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Interaction of a transverse electromagnetic wave with a bounded thermal plasma

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Transmission, reflection and absorption of a transverse electromagnetic wave by a thermal plasma slab of finite width are investigated theoretically. By solving the Maxwell–Boltzmann equations using a mode-analysis method under appropriate boundary conditions, the coefficients of transmission and reflection of the electromagnetic wave and the absorption rate by plasma electrons are obtained as Fourier series. We discuss the relevant physics of the results.

1. Introduction

The interaction of a transverse electromagnetic (EM) wave with plasma electrons is an interesting problem in plasma physics. If the electron collision frequency is sufficiently large that the plasma conductivity becomes local then the problem is easy to treat and is called normal. However, under certain conditions, there is non-negligible thermal motion of the electron, so that the response of the plasma should be described by a non-local conductivity, and we call this case the anomalous skin effect. Moreover, if the plasma is bounded by walls and the electron transit time is comparable to the wave period, a modulation effect by the walls becomes important.

Weibel (1967) studied the anomalous skin effect in a semi-infinite plasma, and Blevin *et al.* (1970, 1973) introduced the bounded plasma effect by assuming a symmetric source configuration. The semi-infinite problem was revisited by Turner (1993), who was concerned with the problem of the collisionless heating mechanism in inductively coupled plasma (ICP) discharge, which is an important plasma source for semiconductor fabrication. Modulation by walls, when one side wall is conducting, was investigated by Yoon *et al.* (1996).

In previous work, the wave source has been represented by an effective surface current at the walls, but there is another useful point of view for the problem of the interaction of the EM wave with a linear material using the concepts of transmitance, reflectance and absorptivity. In this paper, we take a somewhat different point of view from previous workers, adopting a right-going wave boundary condition, and we calculate the reflection, transmission and absorption rate coefficients of the EM wave.

2. Solution of the Maxwell–Boltzmann equations

Let us consider a linearly polarized transverse EM wave incident on a plasma slab (Fig. 1). The EM wave is partly reflected at $z = 0$, partly transmitted and partly

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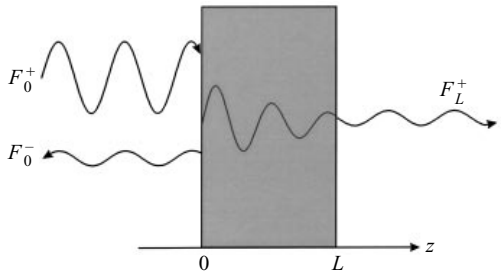


Figure 1. Schematic diagram of wave propagation.

absorbed by the plasma. We assume normal injection of the wave, and that the wave angular frequency ω is much larger than the ion plasma frequency, and thus that the ion motion is negligible. The plasma is assumed to be a linear medium and a weakly ionized gas in which electron collisions with neutral atoms dominate Coulomb collisions. The plasma density and the electron temperature are assumed to be spatially uniform.

The Maxwell equations describing the transverse wave are

$$\frac{1}{c}\frac{\partial B_y(z)}{\partial t} + \frac{\partial E_x(z)}{\partial z} = 0, \tag{2.1a}$$

$$\frac{1}{c}\frac{\partial E_x(z)}{\partial t} + \frac{\partial B_y(z)}{\partial z} = -\eta_0 J_x(z), \tag{2.1b}$$

where E_x and B_y are the electric and magnetic fields, $\eta_0 = 4\pi/c$ is the impedance of free space and c is the velocity of light. The plasma current density J_x is described in general in terms of a non-local plasma conductivity $\Sigma(z, z')$:

$$J_x(z) = \int_0^L \Sigma(z, z') E_x(z')\, dz'. \tag{2.2}$$

The plasma conductivity can be calculated from the linearized Boltzmann equation with the Krook model collision operator:

$$-i\omega f_1 + v_z \frac{\partial f_1}{\partial z} + \frac{eE_x}{T_e} v_x f_0 = -\nu f_1, \tag{2.3}$$

where ν is the collision frequency of electrons with neutral atoms, f_0 is the equilibrium part, f_1 is the perturbed part of the distribution function, and v_x and v_z are the x and z components of the electron velocity. f_0 is assumed to be Maxwellian.

