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Ion acceleration by multiple laser pulses and their spontaneous electromagnetic fields in plasma

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Abstract. The acceleration of ions by multiple laser pulses and their spontaneously generated electric and magnetic fields is investigated by using an analytical model for the latter. The relativistic equations of motion of test charged particles are solved numerically. It is found that the self-generated axial electric field plays an important role in the acceleration, and the energy of heavy test ions can reach several gigaelectronvolts.

1. Introduction

With the rapid development of laser technology, electron acceleration using high-intensity lasers in plasma has become a field of much interest [1–10]. The effect of the strong spontaneously generated electromagnetic fields produced by the laser–plasma interaction has become more and more important as the laser intensity has increased [11–28]. More recently, ion (including proton) acceleration by intense laser fields has also attracted much attention. The latter includes proton bunch production in laser–solid interactions [29,30], proton pulses as a fast ignitor [30,31], etc.

Short-pulse (femtosecond or less) laser interactions with a plasma occur on time and space scales much shorter than that of the plasma, or ion, motion. The plasma response to the pulse itself is thus only made by the plasma electrons. On a much longer time scale the ions follow the fields generated by the initial interaction. This interaction scenario is thus quite different from that of the interaction of the weaker and longer laser pulses with a plasma [7, 8], which is a problem that has been well investigated over the past 40 years. In that situation, the plasma ions have sufficient time to respond before the laser pulse has passed. Furthermore, a short laser pulse contains only a few light wave oscillations, so that the time scales of the wave envelope and the oscillations are the same. Thus, depending on the intensity and polarization, harmonics of the fundamental frequency can appear in the resulting ponderomotive force. It follows that many phenomena, such as particle acceleration and self-generation of electric and magnetic fields, are very different for the two cases [7, 8]. In particular, because of the short time involved, the self-generated fields can influence directly the acceleration of a charged particle by the laser pulse.

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In this paper, using an analytical model [22, 23] for the laser-induced self-generated or spontaneous electric and magnetic fields in an underdense plasma channel, we consider the motion of a test ion by the fields of multiple (for definiteness 10) coherent circularly polarized intense (10^{21} W cm^{-2}) short-pulse lasers, each with an arithmetical frequency difference from the preceding one. The relativistic equations of motion are integrated numerically. It is found that the ion energy can reach several gigaelectronvolts.

2. Model and equations of motion

We assume that n laser pulses of the same polarization δ (=1 or −1, corresponding to right-hand or left-hand circular polarization, respectively) but slightly different frequency ω_j propagate in the same z-direction with phase velocity $v_{\rm ph} = \omega_j/k_j = c$ (where k_j is the wave vector of the jth laser pulse and c is the velocity of light). The vector potential $\mathbf{a}_{L_j}$ of the jth laser pulse is given by

$$\mathbf{a}_{L_j} = a_{L_j x}\mathbf{e}_x + \delta a_{L_j y}\mathbf{e}_y \quad (j = 1, 2, \ldots, n), \tag{1}$$

$$a_{L_j x} = a_0 e^{-(x^2+y^2)/R_{0j}^2} e^{-(w_j t - k_j z + \varphi_j)^2/k_j^2 L_j^2} \cos(\omega_j t - k_j z + \varphi_j), \tag{2}$$

$$a_{L_j y} = a_0 e^{-(x^2+y^2)/R_{0j}^2} e^{-(w_j t - k_j z + \varphi_j)^2/k_j^2 L_j^2} \sin(\omega_j t - k_j z + \varphi_j), \tag{3}$$

$$\omega_j = \omega + j\Delta\omega, \quad \Delta\omega/\omega \ll 1, \tag{4}$$

$$\varphi_j = \varphi + j\Delta\varphi, \tag{5}$$

where the laser pulse width is $L_j = 15\lambda = 30\pi/k$, and the minimum spot size of the pulse is $R_{0j} = 7.5\lambda = 15\pi/k$. We can set $\Delta\varphi = 0$ by choosing suitable initial times for the lasers. The total intensity of the n double-Gaussian laser pulses is then

$$I = I_0 \sin^2(n\Delta\phi/2)/\sin^2(\Delta\phi/2) \quad \text{where } \Delta\phi = \Delta\omega(t - z)$$

