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Arbitrary amplitude dust-acoustic double layers in a non-thermal plasma

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Abstract. The existence of large-amplitude dust-acoustic double layers is investigated in an unmagnetized dusty plasma comprising a negatively charged warm dust fluid, non-thermal ions and Boltzmann electrons. It is found that the non-thermal plasma supports the existence of rarefactive double layers only. The dependence of the double-layer amplitude and Mach number on various parameters such as the non-thermal parameter, dust temperature and electron concentration is numerically examined.

1. Introduction

The study of dusty plasmas is a very exciting area of research that has been developing very rapidly in the last decade due to the widespread occurrence of such plasmas in many space and astrophysical environments, for example in planetary rings, cometary tails and interstellar clouds. The starting point of theoretical studies in such plasmas was the discovery of the low phase velocity dust-acoustic wave (DAW) by Rao et al. [1]. The existence of the DAW, subsequently, was confirmed in many laboratory experiments [2–4].

It is well known that the linear and nonlinear properties of a plasma depend on the velocity distribution functions of the particle constituents of the plasma. In collisionless plasmas, the most commonly used distribution is the Maxwellian velocity distribution, which is a distribution that is in thermal equilibrium. However, various observations of fast ions and electrons in space environments indicate that these particles have velocity distributions that deviate from Maxwellian behaviour. Many instances of fast particles have been observed in space, for example non-thermal ions have been observed in the Earth's bow-shock [5]. The loss of energetic ions from the upper ionosphere of Mars has been observed through satellite observations [6]. More recently, fast protons have been observed in the vicinity of the Moon [7]. Non-thermal ion populations, which could be modelled by the kappa distribution, have been found to occur in the magnetospheres of planets such as Jupiter and Saturn [8, 9]. The kappa distribution (having the spectral index κ) is a velocity distribution that has a high-energy tail but approaches the Maxwellian when $\kappa \rightarrow \infty$. In the light of the discussion above, it appears that space plasmas are more

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realistically modelled by making use of non-thermal velocity distributions. Hence there exists a large body of literature on theoretical wave studies that makes use of non-Maxwellian velocity distribution functions.

Cairns et al. [10] showed that a plasma comprising non-thermal electrons and a cold ion fluid could support the simultaneous existence of solitons with both positive and negative potential. They found that when they considered a thermal plasma, only a single type of soliton was supported. Mamun et al. [11] employed a model comprising non-thermal ions and negatively charged dust and reported that solitons with positive and negative potential can coexist. In an earlier work by Mamun et al. [12], using a model consisting of Boltzmann ions and negatively charged dust particles, it was found that only solitons with negative potential exist.

Gill and Kaur [13] studied nonlinear dust acoustic waves in a two component plasma comprising non-thermal ions and a negatively charged dust component. They found that dust temperature can influence whether rarefactive solitons can simultaneously coexist with compressive solitons in the non-thermal plasma.

Mendoza-Briceno et al. [14] employed a two component model of non-thermal ions and negatively charged dust and used the pseudopotential method to determine, theoretically, the dependence of the required cutoff values of the non-thermal parameter α and Mach number M on dust temperature for compressive and rarefactive solitons to simultaneously exist.

In their investigations, Mahmood and Saleem [15] used a dust–ion and a dust–ion–electron model where the electrons and ions are Boltzmann distributed. Dust temperature effects were not considered. They found that the amplitude of the dust-acoustic soliton structures decreases significantly with an increase in the dust streaming velocity. It was also found that the width of the soliton structure is narrower in the presence of free electrons in the dusty plasma compared with when there are no electrons in the plasma. These observations provide justification for the need for theoretical studies that extend earlier dust–ion models to include electrons and also to study the effects of dust streaming, since we anticipate that both these entities will modify existing results.

Maharaj et al. [16] extended the dust–ion model of Mendoza-Briceno et al. [14] to include electrons and also to consider the case of streaming dust and found that the theoretical conditions that support the coexistence of positive and negative potential solitons in the non-thermal plasma are altered. They found that the plasma supports only compressive solitons if the ions were treated as being Maxwellian rather than non-Maxwellian.

Much of the work on theoretical wave studies that makes use of the non-thermal velocity distribution proposed by Cairns et al. [10] for either the electrons or ions, has up to this point been nonlinear. Maharaj et al. [17] studied linear DAWs in a three component unmagnetized plasma comprising ions that are distributed according to the non-thermal velocity distribution proposed by Cairns et al. [10], Boltzmann electrons and streaming dust particles. The dependence of the real frequency and the instability growth rate on various plasma parameters such as dust drift velocity, particle temperatures and the parameter α that characterizes the non-thermal plasma was studied.

All the investigations described thus far have studied linear modes or nonlinear soliton structures. It is known that potential structures in the form of a double layer (DL) are found to occur in space environments as well as in the laboratory. Several laboratory experiments pertaining to DLs have been reviewed by Hershkowitz [18].

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Moreover, small-amplitude DLs were observed in the auroral plasma by way of electron and ion velocity distributions that indicate the presence of parallel (to the magnetic field) electric fields [19]. These DLs may be largely responsible for the acceleration of particles in the auroral zone.

Weak double layers (WDLs) and solitary waves have been observed by the Viking satellite [20]. Fast solitary waves observed in the form of electron holes by the FAST satellite may be responsible for maintaining the parallel electric field in the downward current region of the auroral zone as mentioned by Ergun et al. [21]. A simulation study by Singh [22] showed that the electron holes observed by the FAST satellite [21] are a feature of a strong DL that occurs below the observing spacecraft. The high-resolution measurements made by the FAST satellite makes it possible to associate observations in the downward current region of the auroral zone with basic plasma modes as outlined in Newman et al. [23]. Simulation studies undertaken by them have yet again established the connection between electron holes and the existence of a DL.

Theoretical studies of DLs undertaken by many authors are vast as is shown by the large body of literature pertaining to the subject. Bharuthram and Shukla [24] studied arbitrary amplitude ion-acoustic DLs in an unmagnetized plasma comprising cold ions and two distinct species of Boltzmann electrons of different temperatures. Both large- and small-amplitude DLs are considered. Large-amplitude, dust-acoustic DLs were studied by Bharuthram and Shukla [25] in an unmagnetized dusty plasma comprising Boltzmann electrons, two separate populations of Boltzmann distributed ions having different temperature and a negatively charged cold dust fluid. A numerical approach was adopted and rarefactive ($\phi_m > 0$) DLs were found to exist. The non-isothermality of the two ion populations was found to support the existence of the DL. Tagare [26] studied solitons and DLs in an unmagnetized dusty plasma comprising a cold dust fluid and two ion populations having different temperatures. A purely analytical approach was adopted and it was reported that both compressive and rarefactive solitons and DLs exist. Roychoudhury and Chatterjee [27] considered a three-component plasma comprising a dust fluid and two ion populations of different temperatures. They incorporated the effects of dust temperature into the model. Analytically they found that both compressive and rarefactive solitons exist but numerical considerations show that only compressive DLs are possible. A finite value for the dust temperature is found to suppress the existence of the DL; however it must be noted that the authors did not look for DL solutions for a range of dust temperature values.