It is assumed that the wave is incident only from the left-hand side, and thus that there is no left-going wave in the region $z > L$. To describe the boundary conditions for the Maxwell equations, it is necessary to define the left- and right-going field quantities. By adding and subtracting (2.1), we have

$$\left(\frac{\partial}{\partial t} \pm c\frac{\partial}{\partial z}\right) F^{\pm}(z) = -\tfrac{1}{2}\eta_0 J_x(z) \tag{2.4}$$

for left-going (F^-) and right-going (F^+) field quantities defined by

$$F^{\pm} \equiv \tfrac{1}{2}(E_x \pm B_y). \tag{2.5}$$

The electromagnetic fields are recovered from $F^{\pm}$ by

$$E_x = F^+ + F^-, \quad B_y = F^+ - F^-. \tag{2.6}$$

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It should be noted that although F^+ and F^- are the right- and left-going quantities only in vacuum, they are very useful in describing the boundary conditions. Since there is a source of the wave only at the left-hand side, we use the following relations as the boundary conditions:

$$F^+(0) = F_0^+, \quad F^-(L) = 0 \equiv F_L^-,$$

or equivalently

$$E_0 + B_0 = 2F_0^+, \quad E_L - B_L = 0, \tag{2.7}$$

where

$$E_0 \equiv E_x(z=0), \quad E_L \equiv E_x(z=L), \tag{2.8a}$$

$$B_0 \equiv B_y(z=0), \quad B_L \equiv B_y(z=L). \tag{2.8b}$$

The set of Maxwell–Boltzmann equations (2.1) and (2.3) with the boundary conditions (2.7) is solved in this work. We note here that the Boltzmann equation also requires a boundary condition for particle reflection when the term $v_z \partial f_1/\partial z$, is not negligible in the Boltzmann equation (2.3).

2.1. Local-conductivity case

When the collision frequency is sufficiently large or the electron transit time across L is much larger than the wave period, the term $v_z \partial f_1/\partial z$ in (2.3) is negligible, and thus the plasma has a local conductivity

$$\Sigma(z, z') = \sigma_l \delta(z - z') = i\frac{\omega_p^2}{4\pi}\frac{1}{\omega + i\nu}\delta(z - z') \tag{2.9}$$

when

$$\frac{v_{\text{th}}}{l} \ll |\omega + i\nu| = (\omega^2 + \nu^2)^{1/2}, \tag{2.10}$$

where σ_l is the local conductivity of the plasma, $\delta(z)$ is the Dirac delta function, and the skin depth

$$l = l_0 \sec \tfrac{1}{2}\epsilon, \tag{2.11}$$

with

$$l_0 = \frac{(1 + \nu^2/\omega^2)^{1/2}}{\omega_p} \quad \epsilon = \tan^{-1}\left(\frac{\nu}{\omega}\right). \tag{2.12}$$

Here ω_p and v_{th} are the plasma frequency and thermal velocity of electrons.

Using the time-varying factor $e^{-i\omega t}$, the Maxwell equations (2.1) with (2.9) give a single wave equation

$$\frac{\partial^2 E_x(z)}{\partial z^2} + \kappa^2 E_x(z) = -i\kappa\eta_0\sigma_l E_x(z), \tag{2.13}$$

where $\kappa \equiv \omega/c$ is the vacuum wavenumber.

The solution of the wave equations, (2.13) can easily be obtained as

$$E_x(z) = i\frac{\kappa/k}{\sinh(kL)}\left\{B_L \cosh(kz) - B_0 \cosh\left[k(L-z)a\right]\right\}, \tag{2.14}$$

where the wavenumber in the plasma becomes

$$k = \sqrt{\tfrac{1}{2}}\kappa[(C + D)^{1/2} - i(C - D)^{1/2}], \tag{2.15}$$

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where

$$C = \left[\frac{Y + (1-X)^2}{1+Y}\right]^{1/2}, \quad D = \frac{X - 1 - Y}{1+Y}, \tag{2.16}$$

with $X \equiv \omega_p^2/\omega^2$ and $Y \equiv \nu^2/\omega^2$.

