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Logic is a Moral Science

HARTLEY SLATER

Abstract
Because of the communicative function of propositions, Logic is a moral science about what ought to be said. The association with morality derives from the connection between Logic and Truth, and the social value of speaking the truth. Propositions are used rather than mentioned sentences, and so are not the platonically abstract objects they have often been taken to be. Instead they are social objects formed by a community's employment of language. So it is the public use of sentences that settles what they mean, and thereby their logic. The matter has a particular relevance to Graham Priest's non-classical views.

1.

Intuitionistic and paraconsistent 'negations' are commonly symbolised using a unary connective read as 'not'. But what if we did the same with variant expressions that are sometimes read as 'if'? For quite common errors arise through confusing 'if' with 'only if' or 'if and only if'. These errors bear on the question: how can Affirming the Consequent (what Susan Haack called '*Modus Morons*')[1] be a fallacy, i.e. be a *bad* form of argument? For someone may argue 'if p then q, but q, so p' and insist that they are quite clear headed. Maybe it is found that they are using 'if' for what is conventionally expressed using 'only if'. Is the person then arguing invalidly? We could introduce a new connective for their 'if' and write the argument 'p > q, q, so p', and no error, it would seem, would then be made. All we would then have would be a novel kind of arrow that obeyed Affirming the Consequent, it might seem. After all, arrows that do not obey *Modus Ponens* are quite common these days, and this one doesn't either. So the inclination may well be to say that nothing is wrong. How could anything be *wrong*? It is just, apparently, that some arrows obey some rules while other arrows obey others. With 'if and only if' as another example, that obeys both rules, *Modus Ponens* and Affirming the Consequent. So a logician's job, it may seem, is just to tabulate which rules which arrows obey.

[1] S. Haack, 'The Justification of Deduction', *Mind* **85** (1976), 112–119.

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But, of course, arrows are not agents that might or might not conform to rules. All of the above overlooks the linguistic facts of the case: the fact that in their home, i.e. in natural language, neither 'only if' nor 'if and only if' is properly read 'if'. It is this that has the consequence that understanding the argument 'if p then q, but q so p' as involving 'only if' or 'if and only if' is *incorrect*. It is *wrong* and the basis for this judgment is simply attention to *the conventions of natural language*. Thinking in the manner of Affirming the Consequent is a form of *disobedience*, in other words. So it follows that Logic is a normative science. Indeed it is even better called a 'moral science', which explains more fully why assessments of *good* and *bad* reasoning are its defining feature. Carnap's Principle of Tolerance ('There are no morals in Logic') may look like it is against this, but that only applies to formal, uninterpreted systems, which can indeed be created entirely at will. It is when an attempt is made to translate the abstract formalism into concrete natural speech that matters of correctness and incorrectness come in.

Writers on non-classical logics do not appreciate the relevance of natural language in this matter, however. Partisans of Paraconsistency and Intuitionism, for example, often can be quite moralistic about their concerns, but would not think that arguments on the issues they dispute about should, or even could be settled simply by reference to natural language. Commonly, though, they use a classical meta-language with which to talk about their non-classical connectives, and regularly that language is natural language, of course. The use of natural language for this purpose has a consequence that is not widely remarked, however. For if there is *communication* using a classical language, then that settles what the standard logical constants, 'and', 'or', 'not', 'if' etc. mean. The point is quite elementary; it only gains a larger significance requiring closer attention in the context of a culture that has abandoned the idea that Truth is a property of propositions, i.e. expressed indicative sentences. For it means that it is wrong to say, for instance, that it is both true that p and true that not-p (for any 'p'), or deny that if it is not true that it is not true that p then it is true that p (again for any 'p').

Yet these things are commonly said when discussing paraconsistent and intuitionistic logics, for instance. If one takes it that it is the *formulas* 'p & ¬p', '¬¬p ⊃ p' that are true or false on this interpretation or that, rather than the propositions expressed when these formulas are used on this interpretation or that, and yet continue to read '&' and '¬' as 'and' and 'not', then it is easy to slip into thinking that the above necessary truths can be denied. Strictly, if what is expressed in a classical language settles what 'not' and the like mean, then some