and

$$I_0 = a_1^2/8\pi = (a_0^2/8\pi)e^{-2r^2/R_0^2} e^{-2(kz-\omega t)^2/k^2L^2}$$

gives the local intensity of each pulse. In the model, the quasi-static electric and magnetic fields generated by the n pulses are approximated by those from an equivalent single pulse with intensity $n^2 I_1$ (see [28]). When the initial electron density $n_{\rm e0}$ is given, these spontaneous quasi-static electric and magnetic fields produced by relativistic intense short-pulse laser–plasma interactions in the underdense plasma channel are then given. Taking $n_{\rm e0} = 0.36 n_{\rm cr}$, where the critical density is $n_{\rm cr} = 1.1 \times 10^{21}$ cm^{-3}, we obtain the following model fields [22, 23]:

$$\Phi = -\tfrac{1}{2}k_{Ex}x^2 - \tfrac{1}{2}k_{Ey}y^2 - \tfrac{1}{2}k_{Ez}(z - t)^2, \tag{6}$$

$$\mathbf{A} = \mathbf{a}_L - \tfrac{1}{2}k_{Bz}(y\mathbf{e}_x - x\mathbf{e}_y) + k_{By}z\mathbf{e}_x - k_{Bx}z\mathbf{e}_y, \tag{7}$$

where $\mathbf{a}_L = \sum_{j=1}^{n} \mathbf{a}_{L_j}$, the laser electric field $\mathbf{E}_L = -\partial_t \mathbf{a}_L$, and

$$k_{Ex} = k_{Ey} = \frac{1+\delta^2}{2}\frac{21\beta_5}{5R_0^2\omega^2\beta_6}|\mathbf{E}_L|^2, \tag{8}$$

$$k_{Ez} = \frac{1+\delta^2}{2}\frac{21\beta_5}{5L^2k^2\beta_6}|\mathbf{E}_L|^2, \tag{9}$$

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$$\frac{\beta_5}{\beta_6} = 0.29e^{-T_e/346} + 0.18e^{-T_e/3145}, \quad T_e = 0.511(\gamma - 1), \tag{10}$$

$$k_{Bz} = B_{sz} = -0.1\frac{\delta}{\omega}|\mathbf{E}_L|^2, \tag{11}$$

$$k_{Bx} = B_{sx} = yB_{s\theta}/r, \tag{12}$$

$$k_{By} = B_{sy} = -xB_{s\theta}/r, \tag{13}$$

$$B_{s\theta} = 0.11\frac{1+\delta^2}{2}\frac{R_0^2}{2r}e^{-(na_0-1.25)/1.35na_0}(e^{2(x^2+y^2)/R_0^2} - 1)|\mathbf{E}_L|^2. \tag{14}$$

The Hamiltonian and the equations of motion for the test particle are

$$H = \sqrt{m_0^2 + \mathbf{p}^2} - m_0 + Q\Phi, \tag{15}$$

$$d\mathbf{p}/dt = -\nabla H - Q\, d\mathbf{A}/dt, \quad d\mathbf{r}/dt = \mathbf{p}/(\gamma m_0) = \mathbf{v}, \tag{16}$$

where m_0, Q and $\mathbf{v}$ denote the ion mass, charge and velocity, respectively, $\mathbf{p}$ is the relativistic momentum and

$$\gamma = \sqrt{m_0^2 + p_x^2 + p_y^2 + p_z^2}/m_0 \tag{17}$$

is the Lorentz factor.

We use the normalizations

$$m_0 = \frac{m_0}{m_e}, \quad \mathbf{a}_L = \frac{e\mathbf{a}_L}{m_e c^2}, \quad \mathbf{A} = \frac{e\mathbf{A}}{m_e c^2}, \quad \Phi = \frac{e\Phi}{m_e c^2}, \quad \mathbf{v} = \frac{\mathbf{v}}{c}, \quad \mathbf{p} = \frac{\mathbf{p}}{m_e c},$$

$$t = \omega t, \quad r = kr, \quad Q = \frac{Q}{e}, \quad k_{Bx} = \frac{ek_{Bx}}{m_e c\omega}, \quad k_{By} = \frac{ek_{By}}{m_e c\omega}, \quad k_{Bz} = \frac{ek_{Bz}}{m_e c\omega}, \tag{18}$$

$$k_{Ex} = \frac{ek_{Ex}}{m_e c^2}, \quad k_{Ey} = \frac{ek_{Ey}}{m_e c^2}, \quad k_{Ez} = \frac{ek_{Ez}}{m_e c^2},$$

where m_e and $-e$ are the mass and charge of an electron.

