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Low-frequency electromagnetic waves in a partially ionized multi-component magnetoplasma

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Abstract. The dispersion properties of low-frequency electromagnetic waves in a partially ionized multi-ion dusty magnetoplasma containing electrons, ions of two different species, charged dust particles and neutrals have been investigated. It has been found that the combined effects of stationary charged dust particles and an additional ion component introduce a new cut-off frequency, and that the polarities of dust particles and the second ion-species significantly modify this new as well as the existing forbidden frequency bands. It has also been observed that the collisions of lighter ion species significantly reduce or completely eliminate both the new and existing forbidden frequency bands, while those of heavier ions reduce or completely eliminate the new forbidden frequency band, but enhance the existing forbidden frequency band.

1. Introduction

A little under fifty years ago, Hines (1957) first considered a plasma with many ions, and examined the heavy-ion effects on audio-frequency radio wave propagation in the ionosphere. He showed that the upper cut-off frequency for the wave propagation transverse to the static magnetic field direction was at a frequency well above the ion gyrofrequency. Buchsbaum (1960) examined the resonance in a magnetoplasma with two ion species, and observed that their presence introduces an additional resonance for the electrostatic wave propagation across the static magnetic field at a frequency between the two ionic gyrofrequencies. Yakimenko (1962) and Gintsburg (1963) considered a plasma with multiple ions and studied the propagation of magnetic field-aligned Alfvén waves. They found different resonance frequencies at different ion gyrofrequencies. Smith and Brice (1964) considered the propagation of low-frequency electromagnetic waves in a multi-ion magnetoplasma containing electrons and multiple positive ions, and showed that for each additional ionic species, there exists a multiple-ion resonance and a multiple-ion cut-off for the waves propagating perpendicular to the magnetic field. These pioneering works (Hines 1957; Buchsbaum 1960; Yakimenko 1962; Gintsburg 1963; Smith and Brice 1964), which clearly show that the presence of the additional ion species does not

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only modify the existing electron–proton plasma waves, but also introduces some new interesting features, motivated several authors (Sauer et al. 1994, 1996, 1998, 2003; Verheest 1995; Verheest and Meuris 1996; Verheest et al. 2002; Pokhotelov et al. 1997, 1998; Boldyrev 1998; Irie and Ohsawa 2003; Jacquinot et al. 1977; Watanabe et al. 1978; Weingarten et al. 2001; Chakrabarti et al. 2002) to study in detail the effects of multi-ion species on the linear and nonlinear properties of electromagnetic waves in space (Sauer et al. 1994, 1996, 1998, 2003; Verheest 1995; Verheest and Meuris 1996; Verheest et al. 2002; Pokhotelov et al. 1997, 1998; Boldyrev 1998; Irie and Ohsawa 2003) and laboratory (Jacquinot et al. 1977; Watanabe et al. 1978; Weingarten et al. 2001; Chakrabarti et al. 2002) plasmas.

Recently, motivated by experimental observations (Jacquinot et al. 1977; Weingarten et al. 2001), Chakrabarti et al. (2002) studied the modified compressional Alfvén waves in a two-ion-species plasma by assuming that the current is carried by electrons and ions moving in a plane perpendicular to the external magnetic field. They showed that the behavior of such a two-ion-species plasma significantly differs from that of a single-ion-species plasma. However, an ensemble of dust particles (Mendis and Rosenberg 1994; Mendis 2002), which are extremely massive (a billion times heavier than the ions), and highly charged (a few thousand electron/proton charges and neutrals, which are significant in numbers: much more than electron or ion number densities in many cases) are found to exist in most space and laboratory plasma situations. The presence of dust particles in a fully ionized electron–ion plasma is found to modify the existing plasma waves as well as to introduce new eigenmodes (Rao et al. 1990; Shukla and Silin 1992; Rao 1993, 1995; Shukla 1996; Verheest 2000; Shukla and Mamun 2002). To the best of our knowledge, no theory has been developed so far for the propagation of low-frequency electromagnetic waves in a multi-ion, partially ionized (Stenflo et al. 1989, 1994) dusty magnetoplasma, which is indeed a more realistic dusty plasma situation. Therefore, in the present work, we consider a multi-ion partially ionized dusty magnetoplasma (composed of electrons, ions of two different species as well as stationary neutrals and charged dust particles), and present a complete theory for the propagation of low-frequency electromagnetic waves. We mainly investigate the effects of the presence of arbitrarily (positively or negatively) charged dust particles, the polarity of the additional ion component, and collisions of both ion species with stationary neutrals on cut-off and resonance frequencies for the low-frequency electromagnetic waves under consideration.

