J. Plasma Physics (2010), *vol.* 76, *parts* 3&4, *pp.* 493–500. © Cambridge University Press 2010
doi:10.1017/S0022377809990729

Avinash–Shukla mass limit for the maximum dust mass supported against gravity by electric fields

K. AVINASH

Department of Physics and Astrophysics, University of Delhi, Delhi, India
Centre for Space Plasma and Aeronomic Research, University of Albama, Hunstville, AL 35899, USA
(ak0005@uah.edu)

(Received 30 November 2009 and accepted 9 December 2009, first published online 15 January 2010)

Abstract. The existence of a new class of astrophysical objects, where gravity is balanced by the shielded electric fields associated with the electric charge on the dust, is shown. Further, a mass limit M_A for the maximum dust mass that can be supported against gravitational collapse by these fields is obtained. If the total mass of the dust in the interstellar cloud $M_D > M_A$, the dust collapses, while if $M_D < M_A$, stable equilibrium may be achieved. Heuristic arguments are given to show that the physics of the mass limit is similar to the Chandrasekar's mass limit for compact objects and the similarity of these dust configurations with neutron and white dwarfs is pointed out. The effect of grain size distribution on the mass limit and strong correlation effects in the core of such objects is discussed. Possible location of these dust configurations inside interstellar clouds is pointed out.

The gravitational collapse of the gas and the dust in interstellar clouds plays an important role in the formation of stars and other planetary bodies [1, 2]. Recent observations from the Hubble telescope have shown the possibility of the existence of dust clouds in the HII region [3]. In the clouds, the interstellar radiation field and other mechanisms ionize the gas and hence grains acquire a significant charge [4, 5]. A pertinent question, which then arises, is what is the effect of the dust charge on the gravitational collapse of the dust? This question was also posed by Alfven sometimes ago (A. Mendis, private communication, 2003). Here we show that dust charge and associated electrostatic (ES) fields modify the gravitational collapse of the dust significantly. The force on the grain due to the electric field behaves like an effective pressure force [6–8]. The ES pressure depends on the dust density. This dependence is characterized by an index Γ. The ES pressure may be balanced with the self-gravity of the dust to obtain a configuration where dust is held in equilibrium. Such dust configurations have been constructed analytically and numerically [9–11]. An energy principle for the stability of these configurations has also been given [12].

In this paper, we show that there is an upper limit on the maximum dust mass that can be supported against gravity by ES pressure [9]. In particular, we study the effect of grain size distribution on the mass limit. The existence of the mass

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press

limit may be explained as follows. Because of the screening of the dust charge by neighboring dust grains, the dust charge decreases with increasing dust density [13–15]. Consequently, the index Γ, which characterizes the dust density dependence of ES pressure, decreases from 2 to 0 at high dust densities. Because of the smaller Γ, the ES pressure is not able to cope with self-gravity with increasing dust density, resulting in a upper limit $M_{\rm A}$ on the total mass of the dust in the cloud $M_{\rm D}$, for the equilibrium. For $M_{\rm D} > M_{\rm A}$, the dust cloud collapses, while for $M_{\rm D} < M_{\rm A}$, there is a stable equilibrium. In dust equilibria near the mass limit, the dust is "compactly" packed at very high densities. These dust clouds, which are a new class of astrophysical objects where gravity is supported by shielded electric fields, may exist inside moderate-sized interstellar clouds. The physics of this mass limit and these objects in general is similar to the physics of compact objects like white dwarfs [16] and neutron stars [17, 18].

Thermodynamically, the notion of the ES pressure P_E may be introduced thermodynamically as

$$P_E = -\frac{\partial}{\partial V}\frac{q_{\rm d}^2}{8\pi\varepsilon_0}\sum\sum\frac{\exp(-\kappa_{\rm d}|r_i - r_j|)}{|r_i - r_j|}\bigg|_T, \tag{1}$$

where V and $q_{\rm d}$ are the volume of the dust cloud and dust charge, respectively, T is the plasma temperature, ith dust grain is located at $r = r_i$, and $\kappa_{\rm d} = 1/\lambda_{\rm d}$. However, in this paper we introduce this notion via a simple dust-charging model. Consider a finite dust cloud, which is embedded, in a much larger background of ionized gas (mostly hydrogen) in the interstellar cloud. The equilibrium charge on the grain is given by the equation $I_{\rm e} + I_{\rm i} + I_{\rm PH} = 0$, where $I_{\rm e}$ and $I_{\rm i}$ are the electron and the ion thermal fluxes impinging on the dust surface and $I_{\rm PH}$ is the flux of photoelectrons [19]. These quantities are given by