Xie et al. [28] employed a four-component plasma comprising Boltzmann electrons, two populations of Boltzmann distributed ions with different temperatures and a negatively charged dust fluid. They studied the case of dust charge variation and found that both compressive and rarefactive solitons exist. It was found that dust charge variation narrows the width of small-amplitude solitons, widens the range of Mach numbers for which large-amplitude solitons exist and decreases the amplitude of solitons when compared with the constant dust charge scenario. The existence of DLs was pointed out but not studied in any detail.

The present work is an extension of the studies on solitons undertaken by Maharaj et al. in [16], to a study of DLs in a plasma in which the non-thermal ions are modelled using the non-thermal distribution function proposed by Mamun et al. [11]. We have included the theory that appeared in [16] for completeness. Our approach is purely numerical and we study the dependence of the DL structure on various

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parameters such as the non-thermal parameter α, dust temperature and electron concentration.

We present our model and the theory in Sec. 2. Numerical results are presented in Sec. 3 and appropriate conclusions are presented in Sec. 4.

2. Theory

We consider, as in Maharaj et al. [16], a three-component unmagnetized dusty plasma comprising a negatively charged drifting dust component, stationary non-thermal ions and Boltzmann electrons. In our analysis, we closely follow the approach of Mendoza-Briceno et al. [14]. For overall charge neutrality at equilibrium, we require

$$n_{\mathrm{i}0} = n_{\mathrm{e}0} + Z_{\mathrm{d}} n_{\mathrm{d}0}, \tag{1}$$

where $n_{\mathrm{i}0}, n_{\mathrm{e}0}$ and $n_{\mathrm{d}0}$ are the equilibrium number densities of the ions, electrons and dust particles, respectively. Z_{d} is the number of electrons that reside on a single dust grain.

For the low phase velocity dust acoustic oscillations, the dust dynamics are governed by the continuity, momentum and pressure balance equations, respectively:

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \nabla \cdot (n_{\mathrm{d}} \mathbf{v}_{\mathrm{d}}) = 0, \tag{2}$$

$$\frac{\partial \mathbf{v}_{\mathrm{d}}}{\partial t} + (\mathbf{v}_{\mathrm{d}} \cdot \nabla)\mathbf{v}_{\mathrm{d}} = \nabla\phi - \frac{\sigma}{n_{\mathrm{d}}} \nabla P, \tag{3}$$

$$\frac{\partial P}{\partial t} + \mathbf{v}_{\mathrm{d}} \cdot \nabla P + \gamma P \cdot \nabla \mathbf{v}_{\mathrm{d}} = 0. \tag{4}$$

The system is closed by Poisson's equation

$$\nabla^2 \phi = n_{\mathrm{e}} + n_{\mathrm{d}} - n_{\mathrm{i}}, \tag{5}$$

where $\sigma = T_{\mathrm{d}}/(Z_{\mathrm{d}} T_{\mathrm{i}})$ is the normalized dust temperature, n_{d} is the dust density normalized by $n_{\mathrm{d}0}$ and $n_{\mathrm{e}}(n_{\mathrm{i}})$ are the electron and ion number densities both normalized by $Z_{\mathrm{d}} n_{\mathrm{d}0}$, $\gamma = (2+N)/N$ where N is the number of degrees of freedom, v_{d} is the dust fluid velocity normalized by the dust acoustic speed $C_{\mathrm{d}} = (Z_{\mathrm{d}} k_{\mathrm{B}} T_{\mathrm{i}}/m_{\mathrm{d}})^{1/2}$, P is the dust pressure normalized by $n_{\mathrm{d}0} k_{\mathrm{B}} T_{\mathrm{d}}$ and ϕ is the electrostatic wave potential normalized by $k_{\mathrm{B}} T_{\mathrm{i}}/\mathrm{e}$ where e is the electronic charge. The space variable is normalized by $\lambda_{\mathrm{d}} = (k_{\mathrm{B}} T_{\mathrm{i}}/4\pi Z_{\mathrm{d}} n_{\mathrm{d}0} e^2)^{1/2}$ and time is normalized by the inverse dust plasma frequency $\omega_{\mathrm{pd}}^{-1} = (m_{\mathrm{d}}/4\pi n_{\mathrm{d}0} Z_{\mathrm{d}}^2 e^2)^{1/2}$. The ions are non-thermal and assumed to follow a distribution given by Mamun et al. [12] that is similar to that used to describe non-thermal electrons by Cairns et al. [10], with the unnormalized ion density given by

$$n_{\mathrm{i}} = n_{\mathrm{i}0}(1 + \beta\phi + \beta\phi^2) e^{-\phi}, \tag{6}$$

where

$$\beta = \frac{4\alpha}{1 + 3\alpha}. \tag{7}$$

The electrons are in thermal equilibrium. Their density is given by the Boltzmann expression

$$n_{\mathrm{e}} = n_{\mathrm{e}0} e^{\delta\phi}, \tag{8}$$

where $\delta = T_{\mathrm{i}}/T_{\mathrm{e}}$.

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The equations in the one-dimensional ($\gamma = 3$) case can be written as

$$\frac{\partial n_{\rm d}}{\partial t} + \frac{\partial (n_{\rm d} v_{\rm d})}{\partial x} = 0, \tag{9}$$

$$\frac{\partial v_{\rm d}}{\partial t} + v_{\rm d}\frac{\partial v_{\rm d}}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{\sigma}{n_{\rm d}}\frac{\partial P}{\partial x}, \tag{10}$$

$$\frac{\partial P}{\partial t} + v_{\rm d}\frac{\partial P}{\partial x} + 3P\frac{\partial v_{\rm d}}{\partial x} = 0, \tag{11}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\rm d} + \frac{n_{\rm e0}}{Z_{\rm d} n_{\rm d0}} e^{\delta\phi} - \frac{n_{\rm i0}}{Z_{\rm d} n_{\rm d0}}(1 + \beta\phi + \beta\phi^2)e^{-\phi}. \tag{12}$$