The above equations are similar to those in [28] in which electron acceleration is considered, except that here $k_{Ez} \neq 0$, since k_{Ez} (or the axial self-generated electric field, associated with the electron cavitation by the laser) is important when the summed laser field is ultra-intense. Also, the mass m_0 and charge Q (normalized by m_e and e, respectively) here are so far unassigned.

3. Numerical results

We integrate the ion equations of motion (16) together with the self-field equations in three dimensions using the Runge–Kutta method. We assume that the trajectory of a test particle starts from the initial position x_0, y_0 and z_0 with initial velocity $\mathbf{v}_0$. We take $z_0 = 4L$ ($L = 15\lambda, \lambda = 1.06\ \mu$m), $x_0 = 0$, $y_0 = 1$ and $\mathbf{v}_0 = 0$ at $t = 0$. The peak of the jth laser pulse with $\varphi_j = 0$ is at $z = 0$ when $t = 0$. We can then obtain the normalized γ described by (17) of an Au ion ($m_{0\mathrm{Au}} = 1836 \times 197 + 47$ and $Q_{\mathrm{Au}} = 50$), proton ($m_{0\mathrm{p}} = 1836$ and $Q_{\mathrm{p}} = 1$) and electron $m_e = 1$, $Q_e = -1$ in dimensionless form as functions of the time t in units of ω^{-1}. The numerical results for these three species of test particles are given in Figs 1–3, respectively. The laser

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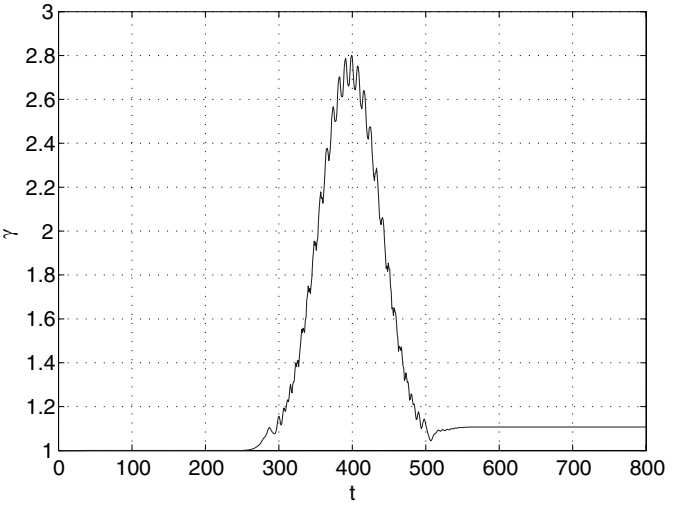


Figure 1. The γ for the Au ion given by (17) for $n = 10$, $a_0 = 45$ and $\Delta\omega = 1/300$.

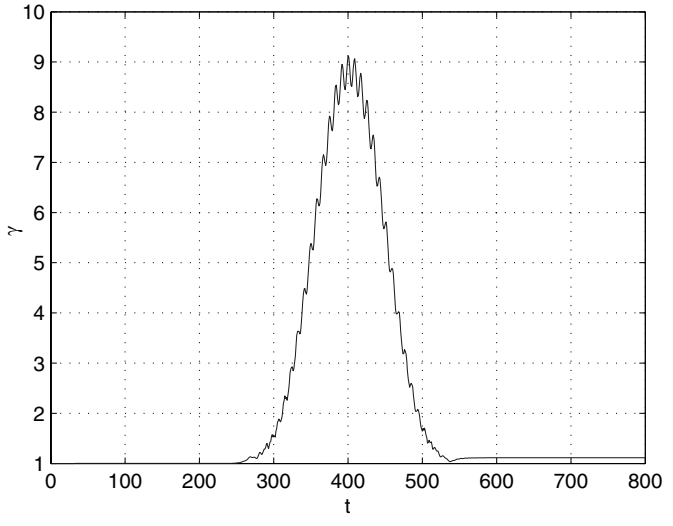


Figure 2. The proton γ given by (17) for $n = 10$, $a_0 = 45$ and $\Delta\omega = 1/300$.

parameters are $n = 10$, $a_0 = 45$ (i.e. $I_1 = 2.53 \times 10^{21}$ W cm^{-2}), and the frequency difference between the jth pulse and $(j + 1)$th pulse laser is $\Delta\omega = 1/300$ in units of the frequency ω.

