10 ε'/ε in the Standard Model

10.1 Preliminaries

We will now discuss the ratio ε'/ε that measures the size of the direct CP violation in $K_L \to \pi\pi$ decays relative to the indirect (mixing induced) CP violation in this decay described by $\varepsilon_K \equiv \varepsilon$. After tremendous efforts of experimentalists over two decades, the present experimental world average from NA48 [644] and KTeV [645, 646] collaborations reads

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}. \tag{10.1}$$

The goal of this chapter is to describe in some detail the calculation of ε'/ε within the SM and to compare it with the measured value given here. In the SM ε' is governed by QCD penguins (QCDP) but receives also an important contribution from electroweak penguins (EWP). In the SM QCDP give in total a positive contribution, while the EWP give a negative one. This partial cancellation between these two contributions implies that already for three decades it was not possible to provide a precise prediction for ε'/ε. But as we will see later, significant progress has been made in calculating this ratio in the last years.

Both contributions are subject to the following uncertainties:

- CKM uncertainties that have been significantly reduced in the last years. We have seen this in Chapter 8. What matters here is $\mathrm{Im}\lambda_t^{(K)}$ to be denoted in this chapter simply by $\mathrm{Im}\lambda_t$.
- Perturbative QCD uncertainties that have been significantly reduced at the beginning of the 1990s through complete renormalization group improved calculations of NLO QCD and electroweak corrections to the Wilson coefficients of QCD and EW penguin operators. We have discussed them in Section 6.4. The dominant NNLO corrections to EWP contributions were calculated already in 1999 [647] and the complete ones to QCDP recently [648, 649].
- Hadronic uncertainties originating in the matrix elements of QCDP and EWP operators. These are in fact the largest uncertainties but also here significant progress has been made recently. We have discussed this progress already in Sections 7.2 and 7.3.
- Isospin breaking effects, which decrease the QCD penguin contribution making the cancellation in question stronger.

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- Final state interactions, which according to ChPT estimates strongly enhance QCDP contribution and suppress the EWP ones, making the cancellation in question weaker and bringing ε'/ε close to the experimental data. But as discussed in Section 7.2, this claim as far as QCDP contributions are concerned is not supported by DQCD. The verdict from LQCD should be known soon.

The partial cancellation of QCDP contributions and EWP contributions combined with the uncertainties just listed remains even today a challenge for theorists, and as we will see at the end of this chapter, the precision of the SM prediction for ε'/ε cannot presently compete with the experimental one, but it appears that the SM value of ε'/ε is significantly below its experimental value in (10.1), opening speculation on which NP could explain this discrepancy. We will describe various suggestions in the BSM parts of this book, but first we will concentrate on ε'/ε in the SM.

Before, going into details it is appropriate to summarize briefly the history of the calculations of ε'/ε, which started in the second half of the 1970s when one still thought that m_t is much smaller than M_W [650–652]. For $m_t \ll M_W$ only QCDP play a substantial role. Over the 1980s these calculations were refined through the inclusion of isospin braking in the quark masses [273, 653], the inclusion of QED penguin effects for $m_t \ll M_W$ [273, 653, 654], and improved estimates of hadronic matrix elements in the framework of the DQCD [221, 227, 228] presented in Section 7.2. This era of ε'/ε culminated in the analyses in [655, 656], where QCDP, EWP (γ and Z^0 penguins), and the relevant box diagrams were included for arbitrary top quark mass. The strong cancellation between QCDP and EWP for $m_t > 150$ GeV found in these papers was confirmed by other authors [657, 658]. In fact, at that time, due to the aforementioned cancellation between QCDP and EWP contributions, the vanishing of ε'/ε in the SM could not be excluded.

However, all these calculations were done in the leading logarithmic approximation (e.g., one-loop anomalous dimensions of the relevant operators) with the exception of the m_t-dependence, which in the analyses [655–657] has been already included at the NLO level. While such a procedure is not fully consistent, m_t dependence enters ε'/ε first at NLO in the RG improved calculation, it allowed for the first time to exhibit the strong m_t-dependence of the EWP contributions, which is not seen in a strict leading logarithmic approximation.

During the 1990s considerable progress was made by calculating complete NLO corrections to ε' [122–125, 146, 659]. This means both QCD and electroweak corrections. First steps toward the inclusion of NNLO corrections have been already made in [200, 647], in particular in [647] dominant NNLO corrections to EWP have been calculated, but only very recently these calculations have been completed for QCDP [648, 649]. Yet, even more important is the significant improvement in the evaluation of the hadronic matrix elements of QCDP and EWP operators. We have already described this progress in Sections 7.2 and 7.3, but for convenience we will briefly summarize the present status. The reviews of ε'/ε prior to the recent progress can be found in [50, 660–662].

After these general remarks let us enter the details.

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10.2 Basic Formulas

The parameter ε' has been defined in (8.28). It is useful to rewrite this formula in terms of the real and imaginary parts of the isospin amplitudes $A_0 \equiv A(K \to (\pi\pi)_{I=0})$ and $A_2 \equiv A(K \to (\pi\pi)_{I=2})$. The most useful for our discussion is the formula derived in [296]. It reads

$$\boxed{\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\,(1-\hat{\Omega}_{\rm eff}) - \frac{1}{a}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]}, \tag{10.2}$$

with (ω_+, a) and $\hat{\Omega}_{\rm eff}$ given in 2019 as follows

$$\omega_+ = a\,\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} = (4.53 \pm 0.02)\times 10^{-2}, \qquad a = 1.017, \qquad \hat{\Omega}_{\rm eff} = 17.0 \pm 9.0 \times 10^{-2}. \tag{10.3}$$

Here a and $\hat{\Omega}_{\rm eff}$ summarize isospin breaking corrections and include strong isospin violation ($m_u \neq m_d$), the correction to the isospin limit coming from $\Delta I = 5/2$ transitions and electromagnetic corrections [663–665, 1339]. $\mathrm{Im}A_0$ receives dominantly contributions from QCD penguins but also from EWP. $\mathrm{Im}A_2$ receives contributions exclusively from EWP.