If we apply the boundary conditions (2.7) to (2.14), we have

$$B_0 = \frac{2}{\eta_s + 1} F_0^+, \tag{2.17a}$$

$$E_0 = \frac{2\eta_s}{\eta_s + 1} F_0^+, \tag{2.17b}$$

$$B_L = E_L = \frac{1}{\cosh(kL) + i(k/\kappa)\sinh(kL)} \frac{2}{\eta_s + 1} F_0^+, \tag{2.17c}$$

where the normalized surface impedance

$$\eta_s \equiv E_0/B_0 = \frac{1 - i(\kappa/k)\tanh(kL)}{1 + i(k/\kappa)\tanh(kL)}. \tag{2.18}$$

The coefficients of reflection (r) and transmission (t) become

$$r \equiv \frac{F_0^-}{F_0^+} = \frac{\eta_s - 1}{\eta_s + 1} = \frac{\kappa^2 + k^2}{\kappa^2 - k^2 + i\kappa k \coth(kL)}, \tag{2.19a}$$

$$t \equiv \frac{F_L^+}{F_0^+} = \frac{4}{\cosh(kL) + i(k/\kappa)\sinh(kL)} = \frac{\kappa k}{\kappa k - i(\kappa^2 - k^2)\tanh(\gamma L)} \frac{1}{\eta_s + 1}. \tag{2.19b}$$

The reflectance R and transmittance T of the EM wave are directly related to the coefficients of reflection and transmission:

$$R \equiv \left|\frac{F_0^-}{F_0^+}\right|^2 = |r|^2, \quad T \equiv \left|\frac{F_0^-}{F_0^+}\right|^2 = |t|^2. \tag{2.20}$$

The absorptivity can be deduced from the power-balance relation:

$$A = 1 - R - T \tag{2.21}$$

2.2. Non-local-conductivity case

Let us consider the case when the thermal motion of the electron cannot be neglected. If

$$\frac{v_{\text{th}}}{l} \not\ll (\omega^2 + \nu^2)^{1/2} \tag{2.22}$$

then the term $v_z \partial f_1/\partial z$ is not negligible in (2.3), and it is necessary to assign a boundary conditon for particle reflection at walls.

As stated by Yoon *et al.* (1996), the perfectly reflecting boundary condition is a good approximation of electron reflection in the usual bounded plasmas, because the electron transition time is much smaller than the global loss time of the plasma as long as $v_{\text{th}} > v_{i,\text{th}}$ where $v_{i,\text{th}}$ is the thermal velocity of ions. Moreover, if we utilize the boundary condition, the bounded-plasma problem can be converted into an infinitely periodic problem. As a result, considering the discontinuities of the magnetic field at the boundaries, the Maxwell equations (2.1) can be combined into

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the form of a wave equation as follows:

$$\frac{\partial^2 E_x(z)}{\partial z^2} + \kappa^2 E_x(z) = -i\kappa\eta_0 \int_{-\infty}^{\infty} \sigma(z - z')E(z')\,dz' + i\kappa\,[B_0\delta(x) - B_L\delta(x - L)]\,, \tag{2.23}$$

where $\sigma(z)$ is the conductivity of an infinite homogeneous plasma.

The solution of the wave equation (2.23) can be obtained as a Fourier series:

$$E_x(z) = \tfrac{1}{2}e_0 + \sum_{n=1}^{\infty} e_n \cos(q_n z), \tag{2.24}$$

where the Fourier component of the electric field

$$e_n = -i\frac{\eta_0}{2L}\frac{B_0 - (-1)^n B_L}{D_n}, \tag{2.25}$$

and

$$D_n = q_n^2 - \kappa^2 - (2\pi)^{1/2}\kappa\eta_0\sigma_n. \tag{2.26}$$

Here $q_n = n\pi/L$, and the Fourier component of $\sigma(z)$ is

$$\sigma_n \equiv \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} \sigma(z)e^{-iq_n z}\,dz = -\frac{i}{\pi^{1/2}}\frac{\omega_p}{8\pi}\frac{q_D}{|q_n|}Z_p\left(\frac{\omega + i\nu}{|q_n|v_{\text{th}}}\right), \tag{2.27}$$

where $q_D \equiv (4\pi n_e^2/T_e)^{1/2}$ is the Debye wave number and Z_p is the plasma dispersion function. Here n_e and T_e are the density and temperature of electrons.