In the linear analysis, the set of equations (6)–(12) may be linearized to produce the dispersion relation, (17),

$$1 = \frac{k^2\lambda_{\rm de}^2}{1 + k^2\lambda_{\rm de}^2}\left[\frac{\omega_{\rm pd}^2}{(\omega - k v_{\rm do})^2 - k^2 v_{\rm td}^2} + \frac{\omega_{\rm pi}^2}{2(1+3\alpha)k^2 v_{\rm ti}^2}\left\{Z'(\xi) + \sqrt{\frac{2}{\pi}}\frac{\alpha}{v_{\rm ti}^3}\frac{d^2 I}{d\mu^2}\right\}\right], \tag{13}$$

which yields the mode with real frequency, given by

$$\omega_{\rm r} = k v_{\rm d0} \pm \frac{k C_{\rm d}}{\sqrt{1 + k^2 \lambda_{\rm D}{}^2}}, \tag{14}$$

where $\lambda_{\rm de} = \sqrt{T_{\rm e}/4\pi n_{\rm eo} e^2}$ is the electron Debye length, $\omega_{\rm pi} = \sqrt{4\pi n_{\rm io} e^2/m_{\rm i}}$ is the plasma frequency of the ions, $v_{\rm td} = \sqrt{T_{\rm d}/m_{\rm d}}$ and $v_{\rm ti} = \sqrt{T_{\rm i}/m_{\rm i}}$ are the thermal speeds of the dust grains and the ions, respectively, $v_{\rm d0}$ is the normalized drift velocity of the dust relative to the ions and electrons, $Z'(\xi)$ is the derivative of the plasma dispersion function of Fried and Conte [30] with argument $\xi = \omega/\sqrt{2}k v_{\rm ti}$, $I = \sqrt{\pi}\sqrt{\mu}Z'(\xi)$, $\mu = 1/2 v_{\rm ti}{}^2$, $C_d = \omega_{\rm pd}\lambda_{\rm D}$ and $\lambda_{\rm D} = \sqrt{T_{\rm e}T_{\rm i}/4\pi e^2\{n_{\rm e0}T_{\rm i} + n_{\rm i0}T_{\rm e}(1-\beta)\}}$. This mode has been studied in detail by Maharaj et al. [17].

For solitary wave solutions, we transform to the stationary frame $\xi = x - Mt$ where M is the Mach number (the solitary wave velocity normalized by the dust acoustic speed $C_{\rm d}$).

The above set of equations can now be written as

$$-M\frac{\partial n_{\rm d}}{\partial \xi} + \frac{\partial (n_{\rm d} v_{\rm d})}{\partial \xi} = 0, \tag{15}$$

$$-M\frac{\partial v_{\rm d}}{\partial \xi} + v_{\rm d}\frac{\partial v_{\rm d}}{\partial \xi} = \frac{\partial \phi}{\partial \xi} - \frac{\sigma}{n_{\rm d}}\frac{\partial P}{\partial \xi}, \tag{16}$$

$$-M\frac{\partial P}{\partial \xi} + v_{\rm d}\frac{\partial P}{\partial \xi} + 3P\frac{\partial v_{\rm d}}{\partial \xi} = 0, \tag{17}$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_{\rm d} - \frac{n_{\rm i0}}{Z_{\rm d} n_{\rm d0}}(1 + \beta\phi + \beta\phi^2)e^{-\phi} + \frac{n_{\rm e0}}{Z_{\rm d} n_{\rm d0}}e^{\delta\phi}. \tag{18}$$

By making use of the boundary conditions $\phi \to 0$, $v_{\rm d} \to v_{\rm d0}$ where $v_{\rm d0}$ is the equilibrium dust drift speed, $P \to 1$ and $n_{\rm d} \to 1$ at $|\xi| \to \pm\infty$, (15) and (17) can be integrated to give

$$n_{\rm d} = \frac{v_{\rm d0} - M}{v_{\rm d} - M} \tag{19}$$

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and

$$P = n_{\mathrm{d}}^3. \tag{20}$$

Combining (16) and (19), one gets

$$-2M\frac{\partial v_{\mathrm{d}}}{\partial \xi} + 2v_{\mathrm{d}}\frac{\partial v_{\mathrm{d}}}{\partial \xi} + 2\sigma\frac{M}{M - v_{\mathrm{d0}}}\frac{\partial P}{\partial \xi} - 2\sigma\frac{v_{\mathrm{d}}}{M - v_{\mathrm{d0}}}\frac{\partial P}{\partial \xi} = 2\frac{\partial \phi}{\partial \xi}. \tag{21}$$

Multiplying (17) by $\sigma/(M - v_{\mathrm{d0}})$, we can write

$$-\frac{M}{M - v_{\mathrm{d0}}}\sigma\frac{\partial P}{\partial \xi} + \frac{v_{\mathrm{d}}}{M - v_{\mathrm{d0}}}\sigma\frac{\partial P}{\partial \xi} + 3P\frac{\sigma}{M - v_{\mathrm{d0}}}\frac{\partial v_{\mathrm{d}}}{\partial \xi} = 0. \tag{22}$$

Subtracting (21) from (22), yields the following differential equation:

$$2M\frac{\partial v_{\mathrm{d}}}{\partial \xi} - 2v_{\mathrm{d}}\frac{\partial v_{\mathrm{d}}}{\partial \xi} - 3\sigma\frac{M}{M - v_{\mathrm{d0}}}\frac{\partial P}{\partial \xi} + \left(\frac{3\sigma}{M - v_{\mathrm{d0}}}\right)\frac{\partial (Pv_{\mathrm{d}})}{\partial \xi} + 2\frac{\partial \phi}{\partial \xi} = 0. \tag{23}$$

Integrating (23), and using the same boundary conditions at $|\xi| \to \pm\infty$, we obtain

$$2M(v_{\mathrm{d}} - v_{\mathrm{d0}}) + (v_{\mathrm{d0}}^2 - v_{\mathrm{d}}^2) - 3P\sigma\frac{M}{M - v_{\mathrm{d0}}} + 3\sigma P\frac{v_{\mathrm{d}}}{M - v_{\mathrm{d0}}} + 3\sigma\frac{M}{M - v_{\mathrm{d0}}} - 3\sigma\frac{v_{\mathrm{d0}}}{M - v_{\mathrm{d0}}} + 2\phi = 0. \tag{24}$$

Substituting for v_{d} and P from (19) and (20), respectively, into this equation, we obtain a quadratic equation in n_{d}^2, viz.

$$3\sigma n_{\mathrm{d}}^4 - [(M - v_{\mathrm{d0}})^2 + 3\sigma + 2\phi]n_{\mathrm{d}}^2 + (M - v_{\mathrm{d0}})^2 = 0. \tag{25}$$

The solution for n_{d} is given by

$$n_{\mathrm{d}} = \frac{\sigma_1}{\sqrt{2}\sigma_0}\left[1 + \frac{2\phi}{(M - v_{\mathrm{d0}})^2\sigma_1^2} - \sqrt{\left(1 + \frac{2\phi}{(M - v_{\mathrm{d0}})^2\sigma_1^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2}, \tag{26}$$

where

$$\sigma_0 = \sqrt{\frac{3\sigma}{(M - v_{\mathrm{d0}})^2}}; \quad \sigma_1 = \sqrt{1 + \sigma_0^2}. \tag{27}$$

It is important to note that when we set the normalized equilibrium dust drift velocity to zero ($v_{\mathrm{d0}} = 0$) in (26) and (27), then our expressions for n_{d}, σ_0 and σ_1 correspond exactly with equations (22) and (23) in [14].

