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Remarks on Euclid's *Elements* I,32 and the Parallel Postulate

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Argument

In this paper I offer some reflections on the thirty-second proposition of Book I of Euclid's *Elements*, the assertion that the three interior angles of a triangle are equal to two right angles, reflections relating to the character of the theorem and the reasoning involved in it, and especially on its historical background. I reject a common view according to which there was at some time a *petitio principii* in the theory of parallels and argue that certain kinds of skepticism about Proclus' historical reports are excessive. The evidence we have makes it reasonable to suppose that the so-called common notions were made explicit in the earlier fourth century BCE and the postulates, including the parallel postulate, somewhat later than that. On the other hand, it seems clear that some proof of I,32 was available by the mid-fifth century. I attempt to describe what such a proof would have been like and reflect on its significance for our understanding of early Greek geometry.

It is well known that the proposition which we express by saying that the sum of the angles of a triangle is 180° is expressed in Euclid's *Elements* (I,32)[1] by saying that "The interior angles of a triangle are equal to two right angles." This difference is standardly explained by saying what is true, namely that the early Greek geometers lacked the metrical concepts needed to express the size of an angle numerically. And, indeed, classical Greek geometry, of which the *Elements* is the classical expression, is not really concerned with measurement in numerical terms at all, but it is importantly concerned with equalities, for the most part of areas and volumes, determined by procedures which involve "cutting and pasting," that is the division of a figure F_1 into other figures f_1, f_2, . . . f_n and the rearrangement of the f_i into a new figure F_2, obviously equal to F_1. Such procedures are common to the elementary geometries of other cultures, what distinguishes the Greek version we know being the extent of axiomatization for proofs of equalities and explicit procedures for constructing figures. I,32 is an apparent exception to this characterization. I,32 does permit the possibility of describing the angle sums of other rectilineal figures by dividing them into triangles, but it itself is not a cut-and-paste theorem in the sense in which that

[1] In this paper I use expressions like "I,32" to refer both to a specific proposition in Euclid's *Elements* and to other formulations of the same truth.

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phrase is ordinarily used, because it involves angle equalities rather than figure equalities. Moreover, so far as I know, we do not find anything like I,32 in the mathematics of any other early culture. In this sense it is a distinctively Greek achievement, and the reasoning that goes into it is distinctively Greek. Finally, I,32 depends on the parallel postulate, perhaps the most striking and influential of all the first principles of Greek mathematics. In this brief essay I want to say some things about the parallel postulate, the proof of I,32, and the notion of being equal to two right angles.

In terms of figure 1 Euclid's fifth postulate, the parallel postulate, says:

> If BAC and DCA are angles on the same side of the straight line AC, and they are together less than two right angles, then if AB and CD are extended past B and D, they will eventually intersect.

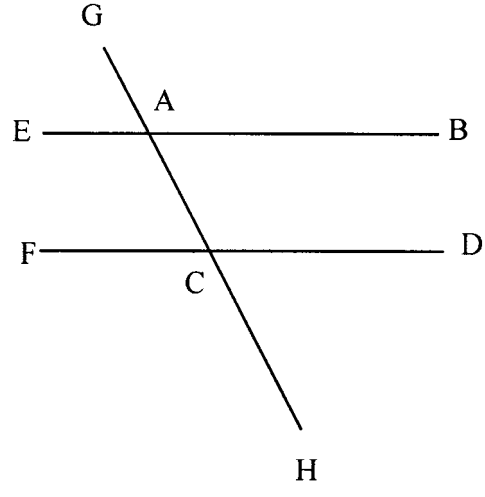


Fig. 1.

Thomas Heath (1949) has argued that Euclid is to be given credit for the introduction of this postulate into geometry, basing himself on a difficult passage in which Aristotle reports the existence of a *petitio principii* in the treatment of parallels:

> **A**. This is what happens with those who think they draw parallels (τὰς παραλλήλους γράφειν); for they unconsciously assume things which it is not possible to demonstrate if parallels do not exist (μὴ οὐσῶν τῶν παραλλήλων) (*Prior Analytics* II.16.65a4–7. Heath 1949 translation).