To complete the calculation, B_0 and B_L should be determined by the boundary conditions. These are

$$E_0 + B_0 = 2F_0^+, \quad E_L - B_L = 0, \tag{2.28}$$

and we now have

$$E_0 = -\frac{2i\kappa}{L}(\xi_1 B_0 - \xi_2 B_L), \quad E_L = -\frac{2i\kappa}{L}(\xi_2 B_0 - \xi_1 B_L), \tag{2.29}$$

where

$$\xi_1 \equiv \frac{1}{2D_0} + \sum_{n=1}^{\infty}\frac{1}{D_n}, \quad \xi_2 \equiv \frac{1}{2D_0} + \sum_{n=1}^{\infty}\frac{(-1)^n}{D_n}. \tag{2.30}$$

The four unknowns E_0, B_0, E_L and B_L can be expressed as functions of F_0^+ by solving the four linear equations (2.7) and (2.29). Also, F_0^- and F_L^+ can be found from (2.4). Finally, we have

$$r = \frac{\eta_s - 1}{\eta_s + 1}, \tag{2.31a}$$

$$t = \frac{\xi_2}{\xi_1 + iL/(2\kappa)}\frac{2}{\eta_s + 1}, \tag{2.31b}$$

$$\eta_s = \frac{2i\kappa}{L}\left(\frac{\xi_2^2}{\xi_1 + iL/2\kappa} - \xi_1\right), \tag{2.31c}$$

and

$$R = \left(\frac{\eta_s - 1}{\eta_s + 1}\right)^2, \quad T = \left(\frac{\xi_2}{\xi_1 + iL/2\kappa}\frac{2}{\eta_s + 1}\right)^2. \tag{2.32}$$

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The absorptivity can be deduced from the power-balance relation as

$$A = 1 - R - T = \mathrm{Re}(\eta_s)\left|\frac{2}{\eta_s + 1}\right|^2 - T. \tag{2.33}$$

It is interesting to investigate some limiting cases. Let us consider the case where the electron transit time is sufficiently long that

$$\frac{L\omega}{\pi v_{\mathrm{th}}}\left[1 + \left(\frac{\nu}{\omega}\right)^2\right]^{1/2} \gg 1. \tag{2.34}$$

If we define

$$\beta \equiv \frac{1}{2}\left(\frac{v_{\mathrm{th}}\omega_p}{c}\right)^2 \frac{\omega}{(\omega + i\nu)^3} \tag{2.35}$$

then the anomalous skin effect becomes important when $|\beta|$ is large (Weibel 1967; Sayasov 1979).

First, let us consider the case where $|\beta| \ll 1$. We can show that

$$D_n \approx (1 + \beta)(q_n^2 + \gamma^2), \tag{2.36}$$

where

$$\gamma \equiv \frac{k}{(1 + \beta)^{1/2}}. \tag{2.37}$$

Using the mathematical relations

$$\begin{aligned} \frac{1}{2\gamma^2} + \sum_{n=1}^{\infty} \frac{1}{q_n^2 + \gamma^2} &= \frac{L}{2\gamma}\coth(\gamma L), \\ \frac{1}{2\gamma^2} + \sum_{n=1}^{\infty} \frac{(-1)^n}{q_n^2 + \gamma^2} &= \frac{L}{2\gamma}\frac{1}{\mathrm{cosech}(\gamma L)}, \end{aligned} \tag{2.38}$$

we have

$$\eta_s \approx \frac{1 - i\,(\kappa/\gamma)\tanh(\gamma L)}{1 + i\,(\gamma/\kappa)\tanh(\gamma L)}, \tag{2.39}$$

$$\frac{\xi_2}{\xi_1 + iL/(2\kappa)} \approx \frac{1}{\cosh(\gamma L) + i\,[\gamma(1 + \beta)/\kappa]\sinh(\gamma L)}. \tag{2.40}$$

Therefore the wave coefficients become

$$r \approx \frac{\kappa^2 + \gamma^2(1 + \beta)^2}{\kappa^2 - \gamma^2(1 + \beta)^2 + i\kappa\gamma(1 + \beta)\coth(\gamma L)}, \tag{2.41a}$$

$$t \approx \frac{\kappa\gamma(1 + \beta)}{\kappa\gamma(1 + \beta) + i\,[\kappa^2 - \gamma^2(1 + \beta)^2]}\tanh(\gamma L). \tag{2.41b}$$

We can see that if $v_{\mathrm{th}} = 0$ then (2.41) go to the local conductivity formula (2.19).