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other words should be introduced for the linguistic expression of the deviant logical constants. But there is a major difficulty with finding other words with which to express paraconsistent and intuitionistic 'negations', for example, since there are no totally exact translations into ordinary speech.[2] Fair approximations are given by 'possibly not' and 'necessarily not' respectively,[3] but that only reinforces the inadequacy of reading either as plain 'not'. Of course, for many readers and writers on these subjects the inadequacy of the plain 'not' is of little moment, since the various deviant logical constants are not themselves commonly used, being merely objects of study. Indeed, as we shall see in a central case later, some are so intent on what they are talking about that they seem to become quite oblivious to the language they are using while talking about it. The prime exception is Graham Priest, who notoriously has insisted that the language he is using to talk about his paraconsistent negation is itself paraconsistent.[4] That has the result that he is happy with the adequacy of the plain translation 'not', and the continued applicability of the term 'contradiction'. But, as has been pointed out before, the contradictory of '$x \leq 0$' is '$x > 0$' and not '$x \geq 0$' as Priest has tried to have it.[5]

These points therefore bring up a central question about the nature of Logic: is it concerned with distinguishing correct from incorrect uses of 'and', 'or', 'not', 'if', etc. or is it merely concerned with the various formal systems setting out the variant uses for something like these words that have been suggested? Without an appreciation that Truth attaches to expressed rather than mentioned sentences, the question of *what is expressed* by the various non-classical connectives loses its interest, and logic even becomes a value-free, third person, mathematical subject simply about different arrangements of formulas. How has this transformation come about? The major problem for recent generations of mathematical logicians, it would seem, is that the basis for the appropriate good-bad evaluations is not mathematical. It is instead, as before, a linguistic matter of remembering, for instance, that neither 'only if', nor 'if and only if'

[2] See, e.g. S. Haack, *Deviant Logic, Fuzzy Logic: Beyond the Formalism*, (Chicago University of Chicago Press, 1996), Ch. 5.

[3] J-Y. Béziau, 'Paraconsistent Logic from a Modal Viewpoint', *Journal of Applied Logic* **3** (2005), 7–14.

[4] G.G. Priest, 'The Logic of Paradox', *Journal of Philosophical Logic* **8** (1979), 219–241.

[5] B.H. Slater, 'Paraconsistent Logics?', *Journal of Philosophical Logic* **24** (1995), 451–454.

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is 'if' and that neither 'possibly not' nor 'necessarily not' is 'not'. It is a matter of being *strict and pedantic* about the natural language meaning of such words. Natural language is central because it this which is in public use, and accurate truthful expression in public is the locus of the relevant moral values.

2.

A pertinent illustration arises towards the end of Francesco Berto's paper 'A Modality Called "Negation"'.[6] But I select the particular passage below because it also brings in the relevance of some recent work by Greg Restall and Graham Priest disputing a principle they call 'BN'. Indeed this dispute relates to some aspects of Priest's thought more generally, since he has argued in other ways against the possibility of 'Boolean negation'. Berto's paper is an intricate and comprehensive systematisation of a variety of 'negations' that have appeared in the philosophical and logical literature over the last century or so. He considers intuitionistic, paraconsistent and classical 'negations' for instance. But Berto does not attempt to tell us which one of these 'negations' is correct, despite the fact that the (unquoted) word 'not', along with its allies and associates, are thick on every page. So for a start it provides a good example of the general point above about the common use of natural language as a meta-language in which the logical constants have their 'classical' sense:

> According to Restall and Priest, once worlds can stand in non-trivial information-inclusion relations, classical negation is not automatically warranted anymore. For insofar as it makes sense to read 'x ||- A' as the claim that world x *includes*, or *conveys,* or *supports* the information that A, then it may well not be the case that x supports the information that ¬A just because it fails to support the information that A, as mandated by BN [which would equate 'x ||- ¬A' with the denial of 'x ||- A']. … The Boolean negation of a proposition may fail to be a proposition, that is, it may not be a hereditary piece of information: there may be no proposition such that *it* is supported by a world just because some proposition is not supported there.

The first question that arises here is: how are we to understand this? What does Berto mean here by 'non-trivial', 'is not automatically

[6] F. Berto, 'A Modality Called "Negation"', *Mind* (forthcoming).

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warranted', 'it may well not be the case', 'it fails to support', 'may fail to be a proposition', etc.? Surely Berto is not allowing that there can be variant non-classical readings of his 'not's? That would make his remarks strictly ambiguous. One presumes, therefore, that he means his 'not's and the like to involve 'Boolean negation'. And Berto would be taking it that his readers would be taking his expressions in the same way, for communication to be achieved.

