J. Plasma Physics (2012), vol. 78, part 6, pp. 601–606. © Cambridge University Press 2012
doi:10.1017/S0022377812000384


# Non-planar Gardner double layers in two-ion-temperature dusty plasma

M. ASADUZZAMAN and A. A. MAMUN

Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh
(asad_phys@yahoo.com)

(Received 17 November 2011; revised 24 March 2012; accepted 26 March 2012; first published online 1 May 2012)

**Abstract.** Non-planar (cylindrical and spherical) double layers (DLs) in two-ion-temperature dusty plasma, whose constituents are inertial negative dust, ions with two distinctive temperatures, and Boltzmann electrons, are studied by employing the reductive perturbation method. The modified Gardner equation describing the nonlinear propagation of dust-acoustic (DA) waves is derived, and its non-planar double layer solutions are analyzed numerically. The parametric regimes for the existence of DA DLs, which are found to be associated with positive potential only, are obtained. The basic features of non-planar DA DLs, which are found to be different from planar ones, are also identified. The implications of our results to different space and laboratory dusty plasma situations are discussed.

## 1. Introduction

The nonlinear structures (viz. solitary, shock, double layers (DLs), etc.) associated with nonlinear propagation of novel dust-acoustic (DA) waves (Rao et al. 1990), whose linear propagation are studied extensively from both theoretical and experimental points of view during the last two decades, has received an enormous interest for understanding electrostatic perturbations in space and laboratory dusty plasma (D'Angelo 1995, Thompson et al. 1997, Verheest 2000, Shukla and Mamun 2002, Merlino and Goree 2004, Shukla and Eliasson 2009).

Rao et al. (1990) in their seminal work have also studied the small amplitude DA solitary waves using the reductive perturbation method (Washimi and Taniuti 1966). Mamun et al. (1996) and Mamun (1999) have then generalized the work of Rao et al. (1990) by employing the pseudo-potential approach (Mamun 1999) to study arbitrary amplitude solitary waves. The nonlinear DA waves have also been rigorously investigated by many authors for different dusty plasma situations theoretically (Shukla and Mamun 2001, Eliasson and Shukla 2004, Moslem 2005, Mamun and Shukla 2009, Tribeche and Merriche 2011) as well as experimentally (Bandyopadhyay et al. 2008, Heinrich et al. 2009). All these above works on nonlinear DA waves are based on the most commonly used dusty plasma model that assumes negatively charged dust. The consideration of negatively charged dust is due to the fact that in low-temperature laboratory plasmas, collection of plasma particles (viz. electrons and ions) is the only important charging process, and the thermal speeds of electrons far exceeds that of ions. The dust grains are usually negatively charged by the absorption of electrons (Rosenberg et al. 1999, Shukla 2000) on to the surface of the dust grain. It has been found that the micron-sized dust particles suspended in a gaseous plasma background of temperature of few electronvolts can be charged up to $10^4 e$.

The theoretical investigations of DLs (which is achieved by bipolar charge distribution) have been made by a number of authors considering unmagnetized dusty plasma models (Bharuthram and Shukla 1992, Tagare 1997, Xie et al. 1999, El-Labany and El-Taibany 2003, Hellberg et al. 2005, Mamun et al. 2009, Alinejad and Mamun 2010) as well as DA solitary waves (SWs). Xie et al. (1999) have pointed out the DA double layer waves that comprise the Boltzmann-distributed two-temperature ions, Boltzmann electrons, and a negatively charged dust fluid. The existence of DLs depends on the density ratio of lower (cool) to higher (hot) positive ions. The work of Xie et al. (1999) is valid for planar geometry and they found both positive and negative potential solitons. Earlier, Tagare (1997) studied the basic features of DA double layer with cold dust particles and isothermal two-temperature ions. Analytical approach was taken by him and both compressive and rarefactive solitons and DLs were also found. He considered significant depletion of electron number density during the charging of dust grains. However, in practice, the electrons can not be sufficiently depleted. The investigation of Tagare (1997) is valid for a planar geometry. EI-Labany and EI-Taibany (2003) investigated the DLs considering the negatively charged dust with arbitrary streaming ion beam. Later, EI-Labany et al. (2004) have studied the DLs with two-temperature-trapped ions. Hellberg et al. (2005) have found the existence regions for DLs using the Sagdeev pseudopotential approach, where both lower (cool) and higher (hot) ions are subsonic. Bharuthram and Shukla (1992) found rarefactive ($\phi_m > 0$) DLs comprising two-temperature

https://doi.org/10.1017/S0022377812000384 Published online by Cambridge University Press

ions, Boltzmann electrons, and a negatively charged cold dust fluid. Recently, Mamun et al. (2009), and Alinejad and Mamun (2010) have studied unmagnetized DLs in a dusty electronegative plasma and DLs with flat-topped electron distribution, respectively. All these works are valid for a planar geometry.