The paper is organized as follows. The governing equations are provided in Sec. 2. A general dispersion relation for the low-frequency electromagnetic waves propagating in a multi-ion partially ionized dusty magnetoplasma is derived in Sec. 3. The effects of polarity of second ion component, dust component, and ion–neutral collisions on their dispersion properties along with resonance and cut-off frequencies are examined in the same section. Finally, a brief discussion is presented in Sec. 4.

2. Governing equations

We consider a multi-ion, partially ionized dusty magnetoplasma composed of electrons, ions of two different species as well as stationary neutrals and charged dust particles. Thus, at equilibrium we have

$$n_{\mathrm{e}}e = q_1 n_1 + q_2 n_2 + q_{\mathrm{d}} n_{\mathrm{d}}, \tag{2.1}$$

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where n_e (n_d) is the equilibrium electron (dust) number density, n_1 and n_2 are the equilibrium ion number densities of two different species, $q_1 = Z_1 e$, $q_2 = sZ_2 e$ and $q_d = pZ_d e$ are first ion-species charge, second ion-species charge and dust particle charge, respectively, e is the proton charge or the magnitude of the electron charge, p (s) is $+1$ when the dust particle (second ion species) is positively charged, while p (s) is -1 when the dust particle (second ion species) is negatively charged.

We are interested in elliptically polarized electromagnetic waves whose frequencies are much smaller than the electron gyrofrequency, i.e. $|\partial_t| \ll \omega_{ce}$, where $\partial_t = \partial/\partial t$, $\omega_{ce} = eB_0/m_e c$, B_0 is the magnitude of the ambient magnetic field $\mathbf{B}_0$, m_e is the electron mass, and c is the speed of light in vacuum. In the presence of such low-frequency electromagnetic waves, the dynamics of electrons and ions of two different species are described by a set of closed linearized equations

$$0 \simeq \mathbf{E} + \frac{1}{c}\mathbf{V}_e \times \mathbf{B}_0, \tag{2.2}$$

$$\hat{D}_1 \mathbf{V}_1 \simeq \frac{Z_1 e}{m_1}\left(\mathbf{E} + \frac{1}{c}\mathbf{V}_1 \times \mathbf{B}_0\right), \tag{2.3}$$

$$\hat{D}_2 \mathbf{V}_2 \simeq \frac{sZ_2 e}{m_2}\left(\mathbf{E} + \frac{1}{c}\mathbf{V}_2 \times \mathbf{B}_0\right), \tag{2.4}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\partial_t \mathbf{B}, \tag{2.5}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}, \tag{2.6}$$

where $\hat{D}_{1,2} = \partial_t + \nu_{1,2}$, $\mathbf{V}_1$ and $\mathbf{V}_2$ are the velocities of the first and second ion species, m_1 (m_2) is the mass of the first (second) ion species, ν_1 (ν_2) is the collision frequency of the first (second) ion species with the stationary neutrals, $\mathbf{E}$ and $\mathbf{B}$ are the perturbed electric and magnetic fields, and $\mathbf{J} = e(n_1\mathbf{V}_1 + sZ_2 n_2 \mathbf{V}_2 - n_e \mathbf{V}_e)$ is the net plasma current. It is obvious from (2.1)–(2.6) that the stationary charged dust particles only play a role in maintaining the equilibrium charge neutrality condition (2.1).

3. Dispersion properties

We now assume that $\mathbf{B}_0$ and $\mathbf{B}$ both point along the z-axis, $\mathbf{E}$ lies in the x–y plane, and $\mathbf{J}$ is almost along the y-axis. We can, therefore, consider an approximated one-dimensional problem in which all perturbed quantities will depend only on x and t. Thus, using (2.1)–(2.6) we have

$$\hat{D}_1 V_{1x} \simeq \omega_{c1}[(1-\mu_1)V_{1y} - s\mu_2 V_{2y}] - \mu_1 V_{A1}^2 \frac{\partial_x B}{B_0}, \tag{3.1}$$

$$\hat{D}_1 V_{1y} \simeq -\omega_{c1}[(1-\mu_1)V_{1x} - s\mu_2 V_{2x}], \tag{3.2}$$

$$\hat{D}_2 V_{2x} \simeq s\omega_{c2}[(1-s\mu_2)V_{2y} - \mu_1 V_{1y}] - s\mu_2 V_{A2}^2 \frac{\partial_x B}{B_0}, \tag{3.3}$$

$$\hat{D}_2 V_{2y} \simeq -s\omega_{c2}[(1-s\mu_2)V_{2x} - \mu_1 V_{1x}], \tag{3.4}$$

$$\mu_1 \partial_x V_{1x} + s\mu_2 \partial_x V_{2x} + \frac{\partial_t B}{B_0} = 0, \tag{3.5}$$