$$I_{\rm e} = -q\pi a^2\left(\frac{8T}{\pi m_{\rm e}}\right)^{1/2} n_{\rm e}\exp(q\psi/T), \quad I_{\rm i} = q\pi a^2\left(\frac{8T}{\pi m_{\rm i}}\right)^{1/2} n_{\rm i}\left(1 - \frac{q\psi}{T}\right),$$

$$I_{\rm PH} = q\pi a^2\Gamma_{\rm PH}, \tag{2}$$

where a is the grain radius, T is the background gas temperature, $n_{\rm e}(n_{\rm i})$ and $m_{\rm e}(m_{\rm i})$ are the electron (ion) density and mass, ψ is the dust surface potential relative to the local plasma potential φ, and $\Gamma_{\rm PH}$ is the flux of photoelectrons per unit area from the dust surface. The effects due to grain size distribution will be discussed later. Since the typical scale of the dust cloud is expected to be much larger than the Debye length, we assume quasineutrality given by $q_{\rm d}n_{\rm d} = q(n_{\rm i} - n_{\rm e})$, where $q_{\rm d}$ is the dust charge related to ψ by $q_{\rm d} = \psi a$ and $n_{\rm d}$ is the dust density.

If the ions and electrons are thermalized within the interstellar cloud, which is a reasonable assumption [13, 14], then $n_{\rm i}$ and $n_{\rm e}$ are given by the Boltzmann's relation: $n_{\rm i} = n_0\exp(-q\varphi/T)$ and $n_{\rm e} = n_0\exp(q\varphi/T)$. The local plasma potential φ is taken to be zero at ∞, where $n_{\rm d} = 0$ and $n_{\rm i} = n_{\rm e} = n_0$.

As shown by Havnes et al. [14], (2) along with Boltzmann electrons and ions and quasineutrality condition can be combined into two equations that can be solved to obtain φ and ψ as functions of $n_{\rm d}$ (for given n_0, a and T), i.e. $\varphi = \varphi(n_{\rm d})$ and $\psi = \psi(n_{\rm d})$. The dust number density $n_{\rm d}$ or the dust mass density ρ is parameterized by a dimensional-less number $p = aTn_{\rm d}/q^2n_0$ called the Havnes parameter. By solving aforementioned equations, Havnes et al. [20] obtained simple polynomial

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press

approximations to φ and ψ as function of p, given by

$$\overline{\psi} = \frac{q\psi}{T} = \frac{a_0 + a_1 p}{1 + b_1 p + b_2 p^2}, \quad \overline{\varphi} = \frac{q\varphi}{T} = \frac{c_1 p + c_2 p^2}{1 + d_1 p + d_2 p^2}, \tag{3}$$

where constants a_j, b_j, c_j and d_j depend on the species of the interstellar gas and $I_{\rm PH}$. These constants are tabulated in Ref. [20]. In the isolated grain limit $p \to 0$, $\overline{\varphi} \to 0$ and $\overline{\psi} \to -2.5$ for H ions and $I_{\rm PH} = 0$. As p increases, $\overline{\psi}$ and $q_{\rm d} \to 0$ while the plasma potential decreases monotonically and saturates, i.e. $\overline{\varphi} \to c_2/d_2 = -1.87$. The dust charge reduction at high dust density is due to the mutual screening of the grains and has been verified in lab experiments and numerical simulations [21, 22].

The dependence of the plasma potential φ on $n_{\rm d}$ implies that the force on a dust grain due to electric field within the cloud acts like an effective pressure force, i.e. $F = n_{\rm d} q_{\rm d} E = -n_{\rm d} q_{\rm d} d\varphi / dn_{\rm d} \nabla n_{\rm d}$. This force expels dust grains from regions of high dust density and has been shown to be responsible for the vertical oscillations and finite thickness of planetary rings [7].