Experimental aspects of existing DLs have been drawn by a number of authors. For example, Hershkowitz (1985) reviewed laboratory experiments on DLs. Small-amplitude DLs were observed in the auroral plasma (Temerin et al. 1982) by the association of electron and ion velocity distributions between the altitudes of 6000 km and 8000 km, which indicates the presence of electric fields parallel to the magnatic field. Particles' acceleration in the auroral zone may have happened due to DLs. Solitary waves and weak DLs have been observed by the Viking satellite (Koskinen et al. 1990). Singh (2000), after a simulation study, has shown that the electron holes observed by the FAST satellite (Ergun et al. 1998) are a feature of strong DLs. Newman et al. (2002) have observed in the downward current the region of the auroral zone with basic plasma modes by the FAST satellite, which made high-resolution measurements. Moreover, fine structure of auroral kilometric radiation may be explained by stationary DLs. The presence of DLs in the auroral and magneto-spheric plasma (Temerin et al. 1982, Boström et al. 1988) are also found by the satellite S3-3.

However, all these theoretical studies are limited to one-dimensional (1D) planar geometry, which may not be the realistic situation in space and laboratory devices, since the waves observed in space (laboratory devices) are certainly not infinite (unbounded) in one dimension. As most of these works of DLs are concerned with planar geometry, here our concern is to add non-planar geometry, which may agree with the experimental existence of DLs. In our model, we derive the modified Gardner (mG) equation. To the best of our knowledge, no attempt has been made on Gardner DLs in a non-planar dusty plasma with two-ion-temperature. Therefore, in our present work, we consider a dusty plasma system (consisting of negatively charged dust fluid, Boltzmann electrons and ions of two distinctive temperatures) and a more general geometry (which is valid for planar, cylindrical and spherical geometries), and theoretically study the basic features of the DA DLs that are found to exist in such a realistic novel dusty plasma system.

The paper is organized as follows. The basic equations governing the dynamics of DA waves are presented in Sec. 2. The mG equation is derived in Sec. 3. The numerical analysis of the mG equation along with a brief discussion is presented in Sec. 4.

## 2. Governing equations

We consider the nonlinear propagation of DA waves in an unmagnetized non-planar dusty plasma system containing negatively charged dust fluid, Boltzmann electrons, and ions of two different temperatures. Thus, at equilibrium we have $n_{i10}+n_{i20} = n_{e0}+Z_d n_{d0}$, where $n_{i10}$ ($n_{i20}$) is the density of unperturbed lower (higher) temperature ion and $n_{e0}$ ($n_{d0}$) is the density of unperturbed electron (dust). The nonlinear dynamics of DA waves, whose phase speed is much smaller than the electron and ion thermal speed in a non-planar geometry is governed by

$$\frac{\partial n_d}{\partial t} + \frac{1}{r^v}\frac{\partial}{\partial r}(r^v n_d u_d) = 0, \tag{2.1}$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial r} = \frac{\partial \phi}{\partial r}, \tag{2.2}$$

$$\frac{1}{r^v}\frac{\partial}{\partial r}\left(r^v\frac{\partial \phi}{\partial r}\right) = -\rho, \tag{2.3}$$

$$\rho = -(\alpha+\mu_{i2}-1)e^{\sigma_1\phi} + \alpha e^{-\phi} + \mu_{i2}e^{-\sigma_2\phi} - n_d, \tag{2.4}$$

