JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS Vol. 54, No. 6, Dec. 2019, pp. 2423–2452
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doi:10.1017/S0022109018001436

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

Volatility-of-Volatility Risk

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Abstract

We show that market volatility of volatility is a significant risk factor that affects index and volatility index option returns, beyond volatility itself. The volatility and volatility of volatility indices, identified model-free as the VIX and VVIX, respectively, are only weakly related to each other. Delta-hedged index and VIX option returns are negative on average and are more negative for strategies that are more exposed to volatility and volatility-of-volatility risks. Further, volatility and volatility of volatility significantly negatively predict future delta-hedged option payoffs. The evidence suggests that volatility and volatility-of-volatility risks are jointly priced and have negative market prices of risk.

I. Introduction

Recent studies show that volatility risks significantly affect asset prices and the macroeconomy.[1] In the data, asset market volatility is often captured by the volatility index (VIX). Calculated in real time from the cross section of Standard & Poor's (S&P) 500 option prices, the VIX provides a risk-neutral forecast of the index volatility over the next 30 days. The VIX exhibits substantial fluctuations, which drive the movements in asset prices and the risk premia in the data and in

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[1]For example, Bansal and Yaron (2004), Bloom (2009), Bansal, Kiku, Shaliastovich, and Yaron (2014), and Fernandez-Villaverde and Rubio-Ramírez (2013) discuss the implications of macroeconomic volatility risks, whereas Coval and Shumway (2001), Bakshi and Kapadia (2003), and Campbell, Giglio, Polk, and Turley (2018) consider market volatility risks.

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

many economic models. Interestingly, the volatility of the VIX itself varies over time. Computed from VIX options in an analogous way to the VIX, the volatility-of-volatility index (VVIX) directly measures the risk-neutral expectations of the volatility of volatility in the financial markets. In the data, we find that the VVIX has separate dynamics from the VIX, and thus fluctuations in volatility of volatility are not directly tied to movements in market volatility. We further show that volatility-of-volatility risks are a significant risk factor that affects the time series and the cross section of index and VIX option returns, above and beyond volatility risks. The evidence is consistent with a no-arbitrage model that features time-varying market volatility and volatility-of-volatility factors that are priced by the investors. In particular, volatility and volatility of volatility have negative market prices of risk, so investors dislike increases in volatility and volatility of volatility, and demand a risk compensation for the exposure to these risks.

Our no-arbitrage model extends the 1-factor stochastic volatility specification of equity returns of Bakshi and Kapadia (2003). Specifically, we introduce a separate time-varying volatility-of-volatility risk factor that drives the conditional variance of the variance of market returns.[2] We use the model to characterize the payoffs to delta-hedged equity and VIX options. The zero-cost, delta-hedged positions represent the gains on a long position in the option, continuously hedged by an offsetting short position in the underlying asset. As argued by Bakshi and Kapadia (2003), delta-hedged option payoffs most cleanly isolate the exposures to volatility risks and thus are very useful to study the pricing of volatility-related risks.[3] Indeed, we show that the expected payoff on the delta-hedged index option positions captures risk compensations for both volatility and volatility-of-volatility risks, whereas for VIX options, the expected gains primarily reflect the compensation for volatility-of-volatility risk. Imposing a standard linear risk-premium assumption, we can further decompose the risk-premia components of the expected gains on the index options into the product of the market price of risk, the risk exposure, and the time-varying quantity of each source of risk. For VIX options, the expected gains do not generally admit a linear factor structure, so we resort to numerical calibration techniques to support our theoretical arguments.

The model delivers clear, testable predictions for expected option returns and their relation to volatility and volatility-of-volatility risks. In the model, if investors dislike volatility and volatility of volatility, and thus the market prices of these risks are negative, the gains for delta-hedged equity and VIX options are negative on average. In the cross section, the average returns are more negative for option strategies that have higher exposure to the volatility and volatility-of-volatility risks. Finally, in the time series, higher volatility and volatility of volatility predict more negative delta-hedged option gains in the future.

We find that the model implications for volatility and volatility-of-volatility risks are strongly supported in the cross section of index and VIX option returns. First, we use the VIX and VVIX to validate our volatility measures, and we

[2] We use the terms *variance* and *volatility* interchangeably unless specified otherwise.

[3] For example, unlike delta-hedged positions, the zero-beta straddles analyzed by Coval and Shumway (2001) are not dynamically rebalanced and may contain a significant time-decay option-premium component.

compare and contrast their dynamics. In our monthly 2006–2016 sample, the VVIX behaves quite differently from the VIX, consistent with a setup of our model that separates market volatility from volatility of volatility. Using predictive regressions, we show that the VIX is a significant predictor of the future realized variance of market returns, whereas the VVIX significantly forecasts future realized variation in the VIX index itself.

On average, the risk-neutral volatilities of the market return and market volatility captured by the VIX and VVIX exceed the realized volatilities of returns and the VIX. The difference between the risk-neutral and physical volatilities of market returns is known as the variance premium (variance-of-variance premium for the VIX), and the findings of positive variance and variance-of-variance premia suggest that investors dislike variance and variance-of-variance risks and demand a premium for being exposed to these risks.

We next turn to the asset-price evidence from the equity index and VIX option markets. In line with our model, we consider discrete-time counterparts to the continuously rebalanced delta-hedged gains; this approach is similar to that of Bakshi and Kapadia (2003)[4]. Consistent with the evidence in previous studies, the average delta-hedged returns on out-of-the-money equity index calls and puts are significantly negative in our sample. The novel evidence in our article is that the average delta-hedged returns on VIX options are also negative at all strikes and statistically significant for all the contracts except for the out-of-the-money puts and in-the-money calls. Estimates of the loss for call options range from -0.25% of the index value for in-the-money VIX calls to -1.20% for out-of-the-money calls. The negative average returns on index and VIX options directly suggest that the market prices of volatility and volatility-of-volatility risks are negative.

We then show that the cross-sectional spreads in average option returns are significantly related to the volatility and volatility-of-volatility risks. In lieu of calculating exact model betas, we compute option exposures using the Black and Scholes (1973) vega and volga. The two quantities vary intuitively with the moneyness of the option in the cross section and help us proxy for the betas of the options to the underlying risks. Empirically, we document that average option returns are significantly and negatively related to our proxies for volatility and volatility-of-volatility risks. Hence, using the cross section of equity index options and VIX options, we find strong evidence for a negative market price of volatility and volatility-of-volatility risks.

Finally, we consider a predictive role of our volatility measures for future option returns. In the model, expected delta-hedged gains are time-varying and are driven by the volatility and volatility of volatility (by volatility of volatility for VIX options). In particular, because option betas are all positive, when the market prices of volatility-related risks are negative, both volatility measures should forecast future returns with a negative sign. This model prediction is consistent with the data.

Our article is most closely related to that of Bakshi and Kapadia (2003), who consider the implications of volatility risk for equity index option markets. We extend their approach to include volatility-of-volatility risk, and we bring evidence

[4]We consider several checks to show the robustness of our results to the measurements of delta.

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

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from VIX options. To help us focus on the volatility-related risks, we consider dynamic delta-hedging strategies where a long position in an option is dynamically hedged by taking an offsetting position in the underlying. Delta-hedged strategies are also used by Bertsimas, Kogan, and Lo (2000), Cao and Han (2013), and Frazzini and Pedersen (2012) and are a standard risk management technique of option traders in the financial industry (Hull (2011)). In an earlier study, Coval and Shumway (2001) consider the returns on zero-beta straddles to identify volatility-risk-sensitive assets.

We also contribute to a growing literature that addresses the role of volatility-of-volatility risks in asset pricing. Zhang and Zhu (2006) and Lu and Zhu (2010) highlight the nature and importance of volatility risks by analyzing the pricing of VIX futures. Related to our findings, Park (2015) shows that the VVIX has predictive power for VIX option returns. Hollstein and Prokopczuk (2018) study the cross section of stocks and find that stocks with a strongly negative return exposure to volatility of volatility have higher returns on average. This is consistent with our evidence for a negative market price of volatility-of-volatility risk. Branger, Hülsbusch, and Kraftschik (2017) analyze the information contained in the VVIX term structure. They find that it predicts S&P 500 and VIX straddle returns and is incremental to the information contained in the VIX term structure, which has been analyzed by Johnson (2017). This is in line with our finding that variance dynamics are richer than those of the square-root process typically considered in the literature and is consistent with the evidence provided by Christoffersen, Jacobs, and Mimouni (2010), Mencía and Sentana (2013), and Branger, Kraftschik, and Völkert (2016).

In a structural approach, Bollerslev, Tauchen, and Zhou (2009) consider a version of the Bansal and Yaron (2004) long-run risks model that features recursive utility and fluctuations in the volatility and volatility of volatility of the aggregate consumption process. They show that in equilibrium, investors require compensation for the exposure to volatility and volatility-of-volatility risks. With a preference for early resolution of uncertainty, the market prices of the two risks are negative. As a result, the variance risk premium is positive on average and can predict future equity returns. Bollerslev et al. (2009) and Drechsler and Yaron (2010) show that the calibrated version of such a model can account for the key features of equity markets and the variance premium in the data. Our empirical results are consistent with the economic intuition in these models and complement the empirical evidence in these studies.

Finally, it is worth noting that in our article, we abstract from jumps in equity returns and focus on diffusive volatilities as the main drivers of asset prices and risk premia. For robustness, we confirm that our predictability results are robust to controlling for measures of jump risk, such as the slope of the implied volatility curve, realized jump intensity (Barndorff-Nielsen and Shephard (2006), Wright and Zhou (2009)), and risk-neutral skewness (Bakshi, Kapadia, and Madan (2003)). Hence, we argue that the VIX and VVIX have a significant impact on option returns even in the presence of stock market and volatility jumps; we leave a formal treatment of jumps for future research. Reduced-form models that highlight the role of jumps include those of Bates (2000), Pan (2002), and

Duffie, Pan, and Singleton (2000), among others. Muravyev (2016) documents that inventory risk faced by market-makers also affects option prices.

Our article proceeds as follows: In Section II we discuss our model that links expected delta-hedged equity and volatility option gains to risk compensations for volatility and volatility-of-volatility risk. We also discuss an illustrative numerical example to show the relation between VIX option gains and the volatility-related risks. Details about this study can be found in the Supplementary Material for our article. In Section III, we describe the construction of both the model-free implied variance measures and the high-frequency realized variance measures, and we summarize their dynamics in the time series. Section IV provides the empirical evidence from option prices by empirically implementing the delta-hedged option strategies in our model. Section V presents robustness tests for alternative measures of variance as well as the robustness of the results in the presence of jump risks. Section VI concludes.

II. Model

In this section we describe our model for stock index returns and for equity and volatility option prices. Our model is an extension of that of Bakshi and Kapadia (2003) and features separate time-variation in the market volatility and volatility of volatility. Both volatility risks are priced and affect the level and time variation of the expected asset payoffs.