Comparison of Figs 1–3 shows that, as expected, the maximum kinetic energy of the Au ion $(\gamma_{\rm Au} - 1)m_{0\rm Au} = (2.8 - 1) \times 361\,739 = 325.6$ GeV is the largest, that of the proton $(\gamma_{\rm p} - 1)m_{0\rm p} = (9 - 1) \times 1836 = 7.34$ GeV is the next largest and that of the electron $(\gamma_{\rm e} - 1)m_{0\rm e} = 4450 = 2.2$ GeV is the least. This is because, despite its large mass, a highly charged particle can be efficiently accelerated. It is also well known that a relativistic electron in the axial electric field experiences accelerating

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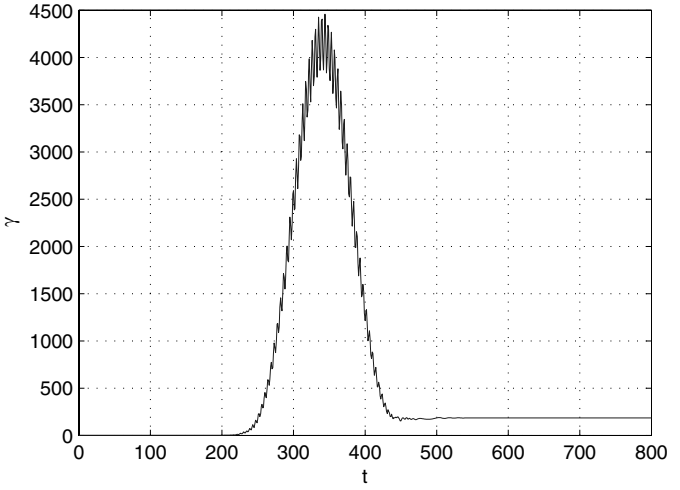


Figure 3. The electron γ given by (17) for $n = 10$, $a_0 = 45$ and $\Delta\omega = 1/300$.

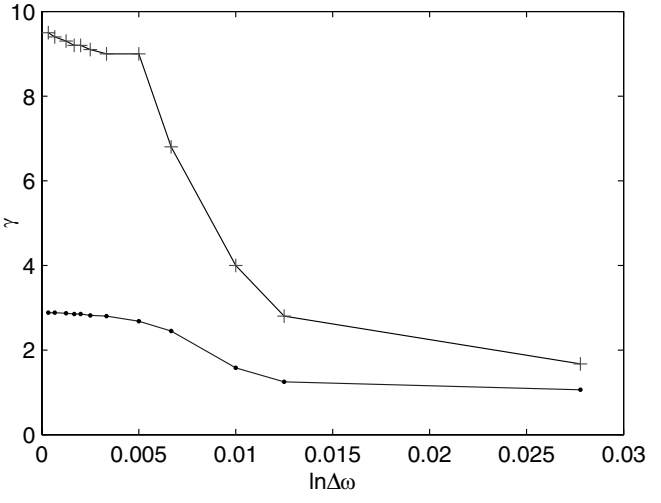


Figure 4. The γ given by (17) as a function of $\ln(\Delta\omega)$. $\Delta\omega$ from $1/36$ to $1/3000$, for $n = 10$ and $a_0 = 45$. The line with the + symbols is for the proton, the line with the dots is for the Au ion.

and focusing forces. The focusing force can be very strong for a highly relativistic electron moving in the direction of the driving laser pulses with $p_z \gg p_\perp$, where p_z and $p_\perp$ are the axial and radial components of the electron momentum, respectively. The action of the focusing force leads to transverse betatron oscillations of the electron about the z-axis. The relativistic electrons undergoing betatron oscillations will emit electromagnetic radiation. The latter can become synchrotron-like when the radial displacement of the oscillations is sufficiently large (or $p_\perp/m_{0e} \gg 1$) and can lead to much radiative energy loss by the electron [31, 32].