where $v = 0$ for 1D planar geometry, and $v = 1$ (2) for a non-planar cylindrical (spherical) geometry; $n_d$ is the dust particle number density normalized by its equilibrium value $n_{d0}$; $u_d$ is the dust fluid speed normalized by $C_d = (Z_d k_B T_{i1}/m_d)^{1/2}$; $\phi$ is the electrostatic wave potential normalized by $k_B T_{i1}/e$; $\rho$ is the net charge density (charge per unit volume); and $\sigma_1 = T_{i1}/T_e$, $\sigma_2 = T_{i1}/T_{i2}$, $\alpha = n_{i10}/Z_d n_{d0}$, $\mu_{i2} = n_{i20}/Z_d n_{d0}$, and $\mu_e = n_{e0}/Z_d n_{d0} = (\alpha+\mu_{i2}-1)$. $Z_d$ is the number of electrons residing on the dust grain surface. It should be mentioned that $T_{i1}$ is the lower ion temperature, $T_{i2}$ is the higher ion temperature, $T_e$ is the electron temperature, $k_B$ is the Boltzmann constant, and $e$ is the magnitude of electron charge. The time variable $t$ is normalized by units of the negative dusty plasma period, $\omega_{pd}^{-1} = (m_d/4\pi e^2 Z_d^2 n_{do})^{1/2}$, and the space variable is normalized by the Debye radius, $\lambda_{Dm} = (k_B T_{i1}/4\pi e^2 Z_d n_{do})^{1/2}$.

## 3. Derivation of mG equation

To study finite amplitude DA DLs by the reductive perturbation method (Washimi and Taniuti 1966, Maxon and Viecelli 1974), we first introduce the stretched coordinates:

$$\zeta = \epsilon(r - V_p t), \tag{3.1}$$

$$\tau = \epsilon^3 t, \tag{3.2}$$

where $\epsilon$ is a small parameter measuring the weakness of dispersion, and $V_p$ (normalized by $C_d$) is the phase speed of the perturbation mode, and expand all the dependent variables (viz. $n_d$, $u_d$, $\phi$, and $\rho$) in power series of $\epsilon$:

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \epsilon^3 n_d^{(3)} + \cdots, \tag{3.3}$$

$$u_d = 0 + \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \epsilon^3 u_d^{(3)} + \cdots, \tag{3.4}$$

$$\phi = 0 + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots, \tag{3.5}$$

$$\rho = 0 + \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \epsilon^3 \rho^{(3)} + \cdots. \tag{3.6}$$

Now, expressing (2.1)–(2.4) in terms of $\zeta$ and $\tau$, and substituting (3.3)–(3.6) into the resulting equations

https://doi.org/10.1017/S0022377812000384 Published online by Cambridge University Press

((2.1)–(2.4) expressed in terms of $\zeta$ and $\tau$), one can easily develop different sets of equations in various powers of $\epsilon$. To the lowest order in $\epsilon$ one obtains

$$u_d^{(1)} = -\frac{1}{V_p}\psi, \quad n_d^{(1)} = -\frac{1}{V_p^2}\psi, \tag{3.7}$$

$$\rho^{(1)} = 0, \quad V_p = \frac{1}{\sqrt{(\alpha+\mu_{i2}-1)\sigma_1+\alpha+\mu_{i2}\sigma_2}}, \tag{3.8}$$

where $\psi = \phi^{(1)}$. The expression for $V_p$ in (3.8) represents the linear dispersion relation for the DA waves propagating in a dusty plasma under consideration. To the next higher order in $\epsilon$, we obtain another set of equations, which, after using (3.7)–(3.8), can be simplified as

$$u_d^{(2)} = \frac{1}{2V_p^3}\psi^2 - \frac{1}{V_p}\phi^{(2)}, \tag{3.9}$$

$$n_d^{(2)} = \frac{3}{2V_p^4}\psi^2 - \frac{1}{V_p^2}\phi^{(2)}, \tag{3.10}$$

$$\rho^{(2)} = -\frac{1}{2}A\psi^2 = 0, \tag{3.11}$$

$$A = \alpha + \mu_{i2}\sigma_2^2 - (\alpha+\mu_{i2}-1)\sigma_1^2 - \frac{3}{V_p^4}. \tag{3.12}$$