A. Dynamics of Equity and Equity Option Prices

Under the physical measure ($\mathbb{P}$), the stock index price S_t evolves according to:

$$
\begin{aligned}
\frac{dS_t}{S_t} &= \mu(S_t, V_t, \eta_t)\,dt + \sqrt{V_t}\,dW_t^1, \\
dV_t &= \theta(V_t)\,dt + \sqrt{\eta_t}\,dW_t^2, \\
d\eta_t &= \gamma(\eta_t)\,dt + \phi\sqrt{\eta_t}\,dW_t^3,
\end{aligned} \tag{1}
$$

where W^i $(i = 1,2,3)$ are the Brownian motions that drive stock returns, the stock return variance, and the variance of variance, respectively. The Brownian components can be correlated (i.e., we assume $dW_t^i\,dW_t^j = \rho_{i,j}\,dt$ for all $i \neq j$). V_t is the variance of instantaneous returns, and η_t is the variance of innovations in V_t. Note that the drift of the variance V_t only depends on itself, not on the stock price S_t or the volatility of volatility η_t. Similarly, the drift of the volatility of volatility η_t is a function of the volatility of volatility η_t.

Under the risk-neutral measure ($\mathbb{Q}$), the stock price S_t follows a similar process, but the drifts are adjusted by the risk compensations for the corresponding risks:

$$
\begin{aligned}
\frac{dS_t}{S_t} &= r_f\,dt + \sqrt{V_t}\,d\widetilde{W}_t^1, \\
dV_t &= (\theta(V_t) - \lambda_t^V)\,dt + \sqrt{\eta_t}\,d\widetilde{W}_t^2, \\
d\eta_t &= (\gamma(\eta_t) - \lambda_t^\eta)\,dt + \phi\sqrt{\eta_t}\,d\widetilde{W}_t^3.
\end{aligned} \tag{2}
$$

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

In this representation, $\widetilde{W}^i$ represents Brownian motions under the risk-neutral measure $\mathbb{Q}$, λ_t^V captures the compensation for variance risk, and λ_t^η reflects the compensation for innovations in the stochastic variance of variance. If investors dislike variance and variance-of-variance risks, the two risk compensations are negative. In this case, the variances have higher drifts under the risk-neutral than under the physical measure.

Let $C_t(K,\tau)$ denote the time t price of a call option on the stock with strike price K and time to maturity τ. Assume the risk-free rate r_f is constant. To simplify the presentation, we further abstract from dividends. Although we focus our discussion on call options, the case of put options follows analogously. Given the specified dynamics of the stock price under the two probability measures, the option price is given by a twice-differentiable function C of the state variables: $C_t(K,\tau)=C(S_t,V_t,\eta_t,t)$. By Itô's Lemma,

$$dC_t = \frac{\partial C}{\partial S}dS_t+\frac{\partial C}{\partial V}dV_t+\frac{\partial C}{\partial \eta}d\eta_t+b_t\,dt, \tag{3}$$

for a certain drift component b_t.

The discounted option price $e^{-r_f t}C_t$ is a martingale under $\mathbb{Q}$ and thus has zero drift. We use Itô's Lemma again to obtain that:

$$\frac{\partial C}{\partial S}S_t r_f+\frac{\partial C}{\partial V}(\theta(V_t)-\lambda_t^V)+\frac{\partial C}{\partial \eta}(\gamma(\eta_t)-\lambda_t^\eta)+b_t-r_f C_t = 0. \tag{4}$$

This implies that:

$$b_t = r_f\left(C_t-\frac{\partial C}{\partial S}S_t\right)-\frac{\partial C}{\partial V}(\theta(V_t)-\lambda_t^V)-\frac{\partial C}{\partial \eta}(\gamma(\eta_t)-\lambda_t^\eta). \tag{5}$$

Let $\Pi_{t,t+\tau}$ stand for the delta-hedged option gain for call options:

$$\Pi_{t,t+\tau} \equiv C_{t+\tau}-C_t-\int_t^{t+\tau}\frac{\partial C}{\partial S}dS_u-\int_t^{t+\tau}r_f\left(C_u-\frac{\partial C}{\partial S}S_u\right)du. \tag{6}$$

The delta-hedged option gain represents the gain on a long position in the option, continuously hedged by an offsetting short position in the stock, with the net balance earning the risk-free rate.

Combining equations (3) and (5) and noting that $C_{t+\tau}=C_t+\int_t^{t+\tau}dC_u$, we see that the delta-hedged gain for call options is given as

$$\begin{aligned}\Pi_{t,t+\tau} &\equiv C_{t+\tau}-C_t-\int_t^{t+\tau}\frac{\partial C}{\partial S}dS_u-\int_t^{t+\tau}r_f\left(C_u-\frac{\partial C}{\partial S}S_u\right)du \\ &= \int_t^{t+\tau}\lambda_u^V\frac{\partial C}{\partial V}du+\int_t^{t+\tau}\lambda_u^\eta\frac{\partial C}{\partial \eta}du \\ &\quad+\int_t^{t+\tau}\frac{\partial C}{\partial V}\sqrt{\eta_u}dW_u^2+\int_t^{t+\tau}\frac{\partial C}{\partial \eta}\phi\sqrt{\eta_u}dW_u^3.\end{aligned} \tag{7}$$

Because the expectation of Itô integrals is 0, the expected delta-hedged equity option gains are given by

$$\mathbb{E}_t[\Pi_{t,t+\tau}] = \mathbb{E}_t\left[\int_t^{t+\tau}\lambda_u^V\frac{\partial C}{\partial V}du\right]+\mathbb{E}_t\left[\int_t^{t+\tau}\lambda_u^\eta\frac{\partial C}{\partial \eta}du\right]. \tag{8}$$

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The expected option gains depend on the risk compensation components for the volatility and volatility-of-volatility risks (λ_t^V and λ_t^η) and the option price exposures to these two sources of risks ($\partial C/\partial V$ and $\partial C/\partial \eta$).

For tractability, following the literature, let us further assume that the risk premium structure is linear:

$$\lambda_t^V = \lambda^V V_t, \quad \lambda_t^\eta = \lambda^\eta \eta_t, \tag{9}$$

where λ^V is the market price of the variance risk, and λ^η is the market price of the variance-of-variance risk. We can further operationalize equation (8) by applying Itô–Taylor expansions (Milstein 1995). This gives us a linear factor structure (see details in the Supplementary Material):

$$\frac{\mathbb{E}_t[\Pi_{t,t+\tau}]}{S_t} = \lambda^V \beta_t^V V_t + \lambda^\eta \beta_t^\eta \eta_t. \tag{10}$$

The sensitivities to the risk factors are given by:

$$\beta_t^V = \sum_{n=0}^{\infty} \frac{\tau^{1+n}}{(1+n)!} \Phi_{t,n}^V > 0, \quad \beta_t^\eta = \sum_{n=0}^{\infty} \frac{\tau^{1+n}}{(1+n)!} \Phi_{t,n}^\eta > 0, \tag{11}$$

where $\Phi_{t,n}^V$ and $\Phi_{t,n}^\eta$ are positive functions that depend on the moneyness of the option and on the sensitivities $\partial C/\partial V$ and $\partial C/\partial \eta$, respectively. Hence, the expected payoff on the delta-hedged option position combines the risk compensations for the volatility and volatility-of-volatility risks. The two risk compensations are given by the product of the market price of risk, the exposure of the asset to the corresponding risk, and the quantity of risk. In particular, options are positive-beta assets with respect to both volatility and volatility-of-volatility risks. Hence, if investors dislike volatility and volatility-of-volatility risks so that their market prices of risks are negative, the expected option payoffs are negative as well.

B. Dynamics of VIX Option Prices

The squared VIX for a time to maturity τ is the annualized risk-neutral expectation of the quadratic variation of returns from time t to $t+\tau$, given by:

$$\text{VIX}_t^2 = \frac{1}{\tau}\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{t+\tau} V_s\, ds\right]. \tag{12}$$

Given our model assumptions, the VIX is a function of the stock market variance: $\text{VIX}_t = \text{VIX}(V_t)$. For example, in a linear model where the variance drift $\theta(V_t)$ is linear in V_t, the squared VIX is a linear function of the stock market variance V_t.

Let F_t be the time t price of a VIX futures contract expiring at $t+\tau$. Under the assumptions of no arbitrage and continuous mark-to-market, F_t is a martingale under the risk-neutral measure $\mathbb{Q}$:

$$F_t = \mathbb{E}_t^{\mathbb{Q}}[\text{VIX}_{t+\tau}] = \mathbb{E}_t^{\mathbb{Q}}[\text{VIX}(V_{t+\tau})]. \tag{13}$$

Under our model structure, the futures price F is a function of the market variance V_t and volatility of volatility η. Under economically plausible scenarios, the

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futures price is monotone in the two volatility processes.[5] Knowing F_t and η_t is sufficient for V_t, so we can rewrite the economic states $[V_t \quad \eta_t]$ in terms of $[F_t \quad \eta_t]$.

Let C_t^* be the time t price of a VIX call option, whose underlying is a VIX forward contract. The option price is given by a twice-differentiable function C^* of the state variables so that $C_t^*(K,\tau) = C^*(F_t, \eta_t, t)$. By Itô's Lemma:

$$dC_t^* = \frac{\partial C^*}{\partial F} dF_t + \frac{\partial C^*}{\partial \eta} d\eta_t + b_t^* dt, \tag{14}$$

for a drift component b_t^*.

Under the risk-neutral measure $\mathbb{Q}$, the discounted VIX option price process $e^{-r_f t} C_t^*$ is a martingale, so it must have 0 drift:

$$\frac{\partial C^*}{\partial F} \mathcal{D}^{\mathbb{Q}}[F_t] + \frac{\partial C^*}{\partial \eta}(\gamma(\eta_t) - \lambda_t^{\eta}) - r_f C_t^* + b_t^* dt = 0, \tag{15}$$

where $\mathcal{D}^{\mathbb{Q}}[F_t]$ is the infinitesimal generator applied to F (i.e., the conditional expectation of dF_t under $\mathbb{Q}$).

This implies that

$$\begin{aligned} b_t^* &= r_f C_t^* + \frac{\partial C^*}{\partial \eta} \lambda_t^{\eta} - \frac{\partial C^*}{\partial \eta} \gamma(\eta_t) - \frac{\partial C^*}{\partial F} \mathcal{D}^{\mathbb{Q}}[F_t] \\ &= r_f C_t^* + \frac{\partial C^*}{\partial \eta} \lambda_t^{\eta} - \frac{\partial C^*}{\partial \eta} \gamma(\eta_t), \end{aligned} \tag{16}$$

where the second line follows from the fact that F_t is a martingale under $\mathbb{Q}$, so its drift is equal to 0.

