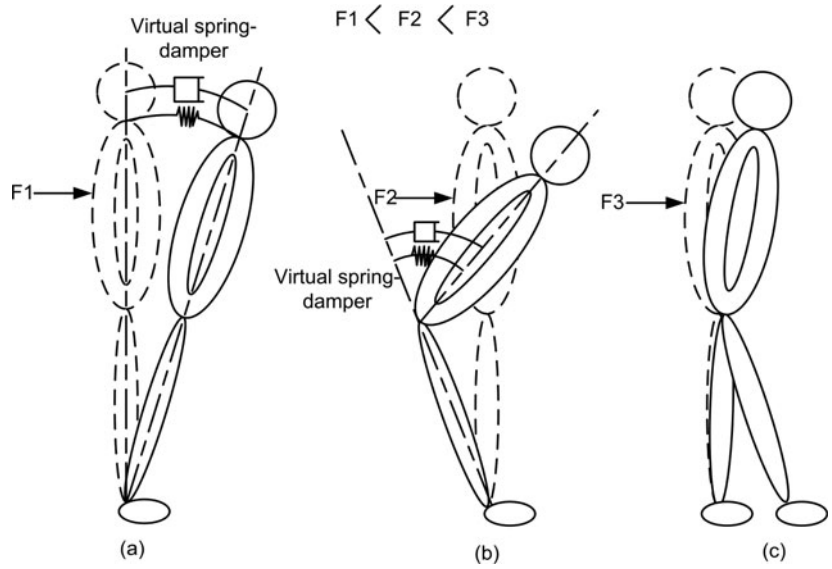


Fig. 7. The familiar balance strategies. (a) Ankle strategy with physical modelling (b) Hip strategy with physical modelling (c) Footstep strategy.[67,135–137]

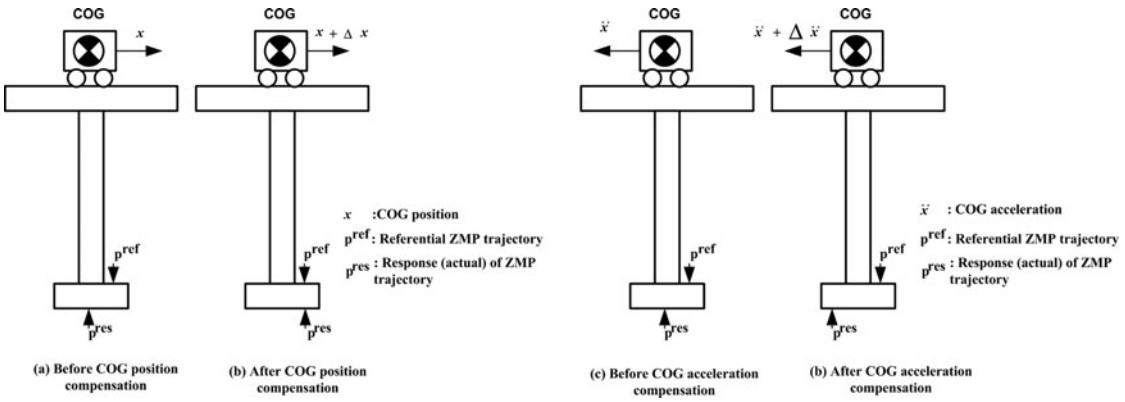


Fig. 8. COG-based compensation. (a) and (b) Before and after COG position compensation respectively (c) and (d) Before and after COG acceleration compensation, respectively.[142]

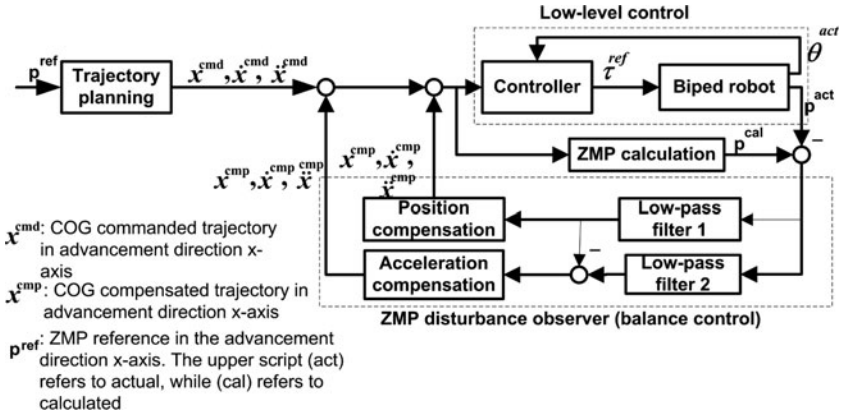


Fig. 9. ZMP disturbance observer.[142]

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Table III. Balance control and stabilization strategies of biped robots.