It is obvious from (3.11) that $A = 0$, since $\phi^{(1)} \neq 0$. The solution of $A = 0$ for $\alpha$ is given by

$$\alpha = \alpha_c = \frac{1}{6(1+C_3)}\Big[1 + 5C_3 - 6C_1(C_2+C_3) \pm\sqrt{1+C_3(10+C_3)+12C_1(C_2-1)(C_2+C_3)}\Big], \tag{3.13}$$

where

$$C_1 = \mu_{i2} = n_{i20}/Z_d n_{d0},$$

$$C_2 = \sigma_2 = T_{i1}/T_{i2},$$

$$C_3 = \sigma_1 = T_{i1}/T_e.$$

It is obvious that (3.11) is satisfied for $\alpha = \alpha_c$. We have numerically shown how $\alpha_c$ varies with $\sigma_2$ and $\sigma_1$ for a fixed $\mu_{i2} = 0.5$. The results are displayed in Fig. 1, which in fact represents the $A = 0$ surface plot, and provides us the parametric regimes (which correspond to above or below the $A = 0$ surface plot) of our present interest. So for $\alpha$ around its critical value ($\alpha_c$), i.e. for $|\alpha - \alpha_c| = \epsilon$ corresponding to $A = A_0$, we can express $A_0$ as

$$A_0 \simeq s\left(\frac{\partial A}{\partial \alpha}\right)_{\alpha=\alpha_c} |\alpha - \alpha_c| = sA_\mu\epsilon, \tag{3.14}$$

where

$$A_\mu = \big[1 - \sigma_1^2 - 6(1+\sigma_1)(\alpha_c\sigma_1 + \mu_{i2}\sigma_1 - \sigma_1 + \alpha_c + \mu_{i2}\sigma_2)\big]. \tag{3.15}$$

We note that in (3.14) we have replaced $(\alpha - \alpha_c)$ by $s|\alpha - \alpha_c|$. This means that $s$ determines the polarity of

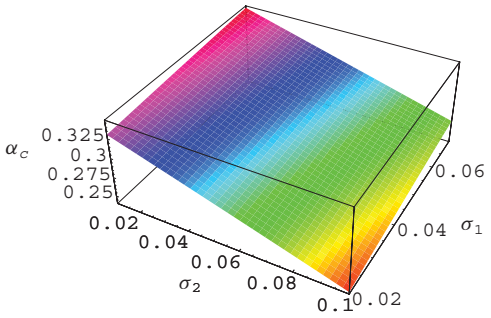



**Figure 1.** (Colour online) Showing how $\alpha_c$ (obtained from $A(\alpha = \alpha_c) = 0$) varies with $\sigma_2$ and $\sigma_1$ for $\mu_{i2} = 0.5$.

the quantity $(\alpha - \alpha_c)$, i.e. $s = 1$ for $\alpha > \alpha_c$ and $s = -1$ for $\alpha < \alpha_c$. So for $\alpha \neq \alpha_c$, using (3.11) and (3.14) we can express $\rho^{(2)}$ as

$$\rho^{(2)} \simeq -\frac{1}{2}s\epsilon A_\mu\psi^2. \tag{3.16}$$

This means that for $\alpha \neq \alpha_c$, $\rho^{(2)}$ must be included in the third-order Poisson's equation. To the next higher order in $\epsilon$, we obtain the third set of equations:

$$\frac{\partial n_d^{(1)}}{\partial\tau} + \frac{v u_d^{(1)}}{V_p\tau} - V_p\frac{\partial n_d^{(3)}}{\partial\zeta} + \frac{\partial F_d}{\partial\zeta} = 0, \tag{3.17}$$

$$\frac{\partial u_d^{(1)}}{\partial\tau} - V_p\frac{\partial u_d^{(3)}}{\partial\zeta} + \frac{\partial}{\partial\zeta}\big[u_d^{(1)}u_d^{(2)}\big] - \frac{\partial\phi^{(3)}}{\partial\zeta} = 0, \tag{3.18}$$

$$\frac{\partial n_d^{(3)}}{\partial\zeta} + \frac{15}{2V_p^6}\psi^2\frac{\partial\psi}{\partial\zeta} - \frac{3}{V_p^4}\frac{\partial}{\partial\zeta}[\psi\phi^{(2)}] + \frac{v}{V_p^3\tau}\psi + \frac{1}{V_p^2}\frac{\partial\phi^{(3)}}{\partial\zeta} + \frac{2}{V_p^3}\frac{\partial\psi}{\partial\tau} = 0, \tag{3.19}$$