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where $\partial_x = \partial/\partial x$, $\mu_1 = 1/(1 + s\mu_i + p\mu_d)$, $\mu_2 = \mu_i/(1 + s\mu_i + p\mu_d)$, $\mu_i = Z_2 n_2/Z_1 n_1$, $\mu_d = Z_d n_d/Z_1 n_1$, $\omega_{c1} = Z_1 eB_0/m_1 c$, $\omega_{c2} = Z_2 eB_0/m_2 c$, $V_{A1} = B_0/\sqrt{4\pi n_1 m_1}$, and $V_{A2} = B_0/\sqrt{4\pi n_2 m_2}$.

Now, substituting (3.2) into (3.1) and (3.4) into (3.3) we can easily eliminate V_{1y} and V_{2y}. Thus, we have

$$\hat{X}_1 V_{1x} - \hat{X}_2 V_{2x} + \mu_1 V_{A1}^2 \hat{D}_1 \hat{D}_2 \frac{\partial_x B}{B_0} \simeq 0, \tag{3.6}$$

$$\hat{Y}_1 V_{1x} - \hat{Y}_2 V_{2x} - s\mu_2 V_{A2}^2 \hat{D}_1 \hat{D}_2 \frac{\partial_x B}{B_0} \simeq 0, \tag{3.7}$$

where

$$\hat{X}_1 = \hat{D}_1^2 \hat{D}_2 + (1-\mu_1)^2 \omega_{c1}^2 \hat{D}_2 + \mu_1 \mu_2 \omega_{c1} \omega_{c2} \hat{D}_1,$$
$$\hat{X}_2 = s(1-\mu_1)\mu_2 \omega_{c1}^2 \hat{D}_2 + (1 - s\mu_2)\mu_2 \omega_{c1} \omega_{c2} \hat{D}_1,$$
$$\hat{Y}_1 = (1 - s\mu_2)\mu_1 \omega_{c2}^2 \hat{D}_1 + s(1-\mu_1)\mu_1 \omega_{c1} \omega_{c2} \hat{D}_2,$$
$$\hat{Y}_2 = \hat{D}_2^2 \hat{D}_1 + (1 - s\mu_2)^2 \omega_{c2}^2 \hat{D}_1 + \mu_1 \mu_2 \omega_{c1} \omega_{c2} \hat{D}_2.$$

We now perform a normal mode analysis by assuming

$$\left.\begin{matrix} V_{1x} \\ V_{2x} \\ B \end{matrix}\right\} \propto \exp[-i(\omega t - kx)] \tag{3.8}$$

in (3.5)–(3.7), and obtain a dispersion relation

$$\left(\Omega_1^2 - \mu_1^2 k^2 V_{A1}^2\right)\left(\Omega_2^2 - \mu_2^2 k^2 V_{A2}^2\right) = \left(\Gamma_1^2 - s\mu_1\mu_2 k^2 V_{A1}^2\right)\left(\Gamma_2^2 - s\mu_1\mu_2 k^2 V_{A2}^2\right), \tag{3.9}$$

where

$$\Omega_1^2 = \omega^2 \alpha_1 - (1-\mu_1)^2 \omega_{c1}^2/\alpha_1 - \mu_1\mu_2\omega_{c1}\omega_{c2}/\alpha_2,$$
$$\Omega_2^2 = \omega^2 \alpha_2 - (1 - s\mu_2)^2 \omega_{c2}^2/\alpha_2 - \mu_1\mu_2\omega_{c1}\omega_{c2}/\alpha_1,$$
$$\Gamma_1^2 = s(1-\mu_1)\mu_2 \omega_{c1}^2/\alpha_1 + (1 - s\mu_2)\mu_2\omega_{c1}\omega_{c2}/\alpha_2,$$
$$\Gamma_2^2 = (1 - s\mu_2)\mu_1 \omega_{c2}^2/\alpha_2 + s(1-\mu_1)\mu_1\omega_{c1}\omega_{c2}/\alpha_1,$$

and $\alpha_{1,2} = 1 + \nu_{1,2}/\omega$. Equation (3.9) is a general dispersion relation for elliptically polarized low-frequency ($\omega \ll \omega_{ce}$) electromagnetic waves propagating in a multi-ion, partially ionized dusty magnetoplasma. In the following, we consider two different situations.