$\hat{\Omega}_{\rm eff}$ differs from $\Omega_{\rm eff}$ in [663, 664, 1339] as it does not include EWP contributions to $\mathrm{Im}A_0$ in contrast to $\Omega_{\rm eff}$. We find it more natural to calculate $\mathrm{Im}A_0$ including both QCDP and EWP contributions, as this allows to keep track of NP contributions to $\mathrm{Im}A_0$. The dominant EWP contribution to ε'/ε is of course present in $\mathrm{Im}A_2$. Strictly speaking, ε'/ε is a complex quantity, and the expression in (10.2) applies to its real part, but its phase is so small that we can drop the symbol "Re" in all expressions that follow to simplify the notation.

The amplitudes $\mathrm{Re}A_{0,2}$ are then extracted from the branching ratios on $K \to \pi\pi$ decays in the isospin limit. Their values are given by [50]

$$\mathrm{Re}A_0 = 27.04(1)\times 10^{-8}\,\mathrm{GeV}, \qquad \mathrm{Re}A_2 = 1.210(2)\times 10^{-8}\,\mathrm{GeV}. \tag{10.4}$$

The main reason for taking these amplitudes from the data is the unpleasant fact that until today no precise values of these two amplitudes are known within the SM, although as discussed in Sections 7.2 and 7.3 significant progress in calculating them has been made. In particular some consensus has been reached through large N and LQCD calculations that this rule is governed to a large extent by low-energy QCD dynamics. Taking these two amplitudes from experiment assumes that they can be fully described within the SM. While this assumption could still be violated at the level of 20% by NP contributions, this strategy can be defended as at the end the values of these two amplitudes must be as listed in (10.4).

We thus give up in the context of the phenomenology of ε'/ε the full explanation of the measured values of $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ and will concentrate in what follows on the imaginary parts of the amplitudes in (10.2). Being related to CP violation and the top quark physics

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in the SM, they should be dominated by short-distance contributions up to hadronic matrix elements of the involved operators.

In what follows it will be useful to state already now that

- $\mathrm{Im}A_0$ being dominated by QCDP is very weakly dependent on m_t. It is suppressed by isospin breaking corrections represented by $\hat{\Omega}_{\rm eff}$.
- $\mathrm{Im}A_2$ being dominated by EWP increases strongly with m_t. As the value of m_t is presently known to a high accuracy, the uncertainty of ε'/ε from m_t is unimportant today but was crucial before the top quark was discovered.
- As seen in (10.2) the $\mathrm{Im}A_0$ contribution to ε'/ε is suppressed by the ratio $\mathrm{Re}A_2/\mathrm{Re}A_0 \approx 1/22$ relative to $\mathrm{Im}A_2$ contributions. This suppression is overcompensated by the ratio of Wilson coefficients of QCDP operators and EWP operators, so that the sign of ε'/ε is governed by $\mathrm{Im}A_0$ contribution.

The calculation of $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$ is then performed using the effective Hamiltonian for $\Delta S = 1$ transitions [122–125, 146, 659]

$$\boxed{\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_{i=1}^{10} \left(z_i(\mu) + \tau\, y_i(\mu)\right) Q_i(\mu), \quad \tau \equiv -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*},} \tag{10.5}$$

which appeared in various forms in previous chapters. But it is convenient to list again the contributing operators:

Current–Current

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}\,(\bar{u}_\beta d_\alpha)_{V-A}, Q_2 = (\bar{s}u)_{V-A}\,(\bar{u}d)_{V-A}, \tag{10.6}$$

QCD Penguins

$$Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V-A}, Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A}, \tag{10.7}$$

$$Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A}, Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A}, \tag{10.8}$$

Electroweak Penguins

$$Q_7 = \frac{3}{2}(\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V+A}, Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V+A}, \tag{10.9}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V-A}, Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V-A}. \tag{10.10}$$

Here, α, β denote color indices and e_q denotes the electric quark charges reflecting the electroweak origin of $Q_7, \ldots, Q_{10}$. Finally, $(\bar{s}d)_{V-A} \equiv \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha$.

We next set[1]

$$\alpha_s(M_Z) = 0.1181, \qquad m_t(M_W) = 173.2\,\mathrm{GeV}, \qquad \mu = m_c = 1.3\,\mathrm{GeV} \tag{10.11}$$

[1] This value of m_t corresponds to $m_t(m_t) = 163.5\,\mathrm{GeV}$. We also set $\mu_W = M_W$.

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to find at NLO in the NDR-$\overline{\text{MS}}$ scheme at $\mu = 1.3\,\text{GeV}$ [1333]

$$\boxed{z_1 = -0.3938, \qquad z_2 = 1.2020,} \tag{10.12}$$

$$\boxed{y_3 = 0.0270, \qquad y_4 = -0.0545, \qquad y_5 = 0.0056, \qquad y_6 = -0.0826,} \tag{10.13}$$

$$\boxed{y_7/\alpha = -0.0241, \qquad y_8/\alpha = 0.1313, \qquad y_9/\alpha = -1.4946, \qquad y_{10}/\alpha = 0.5327.} \tag{10.14}$$

We give several decimal places so that the readers checking our results will not worry about rounding errors. We will use these central values in what follows. The uncertainties in these values are included in the final result for ε'/ε [1333].