Substituting (26) in (18) gives

$$\frac{d^2\phi}{d\xi^2} = \frac{\sigma_1}{\sqrt{2}\sigma_0}\left[1 + \frac{2\phi}{(M - v_{\mathrm{d0}})^2\sigma_1^2} - \sqrt{\left(1 + \frac{2\phi}{(M - v_{\mathrm{d0}})^2\sigma_1^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2} + \frac{n_{\mathrm{e0}}}{Z_{\mathrm{d}}n_{\mathrm{d0}}}e^{\delta\phi} - \frac{n_{\mathrm{i0}}}{Z_{\mathrm{d}}n_{\mathrm{d0}}}(1 + \beta\phi + \beta\phi^2)e^{-\phi}. \tag{28}$$

The nature of solutions of this equation is seen by introducing the Sagdeev potential [29]. Multiplying (28) by $d\phi/d\xi$ and after integrating the resulting expression, we obtain

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{29}$$

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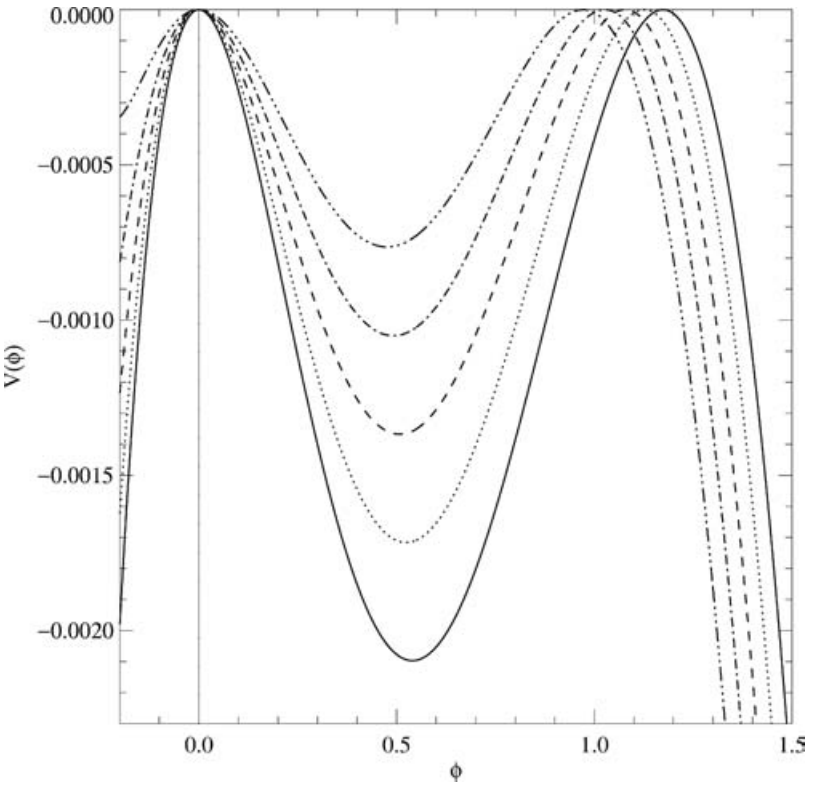


Figure 1. The DL Sagdeev potential as a function of ϕ for different values of the normalized dust temperature. The curves correspond to $\sigma = T_{\rm d}/(Z_{\rm d}T_{\rm i}) = 0$ (–), $\sigma = 0.05$ ($\cdots$), $\sigma = 0.1$ (- -), $\sigma = 0.15$ (— · —) and $\sigma = 0.2$ (— $\cdots$ —) with the corresponding Mach numbers being 1.517, 1.549, 1.580, 1.611 and 1.642, respectively. The fixed parameters are $\alpha = 0.2$, $v_{\rm d0} = 0$, $n_{\rm e0} = 0$ and $\delta = T_{\rm i}/T_{\rm e} = 1$.

where the Sagdeev potential $V(\phi)$ is given by

$$V(\phi) = -\frac{n_{\rm i0}}{Z_{\rm d}n_{\rm d0}}(1 + 3\beta + 3\beta\phi + \beta\phi^2)e^{-\phi} - \frac{n_{\rm e0}}{Z_{\rm d}n_{\rm d0}}\frac{1}{\delta}e^{\delta\phi} - (M - v_{\rm d0})^2\sqrt{\sigma_0}\left[\exp\left(\frac{\theta}{2}\right) + \frac{1}{3}\exp\left(\frac{-3\theta}{2}\right)\right] + C_1 \qquad (30)$$

and C_1 is chosen such that $V(\phi) = 0$ at $\phi = 0$.

As in Mendoza-Briceno et al. [14], to make it possible to consider the limit $\sigma \to 0$ in the Sagdeev potential, we express θ as

$$\theta = \ln\left[\frac{\sigma_1^2}{2\sigma_0^2}\left(1 + \frac{2\phi}{(M - v_{\rm d0})^2\sigma_1^2}\right) + \sqrt{\frac{\sigma_1^4}{4\sigma_0^4}\left(1 + \frac{2\phi}{(M - v_{\rm d0})^2\sigma_1^2}\right)^2 - 1}\right]. \qquad (31)$$