Combining equation (16) with equation (14), we obtain the equation for the delta-hedged VIX option gain:

$$\begin{aligned} \Pi_{t,t+\tau}^* &= C_{t+\tau}^* - C_t^* - \int_t^{t+\tau} \frac{\partial C^*}{\partial F} dF_s - \int_t^{t+\tau} r_f C_s^* ds \\ &= \int_t^{t+\tau} \frac{\partial C^*}{\partial \eta} \lambda_s^{\eta} ds + \int_t^{t+\tau} \frac{\partial C^*}{\partial \eta} \phi \sqrt{\eta_s} dW_s^3. \end{aligned} \tag{17}$$

The delta-hedged VIX option gain in equation (17) is the counterpart to the delta-hedged equity option gain in equation (7). The difference comes from the fact that the short stock position serving as the hedge in the case of equity options is funded at r_f, whereas for VIX futures, the hedging position is 0 cost.

Taking expectations, we can derive the corresponding expected gain on delta-hedged VIX options:

$$\frac{\mathbb{E}_t[\Pi_{t,t+\tau}^*]}{F_t} = \frac{1}{F_t} \int_t^{t+\tau} \mathbb{E}_t \left[\frac{\partial C_s^*}{\partial \eta_s} \lambda_s^{\eta} \right] ds. \tag{18}$$

Similar to index options, the expected gain on delta-hedged VIX options depends on the market price of volatility-related risks and the option exposure to

[5] See also Zhang and Zhu (2006), Lu and Zhu (2010), and Branger et al. (2016) for VIX futures pricing models.

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

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these risks. However, in contrast to index options, at least for short maturities τ, VIX options are mainly exposed to volatility-of-volatility risks. Their exposure to these risks, captured by $\partial C_s^*/\partial \eta_s$, is expected to be positive, so the average level of the expected gains on VIX options and its dependence on the volatility-of-volatility factor are governed by the market price of the volatility risks λ^η. Unlike the expected gains for index options in equation (10), the expected gains for VIX options do not generally admit a linear factor structure because the option price is no longer homogeneous in the underlying asset value. Next, we provide a numerical example to support our arguments, and then we consider the relative importance of the risks in the data.

C. Illustrative Numerical Example

We calibrate the model to match key asset pricing moments in the data. We use numerical methods to calculate model-implied delta-hedged VIX option gains in equation (18) and analyze how the expected VIX option gains depend on the market price of the volatility-of-volatility risks. The Supplementary Material contains a detailed description of the calibration approach and the solution technique.

Using numerical solutions, we find that the sign of the expected option gain is pinned down by the sign of the market price of volatility-of-volatility risk. In particular, we find that the derivative $\partial C^*/\partial \eta$ of the VIX option price with respect to the volatility of volatility is positive for all levels of V and η. As a consequence, the expected gains are negative if and only if the market price of volatility-of-volatility risks λ^η is negative.

We also consider the relation between the expected gains and the level of the volatility of volatility. The option price sensitivity to volatility of volatility, $\partial C^*/\partial \eta$, is always positive. The derivative is mildly decreasing in volatility of volatility, in absolute value. In total, when the market price of volatility-of-volatility risks is negative, the expected gains are more negative at times of high volatility of volatility. In this case, volatility of volatility predicts future VIX option gains with a negative sign.

III. Variance Measures

A. Construction of Variance Measures

The VIX is a model-free, forward-looking measure of implied volatility in the U.S. stock market, published by the Chicago Board Options Exchange (CBOE). The square of the VIX is defined as in equation (12), where $\tau = 30/365$. Carr and Madan (1998), Britten-Jones and Neuberger (2000), and Jiang and Tian (2005) show that VIX^2 can be computed from the prices of call and put options with the same maturity at different strike prices:

$$\text{VIX}_t^2 = \frac{2e^{r_f \tau}}{\tau}\left[\int_0^{S_t^*} \frac{1}{K^2} P_t(K)\, dK + \int_{S_t^*}^{\infty} \frac{1}{K^2} C_t(K)\, dK\right], \tag{19}$$

where K is the strike price, C_t and P_t are the put and call prices, S_t^* is the fair forward price of the S&P 500 index, and r_f is the risk-free rate. The VIX published by the CBOE is discretized, truncated, and interpolated across the two nearest

maturities to achieve a constant 30-day maturity.[6] Jiang and Tian (2005) show, through simulation analysis, that the approximations used in the VIX calculation are quite accurate.

Since Feb. 2006, options on the VIX have been trading on the CBOE, giving investors a way to trade the volatility of volatility. As of 2012:Q3, the open interest in front-month VIX options was approximately 2.5 million contracts, which is similar to the open interest in front-month S&P 500 index option contracts.

We calculate our measure of the implied volatility of volatility using the same method as the VIX, applied to VIX options instead of S&P 500 options. The index, which has since been published by the CBOE as the VVIX in 2012 and back-filled, is calculated as follows:

$$\text{VVIX}_t^2 = \frac{2e^{r_f \tau}}{\tau}\left[\int_0^{F_t} \frac{1}{K^2} P_t^*(K)\,dK + \int_{F_t}^{\infty} \frac{1}{K^2} C_t^*(K)\,dK\right], \tag{20}$$

where F_t is the VIX futures price, and C_t^*, P_t^* are the prices of call and put options on the VIX, respectively.[7] The squared VVIX is calculated from a portfolio of out-of-the-money call and put options on VIX futures contracts. It captures the implied volatility of VIX futures returns over the next 30 days, and is a model-free, forward-looking measure of the implied volatility of volatility.

In addition to the implied volatilities, we can also compute the realized volatilities for the stock market and the VIX. The construction here follows Barndorff-Nielsen and Shephard (2004) using high-frequency, intraday data.[8] Realized variance is defined as the sum of the squared high-frequency log returns over the trading day:

$$\text{RV}_t = \sum_{j=1}^{N} r_{t,j}^2. \tag{21}$$

Barndorff-Nielsen and Shephard (2004) show that RV_t converges to the quadratic variation as $N \to \infty$. We follow the standard approach of considering 5-minute return intervals. A finer sampling frequency results in better asymptotic properties of the realized variance estimator but also introduces more market microstructure noise, such as the bid–ask bounce discussed by Heston, Korajczyk, and Sadka (2010). Liu, Patton, and Sheppard (2015) show that the 5-minute realized variance is very accurate, is difficult to beat in practice, and is typically the ideal sampling choice in most applications, combining accuracy and parsimony.

We estimate two realized variance measures, one for the S&P 500 and one for the VIX. For the S&P 500, we use the S&P 500 futures contract, and the resulting realized variance is denoted RV^{SPX}. For the VIX, we use the spot VIX

[6] More details on the exact implementation of the VIX can be found in the white paper available on the CBOE Web site: http://www.cboe.com/micro/vix.

[7] The official index is back-filled until 2007. We apply the same methodology and construct the index for an additional year back to 2006. The correlation between our measure of the VVIX and the published index is over 99% in the post-2007 sample. Our empirical results remain essentially unchanged if we restrict our sample to only the post-2007 period.

[8] The data are obtained from http://www.tickdata.com.

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and denote the resulting realized variance of the VIX by RV^{VIX}, which is our measure of the physical volatility of volatility. For robustness, we also entertain an alternative measure of the realized variance RV^{VX^*}, which is computed using the 5-minute VIX futures returns. The sample for this measure is shorter and starts in July 2012.

B. Variance Dynamics

All of our variables are at the monthly frequency. The implied variance measures are given by the index values at the end of the month, and the realized variance measures are calculated over the past month and annualized. The sample for the benchmark measures runs from Feb. 2006 to Dec. 2016.

Table 1 presents summary statistics for the implied and realized variance measures. Whereas the average level of the VIX is approximately 22%, the average level of the VVIX is much higher at approximately 88%, which captures the fact that VIX futures returns are much more volatile than market returns: Volatility, itself, is very volatile. The $VVIX^2$ is also much more volatile and less persistent than VIX^2, with a first-order autocorrelation coefficient of 0.33 compared with 0.81 for VIX^2. The VVIX exhibits relatively low correlation with the VIX, with a correlation coefficient of approximately 0.27. The mean of realized variance for S&P 500 futures returns is 0.024, which corresponds to an annualized volatility of 15.5%. S&P 500 realized variance is persistent and quite strongly correlated to the VIX (correlation coefficient = 0.88) and much more weakly correlated to the VVIX (correlation coefficient $\approx$ 0.26). The realized variance of VIX is strongly related to the VVIX (correlation = 0.61) and, to a much lesser extent, the VIX (correlation = 0.18).

TABLE 1
Summary Statistics

Table 1 shows summary statistics for the implied and realized variances of the Standard & Poor's (S&P 500) index and the volatility index (VIX). The implied variances are computed from the option data: VIX^2 corresponds to $(VIX/100)^2$, and $VVIX^2$ stands for (volatility-of-volatility index (VVIX)/100)^2. Realized variances are annualized and computed using 5-minute data on S&P 500 returns (RV^{SPX}), innovations into natural logarithm of the VIX (RV^{VIX}), and VIX futures returns (RV^{VX^*}). The data are monthly from 2006:M2 to 2016:M12; VIX futures high-frequency data are from 2012:M7 to 2016:M12.

Variable	Mean	Std. Dev.	AR(1)	Corr. VIX^2	Corr. $VVIX^2$
VIX^2	0.05	0.05	0.81	1.00	0.27
$VVIX^2$	0.77	0.21	0.33	0.27	1.00
RV^{SPX}	0.02	0.05	0.63	0.88	0.26
RV^{VIX}	0.67	0.73	0.18	0.18	0.61
RV^{VX^*}	0.31	0.18	0.28	0.73	0.64

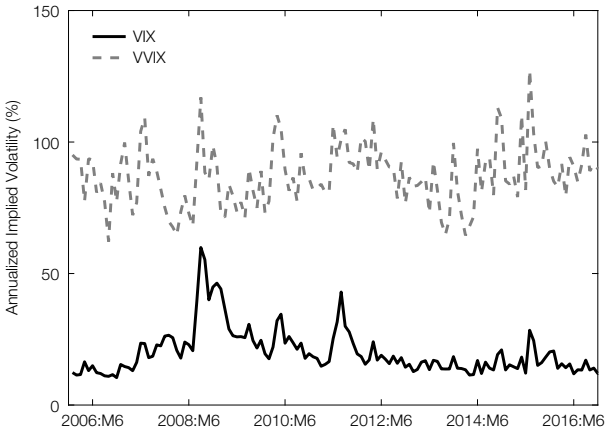
In the last row of Table 1 we also consider the realized variance of VIX futures returns because the VIX futures contract (not the spot VIX) is the underlying asset for VIX options, and there is no simple cost-of-carry relationship between VIX futures and spot VIX. Compared with the spot VIX realized variance, the VIX futures realized variance is lower on average, less volatile, more persistent, and more correlated with VIX. Recall, however, that our data for VIX futures are from 2012:M7 to 2016:M12 because of the availability of tick data.