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Commenting on this passage, Heath says:

> If a general theory of parallels is here alluded to, we infer that it was on the lines of Euclid's propositions on the subject, but that, at some point, it tacitly assumed something equivalent to the very thing to be proved. It is a fair inference that Euclid was the first to get rid of a *petitio principii* in some earlier text-books, by himself formulating the famous Postulate 5. (Heath 1949, 28)

I assume that Heath means that pre-Euclidean treatments of parallel lines made no mention of a Postulate 5 or its equivalent, and that a tacit assumption went into the proof of one of the fundamental propositions Euclid proves using the postulate, namely I,31 (the transitivity of the parallel relation),[2] I, 29 or I,32. I shall formulate I,32 as follows:

I, 32 (see fig. 2). If ACB is a triangle and BC is extended to D and CE is drawn parallel to BA, then (a) $\angle ACD = \angle ABC + \angle BAC$, and (b) angles ACB, ABC, BAC together are equal to two right angles.

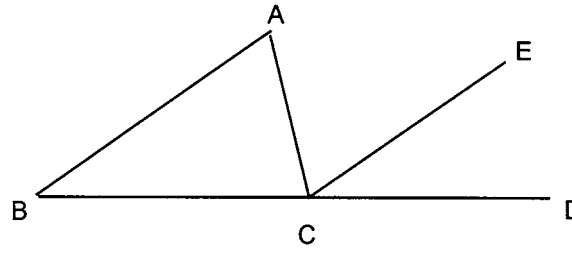


Fig. 2.

What does part b of I,32 add to part a? How little it adds in content is shown by the theorem invoked to justify the move from a to b:

I,13 (see fig. 3). If BCD and AC are two straight lines, then $\angle ACB + \angle ACD =$ 2 right angles.

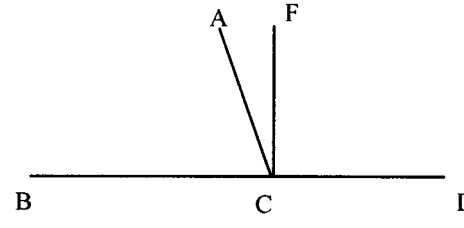


Fig. 3.

[2] The transitivity of parallelism is, of course, equivalent to the uniqueness of the parallel to a straight line through a given point. Vitrac (1990, 307–308) suggests that the *petitio* Aristotle has in mind is the assumption of uniqueness.

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Euclid's proof is little more than the picture:

If ∠ACB = ∠ACD, then by the definition of a right angle (I, definition 10), each is a right angle. Suppose they are unequal and let ∠ACD > ∠ACB.[3]

(i) Let CF be erected at right angles to BCD (I,11).

Then ("obviously")

(ii) ∠FCB = ∠ACB + ∠ACF, and
∠ACD = ∠ACF + ∠FCD,

and so (common notion 2 ("equals added to equals")):

(iii) ∠ACB + ∠ACD = ∠ACB + ∠ACF + ∠FCD, and
∠FCB + ∠FCD = ∠ACB + ∠ACF + ∠FCD.

Therefore, by common notion 1 ("things equal to the same thing"),

(iv) ∠ACB + ∠ACD = ∠FCB + ∠FCD,

but the second pair are together equal to two right angles.

The underlying reasoning here is not so much cutting and pasting as cutting and looking. But in the *Elements* the looking is replaced by inferences involving the common notions. Since Aristotle sometimes mentions Euclid's third common notion ("equals subtracted from equals") as a "common axiom" and speaks of the common axioms in a way which suggests they are a known feature of mathematics ("so-called common axioms" at *Posterior Analytics* I.10.76b14 and "what are called axioms in mathematics" at *Metaphysics* Γ.1.1005a20), it seems likely that by Aristotle's time some mathematicians were being explicit about this sort of general quantitative assumption. But it also seems to me and, I think, most scholars likely that these assumptions were not made explicit much before Aristotle's time. A looking argument would only require the drawing of a perpendicular from point C, and the picture would make the truth of the claim clear. And given the picture the perpendicular would seem to serve more the purpose of saying what is meant by equality to two right angles, than providing a needed presupposition of the proof.