We begin our calculations by deriving an equation of state that relates "electrostatic pressure" P_E to the density $n_{\rm d}$. Since for given n_0, a and T, $(q_{\rm d} n_{\rm d})$ is a function of φ, we may define P_E via the integral $P_E = \int q_{\rm d} n_{\rm d} d\varphi$ with the condition that $P_E = 0$ at $\varphi = 0$. Eliminating $(q_{\rm d} n_{\rm d})$ through the quasineutrality condition we obtain following equation of state for P_E:

$$P_E = n_0 T(\cosh \overline{\varphi} - 1), \tag{4}$$

where P_E is a function of $n_{\rm d}$ via φ in (3) for given n_0, a and T. The force due to this pressure arises essentially due to the mean electric field caused by the inhomogeneous distribution of the dust within the interstellar cloud. In the low-density limit, $P_E \propto p^2$, i.e. the adiabatic index is 2. In the limit $p \to \infty$, P_E is constant due to charge reduction at high dust densities.

Next, we define an electrostatic (ES) potential energy per particle as $U_{\rm E} = \int q_{\rm d} d\varphi$. Using $I_{\rm e} = I_{\rm i}$ and (2), the integral can be done in term of ψ and the result for H ions is

$$U_{\rm E} = \frac{aT^2}{2q^2} \left(4.87 - \frac{(1 + \overline{\psi})^2}{2} + \ln(1 + \overline{\psi}) \right), \tag{5}$$

where we have taken $I_{\rm PH} = 0$ for simplicity. The parameter U_E is also a function of p or ρ via ψ. At low density, $U_{\rm E} \propto p$ and approaches a constant at high density where $\psi \to 0$.

Before proceeding further, we give heuristic arguments to show that there is an upper limit on the total mass of the dust in the cloud for the equilibrium. Consider a homogenous dust ball of mass $M_{\rm D}$ and radius R that is embedded in an ionized background of gas and is undergoing gravitational collapse. The total potential energy per particle is the sum of $U_{\rm E}$ and the gravitational potential energy given by

$$U = \frac{aT^2}{2q^2} f(p) - \frac{3}{5} \frac{G M_{\rm D} m_{\rm d}}{R}, \tag{6}$$

where $f(p)$ stands for the expression in the parenthesis in (5) and is a function of p via ψ. Initially let $U < 0$. Now for small p and given $M_{\rm D}$, $f(p) \propto p \propto R^{-3}$. Hence in the limit $R \to \infty$, gravity dominates and $U \to 0$ through negative values. In the limit $R \to 0$ also, gravity dominates as the first term saturates due to charge reduction at high density. Thus as the dust ball is compressed due to self-gravity, there are

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press

only two possibilities: If $M_{\rm D}$ is large, the first term is near saturation and hence U decreases monotonically, approaches $-\infty$ as $R \to 0$ and there is no equilibrium. In this case the gravity compresses grains and uncharges them before the ES pressure has had a chance to stop gravity. Subsequent collapse then takes place as if the dust is uncharged. On the other hand, if $M_{\rm D}$ is small, then as R decreases the first term will increase and may balance the second term for some R. In this case there will be a minimum of U and a stable equilibrium at R_1 followed by a maximum and an unstable equilibrium at R_2 (where $R_1 > R_2$) before gravity again dominates, i.e. $U \to -\infty$ as $R \to 0$. These arguments indicate that there is an upper limit $M_{\rm A}$ on the total mass of the dust in the cloud for the equilibrium. This limit can be roughly obtained (up to a constant of order unity) by equating the two terms in (6).

The above-stated arguments are similar to the ones given for the mass limit of compact objects [16]. A brief discussion about the similarity of the present problem with that of compact objects is in order here. In case of white dwarfs, as the density is increased, the relativistic corrections reduce the adiabatic index Γ of kinetic pressure from 5/3 to 4/3, which results in a mass limit [16]. When the effects due to the inverse beta decay are included at sufficiently high density, Γ is further reduced to a value below 4/3. This gives rise to a maximum in the mass vs. central density curve, which signals instability due to the gravitational collapse [17, 18]. In the present problem, charge reduction alone reduces the adiabatic index from 2 at small density to 0 at high density, resulting in an analogous mass limit and instability. These heuristic arguments capture the essential features of this problem.