10.3 Hadronic Matrix Elements

10.3.1 Preliminaries

The main source of uncertainty in the calculation of ε'/ε are the hadronic matrix elements of the operators Q_i entering the isospin amplitudes,

$$A_I \equiv \langle(\pi\pi)_I|\mathcal{H}_{\text{eff}}|K\rangle, \tag{10.15}$$

that is

$$\langle Q_i\rangle_I \equiv \langle(\pi\pi)_I\,|Q_i|\,K\rangle, \qquad I = 0, 2. \tag{10.16}$$

They generally depend on the scale μ and on the renormalization scheme used for the operators. These two dependencies are cancelled by those present in the coefficients $C_i(\mu)$ so that the resulting amplitudes do not depend on μ and on the scheme used to renormalize the operators. We will work exclusively in the NDR-$\overline{\text{MS}}$ scheme and for scales $\mu \le m_c$, although in [123] also extensive discussion of scales above m_c can be found.

As we have set the real parts of the isospin amplitudes to the experimental data in (10.4) we may use this information to determine as many matrix elements $\langle Q_i\rangle_I$ as possible or equivalently at least some parts of the imaginary parts of isospin amplitudes governing ε'/ε. The remaining pieces will have to be taken from LQCD and DQCD calculations. This has been suggested already in [123] and recently improved in [296]. Here we will present this strategy following the latter paper.

For $\mu \le m_c$, when the charm quark has been integrated out, only seven operators are independent of each other. Eliminating then Q_4, Q_9 and Q_{10} in terms of the remaining seven operators results in the following important relations in the isospin limit [123]:

$$\langle Q_4\rangle_0 = \langle Q_3\rangle_0 + \langle Q_2\rangle_0 - \langle Q_1\rangle_0, \tag{10.17}$$

$$\langle Q_9\rangle_0 = \frac{3}{2}\langle Q_1\rangle_0 - \frac{1}{2}\langle Q_3\rangle_0, \tag{10.18}$$

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$$\langle Q_{10}\rangle_0 = \langle Q_2\rangle_0 + \frac{1}{2}\langle Q_1\rangle_0 - \frac{1}{2}\langle Q_3\rangle_0, \tag{10.19}$$

$$\langle Q_9\rangle_2 = \langle Q_{10}\rangle_2 = \frac{3}{2}\langle Q_1\rangle_2, \tag{10.20}$$

where we have employed

$$\langle Q_1\rangle_2 = \langle Q_2\rangle_2. \tag{10.21}$$

As stressed in [123], in the NDR-$\overline{\text{MS}}$ scheme, the relation (10.17) receives an $O(\alpha_s)$ correction due to the presence of evanescent operators that have to be taken into account when using Fierz identities in its derivation. The other relations do not receive such corrections. The complete expression for $\langle Q_4\rangle_0$ in the NDR-$\overline{\text{MS}}$ scheme reads [123]

$$\langle Q_4\rangle_0 = \langle Q_3\rangle_0 + \langle Q_2\rangle_0 - \langle Q_1\rangle_0 - \frac{\alpha_s}{4\pi}\Big(\langle Q_6\rangle_0 + \langle Q_4\rangle_0 - \frac{1}{3}\langle Q_3\rangle_0 - \frac{1}{3}\langle Q_5\rangle_0\Big), \tag{10.22}$$

which of course then has to be solved for $\langle Q_4\rangle_0$. However, due to the partial cancellation between the matrix elements $\langle Q_4\rangle_0$ and $\langle Q_6\rangle_0$, and the smallness of the matrix elements of Q_3 and Q_5, this correction affects the determination of $\langle Q_4\rangle_0$ by at most a few percent and can be neglected. This procedure is supported both by the results on hadronic matrix elements from DQCD [244] and from RBC-UKQCD collaboration [276] that we discussed in Sections 7.2 and 7.3.

Setting the contribution of Q_3 to zero[2] and using the operators

$$Q_\pm = \frac{1}{2}(Q_2 \pm Q_1), \tag{10.23}$$

the formulas (10.17)–(10.20) read

$$\langle Q_4\rangle_0 = 2\langle Q_-\rangle_0, \tag{10.24}$$

$$\langle Q_9\rangle_0 = \frac{3}{2}\left(\langle Q_+\rangle_0 - \langle Q_-\rangle_0\right), \tag{10.25}$$

$$\langle Q_{10}\rangle_0 = \frac{3}{2}\langle Q_+\rangle_0 + \frac{1}{2}\langle Q_-\rangle_0, \tag{10.26}$$

$$\langle Q_9\rangle_2 = \langle Q_{10}\rangle_2 = \frac{3}{2}\langle Q_+\rangle_2, \tag{10.27}$$

which reduces the number of independent $(V-A)\otimes(V-A)$ matrix elements entering $\text{Re}A_{0,2}$ and $\text{Im}A_{0,2}$ to three. On the other hand, to an excellent approximation the amplitudes $\text{Re}A_0$ and $\text{Re}A_2$ at $\mu = m_c$ are fully described by the operators Q_- and Q_+, discussed at length in Chapter 5, so that we can write

$$\text{Re}A_0 = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \left(z_+\langle Q_+\rangle_0 + z_-\langle Q_-\rangle_0\right), \tag{10.28}$$

$$\text{Re}A_2 = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \, z_+\langle Q_+\rangle_2, \tag{10.29}$$

with $z_\pm$ being WCs of these two operators.