But look specifically at Restall and Priest's claim 'The Boolean negation of a proposition may fail to be a proposition, that is, it may not be a hereditary piece of information: there may be no proposition such that *it* is supported by a world just because some proposition is not supported there'. What would likely be the reaction from many fellow theorists? The common reaction in this group would surely be not only to find the concept of a world 'supporting' the information that A, and so the associated mathematics of expressions like 'x ||- A', intriguing, but also, by means of such a concentration on the mathematical aspects of the matter, to let pass or sideline the question of whether Restall and Priest's use of the word 'proposition' is right. Does the Boolean negation of a proposition fail to be a proposition in the supposed circumstances? Certainly that would be so if the word 'proposition' were properly used when said to be a matter of what some world 'supports'. On Restall and Priest's definition of '||-' it indeed does not follow that x ||- ¬A given just that it is not the case that x ||- A. But what has that got to do with the Boolean negation of a proposition *not being a proposition*? The Boolean negation of 'A' is 'not-A', and the latter will express a proposition so long as the former does. Boolean negation therefore survives this latest attack; indeed there is no doing away with it. The ultimate irony is that Berto's paper is replete with Boolean negations of propositions that are themselves propositions regardless of whether any world 'supports' them!

A complication is that the general philosophical community has tried to use the term 'proposition' for what 'that'-clauses refer to in general (i.e. for what Kaplan called 'content')[7]. But this is wrong, and not just because 'that-p' and 'the proposition that p' are not interchangeable. Also, 'that'-clauses can refer to beliefs, theorems, conjectures, hypotheses etc. So 'proposition' is not the generic name for this class, but merely the name for one member of it, even if that member has special relations with some of the others. Thus, for example, if one believes that p then one will generally believe the proposition

[7] D. Kaplan, 'Demonstratives' in. J. Almog, J. Perry, and H. Wettstein (eds), *Themes From Kaplan* (Oxford: OUP, 1989).

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that p even though, if one fears that p then it does not follow that one fears the proposition that p.

Nevertheless the case of proposing that p (for some 'p') is a central one to remember in connection with our *using* sentences. So it is this case that one must most helpfully bear in mind in order to understand why it is *propositions* that are properly called true or false, rather than the sentences that express those propositions in this context or that. The standard symbolism of Modern Logic embraces no expression for 'that'-clauses, which is why it has centrally been concerned instead just with mentioned sentences, as in Tarski's T-scheme, and also, for instance, Gödel's Theorems. Certainly 'that'-clauses have been formalized in one or two places,[8] but the absence of 'that'-clauses in mainline twentieth century logic's symbolism has left it severely wanting in expressive power, despite the hope at one time that its language would be 'adequate for science'. Thus it has no ability to express the predicative equivalents of operator expressions like 'It is widely believed that p' (i.e. 'That p is widely believed'), or the associated passive transforms of expressions like 'Peter believes that p' (i.e. 'That p is believed by Peter'). Also it lacks the ability to clearly distinguish conditionals like 'If p then q' from propositional relations like 'That p implies that q', and so helps everybody to think that there are 'paradoxes of material and strict implication'.[9] But it has in all led to complete forgetfulness about much larger swathes of natural language.

Most surprisingly, given the dominance of mathematics in the mainline tradition, proper names and definite descriptions that have the same referents as 'that'-clauses, such as 'Pythagoras' Theorem', 'Goldbach's Conjecture', 'The Riemann Hypothesis' etc., have not been treated in the standard formalism. Amongst other things, that has also left some with deep ontological puzzlement about 'what propositions are', i.e. about what 'that'-clauses might refer to. For surely everyone knows what theorems, conjectures and hypotheses are, as well as (in some cases) what was proposed at the last committee meeting. Even more significantly it has led to widespread ignorance of propositional identity statements such as that in 'Goldbach's Conjecture is that every even integer greater than 2 can be expressed as the sum of two primes, and Goldbach's Conjecture is true, so that every even integer greater than 2 can be expressed as the sum of two primes is true'. This is an argument of the

[8] E.g. G. Bealer, *Quality and Concept* (Oxford: Clarendon, 1982).

[9] B.H. Slater, 'Non-Analytic Logic', *Philosophical Investigations* **37** (2014), 195–207.

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standard form 'x = y, and Px, so Py', even though the replacements for 'x' and 'y' are not of a form to be found in standard logic texts, even today. Because of this, the applicability of Predicate Logic has been taken to be far more limited than it actually is.