Figure 1 shows the time series of the VIX and VVIX from Feb. 2006 to Dec. 2016. There are some common prominent moves in both series, such as a peak during the financial crisis. Notably, however, the VVIX also peaks during other

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FIGURE 1
Volatility and Volatility-of-Volatility Measures

Figure 1 shows the time series of the volatility and volatility-of-volatility measures. The solid line represents the volatility index (VIX), and the dashed line represents the volatility-of-volatility index (VVIX). Data are on a monthly frequency from 2006:M2 to 2016:M12.



times of economic uncertainty, such as the summer of 2007 (quant meltdown, beginning of the subprime crisis), May 2010 (Eurozone debt crisis, flash crash), Aug. 2011 (U.S. debt ceiling crisis), and Aug. 2015 (sell-off driven by the Chinese stock market crash). The movements in VIX during these events are far smaller than the spikes in the VVIX. This suggests that the VVIX captures important uncertainty-related risks in the aggregate market, distinct from the VIX itself.

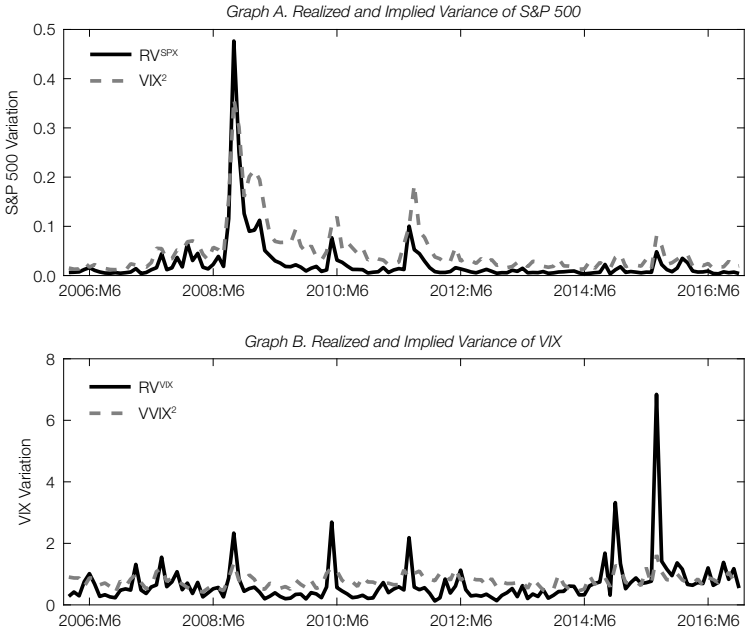
In Figure 2 we present time series plots for both S&P 500 and VIX realized variances. As shown in Graph A, the realized and implied variances of the stock market follow a similar pattern, and S&P 500 realized variance is nearly always below the implied variance. There is a large spike in both series around the financial crisis in Oct. 2008, at which point realized variance exceeded implied variance. The difference between the mean of VIX^2 and RV^{SPX} is typically interpreted as a variance premium, which is the difference between end-of-month model-free, forward-looking implied variance calculated from S&P 500 index options and the realized variance of S&P 500 futures returns over the past month. Unconditionally, the average level of the VIX (22%) is greater than the average level of the S&P 500 realized volatility (15.6%), so the variance premium is positive, consistent with the evidence of Bollerslev et al. (2009) and Drechsler and Yaron (2010). This also implies a negative market price of volatility risk.

Graph B of the Figure 2 shows the time series of the realized and implied volatility of the VIX. Generally, the implied volatility tends to increase at times of pronounced spikes in the realized volatility. The implied volatility is also high during other times of economic distress and uncertainty, such as May 2010 (Eurozone debt crisis and flash crash) and Aug. 2011 (U.S. debt ceiling crisis). During normal times, the VVIX is above the VIX realized variance, although during times of extreme distress, we see that the realized variance of VIX can exceed

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FIGURE 2
Realized- and Implied-Variance Measures

Figure 2 shows the time series of the realized and implied variances for the Standard & Poor's (S&P) 500 index (Graph A) and the volatility index (VIX) (Graph B). The solid lines represent the realized variances, and the dashed lines represent the implied variances. All measures are in annualized variance units. Monthly data from 2006:M2 to 2016:M12.



the VVIX. The average level of the VVIX (87.8%) is greater than the average level of the VIX realized volatility (81.8%), so the volatility-of-volatility premium is also positive.[9] As we see in Table 1, the VIX futures realized volatility is lower than the VIX index realized volatility, so the actual magnitude of the volatility-of-volatility risk premium is even larger when compared with VIX futures. Similar to our discussion of the variance premium, this evidence suggests that investors dislike volatility-of-volatility risks, and the market price of these risks is negative.

In addition to unconditional moments, we can also analyze the conditional dependence of volatility on volatility of volatility. Specifically, we consider the predictability of future realized variances by the VIX and VVIX, in the spirit of Canina and Figlewski (1993), Christensen and Prabhala (1998), and Jiang and Tian (2005), who use option implied volatilities to predict future realized volatilities. Following that approach, we consider both univariate and multivariate encompassing regressions to assess the predictability of future realized variances by the VIX and VVIX.

[9]Song (2013) shows that the average level of his VVIX measure, computed using numerical integration rather than the model-free VIX construction, is lower than the average realized volatility of VIX. One of the key differences between his and our computations is the frequency of returns used in the realized-variance computations. Consistent with the literature, we rely on 5-minute returns to compute the realized variances, whereas Song (2013) uses daily returns.

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In our main specification, the dependent variable is the realized variance (RV) over the next month, for both the S&P 500 and the VIX. Univariate regressions test whether each implied volatility measure (the VIX or the VVIX) can forecast future realized variances; multivariate encompassing regressions compare the relative forecasting importance of the VIX and VVIX and whether one implied-volatility measure subsumes the information content of the other. The univariate regressions are restricted versions of the corresponding multivariate encompassing regression. For the S&P 500, the regression is given as

$$\text{RV}_{t+1}^{\text{SPX}} = \beta_0 + \beta_1 \text{VIX}_t^2 + \beta_2 \text{VVIX}_t^2 + \beta_3 \text{RV}_t^{\text{SPX}} + \epsilon_{t+1}. \tag{22}$$

Similarly, for the VIX, we have:

$$\text{RV}_{t+1}^{\text{VIX}} = \beta_0 + \beta_1 \text{VIX}_t^2 + \beta_2 \text{VVIX}_t^2 + \beta_3 \text{RV}_t^{\text{VIX}} + \epsilon_{t+1}. \tag{23}$$

Our benchmark results are presented for all variables calculated in annualized variance units.

The first regression in Panel A of Table 2 shows that the VIX can forecast the future realized variance of S&P 500 returns. This is consistent with the findings of Jiang and Tian (2005). The VVIX can also forecast the future S&P 500 realized variance somewhat, although the statistical significance is weaker than that of the VIX, and the magnitude of the regression coefficient is several times smaller. In the encompassing regression, we see that the VIX dominates the VVIX in forecasting the future S&P 500 realized variance. A 1-standard-deviation increase in the VIX^2 is associated with a 0.6-standard-deviation increase in the realized variance of S&P 500 returns the next month. The coefficient on the VIX does not change much when we include the VVIX. Including lags of the realized variances themselves do not materially change the results. These results are consistent with our model specification.

Panel B of Table 2 shows our predictability results for VIX realized variance, which is our proxy for the physical volatility of volatility. The VIX is not

TABLE 2
Predictability of Realized Variance Measures

Table 2 shows the evidence from the projections of future realized variances of the Standard & Poor's (S&P 500) index and the volatility index (VIX) on the current implied variances. The numbers in *t*-Stat. column indicate Newey–West *t*-statistics. The data are monthly from 2006:M2 to 2016:M12; VIX futures high-frequency data are from 2012:M7 to 2016:M12.

	VIX^2		VVIX^2		
	Slope	*t*-Stat.	Slope	*t*-Stat.	R^2_{adj}
Panel A. S&P 500					
$\text{RV}_{t,t+1}^{\text{SPX}}$	0.60	4.82			39.42
			0.04	1.48	2.01
	0.61	4.53	−0.00	−0.10	38.95
Panel B. VIX					
$\text{RV}_{t,t+1}^{\text{VIX}}$	−0.61	−0.57			−0.35
			0.61	3.56	2.49
	−1.38	−1.34	0.70	4.05	2.67
Panel C. VIX Futures					
$\text{RV}_{t,t+1}^{\text{VX}^*}$	3.22	3.07			2.55
			0.24	2.24	7.19
	−0.94	−0.46	0.28	2.34	5.49

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significantly related to future VIX realized variation; in fact, the point estimate is negative. The t-statistic is nearly 0, and the adjusted R^2 is below 0. In contrast, the VVIX is a significant predictor of future VIX realized variation. The regression coefficient for the VVIX is approximately 0.6 in a univariate regression, and it is 0.7 in the multivariate regression. A 1-standard-deviation increase in the current value of VVIX^2 is associated with more than a 0.2-standard-deviation increase in next month realized variance of VIX. In Panel C, we use the VIX futures realized return volatility as the proxy for the physical volatility of volatility and also find that VVIX is a significant predictor of future realized variance that drives out the VIX in a multivariate regression.

The empirical evidence suggests that fluctuations in the volatility of volatility are not directly related to the level of the volatility itself. This is consistent with our 2-volatility model specification in Section II. In many reduced-form and structural models, the volatility of volatility is directly linked to the level of the volatility. For example, Heston (1993) models volatility as following a Cox, Ingersoll, and Ross (1985) square-root process. In that case, the level of volatility itself should forecast the future realized volatility of volatility. The evidence in the data does not support this assumption and supports richer dynamics of the volatility process with separate movements in the volatility of volatility.

IV. Evidence from Options

In this section we analyze the implications of equity and VIX option price dynamics for the pricing of volatility and volatility-of-volatility risks in the data. Our model suggests that the market prices of volatility and volatility-of-volatility risks determine the key properties of the cross section and time series of delta-hedged equity and VIX option gains. Specifically, if the market prices of volatility and volatility-of-volatility risks are negative, the average delta-hedged equity and VIX option gains are also negative. In the cross section, the average returns are more negative for option strategies that have higher exposure to the volatility and volatility-of-volatility risks. Finally, in the time series, higher volatility and volatility of volatility predicts more negative gains in the future. We evaluate these model predictions in the data and find strong support that both volatility and volatility-of-volatility risks are priced in the option markets and have negative market prices of risks.