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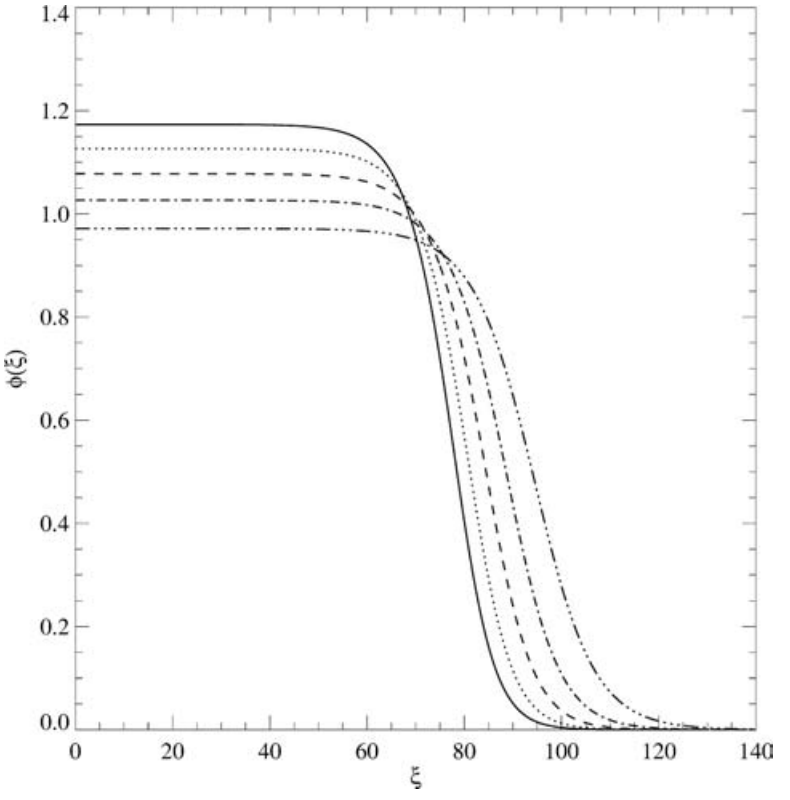


Figure 2. A plot of the DL potential as a function of ξ for the corresponding curves in Fig. 1.

Using (31), the Sagdeev potential can be expressed in the following form:

$$V(\phi) = -\frac{n_{i0}}{Z_d n_{d0}}(1+3\beta+3\beta\phi+\beta\phi^2)e^{-\phi} - \frac{n_{e0}}{Z_d n_{d0}}\frac{1}{\delta}e^{\delta\phi}$$
$$-\frac{(M-v_{d0})^2\sigma_1}{\sqrt{2}}\left[1+\frac{2\phi}{(M-v_{d0})^2\sigma_1^2}+\sqrt{\left(1+\frac{2\phi}{(M-v_{d0})^2\sigma_1^2}\right)^2-4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2}$$
$$-\frac{2\sqrt{2}\sigma}{\sigma_1^3}\left[1+\frac{2\phi}{(M-v_{d0})^2\sigma_1^2}+\sqrt{\left(1+\frac{2\phi}{(M-v_{d0})^2\sigma_1^2}\right)^2-4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{-3/2}+C_1. \tag{32}$$

If we set the electron density n_{e0} and dust drift speed v_{d0} to zero in (32), our expression for the Sagdeev potential agrees with equation (29) in [14].

The requirements for soliton solutions of (29) are

$$V(0)=0, \quad \left[\frac{dV}{d\phi}\right]_{\phi=0}=0, \quad \left[\frac{d^2V}{d\phi^2}\right]_{\phi=0}<0 \tag{33}$$

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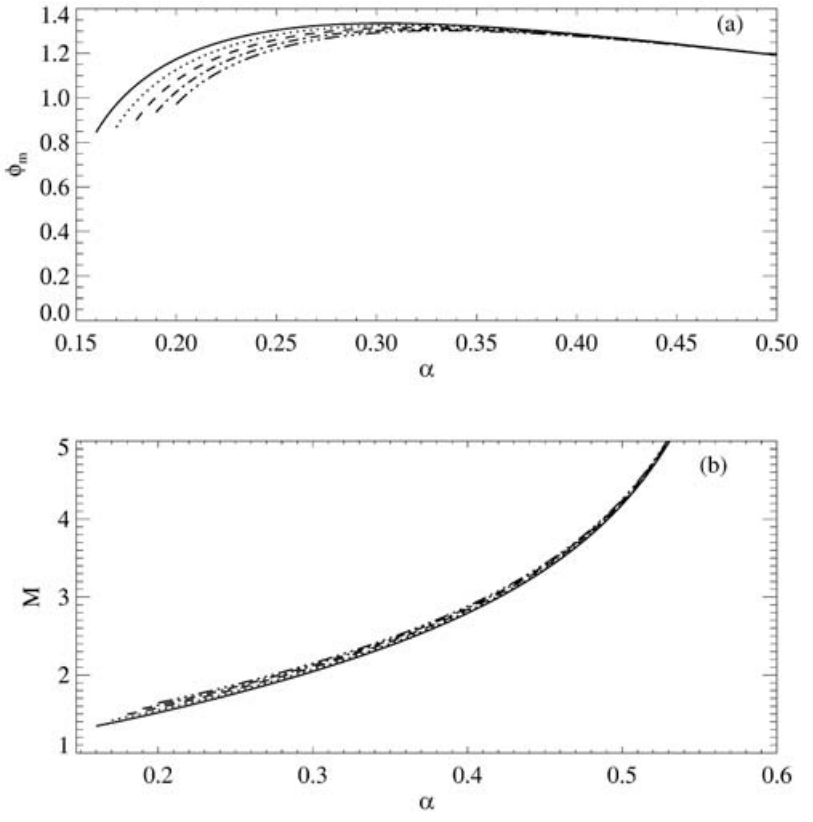


Figure 3. A plot of (a) ϕ_m and (b) M as a function of α for different values of the normalized dust temperature. The curves correspond to $\sigma = 0$ (–), $\sigma = 0.05$ (· · ·), $\sigma = 0.1$ (- -), $\sigma = 0.15$ (— · —) and $\sigma = 0.2$ (— · · · —). The fixed parameters are $v_{d0} = 0$, $n_{e0} = 0$ and $\delta = 1$.

and either

$$V(\phi) < 0 \quad \text{for } 0 < \phi < \phi_m \text{ (if } \phi_m > 0) \tag{34}$$

or

$$V(\phi) < 0 \quad \text{for } \phi_m < \phi < 0 \text{ (if } \phi_m < 0) \tag{35}$$

and

$$V(\phi_m) = 0 \quad \text{for some } \phi = \phi_m. \tag{36}$$

The conditions that need to be satisfied for a DL as mentioned in [24] are the conditions stipulated above as well as

$$\left[\frac{dV}{d\phi}\right]_{\phi=\phi_m} = 0, \quad \left[\frac{d^2V}{d\phi^2}\right]_{\phi=\phi_m} < 0, \tag{37}$$

i.e. the potential reaches a local maximum at some ϕ-value, ϕ_m with $V(\phi_m) = 0$.