But more directly relevant to the theme of this paper is how two distinct sentences come to express the same proposition. The current culture in Formal Logic cannot get anywhere near to the correct account of this because of its attempt to establish a mathematical analysis of such things, when the analysis required is instead linguistic. Thus strict equivalence, i.e. being true in the same possible worlds, has been widely adopted as the definition of propositional identity, despite the well-known problems it generates. For then any proposition is identical with any conjunction of that proposition and a necessary truth. Also all necessary truths are identical, so the Law of the Excluded Middle is the same as the Law of Non-Contradiction, for instance. Also, equally, some distinct contingent remarks, such as 'it is a triangular plane figure' and 'it is a trilateral plane figure', come to express the same proposition. Instead, two sentences express the same proposition simply when they are inter-translatable.[10] If someone speaks in Japanese, for example, and one wants to understand them then their words simply must be translated into one's Mother Tongue. It is a remarkable sociological fact that this banal everyday truth needs to be pointed out at this time in history to the generality of formal logicians.

3.

The poor memory that modern logicians have of the language they are commonly using has had a great number of other, more notorious consequences that need to be corrected. For, as was mentioned before, Priest's attempts to do away with Boolean negation are not limited to the argument above against BN.[11] Priest's arguments against Boolean negation are more generally based on his wish to maintain Tarski's T-scheme, which is about quoted, or mentioned sentences instead of expressed, i.e. used sentences. But, as before, the general truth about Truth is that it is a property of expressed rather than mentioned sentences, which means that 'is true' occurs principally as a predicate of 'that'-clauses.

[10] Op. cit. note 9.

[11] B.H. Slater, 'What Priest (amongst many others) has been missing', *Ratio* **23** (2010), 184–198.

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It is the contrary focus on quoted or mentioned sentences that has caused not just Priest but also a great variety of others such a lot of trouble with the logical paradoxes. One only needs to show that Tarski's T-cheme is widely inapplicable to release the hold that the contrary idea has had on so many people, and so solve the paradoxes. For if an elementary sentence contains reference to a non-specific or fictional individual, i.e. is indexical or contains an 'empty name' (as happens in natural languages), then, while a proposition expressed by the use of such a sentence may be regarded as true, Tarski's T-scheme obviously cannot hold. The truth predicate's application to mentioned sentences, as in Tarski's T-scheme, must be limited just to cases where those sentences express one and only one determinate proposition. As a result, what have come to be thought of as rare exceptions to the scheme in connection with the paradoxes are really far more the rule.

To remedy the presumption inherent in the normal use of the scheme in connection with elementary sentences, it must therefore be strictly restricted to make clear in what conditions it applies. So we get

$$(\exists!x)(\text{'n' refers to x}) \supset (\text{T'Pn'} \equiv \text{Pn}),$$

and then a contradiction on the right hand side only reflects on the falsity of the left hand side in the given case. With the hidden presumption now explicitly expressed, it therefore becomes possible to resolve, and in a uniform way, the main logical paradoxes that have been at issue in the twentieth century. There have been no real paradoxes, in other words, merely far too much confused and illogical thinking derived from not making presumptions of determinacy explicit in Predicate Logic. Inspection of many well-known cases then reveals directly what this *Reductio* argument has established quite generally, but indirectly .[12]

As an illustration I shall comment on a point in another recent paper by Berto.[13] On page 210 of 'The Gödel Paradox and Wittgenstein's Reasons' Berto says:

> Gödel himself pointed out the analogies between his undecidable sentence and such paradoxes as Richard's or the Liar. But it seems clear that, whereas the Liar, 'This sentence is false', produces an

[12] B.H. Slater, 'Consistent Truth', *Ratio* **27** (2014), 247–261.

[13] F. Berto, 'The Gödel Paradox and Wittgenstein's Reasons', *Philosophia Mathematica* **17** (2009), 208–219.

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antinomy, with the Gödel sentence, metamathematically read as 'This sentence is not provable', no contradiction is forthcoming.

But there is no contradiction forthcoming in the Liar case either - even if it is formulated in terms of Gödel numbers – once one carefully separates sentences from the propositions they might be used to make. For the sentence 'this sentence is not true' has many different uses, and so can be used to express many different propositions. For instance, one may express the proposition that 'the earth is square' is not true this way:

This sentence is not true: 'The earth is square'.