On the other hand, when $|\beta| \gg 1$, we use the method of complex integration (Sayasov 1979; Yoon *et al.* 1996). After some calculations, we can show

$$\xi_1 \approx i\frac{4L}{\kappa}\frac{\eta_s}{\eta_0}, \quad \xi_2 \approx 0, \tag{2.42}$$

$$\eta_s \approx \frac{2}{3\pi^{1/6}}\left(\sqrt{\tfrac{1}{3}} - i\right)\left(\frac{\omega^2 v_{\mathrm{th}}}{c\omega_p^2}\right)^{1/3} - \frac{8}{9\pi^{5/6}}\left(\sqrt{\tfrac{1}{3}} + i\right)(\omega - i\nu)\left(\frac{\omega c}{\omega_p^4 v_{\mathrm{th}}}\right)^{1/3}. \tag{2.43}$$

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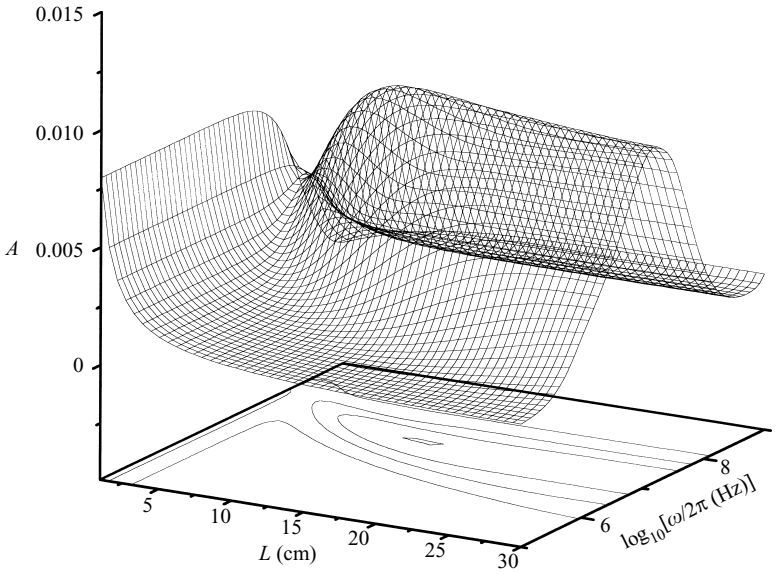


Figure 2. Dependence of absorptivity A on plasma length L and wave frequency $\omega/2\pi$. $n_e = 10^{11}$ cm^{-3}, $T_e = 100$ eV and $\nu/\omega = 0.05$ are assumed.

Therefore in this limit we have

$$r \approx 1, \quad t \approx 0, \tag{2.44}$$

and

$$R \approx 1 - 4\,\mathrm{Re}(\eta_s), \quad T \approx 0, \quad A \approx 4\,\mathrm{Re}(\eta_s), \tag{2.45}$$

up to first order in $1/|\beta|$. Accordingly, we conclude that the wave is almost reflected when the anomalous skin effect is strong and the plasma is sufficiently long.

3. Numerical results and discussion

We present the numerical results for the dependence of the absorptivity A on the various parameters, focusing on the case where the anomalous skin effect is important.

Figure 2 shows the dependence of A on L and $\omega/2\pi$ when $n_e = 10^{11}$ cm^{-3}, $T_e = 100$ eV and $\nu/\omega = 0.05$. The ridge in the large-L region is due to the anomalous skin effect, and the ridge in the small-L region represents the collisional absorption. The maximum in the anomalous-skin-effect regime indicates the bounded-plasma effect due to the wall, and it appears when the electron transit time is equal to the wave half-period.

The dependence of A on L and n_e is presented in Fig. 3, where $T_e = 100$ eV, $\omega/2\pi = 10$ MHz, and $\nu/\omega = 0.01$ are assumed. The ridge in the low-density regime is due to the anomalous skin effect, and the ridge in the high-density regime is the collisional absorption. Similarly to Fig. 2, the maximum in the anomalous-skin-effect regime appears when the electron transit time is equal to the wave half-period.