3. Numerical results

In this section we numerically analyse the pseudo-potential (32) as well as (28). In Fig. 1 we examine the Sagdeev potential of the DL as a function of ϕ for different

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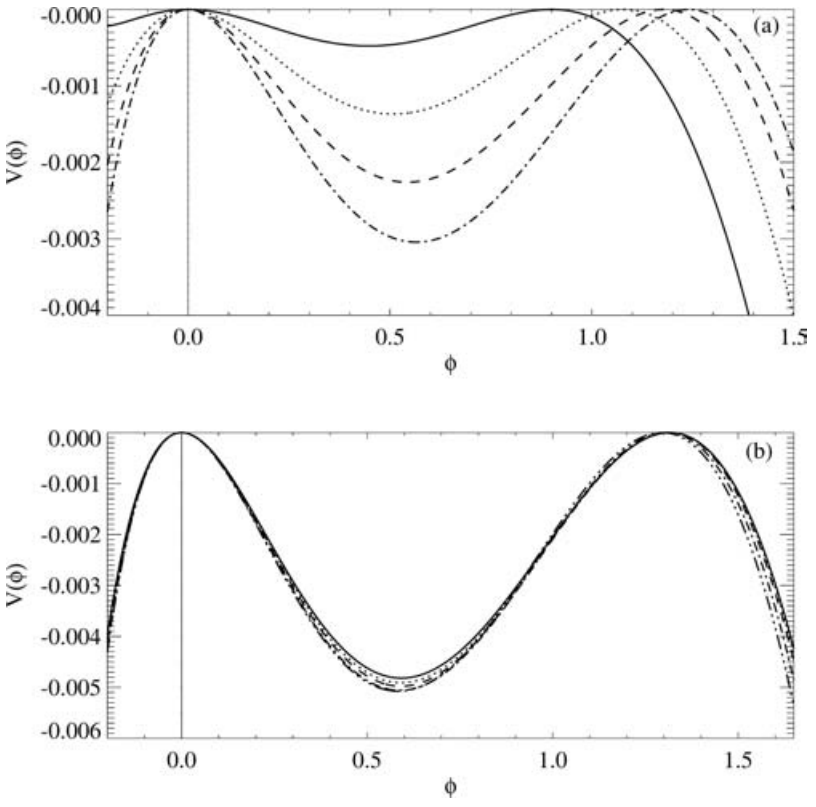


Figure 4. A plot of the DL Sagdeev potential against ϕ for different values of the non-thermal parameter α. The curves (a) correspond to $\alpha = 0.18$ (–), $\alpha = 0.2$ (· · ·), $\alpha = 0.22$ (- -) and $\alpha = 0.24$ (— · —) with the corresponding Mach numbers being 1.495, 1.580, 1.672 and 1.769, respectively. The curves (b) correspond to $\alpha = 0.32$ (–), $\alpha = 0.33$ (···), $\alpha = 0.34$ (- -), $\alpha = 0.36$ (— · —) and $\alpha = 0.38$ (— · · · —) with the corresponding Mach numbers being 2.222, 2.288, 2.356, 2.502 and 2.661, respectively. The fixed parameters are $\sigma = 0.1$, $n_{e0} = 0$, $v_{d0} = 0$ and $\delta = 1$.

values of the dust temperature for $\alpha = 0.2$, normalized $n_{e0} = 0$ and when the dust is stationary (normalized $v_{d0} = 0$). We observe that the non-thermal plasma supports only rarefactive ($\phi_m > 0$) DLs as no such solutions were found for $\phi < 0$. Figure 2 shows the DL potential profiles corresponding to the curves shown in Fig. 1. It is seen that the DL amplitude ϕ_m decreases as we increase the dust temperature, i.e. damping of the DL structure increases with an increase in the dust temperature. A slight increase in the width of the structure is also noted as the dust temperature increases.

In Fig. 3 we examine the dependence of the required Mach number M (which supports the existence of the DL) and the amplitude of the DL on the non-thermal parameter α. We find that the α dependence of ϕ_m and M are both nonlinear. The distinction between the curves corresponding to different dust temperatures is apparent for small values of α but the curves converge and become indistinguishable as α becomes large. The amplitude of the DL for the different dust temperatures increases with increasing α up to $\alpha \approx 0.3$, beyond which the amplitude is seen to decrease with increasing α. The Mach number is found to increase with increasing

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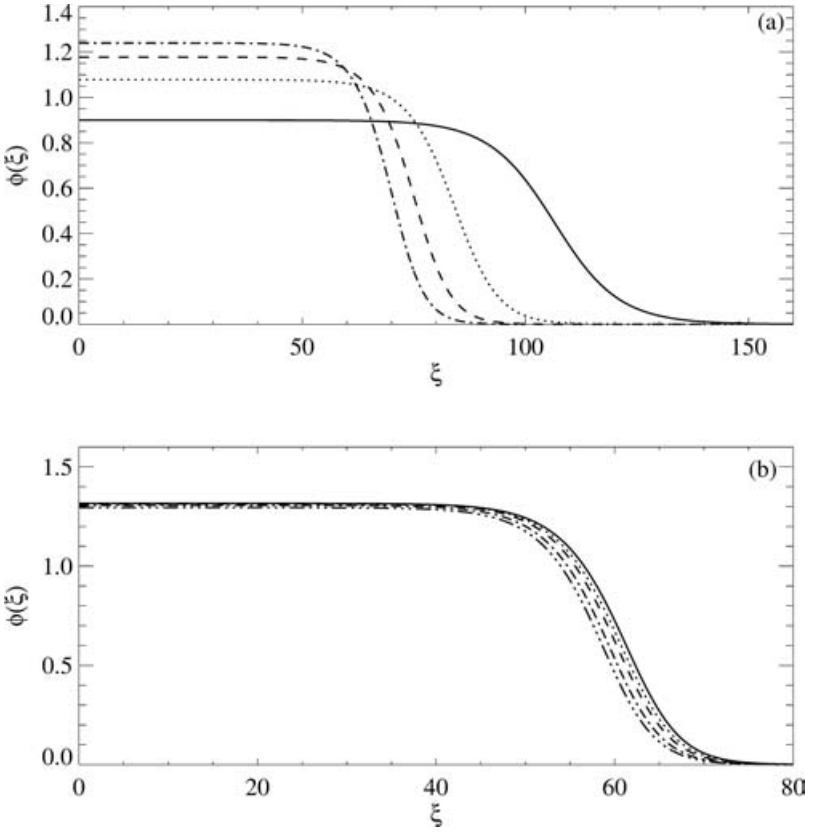


Figure 5. A plot of the DL potential as a function of ξ for the corresponding curves in Fig. 4(a) and 4(b).

α. We observe that for any particular dust temperature, there is a certain minimum value of α, viz. $\alpha_{\rm crit}$, which is required to support the existence of the DL structure. Moreover, this value of $\alpha_{\rm crit}$ increases with dust temperature. From Fig. 3(a) we note that the situation that corresponds to a thermal plasma ($\alpha = 0$), does not support the existence of the DL.