$$\frac{\partial^2\psi}{\partial\zeta^2} - (\alpha+\mu_{i2}-1)\sigma_1\phi^{(3)} - (\alpha+\mu_{i2}-1)\sigma_1^2\psi\phi^{(2)} - \frac{1}{6}(\alpha+\mu_{i2}-1)\sigma_1^3\psi^3 - \alpha\phi^{(3)} + \alpha\psi\phi^{(2)} - \frac{1}{6}\alpha\psi^3 - \mu_{i2}\sigma_2\phi^{(3)} + \mu_{i2}\sigma_2^2\psi\phi^{(2)} - \frac{1}{6}\mu_{i2}\sigma_2^3\psi^3 - n_d^{(3)} + \frac{1}{2}c_1 s\psi^2 = 0, \tag{3.20}$$

where $F_d = n_d^{(1)}u_d^{(2)} + n_d^{(2)}u_d^{(1)} + u_d^{(3)}$. Now, using (3.8)–(3.12) and (3.17)–(3.20), we finally obtain a nonlinear dynamical equation of the following form:

$$\frac{\partial\psi}{\partial\tau} + \frac{v}{2\tau}\psi + p\psi\frac{\partial\psi}{\partial\zeta} + q\psi^2\frac{\partial\psi}{\partial\zeta} + p_0\frac{\partial^3\psi}{\partial\zeta^3} = 0, \tag{3.21}$$

where $p = sA_\mu p_0$, $q = p_0 q_0$, and

$$p_0 = \frac{V_p^3}{2}, \tag{3.22}$$

$$q_0 = \frac{15}{2V_p^6} - \frac{1}{2}\alpha - \frac{1}{2}\mu_{i2}\sigma_2^3 - \frac{1}{2}(\alpha+\mu_{i2}-1)\sigma_1^3. \tag{3.23}$$

https://doi.org/10.1017/S0022377812000384 Published online by Cambridge University Press

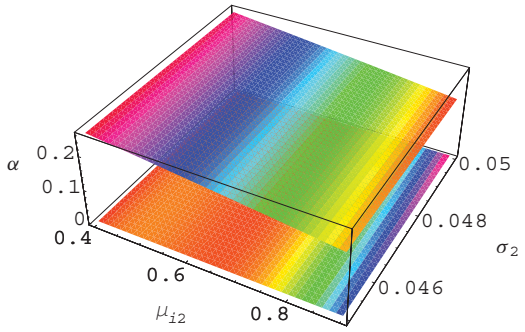



**Figure 2.** (Colour online) Showing the parametric regime for the existence of DLs (obtained from the solutions of ($q_0 = 0$ for $\alpha$) for $\sigma_1 = 0.06$.

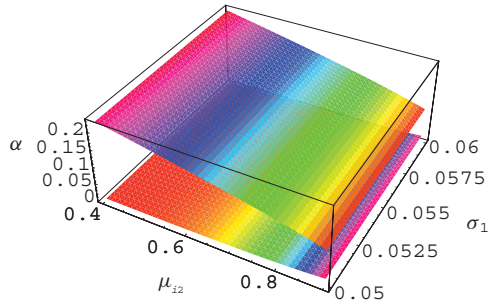



**Figure 3.** (Colour online) Showing the parametric regime for the existence of DLs (obtained from the solutions of ($q_0 = 0$ for $\alpha$) for $\sigma_2 = 0.10$.

Equation (3.21) is an mG equation. The modification is due to the extra term $\frac{\nu}{2\tau}\psi$, which arises due to the effects of non-planar geometry. We have already mentioned that $\nu = 0$ corresponds to a 1D planar geometry, which reduces (3.21) to the standard Gardner (sG) equation. Our aim now is to numerically analyze mG equation. However, for clear understanding, we first briefly discuss the stationary DL solution of this sG equation (i.e. (3.21) with $\nu = 0$). The stationary DL solution of this sG equation is obtained by considering a moving frame (moving with speed $U_0$) $\xi = \zeta - U_0\tau$, and imposing all appropriate boundary conditions for DL solutions, including $\psi \to 0$, $d\psi/d\xi \to 0$, $d^2\psi/d\xi^2 \to 0$ at $\xi \to -\infty$. Thus, one can express the stationary DL solution of this sG equation as