Method	Description	Comments
Ankle-based strategy (reactive mechanism)	• The idea is to make the ankle joint of the stance leg to behave as passive joint. Then the deviations of the desired ZMP could be small as possible.[138] • In general, the compensating ankle torque can be modelled as a rotational spring and damper at foot-ground contact (see Fig. 7 and refs. [78, 138]), or it can be computed from the force sensors located at biped feet (see ref. [139]).	• A compliant ankle joint makes the foot landing easy and gives sufficient contact to the ground.[208] However, adding flexibility to the ankle joint could complicate the required control system because of the possible resonant oscillations (successful control architecture for flexible ankle joint based on ZMP criterion can be found in ref. [141]). • Due to the inherent instability of the biped and other external disturbances, the ankle compensation control could not be enough; therefore, ankle-knee compensation control ref. [137], ankle-hip compensation control[135,136,140] strategies have been proposed.
Hip-based strategy (reactive mechanism)	• This strategy is more robust than the ankle strategy (see Fig. 7). • One possible solution to compensate for the ZMP deviations is modifying the orientation of the large-mass trunk while the lower limbs have a predefine trajectory (see the work of Takanishi and his colleagues).[35,76,107] • To avoid the high energy consumption of the large trunk, COG position and acceleration compensation methods have been proposed (see Figs. 8 and 9). The idea is to modify the position or the acceleration or both of them to compensate for ZMP errors.	• The heavy trunk strategy may lead to high energy consumption with possible difficulties of control of the upper limb.[27] • It has been shown that the COG position compensation method can be applied within low frequency value whereas COG acceleration-based compensation method can work with a high frequency range.[142] • It should be noted that the mentioned COG-based compensation methods do not incorporate control of the ground reaction forces to avoid slipping (see Fig. 10 which incorporates the effect of ground reaction forces).
Whole-body strategy (reactive mechanism)	• This strategy is based on modifying the whole body of the biped (legs, torso, arms and head). It is more robust than the hip-based compensation method. • There are different methods used in the literature for the whole body modification as follows: resolved angular momentum[62] (see Fig. 11), the angular momentum inducing inverted pendulum mode (AMPM),[101,130] Jacobi-based technique,[145–147] filter dynamics,[148–150] passivity theory.[40,152–155]	• The position-based control methods (resolved angular momentum, AMPM, Jacobi technique and filter dynamics) cannot quickly adapt to sudden environment changes, especially unknown external forces.[49] In addition, a position-based compliance control requires force measurements at every expected contact point, which increases the computational load and produces unavoidable time delays[41] (please see refs. [40, 41] for more details). • The ground reaction forces are the key elements through which new control strategies are proposed to provide the level of compliance, adaptation and dynamic stability required for walking in different contexts.[151] Therefore, modification of the biped gait for stabilization purposes combined with control of ground reaction forces is important (see e.g., passivity theory,[40] hybrid position-force control).[151]

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Table III. Continued.

Method	Description	Comments
Footstep strategy (reactive mechanism)	• If the control of COG velocity is impossible, the only possible way to recover balance is take a step forward. • For details on the characteristics of this strategy, see Section 3.3.	• A large external force cannot be easily suppressed by any state feedback, especially if applied statically for a long time, because of the horizontal motion of the trunk. On the other hand, it is difficult for the swing leg to execute a large motion just before landing. Furthermore, if an extra step is used to compensate for a small external force without using any feedback suppression, the robot moves throughout the step and consumes a lot of energy. Therefore, it is logical to distribute the action each way to adequately compensate for the disturbance, according to its frequency or the situation of the support phase.[128]
Predictive control (proactive mechanism)	• The locomotion of the human being on cluttered environments without prescribed reference trajectory in advance motivates the researchers to use the predictive control for biped locomotion.[156] • The model predictive control is a branch of optimal control that needs objective function for the walking patterns, e.g. control effort, to satisfy given constraints, e.g. hip orientation, knee locking, foot trajectory, etc., as shown in refs. [118, 119, 157–161].	• Two problems have been noticed with the application of Trajectory Free -Model Predictive Control (TF-MPC) in real time, which are the computational complexity and the size of the horizon; see refs. [162, 119–121] for solutions of computational complexity. • The MPC is sensitive to any uncertainty in biped model parameters or external disturbances; therefore, the disturbance observer can be incorporated with it as made in refs. [163, 164]. However, the computational complexity in refs. [163, 164] was not considered and therefore its real-time application is questionable.