So the sentence is not self-referential in itself, since its subject phrase has to be given the specific 'self-referential' interpretation in order for this to happen. And the most important thing to notice then is that the specific interpretation then involved makes its subject phrase refer to a sentence that contains a subject phrase without a determinate referent. For in

This sentence is not true: 'This sentence is not true',

the first 'this sentence' has a reference, but the second 'this sentence' has none, because at that second occurrence it is within quotation marks, and so is only mentioned, and not used. So the proposition then expressed is simply true, while the associated sentence is neither true nor false.

What has led many people astray on the above issues has been attachment to what often was their best friend, the standard predicate calculus. In twentieth century formal logic it is presumed that a model for some predicative formulae consists in a non-empty universe where every individual term has a unique referent. That means that no individual term is an 'empty name', or an indexical, context dependent expression. Hence any sentence 'Pa' is taken to express a proposition, and just one such – that a is P. That means that self-reference does occur as a feature of the language, when a = 'Pa', for instance. For the quoted 'a' has to have the same referent as the unquoted one. The result is that truth and provability cannot be defined in the language, i.e. Tarski's and Gödel's theorems hold. But that is no loss since the language so interpreted is unusable in any practical way. The problem is that we can sometimes be wrong when saying things like 'There is a mouse in the room', and even if we are right then there might be more than one thing we could be talking about. So the 'it' in any subsequent remark, like 'It is on the carpet', may have no determinate referent at all, or only one that is yet to be specified. If we were omniscient, and knew beforehand the truth-value of any sentence of the form '$(\exists!x)Px$' we might be able to use

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the formal language in the supposed way, i.e. in line with the standard model. But the humbling fact is that we are not omniscient in this way, making Tarski's and Gödel's theorems entirely academic.

Recognition, on this basis, of the difference between propositions and sentences also cancels one major theoretical argument against the main assumption of this paper. For the judgement that Berto is using Boolean negation in the first passage above would be reversed if, as Priest has thought, natural language itself turns out to be incapable of expressing it. As was briefly mentioned before, a formal argument that Priest has run since at least 1971 (repeated by Berto himself in the second paper above) attempts to show that Gödel's Theorems are applicable to natural language, making that language paraconsistent, and hence incapable of containing Boolean negation. For Priest holds that Gödel's first theorem shows that we can establish things in natural language that a formal system like Russell and Whitehead's *Principia Mathematica* cannot. So, if this theorem held for natural language itself, as Priest supposes, then in that language we would both be able, and yet also be unable to establish the same thing.

But it is easy to see that the fixed-point theorem cannot arise for propositions in a language including empty terms and indexicals since the propositions expressible in such a language are innumerable.[14] Indeed they cannot even be given Gödel numbers, since they are not syntactic entities with common structural parts. The technical impasse is that functions taking empty or indexical terms as arguments do not have calculable values, and so cannot be represented in a formal system. So the supposition that natural language is itself paraconsistent cannot be defended using the Priest-Berto argument. As before, what at one time was thought to be a blemish, or even a defect in natural language – that it contained individual terms that did not have just one referent – turns out to be in fact its saving grace. In addition there is a simple proof that the truth predicate attached to 'that'-clauses is consistent, since, in line with the general equivalence above between appropriate predicative and operator forms, 'that p is true' is the same as 'it is true that p', and 'it is true that' is the vacuous modality in the logic KT, which is consistent.

Above all, appreciating the distinction between propositions (as now understood) and sentences is quite crucial in seeing in full the main conclusion of this paper, i.e. why Logic is not a third person mathematical science, but instead a moral science about what ought or should be said in natural language. For, while sentences are

[14] B.H. Slater, 'Natural Language Consistency', *Logique et Analyse* **215** (2011), 409–420.

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empirical objects, propositions aren't. Most clearly in their proper guise as proposals at public meetings they are social objects produced through the use of sentences for communicative purposes in a common language area. Legalistically holding the meanings of words firm is thus the prime requirement for clarity of expression, exact understanding, and, above all, speaking the truth. And that applies not just in public meetings but also in papers published in learned journals. It is unconcern for the public nature of propositions, or maybe just indifference as to what propositions actually are, so evidently present in the first passage from Berto above, which shows again that it is lack of attention to the use of language that has drawn modern logic into its mathematical, third person studies.

HARTLEY SLATER (hartley.slater@uwa.edu.au) *is a graduate of Cambridge and Kent universities, and is now an Honorary Senior Research Fellow at the University of Western Australia. Current papers of his include, in addition to those in footnotes above, 'Prior's Individuals', and 'Gödel's and other Paradoxes' forthcoming in Synthese and Philosophical Investigations respectively.*

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