As in the theory of stellar configurations [16], the problem of the equilibrium of dust clouds may be formulated in terms of an adiabatic fluid with an equation of state and pressure P_E, which balances gravity according to $\nabla P_E = m_{\rm d} n_{\rm d} \nabla \Phi$, where Φ is the gravitational potential and the force due to the electric field is expressed in terms of P_E. Taking the divergence of this equation, using Poisson's equation for Φ and the normalization described below, the force balance equation in spherical coordinates can be written as

$$\frac{1}{r^2}\frac{d}{dr}\frac{r^2}{p}\frac{dP_E}{dr} = -2c_1^2 p, \tag{7}$$

where we have normalized pressure P_E by $n_0 T$, dust density by p, and the distance r by the scale length L, with $L = \sqrt{2c_1^2 a^2 T^3/4\pi G m_{\rm d}^2 q^4 n_0}$. This parameter gives the typical dimensions of the dust clouds in terms of other parameters. Equation (7) is supplemented with the equation of state given by

$$P_E = (\cosh \overline{\varphi} - 1), \quad \overline{\varphi} = \frac{c_1 p + c_2 p^2}{1 + d_1 p + d_2 p^2}. \tag{8}$$

Equations (7) and (8) may be solved to obtain $P_E(r)$ and $p(r)$ for a given species of interstellar gas and a value of $I_{\rm PH}$. The boundary conditions are $p = p_{\rm c}$ and $\frac{dp}{dr} = 0$ at $r = 0$, where p_c is the normalized central dust density in the cloud. Using these boundary conditions, we construct one-parameter family of spherically symmetric and static solutions characterized by $p_{\rm c}$. The normalized radius R of the configuration is given by the condition $p(R) = 0$ and the total mass $M_{\rm D}$ of the dust cloud is given by $M_{\rm D} = (L^3 m_{\rm d}/\lambda_{\rm d}^2 a) I$, where $I = \int_0^R p r^2 dr$ and $\lambda_{\rm d}$ is the Debye length given by $\lambda_{\rm d}^2 = T/4\pi n_0 q^2$. The integral I, which is a dimensionless number of order unity, is to be evaluated by solving (7) and (8). Before discussing general solutions, we first discuss the low-density limit. In this limit $p_c \ll 1$, and hence the

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press

polytropic approximation is valid, i.e. $\overline{\varphi} \prec 1$ and $P_E \propto p^2$ with polytropic index being one. This case has been analytically solved by Chandrasekar [16] and the solution is given by $p = p_c \sin r/r$, $I = p_c\pi$ and $R = \pi$. These tenuous dust equilibria are characterized by a radius, which is independent of p_c while $M_D \propto \rho_c$ (ρ_c is the central dust mass density). For arbitrary p_c, (7) and (8) may be solved numerically to obtain the static equilibrium solution and the value of the integral I.

Next we show the mass limit M_A. Electric fields are strongest when $I_{PH} = 0$ and all charges are of same sign, which is negative when $I_{PH} = 0$. To show the mass limit we consider this case. From the tables given in Ref. [20], we chose the values of the constants c_j and d_j corresponding to $I_{PH} = 0$ and H ions, i.e. $c_1 = -1.26, c_2 = -0.21, d_1 = 1.04$ and $d_2 = 0.112$. The equations are solved by choosing an initial value of $p_c = 10^{-11}$ (corresponding to a typical value in the interstellar cloud), which is increased up to 100 by increasing n_d keeping n_0, a and T constant. For small values of p_c (<0.01), I increases linearly with p_c due to polytropic approximation. For large values of p_c (>0.01), we find that I saturates with a broad maximum $I_{max} = 0.85$ at $p_c = 2$. The value of R is constant and is equal to π for $p_c < 0.01$. In the range $0.01 < p_c < 2$, it decreases from π to 2.27. After this it decreases very slowly for equilibria on the other side of the maximum. In dust clouds with $p_c \leq 2$, dust is compactly packed at high density, the dust charge is small and electrons are strongly depleted; they are absorbed by the dust leaving behind a plasma, which consists mainly of negatively charged dust and positively charged ions. As stated earlier, equilibrium with $p_c > 2$ are unstable. This is similar to the theory of compact objects where a maximum in the mass vs. central density curve signals instability [23, 24]. The existence of this mass limit has been confirmed in extensive numerical simulations [10]. A full discussion regarding the stability of these configuration is reported in Ref. [12].