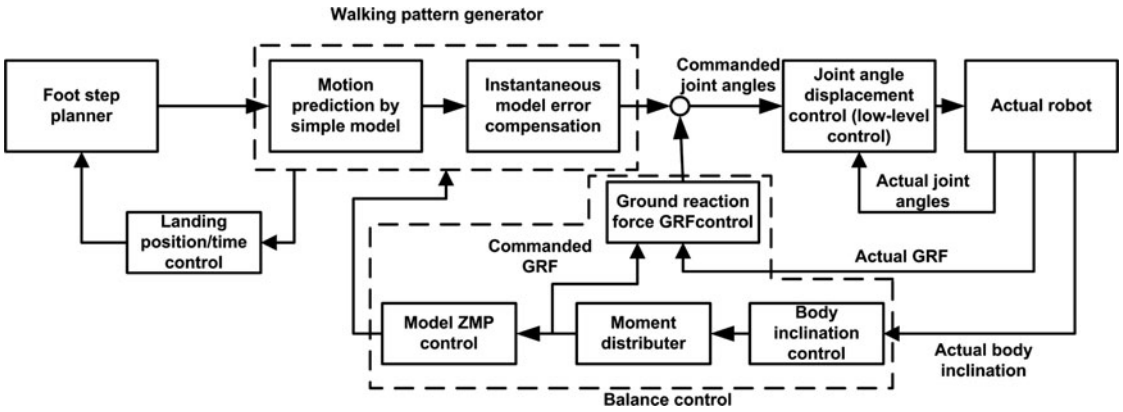


Fig. 10. Multi-level control of ASIMO humanoid robot.[143,144]

(ankle, hip strategies etc.) mentioned above are dependently studied without integration of them. However, some of these balance strategies have been integrated as made in refs. [128, 129, 135–137, 140].

Remark 12. Optimal predictive control combines two balance strategies: a reactive mechanism and a proactive mechanism. In addition, it can directly determine the desired input controls without reference

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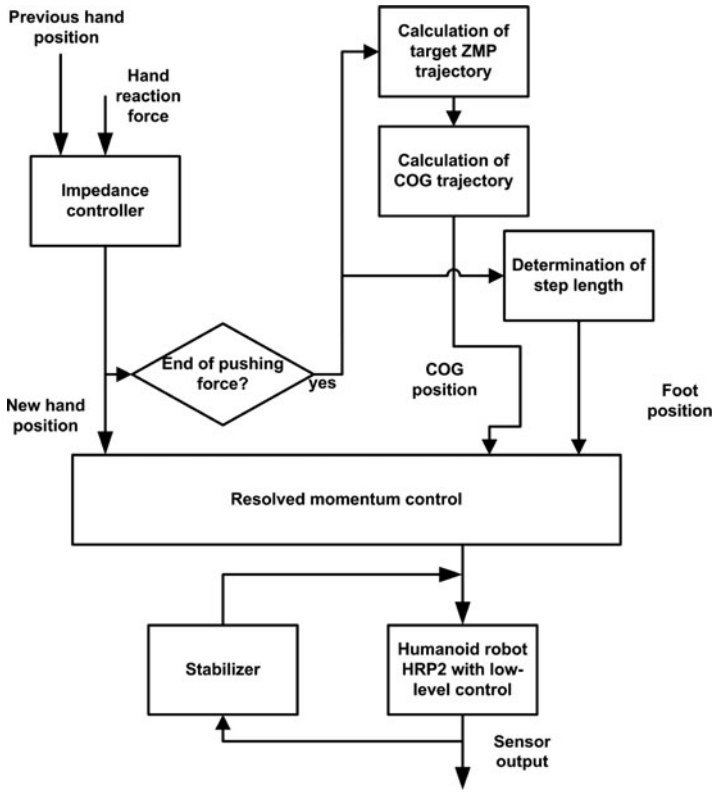


Fig. 11. Multi-level control of HRP2 humanoid robot.[77]

trajectories, incorporating low-level control implicitly. Therefore, it could be a promising strategy for control and balance of biped robots taking the computational complexity into consideration.