3.1. Collisionless plasma

First, we consider a collisionless plasma ($\nu_{1,2} = 0$, i.e. $\alpha_{1,2} = 1$) for which (3.9) can be simplified as

$$\omega^2\left(\omega^2 - \omega_c^2\right) + \omega_d^4 = k^2 V_A^2\left(\omega^2 - \omega_r^2\right). \tag{3.10}$$

Here

$$V_A^2 = V_{A1}^2 \frac{1 + \sigma\mu_i}{(1 + s\mu_i + p\mu_d)^2},$$
$$\omega_c^2 = \omega_{c1}^2 \frac{(\mu_i + \sigma)^2 + \mu_d f_d}{(1 + s\mu_i + p\mu_d)^2},$$
$$\omega_r^2 = \omega_{c1}^2 \frac{\sigma(\mu_i + \sigma)}{(1 + \sigma\mu_i)},$$

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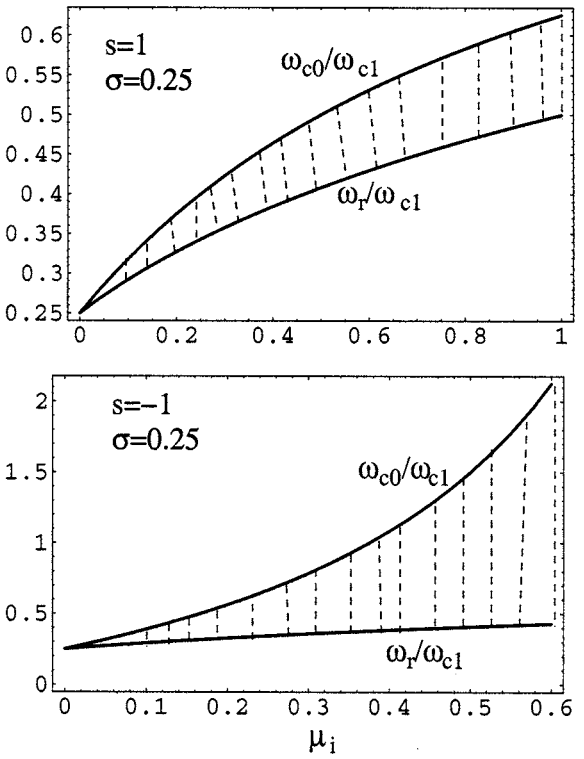


Figure 1. The variation of ω_r/ω_{c1}, ω_c/ω_{c1}, and f_b/ω_{c1} with μ_i for $\sigma = 0.25$ ($Z_2/Z_1 = 4$ and $m_2/m_1 = 12$), $s = 1$ (upper plot) and $s = -1$ (lower plot). The variation of f_b/ω_{c1} is indicated by the dashed lines.

$$\omega_d^4 = \omega_{c1}^4 \frac{\sigma^2 \mu_d^2}{(1 + s\mu_i + p\mu_d)^2},$$

where $\sigma = Z_2 m_1/Z_1 m_2$ and $f_d = \mu_d(1+\sigma^2) + 2p(s\mu_i + \sigma^2)$. It is obvious that if we neglect the presence of the second ion component as well as the background charged dust particles (i.e. we set $\mu_i = 0$ and $\mu_d = 0$ yielding $\omega_c = 0$, $\omega_r = 0$ and $\omega_d = 0$), (3.10) reduces to the dispersion relation for the fast Alfvén waves ($\omega^2/k^2 \simeq V_{A1}^2 = B_0^2/4\pi n_1 m_1$) in which the magnetic pressure $B_0^2/4\pi$ gives rise to the restoring force, and the ion mass density ($n_1 m_1$) provides the inertia. However, when we consider the presence of the second ion component ($\mu_i \neq 0$), but ignore the effect of the background charged dust particles ($\mu_d = 0$), (3.10) becomes