A. Delta-Hedged Option Gains

We consider discrete-time counterparts to the continuously rebalanced delta-hedged gains in equations (7) and (17):

$$\Pi_{t,t+\tau} = \underbrace{C_{t+\tau} - C_t}_{\text{option gain/loss}} - \underbrace{\sum_{n=0}^{N-1} \Delta_{t_n}(S_{t_{n+1}} - S_{t_n})}_{\text{delta hedging gain/loss}} + \underbrace{\sum_{n=0}^{N-1} r_f(\Delta_{t_n} S_{t_n} - C_t)\frac{\tau}{N}}_{\text{risk-free rate}}, \tag{24}$$

$$\Pi^*_{t,t+\tau} = \underbrace{C^*_{t+\tau} - C^*_t}_{\text{option gain/loss}} - \underbrace{\sum_{n=0}^{N-1} \Delta_{t_n}(F_{t_{n+1}} - F_{t_n})}_{\text{delta hedging gain/loss}} - \underbrace{\sum_{n=0}^{N-1} r_f C^*_t \frac{\tau}{N}}_{\text{risk-free rate}}.$$

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where Δ_{t_n} indicates the option delta at time t_n (e.g., for a call option, $\Delta_{t_n} = \partial C_{t_n}/\partial S_{t_n}$), and N is the number of trading days in the month. This discrete delta-hedging scheme is also used by Bakshi and Kapadia (2003) and Bertsimas et al. (2000).

At the close of each option expiration, we look at the prices of all options with nonzero open interest and nonzero trading volume. We take a long position in the option, and we hedge each day using the Δ according to the Black and Scholes (1973) model, with the net investment earning the risk-free interest rate appropriately.[10] To minimize the effect of recording errors, we discard options that have implied volatilities below the 1st percentile or above the 99th percentile. All options have exactly 1 calendar month to maturity; S&P 500 options expire on the third Friday of every month, and VIX options expire on the Wednesday that is 30 days away from the third Friday of the following month.

Table 3 shows the average index and VIX delta-hedged option gains in our sample. We separate options by call or put and group each option into 4 bins

TABLE 3
Delta-Hedged Option Gains

Table 3 shows average delta-hedged option gains on the Standard & Poor's (S&P) 500 and volatility index (VIX) options across their moneyness. Options have 1 month to maturity, are grouped into an equal-weighted portfolio inside the moneyness bin, and are held until expiration. The delta hedge is computed using the Black–Scholes (1973) formula, with daily rebalancing and the margin difference earning the risk-free rate. The delta-hedged option gains Π are scaled by either the index or the option price. The t-statistics are testing the null that the delta-hedged option gain is equal to 0. The "% < 0" column shows the fraction of observations with negative gains. The data are monthly from 2006:M2 to 2016:M12.

	Π/S(%)						Π/C(%)		
Moneyness	Mean	t-Stat.	Median	% < 0	Std. Dev.	AR(1)	Mean	t-Stat.	Median
Panel A. S&P 500 Options									
Call									
0.950–0.975	0.07	3.23	0.00	50%	0.55	0.36	1.61	3.71	0.01
0.975–1.000	0.04	2.63	−0.02	52%	0.50	0.19	2.38	3.72	−0.79
1.000–1.025	−0.04	−2.64	−0.05	57%	0.44	0.09	−2.45	−1.74	−5.90
1.025–1.050	−0.08	−6.51	−0.05	62%	0.38	0.14	−32.98	−9.00	−22.50
Put									
0.950–0.975	−0.14	−7.86	−0.18	81%	0.54	0.35	−19.27	−9.01	−28.74
0.975–1.000	−0.16	−9.89	−0.19	75%	0.50	0.20	−10.24	−7.93	−16.89
1.000–1.025	−0.22	−13.17	−0.22	75%	0.48	0.04	−8.77	−13.99	−10.85
1.025–1.050	−0.25	−12.11	−0.23	78%	0.47	0.05	−5.84	−12.72	−5.97
Panel B. VIX Options									
Call									
0.800–0.900	−0.25	−1.66	−0.59	62%	2.26	−0.03	−1.38	−1.54	−3.12
0.900–1.000	−0.77	−3.83	−1.26	70%	2.95	−0.09	−6.76	−3.80	−12.40
1.000–1.100	−0.84	−3.54	−1.37	71%	3.24	−0.17	−10.03	−3.34	−16.29
1.100–1.200	−1.20	−5.04	−1.52	77%	3.16	−0.13	−19.01	−4.81	−25.49
Put									
0.800–0.900	−0.50	−1.67	−0.90	64%	2.69	−0.06	−12.76	−1.55	−25.18
0.900–1.000	−0.80	−3.73	−1.29	71%	3.05	−0.14	−10.91	−3.08	−21.59
1.000–1.100	−0.84	−3.42	−1.45	70%	3.31	−0.18	−6.13	−3.17	−11.02
1.100–1.200	−1.17	−4.86	−1.31	77%	3.16	−0.14	−5.53	−4.77	−6.45

[10]This requires an estimate of the implied volatility of the option, which may require an option price. We use implied volatilities directly backed out from market prices of options whenever possible; if an option does not have a quoted price on any intermediate date, we fit a cubic polynomial to the implied volatility curve given by options with quoted prices, and we back out the option's implied volatility. This is similar to the typical option-position risk management done by professional traders.

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by moneyness to obtain 8 bins for both S&P 500 and VIX options. The "Π/S" column gives the delta-hedged option gain scaled by the initial index level, and the "Π/C" column gives the delta-hedged option gain scaled by the initial option price, which can be interpreted more readily as a "return" in the traditional sense.

Panel A of Table 3 shows that on average, out-of-the-money delta-hedged S&P 500 call options have significantly negative returns. Likewise, delta-hedged put options on the S&P 500 also have significantly negative returns at all levels of moneyness. To guard against potential outliers in option returns, we show that the results are similar using medians rather than average values; in percentages, delta-hedged index put options are negative approximately 80% of the time. This evidence is largely consistent with Bakshi and Kapadia (2003). In the model, negative average returns on delta-hedged index options imply that volatility and/or volatility-of-volatility risks have a negative market price of risk. S&P 500 option gains display mild positive serial correlation, which we will account for in our time-series predictive regressions in the later sections.

Notably, the average gains for in-the-money index calls are positive, which is puzzling given the evidence for the negative average gains across all the other option contracts.[11] Bakshi and Kapadia (2003) and Chen, Shu, and Zhang (2016) document a similar pattern in different samples. As argued by Bakshi and Kapadia (2003) and Bakshi, Cao, and Chen (2000), deep-in-the-money index call options are quite illiquid, so the market-makers often do not update call prices in response to changes in index levels. In line with them, we also find that in-the-money calls are relatively illiquid compared with out-of-the-money calls or especially out-of-the-money puts. The corresponding out-of-the-money puts are much more liquid and have significantly negative delta-hedged gains.

Panel B of Table 3 shows average returns for delta-hedged VIX options. The average delta-hedged VIX option returns are negative and statistically significant in all bins except for out-of-the-money puts and in-the-money calls, which are marginally significant. Call options lose more money as they become more out of the money, regardless of whether we are scaling by the index or by the option price. Estimates of the loss for call options ranges from -0.25% of the index value for in-the-money VIX calls to -1.20% of the index value for out-of-the-money VIX calls. When viewed as a percentage of the option price, delta-hedged VIX calls return approximately -1% per month at the money and -20% out-of-the-money. The call option returns are negative 60% to 80% of the time. The results for VIX put options are similar. In the model, negative average returns for delta-hedged VIX options imply that investors dislike volatility-of-volatility risks and are systematically paying a premium hedge against increases in the volatility of volatility. This suggests that the price of the volatility-of-volatility risk is negative. VIX option gains exhibit weak negative serial correlation, which we also account for in our time-series predictive regressions in the later sections. For both the S&P 500 and the VIX, the delta-hedged option gains are quite volatile.

[11] The median gains are actually 0 or below 0 for in-the-money calls, so the distribution of gains is right-skewed.

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In the next section, we provide further direct evidence by controlling for the exposures of the delta-hedged option positions to the underlying risks.

B. Cross-Sectional Evidence

As shown in the previous section, the average delta-hedged option gains are negative for S&P 500 and VIX options. Our model further implies (see equations (10) and (18)) that options with higher sensitivity to volatility and volatility-of-volatility risks should have more negative gains. To compute the estimates of option exposures to the underlying risks, we follow the approach of Bakshi and Kapadia (2003), which relies on the use of the Black and Scholes (1973) model to proxy for the true option betas.

Specifically, to compute the proxy for the option beta to volatility risk, we consider the vega of the option:

$$\frac{\partial C}{\partial \sigma} = S\sqrt{\frac{\tau}{2\pi}}e^{-d_1^2/2} \propto e^{-d_1^2/2}, \tag{25}$$

where $d_1 = [\ln(S/K) + (r_f - q + \sigma^2/2)\tau]/(\sigma\sqrt{\tau})$, q is the dividend yield, and σ is the implied volatility of the option. This approach allows us to compute proxies for the exposures of equity options to volatility risks and of VIX options to the volatility-of-volatility risks.

To illustrate the relation between the moneyness of the option and the vega-measured exposure of options to volatility risks, we show the option vega as a function of option moneyness in Figure 3. Vega represents an increase in the value of the option as implied volatility increases by 1 percentage point. Higher volatility translates into higher future profits from delta-hedging due to the convexity effect; hence, both call and put options have strictly positive vegas. Further, because the curvature of the option value is the highest for at-the-money options, at-the-money options have the highest vega in the cross section and thus the largest exposure to volatility risks. An alternative way to proxy for the option sensitivity to volatility risks is to use the "gamma" of the option, which represents the second derivative of the option price to the underlying stock price: $\Gamma = \partial^2 C/\partial S^2$. As shown in Graph A of Figure 3, the shapes of the vega and gamma functions are almost identical; hence, the implied cross-sectional dispersion in volatility betas by moneyness is very similar as well.

To capture the sensitivity of option prices to the volatility of volatility, we compute the Black and Scholes (1973) second partial derivative of the option price with respect to the volatility, which is known in "volga" for "volatility gamma." Volga is calculated as:

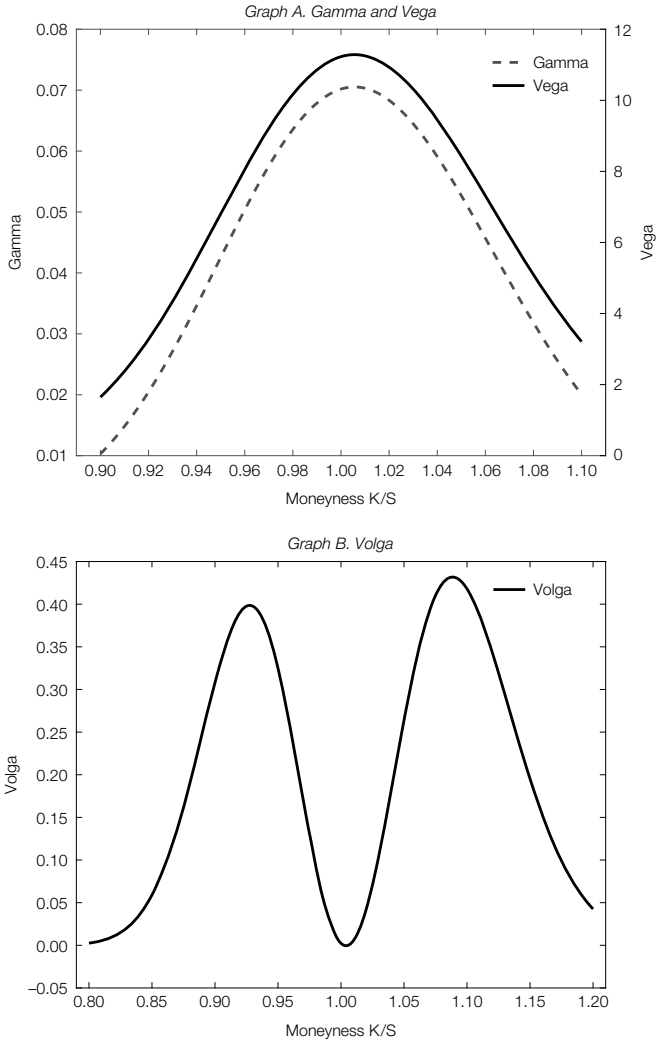
$$\frac{\partial^2 C}{\partial \sigma^2} = S\sqrt{\frac{\tau}{2\pi}}e^{-d_1^2/2}\left(\frac{d_1 d_2}{\sigma}\right) = \frac{\partial C}{\partial \sigma}\left(\frac{d_1 d_2}{\sigma}\right), \tag{26}$$

where $d_2 = d_1 - \sigma\sqrt{\tau}$. Graph B of Figure 3 shows the plot of volga as a function of the moneyness of the option. Volga is positive and exhibits twin peaks with a valley around at the money options. At-the-money options are essentially pure bets on volatility and are approximately linear in volatility (see Stein (1989)). Therefore, the volga is the lowest for at-the-money options. Deep-out-of-the-money options and deep-in-the-money options do not have much sensitivity to the volatility

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FIGURE 3
Gamma, Vega, and Volga by Moneyness

Graph A of Figure 3 shows the Black–Scholes (1973) gamma and vega of the option-by-option moneyness for average levels of volatility. Graph B shows the Black–Scholes volga of the option-by-option moneyness for average levels of volatility.



of volatility either; for the former, it is a pure directional bet, and for the latter, the option value almost entirely comprises intrinsic value. Options that are somewhat away from at-the-money options are most exposed to volatility-of-volatility risks.

Table 4 shows our cross-sectional evidence from the regressions of average option returns on our proxies of options' volatility and volatility-of-volatility betas. Panel A shows univariate and multivariate regressions of delta-hedged S&P 500 option gains scaled by the index on the sensitivities of the options to volatility

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TABLE 4
Delta-Hedged Option Gains by Volatility-Risk Sensitivities

Table 4 shows the evidence from the cross-sectional regressions of the delta-hedged Standard & Poor's (S&P) 500 and VIX option gains on the vega and volga of the options. Regressions include time fixed effects, and option characteristics are computed using the Black–Scholes (1973) formula. The numbers in t-Stat. column indicate Newey–West t-statistics. The data are monthly from 2006:M2 to 2016:M12.

	Vega $= \frac{\partial C}{\partial \sigma}$		Volga $= \frac{\partial^2 C}{\partial \sigma^2}$	
Option Gains	Slope	t-Stat.	Slope	t-Stat.
Panel A. S&P 500 Options				
$\Pi_{t,t+1}/S_t$	0.01	0.61		
			−0.01	−4.28
	−0.10	−4.40	−0.01	−6.21
Panel B. VIX Options				
$\Pi^*_{t,t+1}/F_t$	−1.23	−3.35		

and volatility-of-volatility risks. Our encompassing regression for delta-hedged S&P 500 options is:

$$\text{GAINS}^i_{t,t+\tau} = \frac{\Pi^i_{t,t+\tau}}{S_t} = \tilde{\lambda}_1 \text{VEGA}^i_t + \tilde{\lambda}_2 \text{VOLGA}^i_t + \gamma_t + \epsilon^i_{t,t+\tau}. \tag{27}$$

Because each date includes multiple options, as in Bakshi and Kapadia (2003), we allow for a date-specific component in $\Pi^i_{t,t+1}$ due to the option expirations (i.e., we include time-fixed effects represented by γ_t). Conceptually, our approach is related to Fama and MacBeth (1973) regressions. Instead of estimating risk betas in the first stage, due to the nonlinear structure of option returns, we measure the exposures from economically motivated proxies for the risk sensitivities.[12]

The results in Panel A of Table 4 show that both volatility and the volatility of volatility are priced in the cross section of delta-hedged S&P 500 option returns. Whereas univariate estimates for vega are insignificantly different from 0, univariate estimates for volga are significantly negative at −0.005, with a t-statistic of −4.28. In multivariate regressions, once we control for the effect of volga, the loading on vega becomes significantly negative, as we expect from the volatility premium. This demonstrates that, especially in a period where we observe large spikes in volatility, it is important to control for exposure to the volatility of volatility to properly see the impact of exposure to volatility on delta-hedged S&P 500 option returns.

Panel B of Table 4 presents cross-sectional results for delta-hedged VIX options. As equation (17) shows, delta-hedged VIX options are mainly exposed to volatility-of-volatility risks, and the vega for VIX options captures the sensitivity of VIX options to innovations in the volatility of volatility. The coefficient on vega of −1.23 is negative and statistically significant. Thus, in the cross section of both S&P 500 options and VIX options, we find strong evidence of a negative price of volatility-of-volatility risk.

[12] Song and Xiu (2016) demonstrate an alternative method of estimating risk sensitivities nonparametrically using local linear regression methods.

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

C. Time-Series Evidence

In the model, time variation in the expected delta-hedged option gains is driven by V and η, and the loadings are determined by the market prices of volatility and volatility-of-volatility risk. We group options into the same bins as we used for average returns in Table 3, and we average the scaled gains within each bin so that we have a time series of option returns for each moneyness bin. To examine the contribution of both risks for the time variation in expected index option payoffs, we consider the following regression:

$$(28)\quad \text{GAINS}^i_{t,t+\tau} = \frac{\Pi^i_{t,t+\tau}}{S_t} = \beta_0 + \beta_1 \text{VIX}^2_t + \beta_2 \text{VVIX}^2_t + \gamma \text{GAINS}^i_{t-\tau} + u^i + \epsilon^i_{t+\tau},$$

where we include fixed effects u^i to account for the heterogeneity in the sensitivity of options in different moneyness bins to the underlying risks. We regress the delta-hedged option gain scaled by the index from expiration to expiration on the value of the VIX and VVIX at the end of the earlier expiration; in other words, we run 1-month-ahead predictive regressions of delta-hedged option returns on the VIX and VVIX. We include lagged gains to adjust for serial correlation in the residuals, following Bakshi and Kapadia (2003).

Panel A of Table 5 shows the regression results for the index options. The univariate regression of delta-hedged S&P 500 option gains on the VIX^2 yields a negative coefficient that is statistically significant, consistent with Bakshi and Kapadia (2003). The VVIX^2 also predicts future S&P 500 option gains with a negative coefficient, and in a multivariate regression on both the VIX and VVIX, we find that both loadings are negative and statistically significant.

It is important to point out that the true loadings on the volatility factors are time varying and generally depend on the underlying volatility and

TABLE 5
Predictability of Delta-Hedged Option Gains

Table 5 shows the evidence of predictability of future delta-hedged Standard & Poor's (S&P) 500 and volatility index (VIX) option gains by the implied volatility and volatility-of-volatility measures. The regressions across all moneyness bins include moneyness fixed effects; the regressions for the at-the-money (ATM) options are for the moneyness ranges between 0.975 and 1.025 for the index and between 0.9 and 1.1 for the VIX options. Lag gains are included to correct for serial correlation of the residuals. The numbers in *t*-Stat. column indicate Newey–West *t*-statistics. The data are monthly from 2006:M2 to 2016:M12.

	VIX^2		VVIX^2		GAINS_{t-1}	
Option Gains	Slope	*t*-Stat.	Slope	*t*-Stat.	Slope	*t*-Stat.
Panel A. S&P 500 Options						
$\Pi_{t,t+\tau}/S_t$	−0.69	−3.42			0.24	4.08
			−0.26	−2.87	0.23	4.16
	−0.50	−3.41	−0.23	−2.75	0.26	4.20
Panel B. S&P 500 Options (ATM only)						
$\Pi_{t,t+\tau}/S_t$	−0.53	−2.81			0.16	3.43
			−0.25	−2.41	0.18	3.22
	−0.35	−2.28	−0.24	−2.29	0.19	3.38
Panel C. VIX Options						
$\Pi^*_{t,t+\tau}/F_t$			−1.01	−3.38	−0.10	−7.07
Panel D. VIX Options (ATM only)						
$\Pi^*_{t,t+\tau}/F_t$			−0.77	−4.43	−0.11	−6.89

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

volatility-of-volatility states (see, e.g., equation (11)). As shown by Bakshi and Kapadia (2003), these loadings are insensitive to volatility for at-the-money options. However, away from the money, the loadings may depend on the variance risk factors, so the relation between delta-hedged gains and the variance-related risk factors is nonlinear. To guard against finding a spurious coefficient on the VVIX, we repeat the exercise using only at-the-money options and find very similar results.

Panel B of Table 5 shows the corresponding evidence for VIX options. The VVIX negatively and significantly predicts future VIX option gains. This is consistent with a negative market price of volatility-of-volatility risk. The results remain very similar for only at-the-money options.

V. Robustness

A. Delta Computations

A key input into the computations of delta-hedged option returns is the delta of the option. The delta hedging insulates option returns from changes in the direction of the underlying and allows us to focus on the volatility-related risks. The "true" deltas are not observable and depend on the model specification. It is even more challenging to find the "true" delta for the VIX options because the VIX is a square root of a portfolio of options, and the integrated volatility is not a traded asset. In our article, we have pursued a typical approach in the literature to approximate the unknown "true" deltas by the Black–Scholes (1973) implied ones (see, e.g., Bakshi and Kapadia (2003), Duarte and Jones (2007)). In addition, we consider a variety of checks to show the robustness of our key results to measurements of delta.

From the theoretical perspective, we use numerical simulations of our illustrative model in Section II to compute and compare the "true" and Black–Scholes (1973) deltas, and we evaluate the adequacy of the approximation by the Black–Scholes deltas for the VIX options. In the Supplementary Material, we show that in our model specification, the two variants of delta are quite similar, in particular for at-the-money options.

We also consider several robustness checks in the empirical application. First, we use empirically observed, rather than Black–Scholes (1973) implied, deltas to construct delta-hedged gains. Specifically, we regress changes in option prices on changes in the prices of the underlying assets, option characteristics, and the interactions between the two, and we obtain the estimate of the empirical delta as a time-varying loading on the underlying price change. For example, for index calls, the regression takes the following form:

$$\begin{aligned}(29)\quad \Delta C_{t+1} = C_{t+1} - C_t &= \text{CONST} + \alpha X_t + \beta_0 \Delta S_{t+1} + \beta_1 \Delta S_{t+1} X_t + \epsilon_{t+1} \\ &= \text{CONST} + \alpha X_t + (\beta_0 + \beta_1 X_t)\Delta S_{t+1} + \epsilon_{t+1},\end{aligned}$$

and thus an empirical estimate of delta is given by $\beta_0 + \beta_1 X_t$. We run separate regressions for index and VIX calls and puts, and we include in X_t option characteristics that are known to affect option sensitivity to the underlying, such as its moneyness, implied volatility, and its maturity.

