100.39 An olympiad mathematical problem, proof without words and generalisation

The following problem occurred in a recent Spanish Mathematical Olympiad [1]:

Let a, b be two positive numbers with $a + b = 1$. Prove that

$$\left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right) \geqslant 9.$$

Of course, the problem may be solved in several ways. We give below a proof using a diagram and no words.

Proof: We note that the given inequality may be rewritten as $(1 + a)(1 + b) \geqslant 9ab$. The truth of this inequality is apparent from the following diagram.

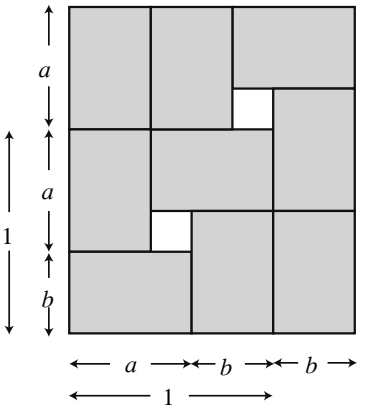


FIGURE 1

This proof is related to that given by Roger B. Nelsen [2], which also appears in [3, p. 62].

If we drop the condition that $a + b = 1$, Figure 1 may also be seen as a proof that $(2a + b)(2b + a) \geqslant 9ab$. Indeed it demonstrates that

$$(2a + b)(2b + a) = 9ab + 2(a - b)^2.$$

It is straightforward to check that this identity may be generalised as follows.

Theorem: Let a, b, m and n be positive real numbers. Then

$$(ma + nb)(mb + na) = (m + n)^2 ab + mn(a - b)^2.$$

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Corollary: Let a, b, m and n be positive real numbers. Then

$$(ma + nb)(mb + na) \geqslant (m + n)^2 ab.$$

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References

1. Problem 3, LII Olimpiada Matemática Española, Melilla, 18, January, 2016.
2. Roger B. Nelsen, Proof without words: the sum of a positive number and its reciprocal is at least two (four proofs), *Math. Magazine*, **67**(5) (1994) p. 374.
3. Roger B. Nelsen, Proof without words: exercises in visual thinking, *The Mathematical Association of America* (1993).

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100.40 The *Mathematical Gazette*, 100 not out

This Note is the final one in the final issue of Volume 100. It seems appropriate to mark this achievement in some way. So the Association has completed 100 volumes but, in reality, there have been 101 volumes! How can that be? Surely mathematicians can at least count to 100. The explanation is that the first six issues had a larger page size and that set of six comprises Volume 0.

Be that as it may, the first issue of *The Mathematical Gazette* appeared in April 1894. It consisted of eight pages and was priced 6d (= $2\frac{1}{2}$ pence). The first item was 'On the Eccentric Circle of Boscovitch', written by the Editor, E. M. Langley. Other items in this issue were: 'Greek Geometers before Euclid'; and 'Approximations and Reductions'. None of these were finished in Issue 1; cliff-hanging was the name of the game. The *Gazette* has now reached Issue 549. I shall not describe them all! The rest of the issues in Volume 0 were published in July and December 1894 and then in February, May and October 1895 and totalled 60 pages. Volume 1, with the same sort of page size that we now have, was made up of 18 issues which appeared at intervals throughout 1896, 1897, 1898, 1899 and 1900. The 18 issues contained a very respectable 422 pages. It must have been quite a challenge for the team to decide what to do and when to do it! I wonder if they ever considered the *Gazette* would still be going strong 100 volumes later! The table below summarises the publication of the *Gazette*. In case this table is unclear, it is in blocks of 4 rows in which are volume numbers (in bold), the years, the issues (in italics) and then the number of pages in each volume.

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