$$\omega^2(\omega^2 - \omega_{c0}^2) = k^2 V_{A0}^2(\omega^2 - \omega_r^2), \tag{3.11}$$

where $V_{A0}^2 = V_{A1}^2(1+\sigma\mu_i)/(1+s\mu_i)^2$ and $\omega_{c0}^2 = \omega_{c1}^2(\mu_i+\sigma)^2/(1+s\mu_i)^2$. Equation (3.11), which for $s = 1$ completely agrees with the dispersion relation of Chakrabarti et al. (2002), implies that the contribution of the second ion component introduces a resonance ($k = \infty$) at $\omega = \omega_r$, and a cut-off ($k = 0$) at $\omega = \omega_{c0}$ as well as a forbidden frequency band (f_b) between these two frequencies, $f_b = \omega_{c0} - \omega_r$.

We have numerically shown how ω_r, ω_{c0}, and f_b vary with μ, σ, and s. The numerical results are displayed in Figs 1 and 2.

The upper plot of Fig. 1 shows that for positive ion species ($s = 1$) as we increase μ_i, the increase in ω_{c0} is slightly more than that in ω_r, i.e. $f_b = \omega_{c0} - \omega_r$ increases

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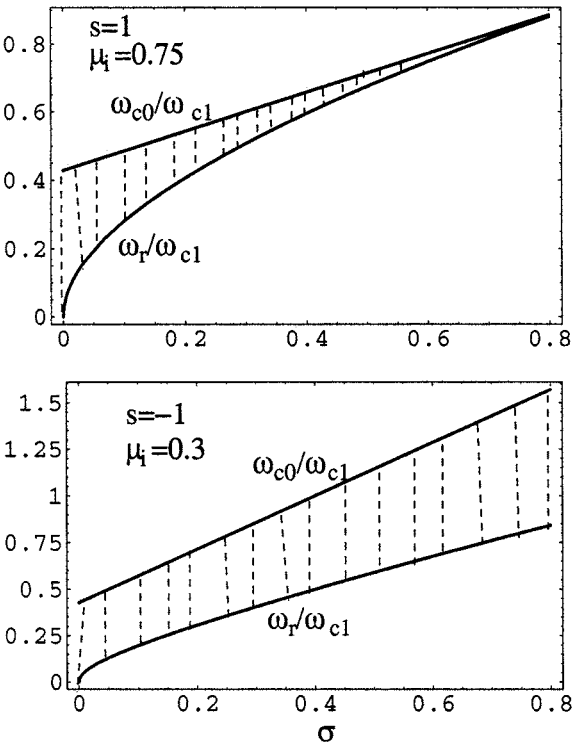


Figure 2. The variation of ω_r/ω_{c1}, ω_{c0}/ω_{c1} and f_b/ω_{c1} with σ for $\mu_i = 0.75$ ($Z_2/Z_1 = 4$ and $n_2/n_1 = 0.75$), $s = 1$ (upper plot), and for $\mu_i = 0.3$ and $s = -1$ (lower plot). The variation of f_b/ω_{c1} is indicated by the dashed lines.

with μ_i. The lower plot of Fig. 1 depicts that for negative ion species ($s = -1$) as we increase μ_i, ω_{c0} (ω_r) increases very slowly (rapidly), i.e. $f_b = \omega_{c0} - \omega_r$ increases very rapidly with μ_i. Figure 2 indicates that the variation of the forbidden frequency band width $f_b = \omega_{c0} - \omega_r$ with σ for positive ion species (cf. upper plot of Fig. 2) is much more rapid than that for negative ion species (cf. lower plot of Fig. 2).

It is obvious from the expressions for ω_{c0}, ω_r and f_b as well as from Figs 1 and 2 that the polarity of the second ion component shifts the cut-off frequency (ω_c), i.e. changes the forbidden frequency bandwidth (f_b) quite significantly. However, the nature of the dispersion curves (solutions of (3.11)) before and after the forbidden frequency band are found to be exactly the same as those given in Chakrabarti et al. (2002) if we normalize ω by ω_{c2} and k by ω_{c2}/V_{A0}.