We examine in Fig. 4 the variation of the DL Sagdeev potential with ϕ for different values of α when the normalized dust temperature is $\sigma = 0.1$, normalized $n_{\rm e0} = 0$ and $v_{\rm d0} = 0$. On examination of Fig. 4(a), we observe an increase in $\phi_{\rm m}$ as α increases for smallish values of α in the range 0.18 to 0.24. The opposite is seen to occur if we consider values of α in the range 0.32 to 0.38 as observed in Fig. 4(b) where we observe a decrease in $\phi_{\rm m}$ with an increase in α. Furthermore, the values of $\phi_{\rm m}$ are very close to each other and the curves corresponding to different α differ slightly from each other in Fig. 4(b) when compared with the curves in Fig. 4(a). To understand this behaviour we refer to Fig. 3(a) where it is seen that, for any particular dust temperature, the gradient of the curve ($\phi_{\rm m}$ versus α) is large in the range of α near $\alpha = 0.2$ but much less (almost zero) near $\alpha = 0.3$. This justifies why the values of $\phi_{\rm m}$ are clearly separated in Fig. 4(a) but very close in Fig. 4(b). For α in the range 0.18 to 0.24, the amplitude of the DL is seen to increase with increasing α as observed in Fig. 5(a). The opposite behaviour is found to be true

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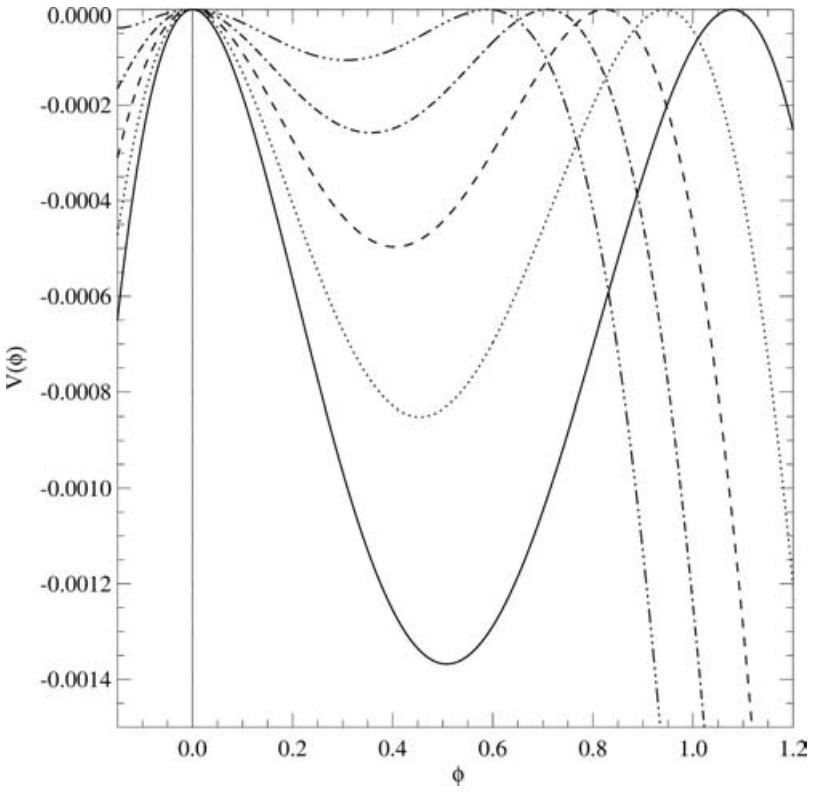


Figure 6. The DL Sagdeev potential as a function of ϕ for different values of the normalized electron concentration. The curves correspond to $n_{e0}/Z_d n_{d0} = 0$ (–), 0.005 (· · ·), 0.01 (- -), 0.015 (— · —) and 0.02 (— · · · —) with the corresponding Mach numbers being 1.580, 1.551, 1.527, 1.505 and 1.487, respectively. The fixed parameters are $\sigma = 0.1$, $\alpha = 0.2$, $\delta = 1$ and $v_{d0} = 0$.

for α in the range $\alpha = 0.32$ to 0.38 as demonstrated in Fig. 5(b) where we observe that ϕ_m decreases with increasing α. In keeping with what is observed in Fig. 4, we find that the values of ϕ_m are extremely close to each other in Fig. 5(b) when compared with the curves in Fig. 5(a).

Next we consider the presence of a finite electron population. Figure 6 shows a plot of the Sagdeev potential for different values of the normalized electron concentration for $\alpha = 0.2$, normalized dust temperature $\sigma = 0.1$ and $v_{d0} = 0$. The presence of electrons in the non-thermal plasma appears to suppress the formation of a DL because as the electron concentration increases, we observe that ϕ_m decreases. A decrease in ϕ_m with increasing electron concentration was also observed by Bharuthram and Shukla [25] but only when $T_c < T_h$, where $T_{c(h)}$ is the temperature of the cold (hot) ion population. The DL potential profiles corresponding to the curves in Fig. 6 are shown in Fig. 7. Our observation of ϕ_m decreasing with increasing normalized n_{e0} is highlighted in Fig. 8(a), where it is clearly seen that for any particular dust temperature, the DL amplitude ϕ_m decreases as normalized n_{e0} increases. We also note that a very small range of very low electron concentration values support the existence of the DL. In addition, Fig. 8(a) reveals that for any fixed value of normalized n_{e0}, ϕ_m decreases with an

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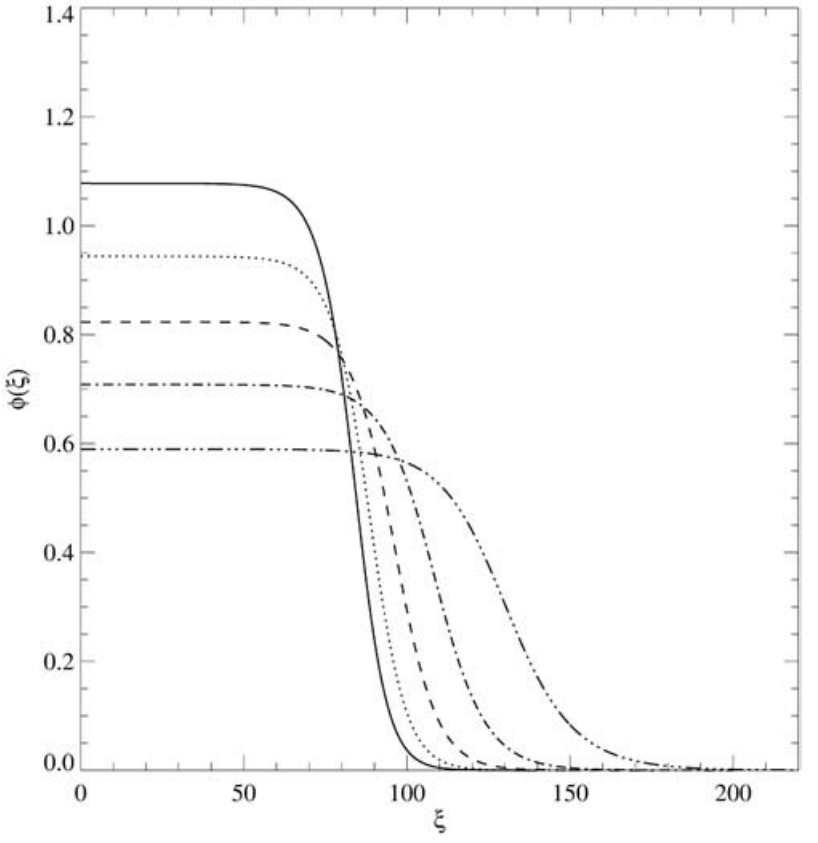