[2] In numerical results all operators are usually taken into account.

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We introduce next the ratio

$$q \equiv \frac{z_+(\mu)\langle Q_+(\mu)\rangle_0}{z_-(\mu)\langle Q_-(\mu)\rangle_0}, \qquad z_\pm = z_2 \pm z_1, \tag{10.30}$$

and to remove the effects of $\hat{\Omega}_{\rm eff} \neq 0$ in EWP contributions to $\mathrm{Im}A_0$, we write

$$\mathrm{Im}A_0 = (\mathrm{Im}A_0)^{\rm QCDP} + b\,(\mathrm{Im}A_0)^{\rm EWP}, \qquad b = \frac{1}{a\,(1-\hat{\Omega}_{\rm eff})} \tag{10.31}$$

with the first term including the contributions from Q_{3-6} and the second from Q_{7-10}. Except for the tiny corrections due to $a \neq 1$, this procedure is equivalent to multiplying the coefficients y_{3-6} by $(1-\hat{\Omega}_{\rm eff})$ leaving y_{7-10} unchanged.

This allows us to express the ratios involving only $(V-A)\otimes(V-A)$ operators that will enter our basic formula for ε'/ε as follows:

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_{V-A} = \mathrm{Im}\tau\,\frac{[4y_4 - b(3y_9 - y_{10})]}{2(1+q)z_-} + b\,\mathrm{Im}\tau\,\frac{3q(y_9+y_{10})}{2(1+q)z_+}, \tag{10.32}$$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{V-A} = \mathrm{Im}\tau\,\frac{3(y_9+y_{10})}{2z_+}. \tag{10.33}$$

Besides the CKM ratio τ, the first ratio depends only on the WCs and the single hadronic ratio q to which we will return later. On the other hand the second ratio is free from hadronic uncertainties, being fully determined by the Wilson coefficients z_+, y_9, y_{10} and by τ.

The remaining contributions to $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$ are due to $(V-A)\otimes(V+A)$ operators and are dominated by the operators Q_6 and Q_8, respectively. We find this time

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_6 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\,y_6\,\frac{\langle Q_6\rangle_0}{\mathrm{Re}A_0}, \tag{10.34}$$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_8 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\,y_8^{\rm eff}\,\frac{\langle Q_8\rangle_2}{\mathrm{Re}A_2}. \tag{10.35}$$

Contributions from Q_3 and Q_5 are very suppressed but can and have been included in our numerical error estimate. We refer to Appendix A in [296]. We have also taken into account the small effect of $\langle Q_7\rangle_2$, for which a relatively precise lattice prediction exists [275], through the substitution

$$y_8 \;\to\; y_8^{\rm eff} \equiv y_8 + p_{72}\,y_7, \tag{10.36}$$

which is included in writing (10.35). Here $p_{72} \equiv \langle Q_7\rangle_2/\langle Q_8\rangle_2 = 0.222$ for central values of [275].

The matrix elements of the Q_6 and Q_8 operators calculated in the context of DQCD are given in terms of the parameters $B_6^{(1/2)}$, $B_8^{(3/2)}$, and $B_8^{(1/2)}$ in (7.44)–(7.46). In the strict large N limit [227, 230, 273]

$$B_6^{(1/2)} = B_8^{(3/2)} = B_8^{(1/2)} = 1, \qquad \text{(large N Limit)}. \tag{10.37}$$

Their values beyond this limit were already discussed in Sections 7.2 and 7.3 and are summarized in the next section.

Finally, the parameter q that enters (10.32) can be estimated on the basis of the present knowledge of matrix elements from DQCD and LQCD approaches with the result [296]

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$$0 \le q \le 0.1. \tag{10.38}$$

While, on the basis of the arguments in [296], this is a credible range, one should stress that our phenomenological results that follow would change very little even if we enlarged this range by a factor of a few: q is simply too small to introduce a large error on ε'/ε.

10.3.2 Present Status of $B_6^{(1/2)}$, $B_8^{(3/2)}$, and $B_8^{(1/2)}$

This status has been already presented in Sections 7.2 and 7.3, but for readers' convenience it is useful to quote them again. They can be extracted from the matrix elements calculated by the RBC-UKQCD collaboration [275, 276] and read [212, 296]

$$B_6^{(1/2)} = 0.57 \pm 0.19, \qquad B_8^{(3/2)} = 0.76 \pm 0.05, \qquad B_8^{(1/2)} = 1.0 \pm 0.2, \tag{10.39}$$

to be compared with their values in the strict large N limit of QCD in (10.37).

As discussed in Section 7.2, these results are compatible with those obtained from the DQCD approach in which one can demonstrate analytically that $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are suppressed below their large-N limit by the meson evolution from low-energy scales $O(m_\pi)$ to scales $O(1\,\text{GeV})$ [244] so that

$$B_6^{(1/2)} < B_8^{(3/2)} < 1 \qquad \text{(DQCD)} \tag{10.40}$$

with $B_6^{(1/2)}$ in the ballpark of 0.6 and $B_8^{(3/2)} = 0.80 \pm 0.10$. While this suppression of both parameters through long-distance QCD dynamics and their decrease with increasing μ in the range from $O(m_\pi)$ to scales $O(1\,\text{GeV})$ is significant, for $\mu \ge 1\,\text{GeV}$, they exhibit a very weak scale dependence [123]. More details can be found in Section 7.2.