In both Figs 2 and 3, the absorption is strong when $|\beta| \approx 1$ provided that the plasma is sufficiently long. If we consider the classical skin depth $\ell \approx c/\omega_p$ and the

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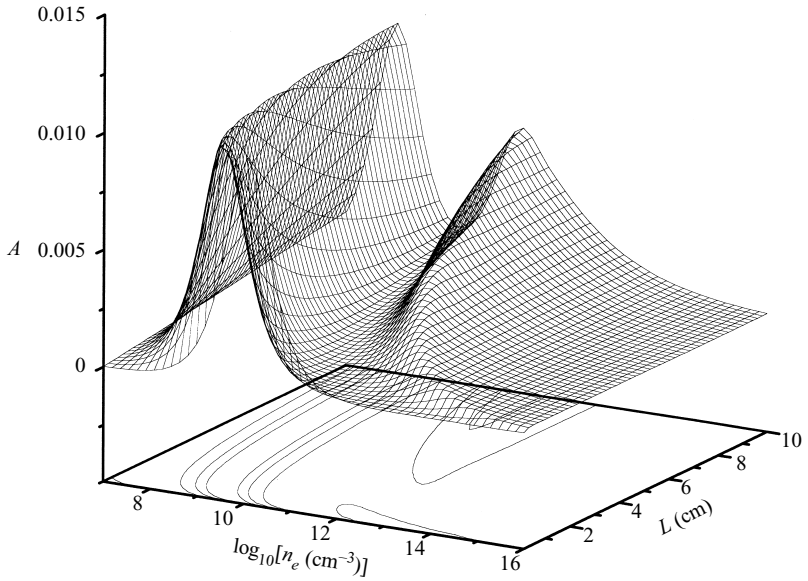


Figure 3. Dependence of absorptivity A on plasma length L and electron density n_e. $T_e = 100$ eV, $\omega/2\pi = 10$ MHz and $\nu/\omega = 0.01$ are assumed.

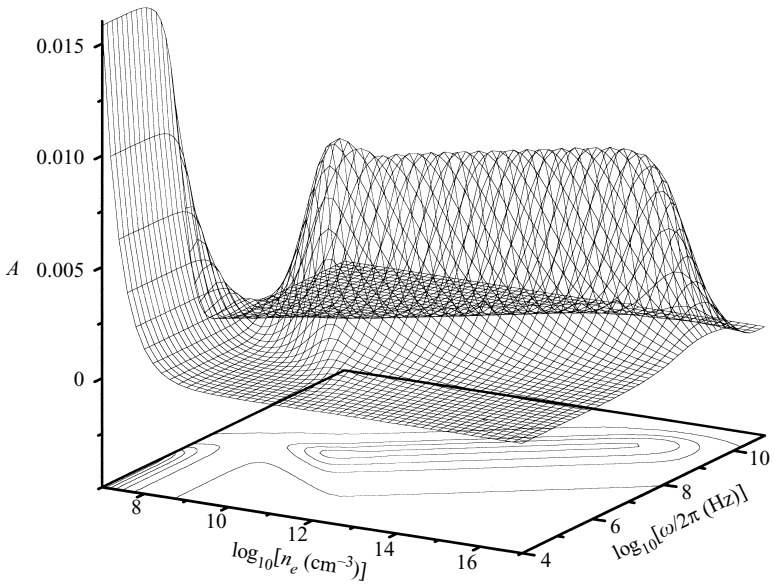


Figure 4. Dependence of absorptivity A on electron density n_e and wave frequency $\omega/2\pi$. $T_e = 100$ eV, $\nu/\omega = 0.001$ and $L = 20$ cm are assumed.

particle path length during the wave period, $s \approx v_{\rm th}/\omega$, we have

$$\beta^{1/2} \approx s/l. \tag{3.1}$$

Therefore we can state that the absorption is strong when the wavelength is nearly equal to the particle path length during the wave period. If $L \lesssim s$, the particle path

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length s should be replaced by L, and accordingly the maximum condition should be found in the higher-density regime (Fig. 3).

Figure 4 shows the dependence of A on n_e and $\omega/2\pi$ in the parameter space when T_e = 100 eV, ν/ω = 0.001 and L = 20 cm. The peak in the high-density indicates the anomalous-skin-effect regime, and the hill in the low-density regime is due to the collisional absorption. This result shows that as the wave frequency increases, higher density is required in order that the anomalous skin effect can occur.

4. Conclusions

Reflectance, transmittance and absorptivity of a transverse electromagnetic wave interacting with a thermal bounded plasma have been calculated by solving the Maxwell–Boltzmann equations with a right-going wave boundary condition. The absorption through the anomalous skin effect is strong when the wavelength in the plasma is nearly equal to the particle path length during the wave period.

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