$$\psi = \frac{\psi_m}{2}\left[1 + \tanh\frac{\xi}{\Delta}\right], \tag{3.24}$$

where the amplitude ($\psi_m$) and the width ($\Delta$) of DLs are as follows:

$$\psi_m = s\frac{6U_0}{A_\mu p_0}, \tag{3.25}$$

$$\Delta = \sqrt{\frac{24}{-\psi_m^2 q_0}}. \tag{3.26}$$

It is clear from (3.24) and (3.26) that DLs exist if and only if $q_0 < 0$, i.e. $\alpha_L < \alpha < \alpha_U$, obtained from $q_0 = 0$, is the lower (upper) limit of $\alpha$ above (below) which DLs exist. We have graphically represented how $\alpha_U$ (upper surface plots of Figs. 2 and 3) and $\alpha_L$ (lower surface plots of Figs. 2 and 3) vary with different parameters $\sigma_1$, $\sigma_2$, and $\mu_{i2}$, obtained from $q_0 = 0$ surface plots. On the other hand, since $p_0 > 0$ and $U_0 > 0$, (3.24) and (3.25) indicate that the DLs are associated with positive potential if $s = -1$, i.e. $\alpha < \alpha_c$ and with negative potential if $s = 1$, i.e. $\alpha > \alpha_c$ as $A_\mu$ is always negative in that case. It is obvious from Figs. 2 and 3 that $\alpha_U < \alpha_c$ (consequently $\alpha_L < \alpha_c$), which confirm us that DLs are associated with positive potential only. The parametric regimes for the existence of positive DLs are bounded by the lower and upper surface plots of Figs. 2 and 3, and DLs exist for parameters corresponding to any point in-between two $q_0 = 0$ surface plots. It may be mentioned here that if we would neglect the higher order nonlinear term

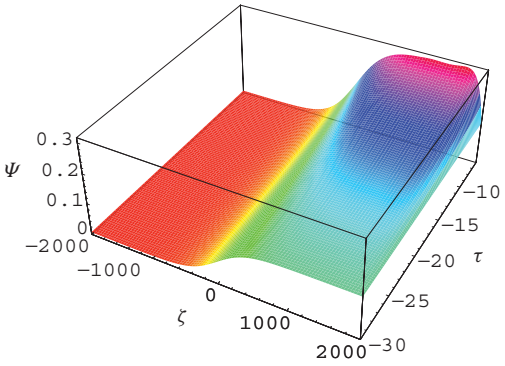



**Figure 4.** (Colour online) Showing the effects of cylindrical geometry on DA positive DLs for $\alpha = 0.24$, $\sigma_1 = 0.07$, $\sigma_2 = 0.05$, $\mu_{i2} = 0.5$, and $U_0 = 0.1$.

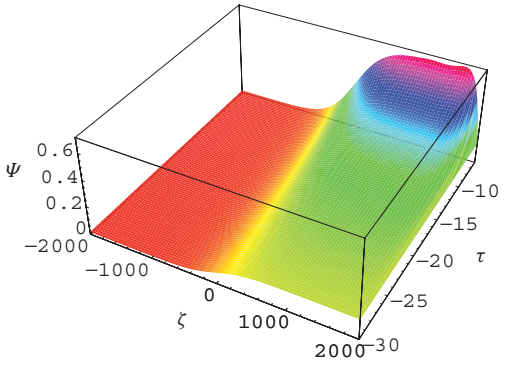



**Figure 5.** (Colour online) Showing the effects of spherical geometry on DA positive DLs for $\alpha = 0.24$, $\sigma_1 = 0.07$, $\sigma_2 = 0.05$, $\mu_{i2} = 0.5$, and $U_0 = 0.1$.

(viz. the fourth term of (3.21) or the term containing $\psi^3$) but keep the lower order nonlinear term (viz. the third term of (3.21) or the term containing $\psi^2$), then we would obtain the solitary structures that are due to the balance between nonlinearity (associated with $\psi^2$ only) and dispersion. On the other hand, in our present work, we have kept both the terms containing $\psi^2$ and $\psi^3$, and have obtained the stationary DL structures associated with positive potential only (shown in Figs. 4 and 5).