Figures 4 and 5 show the Lorentz factor γ of the particle described by (17) as a function of $\ln(\Delta\omega)$, for $\Delta\omega$ from $1/36$ to $1/3000$, $n = 10$ and $a_0 = 45$ (corresponding

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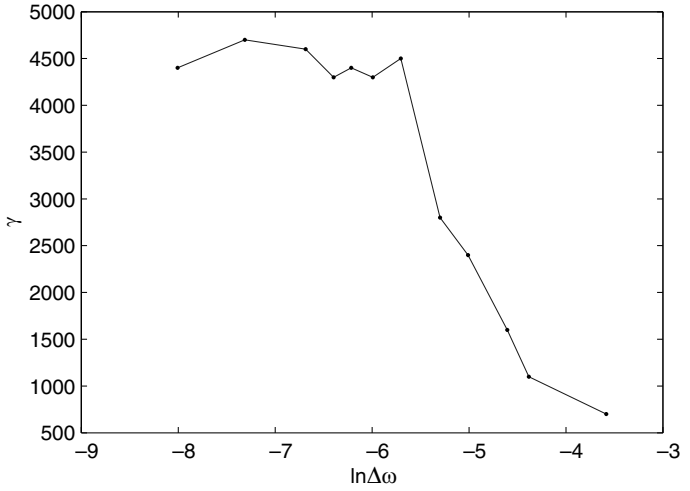


Figure 5. The electron γ given by (17) as function of $\ln(\Delta\omega)$, for $n = 10$ and $a_0 = 45$.

to $I_1 = 2.53 \times 10^{21}$ W cm^{-2}). In Fig. 4, the solid line with the + symbol is for the proton and the solid line with the dot is for the Au ion. The energies $m_0\gamma$ of the Au ion and the proton increase as $\Delta\omega$ decreases. From Fig. 5, one can see that there are several step-like increments of the energy curve, for example at $\Delta\omega = 1/300$, 1/500 and 1/1500. This behaviour can be attributed to stochastic partial Fermi-like resonance that becomes possible when the frequency increment allows for difference frequencies within the range of the cyclotron frequencies of the ions [28].

4. Discussion

The motion of a test charged particle in 10 laser pulses, each with intensity $I_1 = 2.53 \times 10^{21}$ W cm^{-2}, and their strong self-generated magnetic and electric fields are presented. It is shown that the maximum kinetic energy that an electron can reach is 2.2 GeV, that of a proton is 7.34 GeV and that of an Au ion is 325.6 GeV.

If we let $k_{Ez} = 0$, $n = 10$, $a_0 = 45$ and $\Delta\omega = 1/300$, the maximum values of the Lorentz factors $\gamma_{\rm Au} = 1.13 < 2.8$ and $\gamma_{\rm p} = 1.52 < 9$ indicate that the self-generated quasi-static space-charge electric field k_{Ez} plays the most important role in the acceleration of ions by the laser field. In order to see what other effects may also play a role in the acceleration of an ion, we have also calculated three hypothetical cases.

(1) $m_{0\rm Au} \to m_{0\rm Au}/2$ or the replacement $(\gamma_{\rm Au} - 1) \to 2(\gamma_{\rm Au} - 1)$. In this case the increase $(\gamma_{\rm Au} - 1)m_{0\rm Au}$ of the kinetic energy of the Au ion is nearly unchanged for the same $Q_{\rm Au}$.

(2) $Q_{\rm Au} \to -Q_{\rm Au}$. Here we find $\gamma_{\rm Au} = 2.54 \approx 2.8$, so $(2.8 - 2.54) \times m_{0\rm Au} = 0.26 \times 361\,739 = 47.0$ GeV implying that a positive charge is somewhat better than a negative one if the initial conditions are the same.

(3) $Q_{\rm Au} \to 25$. Here $\gamma_{\rm Au} = 1.75$, and the increased kinetic energy $(2.8 - 1.75) \times m_{0\rm Au} = 1.05 \times 361\,739 = 189.9$ GeV is large, so that as expected the charge number is important for the ion acceleration.

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It should be pointed out that the present acceleration is by the laser and the corresponding self-fields, and it differs significantly from acceleration by the laser-created wake plasma–wave fields. On the other hand, at present this seems to be the only approach with which one can identify the initial locations of the few particles that can be optimally accelerated.

Besides the usual restrictions (such as the sensitive dependence of the result on the initial position of the test particle) of the test-particle approach, the actual operation of the present multi-laser scheme will also depend on the validity (for the intense laser field here) of the self-field model [22,23] for near-coherent superposition of many laser pulses to increase the laser strength without significantly changing the interaction properties, which is a problem that may also be relevant for other novel particle acceleration schemes [32–34]. Many of the latter schemes call for lasers of high intensity, high fluence and, most importantly, high controllability. Such laser systems are probably beyond current laser technology, and further research and development are still needed to meet these requirements.

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