In his commentary on Book I of Euclid's *Elements,* Proclus (283.7–10) tells us that the later fifth-century figure Oinopides of Chios first investigated (ἐζήτησεν) the problem of dropping a perpendicular and that he did so because he thought it useful in astronomy; Proclus adds that Oinopides used the word "gomonwise" (κατὰ

[3] It is somewhat surprising that Euclid does not worry about the relative sizes of the two angles, but simply takes for granted that they are as in the picture.

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γνώμονα) because the γνώμον (roughly sundial) is at right angles to the horizon. Later (333.5–6) Proclus, citing Eudemus, ascribes to the same Oinopides the "discovery" of how to construct an angle equal to a given one. Euclid carries this construction out in I,23, and in I,31 he uses it to construct a line through a point parallel to a given line, the construction he uses in I,32 to draw CE (fig. 2). I do not see any reasons which would not fall under the rubric of general skepticism[4] for casting doubt on the things Proclus and others tell us about Oinopides, as there may be when he talks about Pythagoreans or Plato and his associates. It is true that Oinopides and Anaxagoras are introduced together in the Eudemian summary (Proclus 65.7-68.4), where it is pointed out that the two are also mentioned in the *Lovers* (Ἐρασταί), a short dialogue once ascribed to Plato. But this fact hardly entitles one to dismiss the particular attributions of mathematical discoveries made by Oinopides (for which see Gillespie 1970–1990, s.v. "Oenopides"). And it seems to me particularly difficult to explain why elementary "discoveries" of the kind I have mentioned should be assigned to such a late figure as Oinopides rather than, say, to one of the great early Greek geometers, Thales or Pythagoras. I am inclined to think that what mainly Proclus, but also others, tell us allows us to locate the real beginning of axiomatization and the reduction of theorems and constructions to more elementary propositions and constructions in the later fifth century.[5] In particular, Oinopides' near contemporary Hippocrates is said by Proclus to have written an "elements," and to have been interested in the notion of reducing one mathematical question to another.[6] Admittedly we don't have much to go on, but what we do have should be treated as better than nothing.

The last ingredient in the proof of I,32 is the demonstration that $\angle ACE = \angle BAC$ and $\angle ECD = \angle ABC$. For this Euclid invokes:

I,29 (see fig. 1) If EAB and FCD are parallel straight lines, then
(a) the alternate angles EAC and ACD are equal,
(b) the exterior angle GAB is equal to the interior and opposite angle ACD,

[4] As examples of specific skepticism I would mention Dicks (1959) on Proclus' reports on Thales and Burkert (1972, 409–412) on the description of Pythagoras in the Eudemian summary. I would classify Vitrac (1996) under the rubric of general skepticism. Vitrac concentrates on the Eudemian summary, although on page 45 he appears to generalize what he says, mentioning Thales, the Pythagoreans, and Oinopides, but he gives no specific reasons for skepticism about Oinopides, and he suggests that "la contribution d'Eudème prend toute sa valeur lorsqu'il peut disposer de sources écrites, en tout état de cause à partir d'Hippocrate de Chio." So far as I know, no one supposes any significant chronological difference between Hippocrates and his fellow Chian Oinopides. Although Proclus refers to Hippocrates as coming after Oinopides, modern scholars (e.g., Morrow 1992, 54, n. 34) sometimes refer to them as contemporaries. The *Dictionary of Scientific Biography* (Gillespie 1970–1990), locates the active life of Oinopides "in the second third of the fifth century" (vol. X, 179), and that of Hippocrates "in the second half of the fifth century" (vol. VI, 410). In this paper I am not interested in any period prior to the mid-fifth century.

[5] I have argued for this claim in more detail in Mueller 1997.

[6] On Hippocrates see Gillespie 1970–1990, s.v., "Hippocrates of Chios."

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(c) the interior angles on the same side BAC, DCA together are equal to two right angles.