## 4. Numerical analysis and discussion

We now turn to (3.21) with the term $(\nu/2\tau)\psi$, which is due to the effects of non-planar (cylindrical or spherical)

https://doi.org/10.1017/S0022377812000384 Published online by Cambridge University Press

geometry. An exact analytic solution of (3.21) is not possible. Therefore, we have numerically solved (3.21), and have studied the effects of cylindrical and spherical geometries on time-dependent DA DLs. The results are depicted in Figs. 4 and 5. The initial condition that we have used in our numerical analysis is in the form of a stationary solution of (3.21) without the term $(\nu/2\tau)\psi$. Figures 4 and 5 show how the effects of cylindrical (spherical) geometry modify DA DLs.

The numerical solutions of (3.21) (displayed in Figs. 4 and 5) reveal that for a large value of $\tau$ (e.g. $\tau = -30$), the cylindrical ($\nu = 1$) and spherical ($\nu = 2$) DLs are almost similar to 1D planar ($\nu = 0$) structures. This is because for a large value of $\tau$, the term $(\nu/2\tau)\psi$, which is due to the effects of cylindrical or spherical geometry, is no longer dominant. However, as the value of $\tau$ decreases, the term $(\nu/2\tau)\psi$ becomes dominant, and spherical and cylindrical DL structures differ from 1D planar ones. It is found that as the value of $\tau$ decreases, the amplitude of these localized pulses increases. It is also found that the amplitude of cylindrical DA DL structures is larger than those of 1D planar ones, but smaller than that of spherical ones. The amplitude of DLs increases but width decreases with increase of $U_0$. With gradual increment of $\alpha$ below $\alpha_c$, the amplitude of the DLs increases but width decreases with increase of $\alpha$.

We have used a wide range of the dusty plasma parameters (viz. $\mu_{i2} = 0.4-0.9$, $\sigma_1 = 0.05-0.06$, and $\sigma_2 = 0.045-0.05$) in our numerical analysis. It may be stressed here that the results of this investigation should be useful for understanding the nonlinear features of electrostatic disturbances in laboratory and space plasmas, i.e. laser fusion, solar physics, plasma processing, intense beam production, etc. are the extensive areas of DL interest, in which charged dust fluid, electrons, and ions of two different temperatures (lower and higher) are the major plasma species. It may also be added here that our present investigation is valid for small amplitude DA Gardner DLs in non-planar dusty plasma system. Thus, the dusty plasma parameters are within appropriate ranges for both space environments (Ellis and Neff 1991, Horányi et al. 1993, Havnes et al. 1996, Farrell et al. 2004, Merrison et al. 2004) and laboratory devices (Ali et al. 1998, Trigwell et al. 2003, Zhao et al. 2003). The value of $\alpha$ for which the existence of DLs is found is also within the range of the dusty plasma parameters corresponding to dusty plasma parameters for both space environments (Ellis and Neff 1991, Horányi et al. 1993, Havnes et al. 1996, Farrell et al. 2004, Merrison et al. 2004) and laboratory devices (Ali et al. 1998, Trigwell et al. 2003, Zhao et al. 2003).

## Acknowledgements

The research grant for research equipment from the Third World Academy of Sciences (TWAS), ICTP, Trieste, Italy is gratefully acknowledged.

## References

Ali, F. S., Ali, M. A., Ali, R. A. and Lncullet, I. I. 1998 *J. Electrost.* **45**, 139.

Alinejad, H. and Mamun, A. A. 2010 *Phys. Plasmas* **17**, 123704.

Baisong, X., Kaifen, H. and Zuqia, H. 1999 *Phys. Plasmas* **6**, 3808.

Bandyopadhyay, P., Prasad, G., Sen, A. and Kaw, P. K. 2008 *Phys. Rev. Lett.* **101**, 065006.

Bharuthram, R. and Shukla, P. K. 1992 *Planet. Space Sci.* **40**, 465.

Boström, R., Gustafsson, G., Holback, B., Holmgren, G. and Koskinen, H. 1988 *Phys. Rev. Lett.* **61**, 82.

D'Angelo, N. 1995 *J. Phys.* D **28**, 1009.

Eliasson, B. and Shukla, P. K. 2004 *Phys. Rev.* E **69**, 067401.