We are now interested in examining the role of the stationary (background) charged dust particles on the resonance frequency, the cut-off frequency, and the forbidden frequency bandwidth by analyzing (3.10). It is obvious that the last term (ω_d^4) on the right-hand side of (3.10) arises due to the presence of stationary charged dust particles. Due to this extra term (ω_d^4), the cut-off frequency has been significantly modified, and can be determined by $\omega_{c\star}^4 - \omega_c^2\omega_{c\star}^2 + \omega_d^4 = 0$, i.e.

$$\omega_{c\star 1,2}^2 = \tfrac{1}{2}\left(\omega_c^2 \pm \sqrt{\omega_c^4 - 4\omega_d^4}\right), \tag{3.12}$$

which represents two cut-off frequencies, one is $\omega_{c\star 1}$ corresponding to + sign and the other is $\omega_{c\star 2}$ corresponding to − sign. It is obvious that $\omega_{c\star 1} = \omega_c$ and $\omega_{c\star 2} = 0$

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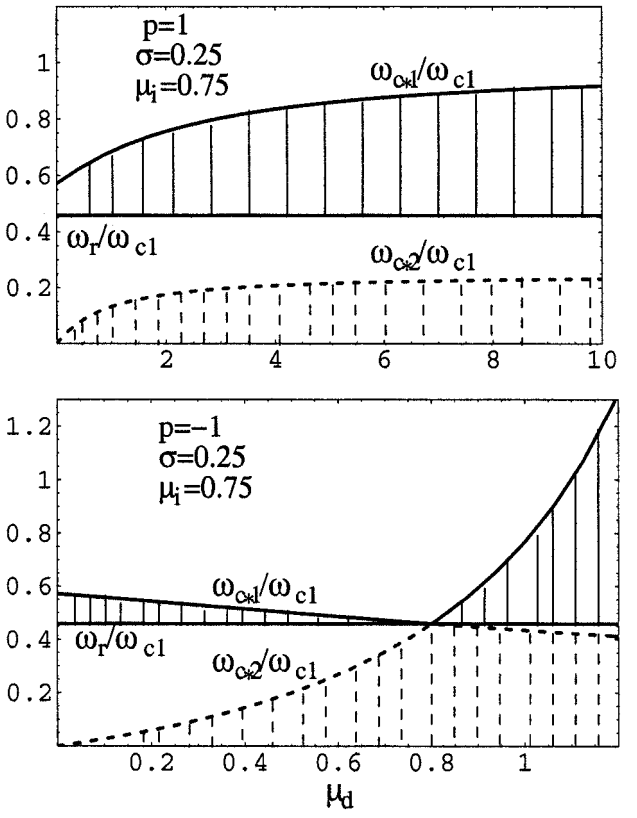


Figure 3. The variation of $\omega_{c\star1}/\omega_{c1}$, $\omega_{c\star2}/\omega_{c1}$, f_{b1}/ω_{c1}, and f_{b2}/ω_{c1} with μ_d for $\mu_i = 0.75$, $\sigma = 0.25$, $p = 1$ (upper plot), and $p = -1$ (lower plot). The variation of f_{b1} (f_{b2}) is indicated by solid (dashed) lines.

for $\mu_d = 0$. This means that the cut-off frequency is due to the combined effects of the second ion species and the background charged dust particles. We have shown the variation of $\omega_{c\star1}$ and $\omega_{c\star2}$ with μ_d for both positively (upper plot of Fig. 3) and negatively (lower plot of Fig. 3) charged dust particles. It is obvious from Fig. 3 that (3.10) has one resonance frequency at $\omega = \omega_r$ and two cut-off frequencies at $\omega = \omega_{c\star1}$ and $\omega = \omega_{c\star2}$, respectively. The two cut-off frequencies correspond to two forbidden frequency bands, namely $f_{b1} = \omega_{c\star2} - \omega_r$ and $f_{b2} = \omega_{c\star1} - \omega_r$. We have also numerically solved (3.10) and shown the dispersion properties of the low-frequency electromagnetic waves along with the cut-off frequencies ($\omega_{c\star1}$ and $\omega_{c\star2}$) and the resonance frequency (ω_r) for a particular set of dusty plasma parameters, namely $\mu_i = 0.75$, $\sigma = 0.25$, $\mu_d = 0.4$, $p = 1$ (upper plot of Fig. 4) and $p = -1$ (lower plot of Fig. 4). The numerical results for the cut-off and resonance frequencies shown in the upper and lower plots of Fig. 4 completely agree with those shown in the upper and lower plots of Fig. 3.