10.4 A Convenient Formula for ε'/ε in the SM

Before turning to quantitative phenomenology, to make easier connection with the phenomenological literature and aid discussion of our results, we summarize the discussion so far in a concise formula (derived first in [123]) for ε'/ε that exhibits the sensitivity to the two most important hadronic matrix elements $B_6^{(1/2)}$ and $B_8^{(3/2)}$ transparently.

Using the effective Hamiltonian (10.5) and the experimental data for ω, $\text{Re}A_0$ and ε_K, we find [296]

$$\boxed{\frac{\varepsilon'}{\varepsilon} = \text{Im}\lambda_t \cdot \left[a(1-\hat{\Omega}_{\text{eff}})\, P^{(1/2)} - P^{(3/2)} \right],} \tag{10.41}$$

where

$$P^{(1/2)} = \sum P_i^{(1/2)} = r \sum y_i \langle Q_i \rangle_0, \tag{10.42}$$

$$P^{(3/2)} = \sum P_i^{(3/2)} = \frac{r}{\omega} \sum y_i \langle Q_i \rangle_2, \tag{10.43}$$

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with

$$r = \frac{G_F\,\omega}{2\,|\varepsilon_K|\,\mathrm{Re}A_0}, \qquad \omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}. \tag{10.44}$$

In (10.42) and (10.43) the sums run over all contributing operators. Therefore in $P^{(1/2)}$ in the case of EWP contributions we have to take into account the correction $b \neq 1$ defined in (10.31). It should be mentioned that $P^{(1/2)}$ and $P^{(3/2)}$ do not depend on h, the normalization factor, discussed after (7.45). We stress it because in [296] and in RBC-UKQCD papers $h = \sqrt{3/2}$ has been used and in this book we set $h = 1$.

Writing then

$$P^{(1/2)} = a_0^{(1/2)} + a_6^{(1/2)}\,B_6^{(1/2)} \equiv a_4^{(1/2)} + b a_{\mathrm{EWP}}^{(1/2)} + a_6^{(1/2)}\,B_6^{(1/2)}\,, \tag{10.45}$$

$$P^{(3/2)} = a_0^{(3/2)} + a_8^{(3/2)}\,B_8^{(3/2)}\,, \tag{10.46}$$

with the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ taken at $\mu = m_c$ and using the expressions (10.32)–(10.35), we find:

$$a_0^{(1/2)} = r_1\left[\frac{[4y_4 - b(3y_9 - y_{10})]}{2(1+q)z_-} + b\,\frac{3q(y_9+y_{10})}{2(1+q)z_+}\right] + r_2\,b\,y_8\,\frac{\langle Q_8\rangle_0}{\mathrm{Re}A_0}, \tag{10.47}$$

$$a_6^{(1/2)} = r_2\,y_6\,\frac{\langle Q_6\rangle_0}{B_6^{(1/2)}\mathrm{Re}A_0}, \tag{10.48}$$

$$a_0^{(3/2)} = r_1\,\frac{3(y_9+y_{10})}{2z_+}, \tag{10.49}$$

$$a_8^{(3/2)} = r_2\,y_8^{\mathrm{eff}}\,\frac{\langle Q_8\rangle_2}{B_8^{(3/2)}\mathrm{Re}A_2}, \tag{10.50}$$

where

$$r_1 = \frac{\omega}{\sqrt{2}|\varepsilon_K|}\frac{1}{V_{ud}V_{us}^*}, \qquad r_2 = \frac{\omega}{2|\varepsilon_K|}G_F. \tag{10.51}$$

Further details on these formulas can be found in [296], in particular in an Appendix of that paper.

With the values in (10.12)–(10.14), we find at NLO in the NDR-$\overline{\mathrm{MS}}$ scheme [1333]

$$\boxed{a_0^{(1/2)} = -4.19, \qquad a_6^{(1/2)} = 17.68, \qquad a_0^{(3/2)} - a_{\mathrm{EWP}}^{(1/2)} = -2.00, \qquad a_8^{(3/2)} = 8.25.} \tag{10.52}$$

We observe that the ratio ε'/ε is governed in the SM by the following four contributions:

i) The term $a_0^{(1/2)}$ in $P^{(1/2)}$ in (10.45). As seen in (10.47) this term is governed by the contribution of the operator Q_4, denoted by $a_4^{(1/2)}$ and includes also smaller contributions from EWP operators Q_8, Q_9 and Q_{10} summarized by $b a_{\mathrm{EWP}}^{(1/2)}$. We find that this term is *negative* and only weakly dependent on q. Also the dependences on α_s and renormalisation scheme (see [123]) are weak. These weak dependences originate from the fact that in our approach the matrix elements entering the first term in $P^{(1/2)}$ cancel out. The dependence on m_t resulting from the contributions of sub-leading EWP operators turns out to be very weak [296].

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ii) The contribution of $(V-A)\otimes(V+A)$ QCDP operators to $P^{(1/2)}$, given by the second term in (10.45). This contribution is large and *positive* and is dominated by the operator Q_6.

iii) The contribution of the $(V-A)\otimes(V-A)$ EWP operators Q_9 and Q_{10} to $P^{(3/2)}$, represented by the first term in $P^{(3/2)}$. As in the case of the contribution i), the matrix elements contributing to $a_0^{(3/2)}$ cancel out in the SM. Consequently, the scheme and α_s dependences of $a_0^{(3/2)}$ are weak. As seen in (10.49) the m_t-dependence of $a_0^{(3/2)}$ results from the corresponding dependence of y_9+y_{10}, but the precision on m_t increased by much in the last two decades. $a_0^{(3/2)}$ contributes *positively* to ε'/ε.

iv) The contribution of the $(V-A)\otimes(V+A)$ EWP operators Q_7 and Q_8 to $P^{(3/2)}$, represented by the second term in (10.46). This contribution is dominated by Q_8. It contributes *negatively* to ε'/ε.