Remark 13. Intelligent control techniques have attracted the attention of many researchers wishing to generate and stabilize biped walking patterns. Katic and Vukobratovic[165] performed a useful survey about these useful techniques for controlling biped humanoid robots.

The results presented in Section 5 can be summarized as follows:

1. If the biped robot moves without ZMP feedback, any perturbation could lead to imbalance.[166]
2. The essential idea is to keep the ZMP inside support polygon. In order to control it there are several possibilities: (1) change the ankle torque directly, (2) change the hip motion or flap with arms around (like we do when balancing on a cord), (3) combination of points (1) and (2) which is called whole body control. (4) footstep strategy (If the ZMP tends to go outside support polygon, the above strategies could be insufficient anymore; so, we need to make a step to recover).
3. It has been observed that it is possible for a rotation of the whole body of biped mechanism while walking with zero spin angular momentum, or having non-zero balancing angular momentum while walking stably. With these observations, the assumptions of the authors of refs. [53, 54, 58–61] to balance biped mechanism by tracking zero angular momentum is questionable; instable biped locomotion could be come from that assumption. On the other hand, tracking specific angular momentum as made in refs. [62, 63] could be acceptable and accompanied by difficulties in selection of this non-zero value. Therefore, varying step length of biped walking (footstep) or walking speed could be safe strategy (in some circumstances) for stabilization and balance purposes.[167]
4. The footstep technique can make instability problems if the biped must move on certain locations (e.g. stepping on stones).[166]
5. In some situations, controlling the ZMP trajectory is not sufficient to maintain dynamic stability of the biped mechanism such as moving on smooth surfaces. In that case, it is important to control

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Table IV. Low-level control during different walking phases of the biped locomotion.