The equivalence of a, b, c, and the parallel postulate is guaranteed by I,13 and:

I,15 (see fig. 4) Vertex angles such as GAB and EAC are equal.

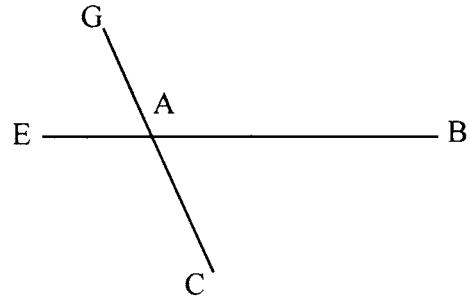


Fig. 4.

I shall discuss the proof of I,15 later, but now I want to bring in Proclus again. In his discussion of I,32 (379.2–16) he tells us that Eudemus credits the Pythagoreans with the discovery of the theorem that the interior angles of a triangle are equal to two right angles, and gives the Pythagorean proof, a Euclidean-style demonstration, but avoiding the preliminary assertion of the equality of the exterior angle ACD to the interior and opposite angles ABC, BAC. The proof does not differ in any interesting way from Euclid's. In fig. 5, FAG is parallel to BC,

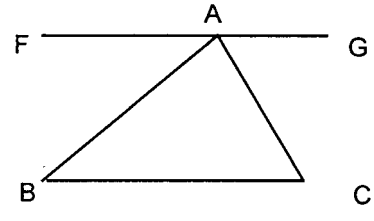


Fig. 5.

and the result is inferred from the equalities (given by I,29(a)) of the alternate angles GAC, ACB, and FAB, ABC; again I,13 would be used to get the equality to two right angles.

There will probably always be questions about the reliability of what Proclus says about the Pythagoreans, but I do not know any good, specific reason for rejecting the view that the proof he gives and hence the theorem dates from around the middle of

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the fifth century BCE.[7] That, of course, tells us very little, since we have no way of inferring how exactly the "Pythagoreans" argued. Proclus pretty clearly takes for granted that they made use of propositions like those found in Euclid, and Eudemus perhaps represented the proofs he describes in terms of some other similar formulation of elementary geometry known to him. We do not have adequate reasons to take for granted that Proclus is right in this regard or that Eudemus' representations are intended to be something like quotations.

Aristotle provides us with quite clear evidence that I,32 was something to talk about, at least in his circles, in the mid-fourth century. It is his favorite example of a mathematical proposition,[8] and one Aristotelian passage shows a knowledge of the relations between parallelism and important results relating to angles. In the passage, Aristotle invokes parallels to show that a false conclusion can result from different false assumptions:

> **B.** It is not perhaps absurd that the same false result should follow from several suppositions, e.g., that parallels meet, both on the assumption (i) that the interior angle is greater than the exterior and on the assumption (ii) that a triangle contains more than two right angles. (*Posterior Analytics* II.17.66a11–16; Barnes 1984 translation)

Consequently I think we can be reasonably confident that I,32 was proved as early as 400, and Proclus suggests that we are entitled to move it earlier. I also think it likely that the pre-Aristotelian proof did not use Euclid's parallel postulate or any other explicit assumption equivalent to it, but my reasons are primarily that, although Aristotle's discussions of first principles, which are fairly detailed, show clear recognition of the definitions and common notions we find in the *Elements*, there is no acknowledgment of anything like Euclid's postulates.[9]

Heath's own inference from Text A that there was a *petitio principii* in earlier treatments of parallels seems to me unwarranted since it relies on questionable interpretations of γράφειν τὰς παραλλήλους and μὴ οὐσῶν τῶν παραλλήλων.[10] Heath proposes to construe "think they draw parallels" as "establish the theory of parallels" or "write on parallels," as if Aristotle had written γράφειν περί or γράφειν ὑπό. It is true that in geometrical writings γράφειν is generally reserved for circles and other non-straight lines, and that the most common word for drawing a straight

[7] As far as I know, no one has raised skeptical objections to this passage in particular. It appears not to be mentioned by Burkert, who did so much to discredit the idea of Pythagoras the mathematician.
[8] For a list of some twenty Aristotelian references to it, see Heiberg 1904, 19–20.
[9] For the argument, see Mueller 1991, 66–81.
[10] Vitrac (1990, 306, n. 15), who appears to follow Heath in this matter, proposes 'prouver géométriquement que deux droites sont parallèles,' and remarks that the words 'parallels exist' "peut aussi être diversement interprété."