El-Labany, S. K. and El-Taibany, W. F. 2003 *Phys. Plasmas* **10**, 989.

El-Labany, S. K., El-Taibany, W. F., Mamun, A. A. and Moslem, W. M. 2004 *Phys. Plasmas* **11**, 926.

Ellis, T. A. and Neff, J. S. 1991 *Icarus* **91**, 280.

Ergun, R. E. et al. 1998 *J. Geophys. Res.* **25**, 2041.

Farrell, V. M. et al. 2004 *J. Geophys. Res.* **65**, 3504.

Havnes, O. et al. 1996 *J. Geophys. Res.* **101**, 1039.

Heinrich, J., Kim, S. H. and Merlino, R. 2009 *Phys. Rev. Lett.* **103**, 115002.

Hellberg Manfred, A., Verheest, F. and Cattaert, T. 2005 *AIP Conf. Proc.* **799**, 498.

Hershkowitz, N. 1985 *Space Sci. Rev.* **41**, 351.

Horányi, M., Morfill, G. E. and Grün, E. 1993 *Nature* **363**, 144.

Koskinen, H. E. J., Lundlin, R. and Holback, B. 1990 *J. Geophys. Res.* **95**, 5921.

Mamun, A. A. 1999 *Astrophys. Space Sci.* **268**, 443.

Mamun, A. A., Cairns, R. A. and Shukla, P. K. 1996 *Phys. Plasmas* **3**, 702.

Mamun, A. A. and Shukla, P. K. 2009 *Europhys. Lett.* **87**, 2500.

Mamun, A. A., Shukla, P. K. and Eliasson, B. 2009 *Phys. Rev.* E **80**, 046406.

Maxon, S. and Viecelli, J. 1974 *Phys. Rev. Lett.* **32**, 4.

Merlino, R. L. and Goree, J. 2004 *Phys. Today* **57**(7), 32.

Merrison, J., Jensen, J., Kinch, K., Mugfor, R. and Nornberg, P. 2004 *Planet. Space Sci.* **52**, 279.

Moslem, W. M. 2005 *Chaos Solitons Fractals* **23**, 939.

Newman, D. L., Goldman, M. V. and Ergun, R. E. 2002 *Phys. Plasmas* **9**, 2337.

Rao, N. N., Shukla, P. K. and Yu, M. Y. 1990 *Planet. Space Sci.* **38**, 543.

Rosenberg, M., Mendis, D. A. and Sheehan, D. P. 1999 *IEEE Trans. Plasma Sci.* **27**, 239.

Shukla, P. K. 2000 *Phys. Rev.* E **61**, 7249.

Shukla, P. K. and Eliasson, B. 2009 *Rev. Mod. Phys.* **81**, 23.

Shukla, P. K. and Mamun, A. A. 2001 *IEEE Trans. Plasma Sci.* **29**, 221.

Shukla, P. K. and Mamun, A. A. 2002 *Introduction to Dusty Plasma Physics.* Bristol, UK: IoP, pp. 8–27.

Singh, N. 2000 *Geophys. Res. Lett.* **27**, 927.

Tagare, S. G. 1997 *Phys. Plasmas* **4**, 3167.

Temerin, M., Cerny, K., Lotko, W. and Mozer, F. S. 1982 *Phys. Rev. Lett.* **48**, 1175.

https://doi.org/10.1017/S0022377812000384 Published online by Cambridge University Press

Thompson, C., Barkan, A., D'Angelo, N. and Merlino, R. L. 1997 *Phys. Plasmas* **4**, 2331.

Tribeche, M. and Merriche, A. 2011 *Phys. Plasmas* **18**, 034502.

Trigwell, S., Grable, N., Yurreri, C. U., Sharma, R. and Mazumder, M. K. 2003 *IEEE Trans. Ind. Appl.* **39**, 79.

Verheest, F. 2000 *Waves in Dusty Plasmas.* (Dordrecht, Netherlands: Kluwer.

Washimi, H. and Taniuti, T. 1966 *Phys. Rev. Lett*. **17**, 996.

Zhao, H., Castle, G. S. P., Lnculet, I. I. and Bailey, A. G. 2003 *IEEE Trans. Ind. Appl.* **39**, 612.

https://doi.org/10.1017/S0022377812000384 Published online by Cambridge University Press