From the value of I and p_c obtained above, we may plot a universal equilibrium curve in terms of normalized variables or a specific curve in terms of physical units of M_D and ρ_c. However, before we do this we must relax the assumption of constant grain size and incorporate the effects due to finite grain size distribution in our model. These effects, especially due to smaller grains, are important [25] and their inclusion is essential for a realistic evaluation of M_A and L. This may done by redefining the parameter p as $p = \langle a\rangle T n_d/q^2 n_0$, where $\langle a\rangle$ is the average grain size given by $\langle a\rangle = c\int_{a_{min}}^{a_{max}} a dn_d$, c is constant of normalization and $dn_d = a^{-s}da$ is the density of particles with size between a and $(a + da)$. The ensuing calculations can be carried out in a similar fashion and the net result is that in the expressions for M_A and L, a is replaced by $\langle a\rangle$ and m_d is replaced by $\langle m_d\rangle = c\int_{a_{min}}^{a_{max}} m_d dn_d$. For our calculation we use $s = -3.5$, which is the value inferred for the interstellar grains; for planetary system, $s \sim 0.9$–4.5 [26, 27]. In this case $\langle a\rangle \approx a_{min}$ and $\langle m_d\rangle \propto a_{max}^{0.5} a_{min}^{2.5}$. Clearly, averages are dominated by small sizes. The corresponding expressions for M_A and L are given by

$$M_A = 4.25 \times 10^{-3} \frac{c_1^3 T^{7/2}}{G^{3/2} q^4 n_H^{1/2}} a_{max}^{-1} a_{min}^{-3}, \quad L = 0.033 \frac{c_1 T^{3/2}}{G^{1/2} q^2 n_H^{1/2}} a_{max}^{-0.5} a_{min}^{-1.5}, \quad (9)$$

where the value of the integral (=0.85) has been absorbed in other constants. The important scaling of M_A and L are explicitly shown. The strong dependence on a_{min} is important, as any error in a_{min} will severely affect the values of M_A and L. For example, the dust sizes smaller than a few tenth of a micron are not easily

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press

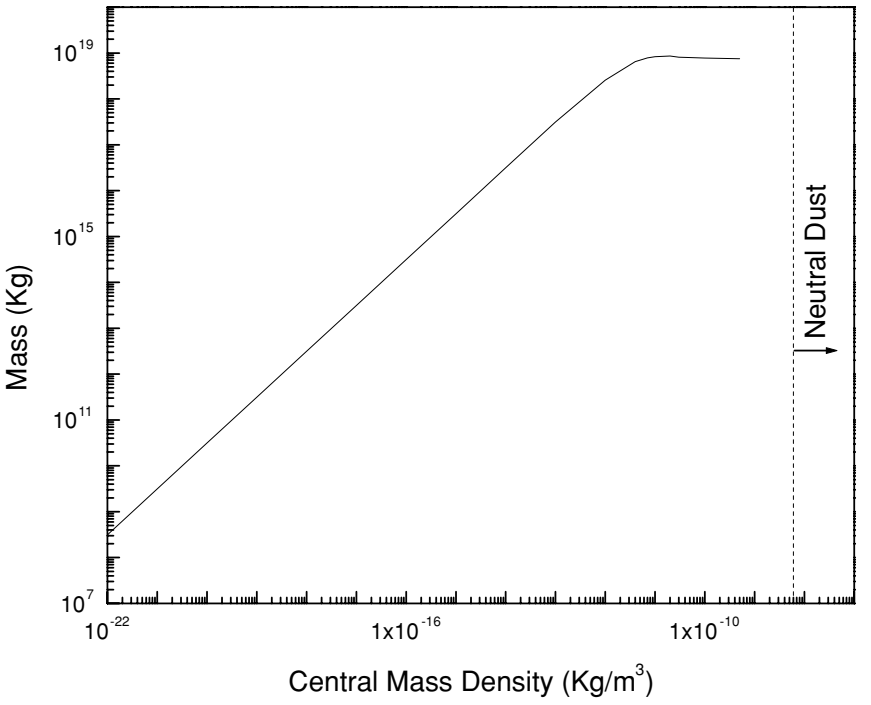


Figure 1. The equilibrium curve $M_{\rm d}$ vs. $\rho_{\rm c}$ showing the mass limit $M_{\rm A}$ for constant grain size. The equilibrium curve with grain size distribution is obtained by rescaling the axis as $M_{\rm D} \rightarrow M_{\rm D}(0.1)(\frac{a_{\rm max}}{a_{\rm min}})^3$ and $\rho_{\rm c} \rightarrow \rho_{\rm c} 3(\frac{a_{\rm min}}{a_{\rm max}})^{1.5}$. The corresponding mass limit with grain size distribution is $M_{\rm A} \approx 2 \times 10^{25}$ kg.