The competition between these four contributions is the reason why it is difficult to predict ε'/ε precisely. In this context, one should appreciate the virtue of our approach: The contributions i) and iii) can be determined rather precisely by CP-conserving data so that the dominant uncertainty in our approach in predicting ε'/ε resides in the values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$, which for scales $\mu \geq 1$ GeV are practically μ independent.

The expression (10.41) can be put into a formula that shows explicitly the dependence on m_t and m_s. It can be found in Appendix B of [296]. See also [666, 667]. Here we would like just to insert the values in (10.52) into (10.41), which will allow us to see various effects transparently. The final numerical results in [1333] are based on the formulas listed here with all uncertainties taken into account.

We find then [1333]

$$\boxed{\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} = 10^{-4}\left[\frac{{\rm Im}\lambda_{\rm t}}{1.4\cdot 10^{-4}}\right]\left[a(1-\hat{\Omega}_{\rm eff})(-5.9+24.7\,B_6^{(1/2)})+2.9-11.5\,B_8^{(3/2)}\right].} \tag{10.53}$$

It is an update of a similar formula presented in [296].

Before proceeding, it should be emphasized that although the NLO analysis of ε'/ε reduces renormalization scheme dependence in the QCDP sector, it is still insensitive to the choice of μ_t in $m_t(\mu_t)$. This dependence can only be removed through NNLO calculations but in the QCDP sector it is already weak at the NLO level because of the weak dependence of the QCDP contributions on m_t. On the other hand, as pointed out already in [647], the EWP contributions at the NLO level suffer from a number of unphysical dependences.

- First of all there is the renormalization scheme dependence with ε'/ε in the HV scheme generally smaller than in the NDR scheme.
- The dependence on μ_t, which is much larger than in the QCDP sector because the EWP contributions exhibit much stronger dependence on m_t. Increasing μ_t makes the value of m_t smaller, decreasing the EWP contribution and thereby making ε'/ε larger. At NLO there is no QCD correction that could cancel this effect.
- The dependence on the choice of the matching scale μ_W. It turns out that with increasing μ_W in the EWP contribution, the value of ε'/ε decreases.

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We have discussed such uncertainties and their reduction in Section 6.3.3 in the context of the NLO analysis of $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing. But in the case of ε'/ε such discussion is only possible at the NNLO level. In particular the scales μ_W and μ_t can be chosen to be equal or different, and they could be varied independently in the ranges $60\,\text{GeV} \le \mu_W \le 160\,\text{GeV}$ and $80\,\text{GeV} \le \mu_t \le 300\,\text{GeV}$ implying significant uncertainties in the NLO prediction for ε'/ε as demonstrated in [647, 1333]. We refer to these papers for details. In particular the leftover dependence of ε'/ε on μ_t is bothering. In obtaining the NLO formula (10.53) we have set $\mu_W = \mu_t = M_W$.

Fortunately all these uncertainties have been practically removed in the NNLO matching at the electroweak scale performed in [647]. In the NDR scheme, used in all recent analyses, these corrections enhance EWP contribution by roughly 8%, thereby leading to a *negative* shift in ε'/ε that depends on $B_8^{(3/2)}$ and $\text{Im}\lambda_t$ and in fact as demonstrated at the end of this chapter on the chosen values of μ_t and μ_W in the NLO expressions. The shift of 8% applies to $\mu_W = \mu_t = M_W$.

Let us find out how our formula in (10.53) is modified through these corrections. To this end we will set $\mu_t = \mu_W = M_W$ as at NNLO any choice will lead practically to the same result for ε'/ε. This is not the case at NLO as illustrated in [647, 1333] and shown briefly next.

Inspecting the results in [647], this NNLO effect implies the modification of the values of the WCs of EWP operators so that we find now instead of (10.14) [1333]

$$\boxed{y_7/\alpha = -0.0258, \qquad y_8/\alpha = 0.1405, \qquad y_9/\alpha = -1.3302, \qquad y_{10}/\alpha = 0.4049.} \tag{10.54}$$

This in turn implies shifts in the coefficients in (10.52) so that we find now [1333]

$$\boxed{a_4^{(1/2)} = -4.19\,, \qquad a_6^{(1/2)} = 17.68\,, \qquad a_0^{(3/2)} - a_{\text{EWP}}^{(1/2)} = -2.00\,, \qquad a_8^{(3/2)} = 8.82\,.} \tag{10.55}$$

The coefficients $a_4^{(1/2)}$ and $a_6^{(1/2)}$ are expected to be modified slightly by the NNLO QCD corrections to QCDP. Consequently the formula (10.53) now reads [1333]

$$\boxed{\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = 10^{-4}\left[\frac{\text{Im}\lambda_t}{1.4\cdot 10^{-4}}\right]\left[\,a(1-\hat{\Omega}_{\text{eff}})(-5.9 + 24.7\,B_6^{(1/2)}) + 2.8 - 12.3\,B_8^{(3/2)}\,\right],} \tag{10.56}$$

implying an additional downward shift of ε'/ε relative to the NLO value for $\mu_t = \mu_W = M_W$ given by

$$\Delta\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = -0.8\left[\frac{\text{Im}\lambda_t}{1.4\cdot 10^{-4}}\right]B_8^{(3/2)}\,10^{-4}. \tag{10.57}$$

For $B_8^{(3/2)} = 0.76$ this amounts to the shift of approximately -0.6×10^{-4}. But choosing $\mu_W = M_W$ and $\mu_t = m_t = 163\,\text{GeV}$ would increase the difference between NLO and NLLO to roughly -1.4×10^{-4}. This could appear small in view of other uncertainties.