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Overall, we find that although empirical estimation is noisy, the main results are quite similar to the benchmark. The correlation between the empirical and the Black–Scholes (1973) deltas is approximately 90% across all the contracts. As shown in Table 6, the results for the average delta-hedged gains are quite similar to the benchmark specification. Similar to the Black–Scholes delta hedging, all the gains, with the exception of in-the-money calls, are negative; nearly all are significant; and the magnitudes are quite similar across the two specifications.

TABLE 6
Delta-Hedged Option Gains: Empirical Delta

Table 6 shows average delta-hedged option gains on the Standard & Poor's (S&P) 500 and volatility index (VIX) options across their moneyness constructed using the empirical, regression-based estimates of the option deltas. Delta-hedging is done with daily rebalancing and the margin difference earning the risk-free rate. The delta-hedged option gains Π are scaled by either the index or the option price. The t-statistics are testing the null that the delta-hedged option gain is equal to 0. The "% < 0" column shows the fraction of observations with negative gains. The data are monthly from 2006:M2 to 2016:M12.

	Π/S(%)						Π/C(%)		
Moneyness	Mean	*t*-Stat.	Median	% < 0	Std. Dev.	AR(1)	Mean	*t*-Stat.	Median
Panel A. S&P 500 Options									
Call									
0.950–0.975	0.09	1.89	0.08	47%	1.24	0.10	1.99	1.97	1.61
0.975–1.000	0.02	0.44	−0.08	54%	1.17	0.13	0.47	0.31	−3.99
1.000–1.025	−0.11	−2.48	−0.40	64%	1.40	0.00	−3.73	−0.50	−39.12
1.025–1.050	−0.27	−4.97	−0.69	66%	1.63	−0.03	−129.30	−3.38	−142.13
Put									
0.950–0.975	−0.10	−2.34	−0.06	52%	1.31	0.13	−21.93	−2.94	−5.67
0.975–1.000	−0.16	−4.11	−0.27	59%	1.19	0.18	−12.36	−4.24	−21.07
1.000–1.025	−0.26	−5.81	−0.45	67%	1.33	0.03	−9.84	−5.61	−21.78
1.025–1.050	−0.43	−6.28	−0.74	72%	1.56	−0.03	−9.41	−5.93	−18.69
Panel B. VIX Options									
Call									
0.800–0.900	−0.44	−1.72	−1.03	62%	3.84	−0.12	−2.43	−1.64	−5.94
0.900–1.000	−0.70	−2.33	−1.33	60%	4.43	−0.14	−6.18	−2.29	−11.05
1.000–1.100	−0.24	−0.66	−0.15	51%	4.95	−0.10	−1.82	−0.38	−1.66
1.100–1.200	−0.55	−1.44	0.39	48%	5.09	−0.07	−7.85	−1.15	6.42
Put									
0.800–0.900	−2.44	−3.24	−3.58	77%	6.76	0.37	−67.33	−3.07	−93.83
0.900–1.000	−1.42	−2.92	−2.98	69%	6.91	−0.06	−19.25	−2.16	−45.96
1.000–1.100	−0.86	−1.81	−1.45	58%	6.44	−0.01	−6.76	−1.72	−11.84
1.100–1.200	−1.37	−3.08	−1.40	60%	5.82	−0.03	−6.62	−3.05	−7.10

To provide additional robustness checks, we also considered an exercise similar to that of Coval and Shumway (2001) by perturbing the Black–Scholes deltas used in the option hedging. We set the deltas to 0.95 and 1.05 times the value from the Black–Scholes (1973) model, respectively. Our main results are not sensitive to the choice of using Black–Scholes deltas for option hedging and remain quite robust. The results are provided in the Supplementary Material.

B. Sensitivity to Jump-Risk Measures

The evidence in our article highlights the roles of the volatility and volatility-of-volatility factors, which are driven by smooth Brownian-motion shocks. In principle, the losses on delta-hedged option portfolios can also be attributed to large, discontinuous movements (jumps) in the stock market and in the market volatility. In this section we verify that our empirical evidence for the importance

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

of the volatility-related factors is robust to the inclusion of jump measures considered in the literature.

Specifically, we consider three measures of jump risks, which we construct for the S&P 500 and the VIX. Our first jump measure corresponds to the slope of the implied volatility curve:

$$\begin{aligned} \text{SLOPE}^{\text{SPX}} &= \sigma^{\text{SPX}}_{\text{OTM}} - \sigma^{\text{SPX}}_{\text{ATM}}, \\ \text{SLOPE}^{\text{VIX}} &= \sigma^{\text{VIX}}_{\text{OTM}} - \sigma^{\text{VIX}}_{\text{ATM}}. \end{aligned} \tag{30}$$

The out-of-the-money (OTM) contract for the S&P 500 options is defined as a put option with a moneyness closest to 0.9, and for VIX options, it is defined as a call option with a moneyness closest to 1.1. In both cases, the at-the-money (ATM) option has moneyness of 1. These slopes are positive for both index and VIX options. A positive slope of the index volatility smile is consistent with the notion of negative jumps in market returns (see, e.g., Bates (2000), Pan (2002), and Eraker, Johannes, and Polson (2003)), whereas the fact that the implied volatility curve for VIX options slopes upward is consistent with positive volatility jumps (Drechsler and Yaron (2010), Eraker and Shaliastovich (2008), among others). In this sense, these slope measures help capture the variation in the market and volatility jumps in the economy.

Our second jump measure incorporates the whole cross section of option prices, beyond just the slope of the smile. It is based on the model-free risk-neutral skewness of Bakshi et al. (2003):

$$\text{SKEW}(t,t+\tau) = \frac{e^{r_f\tau}W_{t,t+\tau} - 3\mu_{t,t+\tau}e^{r_f\tau}V_{t,t+\tau} + 2\mu^3_{t,t+\tau}}{[e^{r_f\tau}V_{t,t+\tau} - \mu^2_{t,t+\tau}]^{3/2}}, \tag{31}$$

where $V_{t,t+\tau}, W_{t,t+\tau}, X_{t,t+\tau}$ are given by the prices of the volatility, cubic, and quartic contracts, and $\mu_{t,t+\tau}$ is a function of them. Importantly, these measures are computed model-free using only observed option prices. The details for the computations are provided in the Supplementary Material.

Finally, our third measure of jump risks is based on high-frequency index and VIX data, rather than option prices. It corresponds to the realized jump intensity and relies on the bipower variation methods of Barndorff-Nielsen and Shephard (2004), Huang and Tauchen (2005), and Wright and Zhou (2009). Specifically, whereas the realized variance defined in equation (21) captures both the continuous and jump variation, the bipower variation, defined as

$$\text{BV}_t = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}|r_{t,j-1}||r_{t,j}|, \tag{32}$$

measures the amount of continuous variation returns. Hence, we can use the following test statistic to determine if there is a jump on any given day:

$$J_t = \frac{\dfrac{\text{RV}_t - \text{BV}_t}{\text{RV}_t}}{\sqrt{\dfrac{\theta}{M}\max\left(1, \dfrac{\text{QV}_t}{\text{BV}_t^2}\right)}}, \tag{33}$$

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where $\theta = (\pi/2)^2 + \pi - 5$, and QV_t is the quad-power quarticity defined by Huang and Tauchen (2005) and Barndorff-Nielsen and Shephard (2004). The test statistic is standard normally distributed. We flag the day as having a jump if the probability exceeds 99.9% both for index returns and for the VIX. These cut-offs imply an average frequency of jumps of once every 2 months for the index and approximately three jumps a month for the VIX. This is broadly consistent with the findings of Tauchen and Todorov (2011), who find that VIX jumps tend to happen much more frequently than S&P 500 jumps. Over a month, we sum up all the days where we have a jump, and we define our jump-intensity measure on a monthly level as follows:

$$RJ_t = \frac{1}{T}\sum_{i=0}^{T-1} J_{t+i}, \tag{34}$$

where T is the number of trading days in the given month.

We use the jump statistics to document the robustness of the link between the volatility and volatility-of-volatility factors and options gains. We consider a regression of the form

$$\begin{aligned} GAINS^i_{t,t+\tau} = {} & \beta_0 + \beta_1 VIX_t^2 + \beta_2 VVIX_t^2 + \beta_3 JUMP_t \\ & + \gamma GAINS^i_{t-\tau} + u^i + \epsilon^i_{t+\tau}, \end{aligned} \tag{35}$$

where $JUMP_t$ is one of the aforementioned proxies for jump risk. We use index jump measures for index gains and VIX jump measures for VIX gains.

Table 7 displays our results. For both S&P 500 and VIX options, controlling for SLOPE does not change the ability of the VIX and VVIX to predict future

TABLE 7

Predictability of Delta-Hedged Option Gains: Robustness to Jump Measures

Table 7 shows the evidence of predictability of future delta-hedged Standard & Poor's (S&P) 500 and volatility index (VIX) option gains by the implied volatility and volatility-of-volatility measures, controlling for the jump-risk measures. The cross-sectional regressions include moneyness fixed effects, and lagged gains are included to correct for serial correlation of the residuals. The jump measures are as follows: SLOPE is the slope of the implied-volatility smile, calculated as the Black–Scholes (1973) implied volatility of an out-of-the-money put option ($K/S = 0.9$) minus the implied volatility of an at-the-money put option ($K/S = 1$) for S&P 500 options and between a $K/S = 1.1$ call option and an at-the-money call option for the VIX options. RJ is the realized jump variation, calculated using high-frequency S&P 500 futures tick data for the S&P 500 and VIX tick data for the VIX. SKEW is the model-free measure of risk-neutral skewness. The numbers in t-Stat. column indicate Newey–West t-statistics. The data are monthly from 2006:M2 to 2016:M12.

	VIX^2		$VVIX^2$		SLOPE		SKEW		RJ		$GAINS_{t-1}$	
Option Gains	Slope	t-Stat.	Slope	t-Stat.	Slope	t-Stat.	Slope	t-Stat.	Slope	t-Stat.	Slope	t-Stat.
Panel A. S&P 500 Options												
$\Pi_{t,t+\tau}/S_t$	−0.64	−2.49	−0.21	−3.25	−1.44	−1.02					0.27	4.00
	−0.49	−2.58	−0.23	−2.92			−0.01	−0.15			0.26	4.06
	−0.49	−3.44	−0.23	−2.75					0.01	1.81	0.26	4.20
Panel B. VIX Options												
$\Pi^*_{t,t+\tau}/F_t$			−0.98	−3.19	2.55	2.60					−0.10	−7.14
			−0.76	−2.17			0.34	3.64			−0.10	−7.58
			−1.39	−3.59					0.03	1.12	−0.08	−3.55

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

delta-hedged option gains. Both factors are still significant, and the point estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ remain largely unchanged. SLOPE itself is not significant at conventional levels for S&P 500 options but is significantly positive for VIX options. The risk-neutral skewness also does not affect the predictive ability of the VIX and VVIX, although, again, it is significant for VIX options. The two estimates of β_3 have the correct signs because skewness is negative for S&P 500 options and positive for VIX options; this is broadly similar to the findings of Bakshi and Kapadia (2003). Finally, we control for realized jump intensity (RJ), and we see a similar result where the statistical significance of the VIX and VVIX and their point estimates are largely unchanged. RJ seems to marginally predict future S&P 500 option gains; for VIX options, however, RJ does not seem to predict future VIX option gains.