measurable while $a_{\rm min}$ could well be below this value. In situ measurement of comet Halley have revealed the presence of grains as small as $\sim 10^{-7}$cm [28]. In addition, when considering very small sizes one must take into account the disruption of the grain due to electrostatic forces [19]. However, for small sizes ($\geq 10^{-7}$cm) it has been shown that the disruption is stabilized by the field emission of charged particles [19]. Based on these consideration it may not be unreasonable to take $a_{\rm min} \sim 10^{-7}$cm.

To plot the equilibrium curve in terms of $M_{\rm D}$ and $\rho_{\rm c}$, we chose the following parameters which characterize a standard interstellar cloud in HII region [5]: $n_0 = n_{\rm H} = 10^7 {\rm m}^{-3}$ and $T = 5 \times 10^3 {}^\circ$K. For size distribution we take $a_{\rm max} = 3 \times 10^{-5}$cm, $a_{\rm min} \sim 10^{-7}$ cm and $s = -3.5$. To show the large effect of size distribution, we first plot the equilibrium curve for uniform dust size with $a = 3 \times 10^{-5}$cm. For these parameters, $L = 5 \times 10^6$ km and in Fig. 1 there is broad maximum at $M_{\rm D} \approx 8 \times 10^{18}$kg, which is the value of the mass limit for constant grain size. To incorporate the grain size effects in Fig. 1, we simply rescale the $M_{\rm D}$ and $\rho_{\rm c}$ axis as $M_{\rm D} \rightarrow M_{\rm D}(0.1)(\frac{a_{\rm max}}{a_{\rm min}})^3$ and central density $\rho_{\rm c} \rightarrow \rho_{\rm c} 3(\frac{a_{\rm min}}{a_{\rm max}})^{1.5}$. The corresponding value of the mass limit with size distribution is $M_{\rm A} \approx 2 \times 10^{25}$ kg, which is seven orders of magnitude larger than the value for constant size. It is roughly equal to the mass of a planet like Earth, Saturn or Neptune. The scale length with size distribution is $L \approx 10^{10}$ km ~ 100 AU. This gives an order of magnitude estimate of the total mass of the dust and the overall dimension of the dust clouds near the

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press

mass limit. Of course these values are sensitive to the value of $a_{\min}$. The core of such equilibrium is compact with high central density $n_{\rm dc} \sim 6 \times 10^7 {\rm m}^{-3}$, which is about 13 orders of magnitude higher than the average interstellar grain density $\sim 10^{-6} m^{-3}$.

Effects of Coulomb correlations defined by the parameter $\Gamma_{\rm c} = q_d^2/dT_{\rm d}$ (d is the mean inter-particle distance and $T_{\rm d}$ is the kinetic temperature associated with the random motion of the dust grain) may become significant in the compact dust equilibrium. As a function of p, $\Gamma_{\rm c}$ has a maximum at $p = 0.2$. Depending on whether we take T_d to be equal to the gas temperature or lower, $\Gamma_{\rm c}^{\rm max} \sim 5$ or higher. Thus a significant portion of compact dust equilibrium (with p_c in the range 0.1–2) may be in a strongly correlated Coulomb liquid or even a crystalline state.