Walking phase	Description	Comments
The SSP	• During the SSP, the biped robot behaves as an open-chain mechanism; therefore, the Lagrangian (or Newtonian Euler[171,206]) dynamic equation can be written as $\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{g} = \boldsymbol{A}\boldsymbol{\tau}$, (14) where $\boldsymbol{M} \in \mathbb{R}^{n_q \times n_q}$ is the mass robot matrix, $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}} \in \mathbb{R}^{n_q}$ are the absolute angular displacement, velocity and acceleration of the robot links, $\boldsymbol{C} \in \mathbb{R}^{n_q \times n_q}$ represents the Coriolis and centripetal robot matrix, $\boldsymbol{g} \in \mathbb{R}^{n_q}$ is the gravity vector, $\boldsymbol{A} \in \mathbb{R}^{n_q \times n_\tau}$ is a mapping matrix derived by the principle of the virtual work,[214,215] $\boldsymbol{\tau} \in \mathbb{R}^{n_\tau}$ is the actuating torque vector and n_τ represents the number of actuators. • Most ZMP-based biped robots are fully actuated during the SSP, therefore, all classical control methods used for robotic manipulators can be used successfully (see ref. [168]). • The control methods used in the literature are: linearization,[16,90] PID family control,[19,90] computed torque control,[19] sliding mode control and its extensions,[19,173–175,176–178] learning control.[179–181]	• The disadvantage of the linearization-based control is that the solution is valid only in small regions around the operating points (nominal trajectory).[216] • During motion of any walking robot, a number of sudden geometric constraints are imposed, e.g. stepping on the ground, knee locking, etc. These inherent constraints give rise to impulse-like disturbances that make control by a standard PID controller extremely difficult[19] (if the uncertainty level is very high (more than 80%) it may not be possible to maintain stable gait with the usual PID). • Most researchers avoid using adaptive control in complex biped mechanisms because the classical adaptive control could depend on a regressor matrix which is applicable to 6-DOFs or fewer. The virtual decomposition control (VDC) proposed by Wen-Hong Zhu[20] is a promising solution to this problem; it virtually decomposes every link of the complex system and applies a similar classical adaptive control to each link subsystem.[128]
The impact phase (initial contact)	• During the impact phase, the robot configuration remains unchanged with abrupt change in the joint velocity. The dynamic equation can be written as follows: $\boldsymbol{M}(\boldsymbol{q})(\dot{\boldsymbol{q}}^+ - \dot{\boldsymbol{q}}^-) = \boldsymbol{J}^T \int_{t_o}^{t_o+\Delta t} \boldsymbol{\lambda} dt$, (15) $\boldsymbol{J}^T(\dot{\boldsymbol{q}}^+ - \dot{\boldsymbol{q}}^-) = 0$, (16) where $\dot{\boldsymbol{q}}^+$ and $\dot{\boldsymbol{q}}^-$ refer to the angular velocity vector of the biped mechanism after and before impact respectively, $\boldsymbol{J} \in \mathbb{R}^{n_\lambda \times n_q}$ represents the Jacobian matrix resulting from the constrained motion of the biped robot, $\boldsymbol{\lambda} \in \mathbb{R}^{n_\lambda}$ is the constrained force/moment vector at the foot-ground interaction with dimension of n_λ, and t is the time. • The human has amazing compliance control to attenuate the impact effects; her/his muscles are relaxed to absorb the impact/contact just before her/his landing foot contacts the ground while her/his muscles are hardened to maintain balance after landing.[183] • The mechanisms are usually designed so that the impact forces are reduced by minimizing the impact velocity and the mass of the impacting bodies, and by suitably designing the mechanism so that a minimum stiffness is located in a neighbourhood of the point of impact, but these designs seem to be more useful for reducing the impact effects than precise control.[187]	• If the impact dynamics is not properly modelled and controlled, the impact forces could result in poor system performance and instability. • The difficulties of controlling systems subject to impact include: (1) the dynamics is different during system transition from a non-contact state to a contact state, (2) measuring the impact force, which depends on the geometry of the robot, (3) geometry of the environment and (4) type of contact.[186] • One possible solution to avoid impact is generation of reference trajectory such that the swing foot has zero impact velocity at the impact instance, avoiding impact phenomena.[195–197] However, this strategy and the rigid approach may need high power actuators whereas the compliant contact model can alleviate the energy consumption resulting from impact. Therefore it is used for simulating most designed humanoid biped robots such as Saika-3,[192] Johnnie,[193] and OpenHRP.[194]

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Table IV. Continued.

Walking phase	Description	Comments
	• Modelling of feet-ground interaction can be divided into two main groups: (1) rigid approach (constraint-based approach/non-smooth dynamics formulation),[188] and (2) complaint contact (spring-damper) model (penalty-based approach/regularized approach).[148,189,188] • In the rigid approach, the colliding bodies are hard enough to resist deformation.[190] The equation of system motion is complemented by constraint equations associated with the foot position;[191] the impact event (if it exists) should be modelled as a discontinuous event complicating the control solutions (see refs. [188, 189] for investigating disadvantages of this method). • The complaint contact model (second approach) uses spring-damper elements between the feet and the ground for modelling contact.[191]	• The main disadvantages associated with the complaint contact model are parameter tuning and multiple unilateral contacts;[189] efficient and precise simulation of collisions and contacts still remains an open research issue.[148] Other problem is the rubber foot sole of many bipeds that makes the robot turn slightly to the front so that the foot touches earlier than expected. • Impedance control can be powerful strategy to control biped robot with compliance (see refs. [198, 172, 184, 185]).
The DSP	• During the DSP, the biped constitutes a closed chain mechanism with redundant co-ordinates. Therefore, the following Lagrange formulation of the 1nd kind, which can deals with constraints, is needed for dynamic modelling of the constrained biped. $\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{g} = \boldsymbol{A}\boldsymbol{\tau} + \boldsymbol{J}^T\boldsymbol{\lambda}$, (17) $\boldsymbol{\phi}(\boldsymbol{q}) = \boldsymbol{0}$, (18) where $\boldsymbol{\phi}(.)$ represents the constraint equation, with the same notations mentioned earlier. • Equations (17) and (18) can be solved using two well-known techniques:[203,204] (1) the redundant coordinates-based techniques which are used mainly in commercial software such as MSC ADAMS, and (2) the minimum coordinates-based techniques which could be, to some extent, suitable for control strategies and real-time applications.	• If the robotic mechanism contacts with environment, it will lose some DoFs. Consequently, the generalized coordinates of the target robot could be larger than its DOFs due to its constrained motion. • Many researchers have preferred the minimum coordinates-based technique due to its simplicity and ease of derivation at the expense of difficulties of numerical methods encountered in the solution.[203] Consequently, this motivates the researchers to investigate the second technique which includes eliminating the constraint equations (Lagrange multipliers) from Eq. (18) to result in constraint-free differential equations.[203] This can be implemented using one of orthogonalization methods which are:[204] coordinate partitioning method, zero-eigenvalue method, singular value decomposition (SVD), QR decomposition, Udwadia-Kabala formulation, PUTD method, and Schur decomposition. • There are several approaches proposed for control of robotic manipulators with constrained motion; for details, refer to ref. [205]. • To ensure continuous dynamic response, some researchers proposed a linear shift function for the distribution of the ground reaction forces such that there is a gradual shift of the body from the rear foot to the front foot.[37,66,200,201,206]