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line is ἄγειν.[11] But it seems to me simpler to suppose that Aristotle here diverges from this practice than to suppose that he uses γράφειν τὰς παραλλήλους to mean something like "write about parallels." And certainly it is true that Aristotle could use an expression like μὴ οὐσῶν τῶν παραλλήλων as shorthand for a variety of things. But it seems to me preferable to start from the most direct rendering of the phrase, the kind which Heath himself offers in his translation, than to start by supposing that the phrase means something broader than what it literally says.

The only ancient Greek commentary we have on this passage, written by John Philoponus, cannot be relied on for what it suggests about the specific geometric content behind Aristotle's remark, but there is no question that Philoponus understands the Greek constructions in the straightforward way I am proposing. He writes:

> Those who draw parallel lines (οἱ τὰς παραλλήλους γράφοντες) also do the same thing: they beg the question. For they claim that it is possible to draw (καταγράψαι) parallel straight lines from the meridian circle, and they take a point on the plane of the circle and in this way they extend the straight lines – and they have assumed what was sought, since the person who does not concede the existence of parallels will not concede the existence of that point either. (Philoponus 454.4–9)

The details of the argument introduced by Philoponus are uncertain, but obviously he takes it to be one in which a parallel line is drawn by postulating a point which exists if and only if the line does.

We cannot know what sort of situation Aristotle is envisaging, but I would suppose it is one in which a geometer says in the course of an argument, "Let the straight line CE be drawn through C parallel to BA." Obviously this kind of reasoning has to take for granted that parallel lines exist – in some sense of exist. In the only Greek geometry we know the existence of such lines would be established by assuming or proving that such lines can be drawn. I am inclined, then, to think that Aristotle is referring to arguments like the proof of I,32, but in which the possibility of drawing parallels was not demonstrated, but simply taken for granted. We have no way of knowing whether Aristotle is explicitly thinking about I,32, but it is obviously unlikely that the possibility would be demonstrated in one case and taken for granted in another. And the idea that he is thinking about this case, the report that the Pythagoreans produced a proof of I,32, and the multiply confirmed suggestion that elementary constructions were first studied in the later fifth century all fit together nicely.[12]

[11] See Mugler 1958, 39–40 and 107–109, where there are passages from Aristotle conforming to this standard usage. I thank Bernard Vitrac for making this point about the words for "draw" in a discussion with me of an earlier version of this paper.

[12] Text A is in the present tense, but Aristotle need not be referring to anything about the geometrical practice of his contemporaries, since he frequently discusses views of his predecessors in the present tense.

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I have argued that certain elementary constructions, notably I,23 or I,31 and I,11, and the first three common notions might well have been taken for granted in an earlier version of the proof of I,32. It is also reasonable to assume that the same is true of the first two Euclidean postulates, which allow one to draw the straight line connecting two points and to extend a straight line. It also appears that the parallel postulate was not enunciated as a postulate by the mid-fourth century, but any proof of I,32 would have had to in some way assume or take for granted it or one of its equivalents, I,29(a), (b), or (c). To me, for what it is worth, I,29(b) is the most intuitively "obvious" of these. But I,29(b) will not suffice without I,29(a), and I am very doubtful that it is helpful to call I,29(a) obvious. Moreover, I,29(a) and (b) only get us to I,32(a). For the assertion that the angles of the triangle are equal to two right angles requires I,13, which itself seems so obvious that its very assertion seems to require the kind of sophistication one associates with the *Elements*. One can imagine someone pointing to fig. 2 and saying that the angles of triangle ABC are equal to *those* three angles at point C, but the reformulation of that claim in terms of equality to two right angles suggests the kind of general or systematic interest which we find in fully developed Greek mathematics.