Hence, our evidence suggests that the VIX and VVIX have a significant impact on option returns even in the presence of stock market and volatility jumps. We leave a formal treatment of jumps for future research.

C. Subsamples

To further investigate the robustness of our results, we split our data into 2 subperiods of roughly equal length, a pre-2012 and a post-2012 period. Tables 8 and 9 show the results in the 2 subperiods. As we can see from the tables, the

TABLE 8
Delta-Hedged Option Gains: Pre-2012 Subsample

Table 8 shows average delta-hedged option gains on the Standard & Poor's (S&P 500) and volatility index (VIX) options across their moneyness in the pre-2012 subsample. Options have 1 month to maturity, are grouped into an equal-weighted portfolio inside the moneyness bin, and are held until expiration. The delta hedge is computed using the Black–Scholes (1973) formula, with daily rebalancing and the margin difference earning the risk-free rate. The delta-hedged option gains Π are scaled by either the index or the option price. The t-statistics are testing the null that the delta-hedged option gain is equal to 0. The "% < 0" column shows the fraction of observations with negative gains. The data are monthly from 2006:M2 to 2011:M12.

	Π/S(%)						Π/C(%)		
Moneyness	Mean	t-Stat.	Median	% < 0	Std. Dev.	AR(1)	Mean	t-Stat.	Median
Panel A. S&P 500 Options									
Call									
0.950–0.975	0.12	2.67	−0.04	54%	0.76	0.40	2.48	2.94	−0.85
0.975–1.000	0.04	1.12	−0.02	53%	0.64	0.26	1.61	1.52	−0.94
1.000–1.025	−0.08	−2.92	−0.11	61%	0.58	0.11	−5.83	−3.00	−11.16
1.025–1.050	−0.13	−4.92	−0.07	63%	0.51	0.12	−28.22	−5.74	−17.53
Put									
0.950–0.975	−0.12	−3.31	−0.25	81%	0.77	0.38	−12.24	−3.23	−28.10
0.975–1.000	−0.19	−5.98	−0.31	75%	0.66	0.24	−9.36	−4.68	−18.05
1.000–1.025	−0.27	−8.64	−0.34	72%	0.62	0.05	−9.64	−9.24	−12.92
1.025–1.050	−0.30	−8.58	−0.30	76%	0.57	0.06	−6.51	−8.80	−7.33
Panel B. VIX Options									
Call									
0.800–0.900	−0.62	−2.96	−0.65	68%	2.16	−0.10	−3.74	−3.10	−3.66
0.900–1.000	−1.49	−5.63	−2.01	75%	2.62	−0.21	−13.00	−5.48	−15.61
1.000–1.100	−1.02	−3.16	−1.46	73%	2.90	−0.10	−11.97	−2.72	−20.79
1.100–1.200	−1.38	−4.25	−1.60	80%	2.89	−0.08	−22.24	−3.72	−29.28
Put									
0.800–0.900	−0.65	−1.75	−1.37	64%	2.64	−0.10	−14.14	−1.28	−34.73
0.900–1.000	−1.40	−4.94	−1.96	74%	2.77	−0.21	−19.34	−3.98	−32.81
1.000–1.100	−0.97	−2.82	−1.56	74%	3.05	−0.13	−7.01	−2.48	−12.75
1.100–1.200	−1.29	−3.88	−1.38	79%	2.87	−0.07	−6.12	−3.68	−7.49

TABLE 9

Delta-Hedged Option Gains: Post-2012 Subsample

Table 9 shows average delta-hedged option gains on the Standard & Poor's (S&P) 500 and volatility index (VIX) options across their moneyness in the post-2012 subsample. Options have 1 month to maturity, are grouped into an equal-weighted portfolio inside the moneyness bin, and are held until expiration. The delta hedge is computed using the Black–Scholes (1973) formula, with daily rebalancing and the margin difference earning the risk-free rate. The delta-hedged option gains Π are scaled by either the index or the option price. The t-statistics are testing the null that the delta-hedged option gain is equal to 0. The "% <0" column shows the fraction of observations with negative gains. The data are monthly from 2012:M1 to 2016:M12.

	Π/S(%)						Π/C(%)		
Moneyness	Mean	t-Stat.	Median	% <0	Std. Dev.	AR(1)	Mean	t-Stat.	Median
Panel A. S&P 500 Options									
Call									
0.950–0.975	0.03	2.05	0.01	46%	0.27	−0.07	0.88	2.40	0.31
0.975–1.000	0.05	3.21	−0.01	51%	0.35	−0.14	3.00	3.83	−0.40
1.000–1.025	0.00	0.21	−0.03	54%	0.29	−0.08	0.22	0.11	−3.70
1.025–1.050	−0.05	−5.08	−0.04	62%	0.20	0.17	−36.85	−6.95	−32.35
Put									
0.950–0.975	−0.15	−13.37	−0.14	81%	0.26	0.02	−24.72	−10.37	−29.33
0.975–1.000	−0.14	−9.75	−0.15	75%	0.32	−0.03	−10.96	−6.49	−15.59
1.000–1.025	−0.17	−11.76	−0.18	77%	0.31	−0.11	−8.06	−10.69	−9.39
1.025–1.050	−0.20	−9.91	−0.21	82%	0.31	−0.01	−5.11	−9.87	−5.39
Panel B. VIX Options									
Call									
0.800–0.900	0.07	0.36	−0.25	57%	2.30	−0.05	0.64	0.50	−1.57
0.900–1.000	−0.17	−0.61	−0.87	66%	3.08	−0.08	−1.63	−0.65	−6.79
1.000–1.100	−0.71	−2.07	−1.21	69%	3.49	−0.23	−8.52	−2.07	−13.85
1.100–1.200	−1.05	−3.06	−1.32	75%	3.36	−0.19	−16.37	−3.10	−19.43
Put									
0.800–0.900	−0.25	−0.50	−0.86	65%	2.79	0.22	−10.54	−0.85	−20.76
0.900–1.000	−0.26	−0.85	−0.90	68%	3.19	−0.13	−3.43	−0.68	−12.19
1.000–1.100	−0.74	−2.15	−1.18	66%	3.50	−0.22	−5.46	−2.06	−8.49
1.100–1.200	−1.08	−3.15	−1.26	76%	3.38	−0.19	−5.08	−3.15	−5.49

subperiod results are very similar to the full-sample results in Table 3 and show large negative average S&P 500 and VIX option gains, consistent with investors pricing volatility and volatility-of-volatility risks with negative market prices of risk.

D. Alternative Variance Specifications

Our results for the predictability of realized by implied variance are robust to alternative specifications of volatility. Specifically, we consider standard deviation and natural logarithm units, rather than the variance. The robustness regressions take the following form:

$$\begin{aligned} \sqrt{\mathrm{RV}^{x}_{t,t+1}} &= \beta_0 + \beta_1 \mathrm{VIX}_t + \beta_2 \mathrm{VVIX}_t + \epsilon_{t+1}, \\ \ln\left(\sqrt{\mathrm{RV}^{x}_{t,t+1}}\right) &= \beta_0 + \beta_1 \ln(\mathrm{VIX}_t) + \beta_2 \ln(\mathrm{VVIX}_t) + \epsilon_{t+1}, \end{aligned} \tag{36}$$

where x refers to the SPX or VIX.

In Table 10, we see that the point estimates and significance are very close to our baseline specification in variance units.

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

TABLE 10

Predictability of Realized Variance Measures: Alternative Specifications

Table 10 shows the evidence from the alternative projections of the future realized variances of the Standard & Poor's (S&P) 500 index and the volatility index (VIX) on the current implied volatility and volatility-of-volatility measures. The realized and implied variances are modified to be expressed in standard deviation or natural logarithm units. The numbers in *t*-Stat. column indicate Newey–West *t*-statistics. The data are monthly from 2006:M2 to 2016:M12.

	VIX		VVIX		$\ln(VIX^2)$		$\ln(VVIX^2)$	
Variable	Slope	*t*-Stat.	Slope	*t*-Stat.	Slope	*t*-Stat.	Slope	*t*-Stat.
Panel A. S&P 500								
$\sqrt{RV^{SPX}_{t,t+1}}$	0.72	6.37						
			0.13	1.63				
	0.73	5.98	−0.02	−0.52				
$\ln(RV^{SPX}_{t,t+1})$					0.96	8.70		
							0.53	1.38
					0.99	8.26	−0.22	−1.07
Panel B. VIX								
$\sqrt{RV^{VIX}_{t,t+1}}$	0.20	−0.57						
			0.69	5.10				
	−0.50	−1.42	0.79	5.15				
$\ln(RV^{VIX}_{t,t+1})$					−0.07	−0.68		
							0.75	5.18
					−0.16	−1.57	0.87	4.80

VI. Conclusion

Using S&P 500 and VIX options data, we show that the time-varying volatility of volatility is a separate risk factor that affects option returns, above and beyond market volatility risks. We measure volatility risks using the VIX, and we measure the volatility-of-volatility risk using the VVIX. The VIX and VVIX have separate dynamics and are only weakly related in the data: The correlation between the two series is under 30%. On average, the risk-neutral volatilities identified by the VIX and VVIX exceed the realized physical volatilities of the corresponding variables in the data. Hence, the variance premium and variance-of-variance premium for the VIX are positive, which suggests that investors dislike variance and variance-of-variance risks.

We show the pricing implications of volatility and volatility-of-volatility risks using options market data. Average delta-hedged option gains are negative, which suggests that investors pay a premium to hedge against innovations in not only volatility but also the volatility of volatility. In the cross section of both delta-hedged S&P 500 options and VIX options, options with higher sensitivities to volatility-of-volatility risk earn more negative returns. In the time series, higher values of the VVIX predict more negative delta-hedged option returns, for both S&P 500 and VIX options.

Our findings are consistent with a no-arbitrage model that features time-varying market volatility and volatility-of-volatility factors. The volatility factors are priced by the investors, and in particular, volatility and volatility of volatility have negative market prices of risk.

https://doi.org/10.1017/S0022109018001436 Published online by Cambridge University Press

Supplementary Material

Supplementary Material for this article is available at https://doi.org/10.1017/S0022109018001436.

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