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the ground reaction wrench (forces and moments) as well as ZMP trajectory together to ensure stable and non-sliding motion.[143,144,166]
6. Optimal predictive control considering computational complexity can be used as global approach for stabilization of biped robots.

6. Low-Level Control

Tracking the desired angular joint trajectories of the biped mechanism is the task of the low-level control. As mentioned earlier, the notion of the ZMP criterion assumes a fully actuated biped robot during the SSP to track the desired trajectory of the ZMP or satisfy it and over-actuated biped during the DSP. Consequently, the stance foot of the biped robot is assumed to be fixed during the SSP and the conventional control techniques used for controlling manipulators can be used successfully. In effect, most of the control methods investigated in the excellent work by the authors of refs. [168–170] can be used sufficiently to control ZMP-based gait. The designer should, however, consider the high DOFs inherent in the biped humanoid robot. Therefore, most humanoid robots have simple local PID controllers for tracking purposes.[23] Below we investigate the control techniques used in biped robots according to their walking phases. Table IV shows with details the control systems used during the walking phases of biped robots.

The results presented in Section 6 can be summarized as follows:

1. Although most researchers have used local controller (decentralized controller) such as PID family for controlling complex dynamic systems, the problem is that the independent joint control considers the coupling effects of the other bodies or links as disturbances, which is unrealistic.[20] Dallali *et al.*[202] have made a comparison between multivariable and decentralized control strategies for 10-DOF humanoid robot. It was shown that the multivariable linear quadratic regulator (LQR), which takes the entire system dynamics into account, has better robustness and energy efficiency.
2. There are two interesting control techniques dealing with uncertain plant: robust control and adaptive control. The robust control law tries to stabilize the target system within assumed bounds; therefore, it may lead to saturation problem. Whereas, adaptive control tries to estimate the unknown parameters to deal with uncertainty problem. Applying these techniques to complex robotic system such as biped robot is difficult due to inherently high DOFs. So, recursive techniques should be used to resolve this dilemma. A promising control system is the virtual decomposition control (VDC) based on adaptive technique which deals with every subsystem alone; it can be used as local controllers considering the full dynamics of the complex robotic system.
3. Most humanoid robots prefer using compliant model of foot-ground interaction to solve the problem of over-actuation during the DSP and to exploit it in balance control.

7. Conclusions

From the many miscellaneous methods of modelling and controlling biped robots, we have attempted to collect the multi-level control strategies used for ZMP-based biped/humanoid robots and introduce them systematically. The designer should take into account the computational complexity of the target control schemes owed to the large DOFs of the target biped/humanoid robot. The efficacy of ASIMO humanoid robots may motivate researchers to investigate the control architecture of Fig. 10 in detail. It is clear that ASIMO designers depend on approximate models for walking generators and balance control combined with force/moment distributors and control. In fact, a thorough comparative study is needed to compare the familiar control architectures to determine the best control system for designing biped robots.

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