A similar point can be made about I,15, which, as I indicated, is used in the proof of I,29 and would be needed in any case unless both I,29(a) and (b) were taken for granted. I would not call I,15 obvious. Euclid's proof uses I,13, according to which (see fig. 4)

(i) $\angle GAB + \angle BAC = 2$ right angles, and
$\angle EAC + \angle BAC = 2$ right angles.

Euclid does not explain the inference from i to:

(ii) $\angle GAB + \angle BAC = \angle EAC + \angle BAC$,

but it apparently depends on postulate 4, according to which all right angles are equal, and common notion 1.[13] The theorem results by applying common notion 3 to ii.

The fourth postulate is the most perplexing of the five. Proclus points out that it is provable:

> Let us assume two right angles ABC and DEF. I say they are equal. For, if they are not, one of them will be greater than the other. Let this be the angle at B. Then if (fig. 6) DE be made to coincide with AB, EF will fall within the angle as BG. Let BC be extended to H. Since ABC is a right angle, so also is ABH and they are equal to one another (for

[13] So Heath (1926, 278), but normally the common notion is applied to "things," specific magnitudes like lines or angles or triangles, and not to more abstract quantities like "two right angles." It seems to me possible that Euclid is simply taking for granted that if all right angles are equal then things equal to two right angles are equal to each other. But nothing I say in this essay turns on this question.

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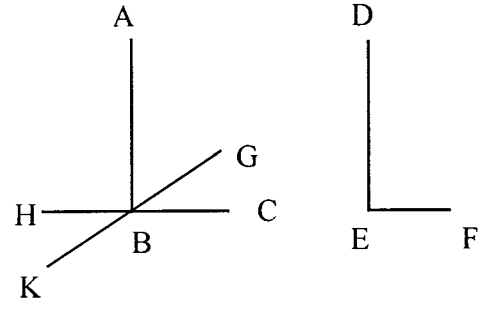


Fig. 6.

in the definitions it says that a right angle is equal to its adjacent angle). Therefore, angle ABH is greater than angle ABG. Now let BG be extended to K. Since ABG is a right angle, its adjacent angle will also be a right angle and equal to ABG. So angle ABK is equal to angle ABG, so that angle ABH is less than ABG. But it is also greater, which is impossible. (188.2–189.9; Morrow 1992 translation slightly revised)

We do not know why Euclid includes the fourth postulate as a postulate (see Mueller 1981, 29–30), but it does not seem unreasonable to assume that the postulate was at some time either taken for granted or justified in something like the manner of Proclus' argument.[14] But even if that is so and the common notions, the possibility of the relevant elementary constructions, and I,29(b) were also used without proof or explicit assertion, a proof of I,32 still requires the criterion of equality to two right angles brought out in I,13 and something like I,15. And, although the proof of I,15, is elegantly simple, it does involve the abstract step of subtracting angle BAC from the two quantities equal to two right angles. And without that abstract step one cannot "see" the equality of angles GAB and EAC.

I have attempted to construct a pre-Euclidean proof of I,32 on the assumption that an earlier proof would have been more intuitive and less rigorous than the one we find in the *Elements*, but that the idea of the proof would not have differed radically from the idea of Euclid's proof. I have used various evidence from Aristotle and Proclus to argue that there was a fifth-century proof and to bring out the ingredients of the Euclidean proof which might have been used earlier without proof or explicit statement. But I do not see how to deal with either the criterion of equality to two right angles brought to the forefront in I,13, or its use in the proof of I,15 (or equivalently I,29(a)), which seem to me as abstract and rigorous as anything in the *Elements*. Some might see this as a reason to doubt that I,32 was a part of fifth-century Greek geometry, whatever Proclus and Eudemus say to the contrary. Others might see it as a reason to think that the style of fifth-century Greek geometry was more like Euclid's than one might otherwise suppose. I incline to side with this second group.

[14] For a general discussion bearing on this question, see von Fritz 1955, 76–89.

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It is an honor and a pleasure to offer this small essay to Sabetai Unguru, who is for me the model of a courageous and forthright scholar and human being.

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