The overall dimensions of the dust cloud ranges from 0.01 to 50 pc with most frequent size being ∼5 pc [5]. The dust in these clouds is typically silicate or 'PAH' (polycyclic aromatic hydrocarbon) distributed inhomogeneously. Now, if as a conservative estimate, we take the average interstellar dust mass density $\sim 3 \times 10^{-28}\,{\rm kg/m}^{-3}$, then in a moderate-sized (∼5 pc) dust cloud the total dust mass is $M_{\rm D} \sim 4 \times 10^{24}$ kg, which is roughly in the same range as $M_{\rm A} \sim 2 \times 10^{25}$ kg. This implies that in such clouds the electric fields are sufficiently strong to hold the dust against gravity in a stable equilibrium of dimensions ∼100 AU. These numbers may be somewhat reduced by a finite $I_{\rm PH}$, which is a function of the interstellar UV radiation field and the work function of the grain material. Compact dust equilibria are thus predicted to exit inside a few pc-sized interstellar clouds. Smaller clouds may contain the tenuous dust equilibria described earlier.

Acknowledgements

Discussions with Dr. A. Mendis are gratefully acknowledged.

References

[1] Hartmann, W. 1983 *Moons and Planets*. California: Wadsworth.
[2] Harpaz, A. 1994 *Stellar Evolution*. Wellesley, MA: A. K. Peters.
[3] Fischer, D. and Duerbeck, H. 1998 *Hubble Revisted*. New York: Springer Verlag.
[4] Spitzer, L. 1968 *Diffuse Matter in Space*. New York: Interscience Publisher.
[5] Hoyle, F. and Wikcramsinghe, N. C. 1991 *Theory of Cosmic Grains*. Boston: Kluwer Academic Publishers.
[6] Avinash, K. 2006 *Phys. Plasmas*. **13**, 012109.
[7] Goertz, C. K., Shan, L. and Havnes, O. 1987 *Geophys. Res. Lett.* **15**, 84; Melandso, J. and Havnes, O. 1991 *J. Geophys. Res.* **95**, 5837.
[8] Avinash, K. and Shukla, P. K. 2000 *Phys. Plasmas*. **7**(10), 2763.
[9] Avinash, K. and Shukla, P. K. 2006 *New J. Phys.* **8**, 2.
[10] Avinash, K., Eliasson B. and Shukla, P. K. 2006 *Phys. Lett.* A **353**, 105.
[11] Avinash, K. 2007 *Phys. Plasmas*. **14**, 012904.
[12] Avinash, K. 2007 *Phys. Plasmas*. **14**, 093701.
[13] Whipple, E. C., Northrop, T. G. and Mendis, A. 1985 *J. Geophys. Res.* **96**, 7405.
[14] Havnes, O., Goertz, C. K., Morfill, G. E. et al. 1987 *J. Geophys. Res.* **92**, 2281.
[15] Avinash, K., Bhattacharjee, A. and Merlino, R. 2003 *Phys. Plasmas*. **10**, 2663.
[16] Chandrasekar, S. 1957 *An Introduction to the Study of Stellar Structure*. Chicago: University of Chicago Press.

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press

[17] Shapiro, L. and Teukolsky, S. A. 1983 *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects*. New York: John Wiley & Sons.

[18] Oppenheimar, J. R. and Serber, R. 1938 *Phys. Rev.* **54**, 540.

[19] Shukla, P. K. and Mamun, A. A. 2002 *An Introduction to Dusty Plasma Physics*. Bristol: Institute of Physics Publishing.

[20] Havnes, O., Aanesen, T. K. and Melandso, F. 1990 *J. Geophys. Res.* **95**, 6581.

[21] Barkan, A., D'Angelo, N. and Merlino, R. L. 1994 *Phys. Rev. Lett.* **73**, 3093.

[22] Young, B., Cravens, T. E., Armstrong, T. P. et al. 1994 *J. Geophys. Res.* **99**, 2255.

[23] Baym, G., Pethick, C. and Sutherland, P. 1971 *Astrophys. J.* **170**, 299.

[24] Hamada, T. and Salpeter, E. E. 1961 *Astrophys. J.* **134**, 683.

[25] Krugel, E. 2003 *The Physics of Interstellar Dust*. Philadelphia, PA: IOP.

[26] Methis, J. S., Rumpl, W. and Nordsieck, K. N. 1977 *Ap. J.* **227**, 425.

[27] Burns, J. A., Showalter, M. R. and Morfill, G. M. 1984 *Planetary Rings* (ed. R. Greenberg and A. Brahic). Tucson: University of Arizona Press.

[28] Mcdonnell, J. A. et al. 1987 *Astron. Astrophys.* **187**, 719.

https://doi.org/10.1017/S0022377809990729 Published online by Cambridge University Press