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However, on the one hand, potential scale and renormalization scheme uncertainties have been removed in this manner, and on the other hand, one day such corrections could turn out to be relevant.

10.5 Numerical Analysis of ε'/ε

10.5.1 2015 Hints for the ε'/ε Anomaly

One can check using the preceding expressions that with $B_6^{(1/2)} = B_8^{(3/2)} = 1$, as found in the large N limit, the resulting ε'/ε is roughly by a factor of two below the central experimental value in (10.1). In fact this was a typical value obtained by the Rome [668] and Munich [669] groups in the 1990s, which used $B_6^{(1/2)} = B_8^{(3/2)} = 1$.

However, the authors in [280–286] using the ideas from ChPT made a strong claim that final state interactions (FSI) enhance $B_6^{(1/2)}$ significantly above unity and suppress $B_8^{(3/2)}$ below it so that the SM value for ε'/ε according to them is fully consistent with experiment. Albeit only a very inaccurate value $(17 \pm 9) \times 10^{-4}$ [286] could be obtained. As there are some different views in the literature what is meant by FSI, let me call the effects pointed out in these papers simply *pion loops*. In fact this is the wording used in recent papers by some of these authors.

Therefore, it was a surprise when in 2015 the RBC-UKQCD collaboration [275, 276] presented their first results for $K \to \pi\pi$ hadronic matrix elements summarized in (10.39). Using these values, the 2015 NLO WCs and including isospin breaking (I.B.) corrections we have found[3]

$$(\varepsilon'/\varepsilon)_{\rm SM} = (1.9 \pm 4.5) \times 10^{-4}, \qquad \text{(BGJJ)}, \tag{10.58}$$

that is one order of magnitude below the experimental value. This result has been confirmed within the uncertainties in [670]

$$(\varepsilon'/\varepsilon)_{\rm SM} = (0.96 \pm 4.96) \times 10^{-4}, \qquad \text{(KNT)}. \tag{10.59}$$

Moreover, an upper bound from DQCD has been derived [244, 296]

$$(\varepsilon'/\varepsilon)_{\rm SM} \le (6.0 \pm 2.4) \times 10^{-4}, \qquad \text{(DQCD)} \tag{10.60}$$

that follows from $B_6^{(1/2)} \le B_8^{(3/2)}$ with $B_8^{(3/2)}$ taken from the 2015 result of RBC-UKQCD collaboration in (7.61).

It should be stressed that the RBC-UKQCD lattice collaboration, calculating directly hadronic matrix elements of all operators, but not including I.B. effects, found in 2015 [275, 276]

$$(\varepsilon'/\varepsilon)_{\rm SM} = (1.38 \pm 6.90) \times 10^{-4}, \qquad \text{(RBC-UKQCD) 2015}. \tag{10.61}$$

[3] It should be emphasized that in contrast to incorrect statements about this analysis in the literature, in particular by the authors of [247], this result, based on LQCD matrix elements, included automatically FSI effects, even if smaller than extracted from the data. See following.

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The larger error than in (10.58) and (10.59) is related to the fact that RBC-UKQCD collaboration did not use experimental data for $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ for the extraction of the $(V-A)\times(V-A)$ contributions. Otherwise the error in (10.61) would be smaller. The comparison of RBC-UKQCD value, which does not include I.B. effects, with the values in (10.58) and (10.59) would seemingly indicate that such effects are small. But such a conclusion would be wrong. I.B. effects definitely suppress significantly ε'/ε, as we can see in (10.53), but the first negative term in this formula, that is dominated by the QCDP operator Q_4, was found by RBC-UKQCD to be significantly larger in magnitude than extracted from the experimental value of $\mathrm{Re}A_0$. This difference suppresses ε'/ε at a similar level as I.B. effects.

Now, already in 2016 ChPT experts expressed serious doubts about the RBC-UKQCD result in (10.61) because the $(\pi\pi)_I$ phase shift $\delta_0 = (24 \pm 5)^\circ$ obtained by RBC-UKQCD collaboration disagreed with $\delta_0 \approx 34^\circ$ obtained by combining dispersion theory with experimental input [671]. This criticism appeared later in print [247]. Updating their earlier estimates of ε'/ε they found $(14 \pm 5)\times 10^{-4}$ [1339] fully consistent with the experimental value.

The weakness of this estimate and of earlier estimates of ε'/ε in ChPT is that to reach this result various inputs outside the ChPT framework have to be used that are presently poorly known. But the main problem of this estimate is the omission of the meson evolution that suppresses $B_6^{(1/2)}$. This was already stressed in [245], where we have argued that the suppression of $B_6^{(1/2)}$ by the meson evolution must be stronger than its possible enhancement by pion loops. Moreover, it is unlikely that the increase of δ_0 from $(24 \pm 5)^\circ$ to 34° will shift ε'/ε upward by one order of magnitude. In the meantime a new result for δ_0 has been presented by the RBC-UKQCD collaboration that with $\delta_0 = (30.9 \pm 3.4)^\circ$ is within 1σ from its dispersive value [672, 673]. The implications of this shift for ε'/ε should be known soon.