3.2. Collisional plasma

We now examine the effects of ion–neutral collisions ($\nu_{1,2} \neq 0$) on the dispersion properties of the low-frequency electromagnetic waves as well as on their cut-off and resonance frequencies by a numerical analysis of (3.9). The numerical results

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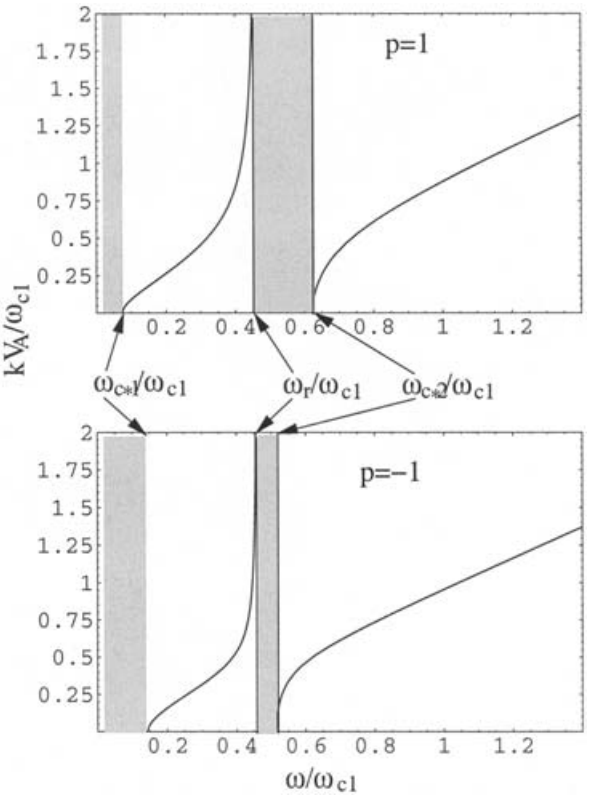


Figure 4. The dispersion properties of the low-frequency electromagnetic wave branches along with two cut-off and resonance frequencies for $\mu_i = 0.75$, $\sigma = 0.25$, $\mu_d = 0.4$, $p = 1$ (upper plot) and $p = -1$ (lower plot).

are displayed in Figs 5 and 6. Figure 5 shows that keeping other parameters constant as we increase ν_1, the new (introduced by the combined effects of charged dust particles and second ion species) and existing (introduced by the additional ion species only) forbidden frequency bands are decreased. It also implies that both of these forbidden frequency bands are completely destroyed for $\nu_1 > 0.6\omega_{c1}$ even with $\nu_2 = 0$. Figure 6 shows that keeping other parameters constant, as we increase ν_2, the new forbidden frequency band is decreased, but the existing one is increased and shifted to a lower frequency range. It is also obvious from Fig. 6 that the new forbidden frequency band is completely destroyed for $\nu_1 = 0.1\omega_{c1}$ and for $\nu_2 \geqslant 0.3\omega_{c1}$.

4. Discussion

We have presented a complete theory for the low-frequency electromagnetic waves in a multi-ion partially ionized dusty magnetoplasma composed of electrons, ions of two different species as well as stationary neutrals and charged dust particles. We have mainly investigated the effects of the presence of arbitrarily (positively or negatively) charged dust particles, the polarity of the second ion component, and collisions of both ion species with stationary neutrals on the cut-off and resonance

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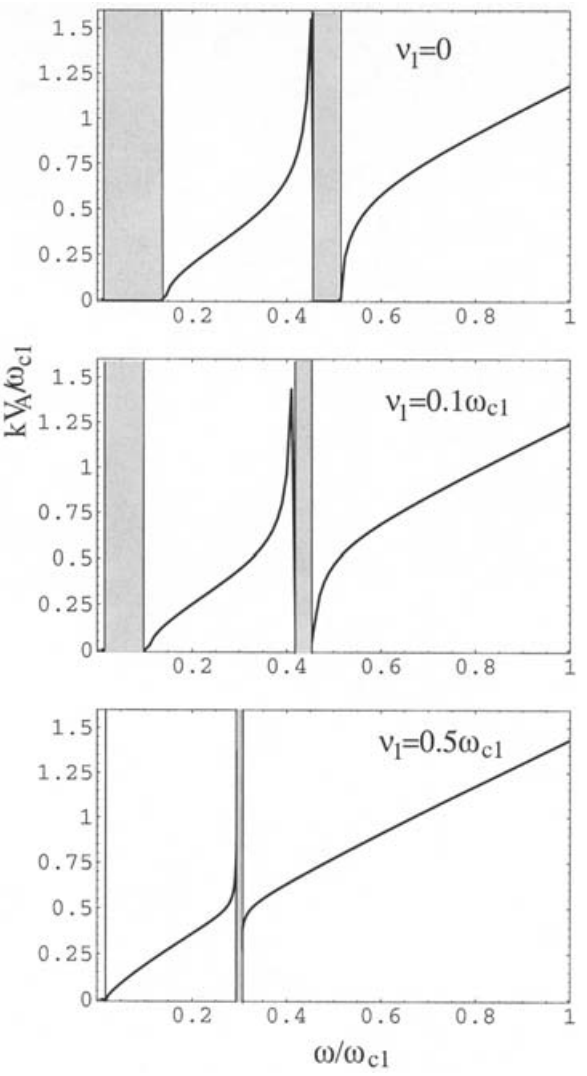