Figure 7. A plot of the DL potential as a function of ξ for the corresponding curves in Fig. 6.

increase in the dust temperature as found in the absence of electrons. The range of electron concentrations over which the DL exists, is seen (Fig. 8(a)) to decrease significantly with increasing dust temperature, as does the range of Mach numbers (Fig. 8(b)). An upper limit for the equilibrium electron concentration for which DLs are supported was also observed by Bharuthram and Shukla [25].

4. Conclusions

In this paper, we have investigated the existence of arbitrary amplitude dust-acoustic DLs in an unmagnetized dusty plasma comprising Boltzmann electrons, non-thermal ions and negatively charged dust particles that are drifting relative to the other two species. We have undertaken a numerical study to examine the dependence of the DL amplitude ϕ_m and the Mach number (which supports the existence of the DL) on various plasma parameters such as the non-thermal parameter α, dust temperature and electron concentration. Our primary finding is that for the other fixed plasma parameters, there is a minimum value of the ion non-thermal parameter, viz., α_{crit}, that is required for the existence of a DL. A plasma with thermal ions ($\alpha = 0$) does not support the existence of DLs.

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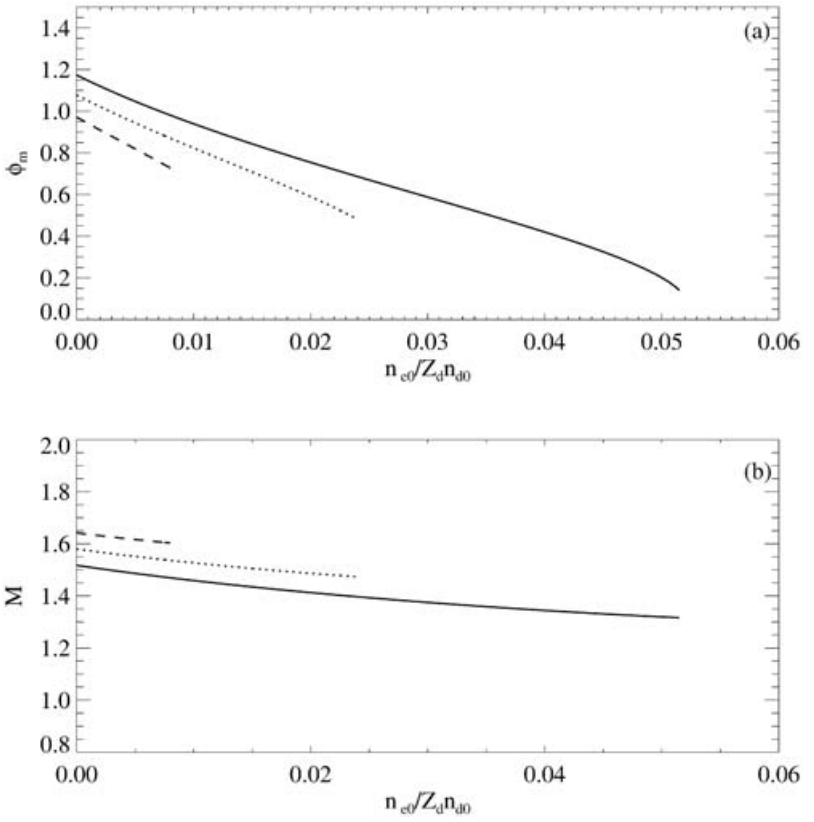


Figure 8. A plot of (a) ϕ_m and (b) M as a function of the normalized electron concentration for different values of the normalized dust temperature. The curves are correspond to $\sigma = 0$ (–), $\sigma = 0.1$ ($\cdots$) and $\sigma = 0.2$ (- -) . The fixed parameters are $\alpha = 0.2$, $v_{d0} = 0$ and $\delta = 1$.

Our results indicate that only rarefactive ($\phi_m > 0$) DLs are possible. We find that increasing the dust temperature results in a decrease in the amplitude of the DL for a fixed value of α, when there are no electrons present in the plasma and there is no dust streaming.

The calculations show that ϕ_m remains constant with increasing dust drift velocity whilst the Mach number is found to increase linearly with increasing dust drift velocity for fixed values of α, dust temperature and when there are no electrons present in the plasma. On the other hand, a decrease (increase) in ϕ_m (M) with increasing dust temperature is observed for fixed values of the dust drift velocity, α and in the absence of electrons.

When we increase the electron concentration, we observe that ϕ_m decreases for a fixed value of α and the dust temperature σ in the absence of dust streaming ($v_{d0} = 0$). It is found that a very small range of electron concentrations supports the existence of the DL. The range of n_{e0} values and Mach numbers which supports the existence of the DL is seen to decrease with increasing dust temperature.

The dependence of the amplitude and the Mach number of the DL on the non-thermal parameter α is found to be nonlinear. Our results show that ϕ_m increases initially rapidly with increasing α up to some maximum value of α, beyond which

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ϕ_m decreases very gradually with increasing α. The Mach number that supports the existence of the DL is seen to increase as α increases.

For a particular dust temperature, there is a minimum value of α, α_{crit}, that is required to support the existence of the DL. We find that this value of α_{crit} increases with increasing dust temperature. Our results indicate that a plasma with Maxwellian ions ($\alpha = 0$) does not support the existence of the DL. The requirement that the plasma has one or more species of non-Maxwellian particles is crucial for the plasma to admit DL potential structures.

The findings in this paper are important in understanding nonlinear wave phenomena in space plasmas where observations indicate that non-Maxwellian particle velocity distributions [8,9] and DLs [19,20] appear to be a common feature. A region in the Earth's magnetosphere of particular importance is the auroral acceleration region.

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