Further details on these issues and the status of ε'/ε within the SM, seen from my point of view in 2018, can be found in my 2018 Christmas story [674], which I hope some readers will find amusing. As a result of the latter analysis I presented my expectation for the final result for ε'/ε within the SM, which should include in addition to the meson evolution, isospin breaking effects, NNLO QCD corrections to both QCDP and EWP contributions and final state interactions. It reads

$$\boxed{(\varepsilon'/\varepsilon)_{\mathrm{SM}} = (5 \pm 2)\cdot 10^{-4}, \qquad \text{(AJB 2018).}} \tag{10.62}$$

It will be interesting to see whether these expectations will be confirmed in the first half of 2020 when new results for $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from RBC-UKQCD collaboration, improved estimate of $\hat{\Omega}_{\mathrm{eff}}$ and the NNLO results for QCDP contributions will be available.

If confirmed, this would be the largest anomaly in flavor physics present in a single observable and comparable to the anomaly in the flavor conserving $(g-2)_\mu$, which we will discuss in Section 17.4. Moreover, this would be presently the only significant anomaly as far as CP-violation is concerned. The implications of these expectations for NP models are discussed in the BSM chapters of this book, in particular in Section 14.7.

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Table 10.1 The ratio ε'/ε in units of 10^{-4} for different values of B_6, different isospin corrections and with NNLO EWP corrections. $B_8^{(3/2)} = 0.76$. From [1333]. Last row includes only NLO corrections.

$\hat{\Omega}_{\rm eff}/B_6$	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
0.00	0.02	1.28	2.54	3.80	5.06	6.32	7.58	8.83	10.09	11.35	12.61
0.10	−0.64	0.50	1.63	2.76	3.89	5.03	6.16	7.29	8.42	9.56	10.69
0.15	−0.97	0.10	1.17	2.24	3.31	4.38	5.45	6.52	7.59	8.66	9.73
0.20	−1.30	−0.29	0.71	1.72	2.73	3.74	4.74	5.75	6.76	7.76	8.77
0.25	−1.63	−0.69	0.26	1.20	2.15	3.09	4.03	4.98	5.92	6.87	7.81
0.30	−1.96	−1.08	−0.20	0.68	1.56	2.44	3.33	4.21	5.09	5.97	6.84
0.00	0.75	2.01	3.27	4.53	5.79	7.05	8.30	9.56	10.82	12.08	13.34

10.5.2 Waiting for New Advances in 2020

In view of still sizable uncertainties in ε'/ε coming from $B_6^{(1/2)}$ and $\hat{\Omega}_{\rm eff}$, we give the central value of ε'/ε as a function of these two parameters in Table 10.1. To this end we have set $B_8^{(3/2)} = 0.76$ and $\mathrm{Im}\lambda_t = 1.4 \times 10^{-4}$. We give the values based on (10.56) that include NNLO QCD corrections to EWP. We expect that NNLO corrections to QCDP will further reduce the value of ε'/ε in the SM. This table should facilitate monitoring the values of ε'/ε in the SM when the LQCD calculations of hadronic matrix elements including isospin breaking corrections and QED effects will improve with time. We observe a large sensitivity of ε'/ε to $B_6^{(1/2)}$, but for $B_6^{(1/2)} \geq 0.7$ also the dependence on $\hat{\Omega}_{\rm eff}$ is significant.

Let us illustrate the usefulness of this table. Let us assume that the new values of $B_6^{(1/2)}(m_c)$ and $B_8^{(3/2)}(m_c)$ from RBC-UKQCD will be

$$B_6^{(1/2)}(m_c) = 0.80 \pm 0.08, \qquad B_8^{(3/2)}(m_c) = 0.76 \pm 0.04, \tag{10.63}$$

and use $\hat{\Omega}_{\rm eff} = 0.17 \pm 0.09$ from [1339].

Now RBC-UKQCD will not yet include in their 2019 result I.B. corrections so that in their case $\hat{\Omega}_{\rm eff} = 0$ and the NNLO EWP corrections will also be missing in their result. Therefore assuming for a moment that they will set $\mu_W = \mu_t = M_W$ in the WCs they will find according to the last row in Table 10.1

$$(\varepsilon'/\varepsilon)_{\rm SM} = (8.3 \pm 3.5) \cdot 10^{-4}, \tag{10.64}$$

that is $2.0\,\sigma$ below the experimental value in (10.1).

But using $\hat{\Omega}_{\rm eff} = 0.17 \pm 0.09$ and including NNLO corrections to EWP we would find

$$(\varepsilon'/\varepsilon)_{\rm SM} = (5.6 \pm 2.4) \cdot 10^{-4}, \qquad (2019) \tag{10.65}$$

that is ε'/ε anomaly at the level of $3.3\,\sigma$ and quite consistent with the expectation in (10.62). Such a result would be truly exciting because the inclusion of NNLO QCD corrections to QCDP would further increase the anomaly.

Now, it should be emphasized that the NLO value in (10.64) corresponds to $\mu_W = \mu_t = M_W$. For $\mu_W = 80\,\mathrm{GeV}$ and $\mu_t = 163\,\mathrm{GeV}$ the central value in (10.64) would go up to approximately $9.3 \cdot 10^{-4}$, demonstrating the importance of NNLO corrections. The NNLO value in (10.65) depends only weakly on these changes. See Figure 1 in [1333].

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