Figure 5. The effects of collision frequency ν_1 on the cut-off and resonance frequencies for $\mu_i = 0.75$, $\sigma = 0.25$, $\mu_d = 0.4$, $s = 1$, $p = -1$, $\nu_2 = 0$, $\nu_1 = 0$ (upper plot), $\nu_1 = 0.3\omega_{c1}$ (middle plot) and $\nu_1 = 0.5\omega_{c1}$ (lower plot).

frequencies of the modified compressional Alfvén waves. We have found that the combined effects of stationary charged dust particles and the additional ion component introduce a new cut-off frequency, and that the polarities of dust particles and the second ion species significantly modify the new (introduced by the combined effects of charged dust particles and second ion species) and existing (introduced by the additional ion species only) forbidden frequency bands. It has also been observed that the collisions of lighter ion species significantly reduce or completely eliminate both the new and existing forbidden frequency bands, while those of heavier ion species reduce or completely eliminate the new forbidden frequency band, but enhance the existing forbidden frequency band.

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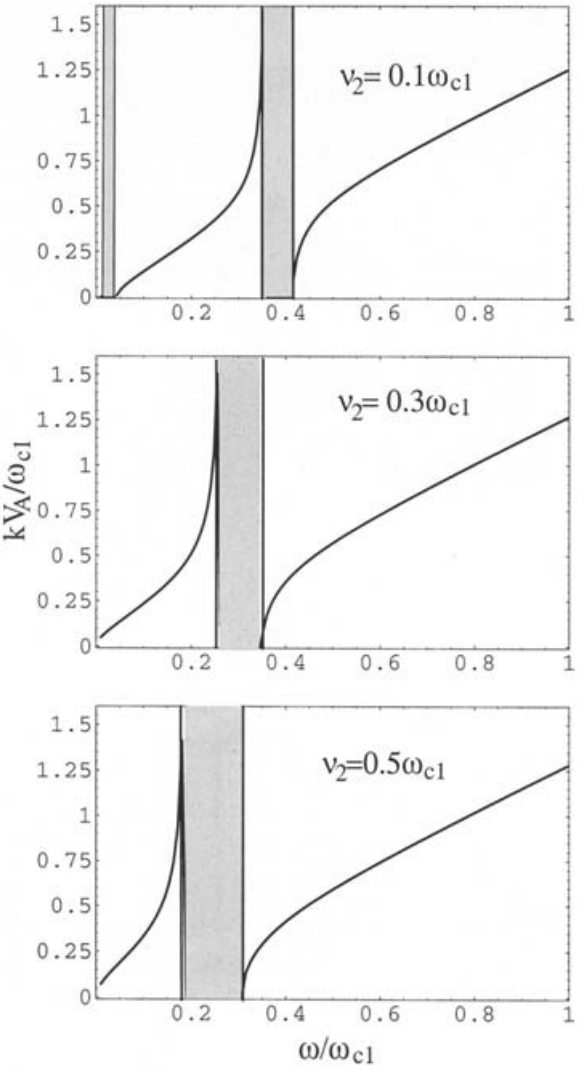


Figure 6. The effects of collision frequency ν_2 on the cut-off and resonance frequencies for $\sigma = 0.25$, $\mu_i = 0.75$, $\mu_d = 0.4$, $s = 1$, $p = -1$, $\nu_1 = 0.1\omega_{c1}$, $\nu_1 = 0.1\omega_{c1}$ (upper plot), $\nu_1 = 0.3\omega_{c1}$ (middle plot) and $\nu_1 = 0.5\omega_{c1}$ (lower plot).

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