# Empirical Adequacy in the Received View

Sebastian Lutz*†

I show that the central notion of Constructive Empiricism, empirical adequacy, can be expressed syntactically and specifically in the Received View of the logical empiricists. The formalization shows that the Received View is superior to Constructive Empiricism in the treatment of theories involving constants or functions from observable to unobservable objects. It also suggests a formalization of 'full empirical informativeness' in Constructive Empiricism.

**1. Introduction.** The so-called Received View in the philosophy of science (RV), developed by the logical empiricists and especially by Carnap and Hempel, assumes the formalization of scientific theories in predicate logic and a bipartition of the nonlogical vocabulary into observational and theoretical terms. While it had never been generally accepted (Richardson 2007, 362–65), it suffered a complete fall from grace (and then some) in the second half of the twentieth century (see Lutz 2012, sec. 1). Only recently have there been defenses of Carnap's general philosophical method (Carus 2007; Maher 2007; Lutz 2009; Justus 2012) and of the use of predicate logic or more generally syntactic means for the analysis of scientific theories (Halvorson 2012, 2013; Lutz, forthcoming). The RV itself, however, has so far mainly seen historical defenses (Mormann 2007; Lutz 2012). It is the aim of this article to provide a conceptual defense relative to one of its main competitors, Constructive Empiricism (CE; van Fraassen 1980). Specifically, I argue that the relation between theory and observations that is described in the CE-

*To contact the author, please write to: Munich Center for Mathematical Philosophy, Ludwig-Maximilians-Universität München; e-mail: sebastian.lutz@gmx.net.

†Early versions of this article have been presented at the Lunch Talk of the Theoretical Philosophy Unit at Utrecht University and at the PhDs in Logic conference at Ghent University in 2009. I thank the audiences, the audience at the Twenty-Third Biennial Meeting of the Philosophy of Science Association, Albert Visser, Thomas Müller, and F. A. Muller for helpful discussions. Research for this article was in part supported by the Alexander von Humboldt Foundation.

Philosophy of Science, 81 (December 2014) pp. 1171–1183. 0031-8248/2014/8105-0037$10.00
Copyright 2014 by the Philosophy of Science Association. All rights reserved.



https://doi.org/10.1086/677886 Published online by Cambridge University Press

formalism can also be described in the RV-formalism, while the converse does not hold.

The only previous defense of the RV-formalism relative to CE (Friedman 1982, 276–77) consists of a valuable but very short outline of an argument and furthermore assumes a restriction to first-order logic. Other relative defenses have focused on syntactic approaches in general (Worrall 1984, sec. 2; Turney 1990). Turney provides the technically most elaborate defense. After proving the equivalence of van Fraassen's notion of embedding and his syntactic notion of implanting (under specific assumptions), Turney concludes: "I agree with van Fraassen, that the positivist account does not yield good definitions for empirical equivalence and empirical adequacy. However, I do not agree with his claim that this is because the positivist account is syntactic, while his is semantic. . . . The relevant distinction here . . . is between two ways of linking theory and observation: Correspondence rules versus embedding/implanting" (1990, 448–49). Turney provides a detailed argument for the equivalence of his syntactic and van Fraassen's semantic account, but that van Fraassen's embeddings are an improvement over correspondence rules remains a conjecture.

To disprove Turney's (and van Fraassen's) conjecture, I provide a definition of the RV-formalism and a definition of the formal core of CE, empirical adequacy. The latter is in conflict with Turney's definition, which I show to be inadequate (sec. 2). Next, I provide a syntactic description of empirical adequacy (sec. 3), which forms the basis for a description of empirical adequacy on the basis of correspondence rules (sec. 4). This shows that whatever the relation between theory and observations is, if it can be described in the CE-formalism, then it can be described in the RV-formalism as well. The converse does not hold without significantly rewriting scientific theories, however, and I show that this puts CE at a disadvantage (sec. 5). Finally, I give a precise description of a problem first pointed out by Worrall (1984, sec. 3) and show how it can be solved in CE by taking a cue from the RV.

**2. Two Accounts of the Relation between Theory and Observation.** Like the CE-formalism, the RV-formalism provides a framework for analyzing scientific theories and their relation to observations. The most important reconstructions of theories and most important tools developed in the RV assume a formalization of theories in higher-order predicate logic with a vocabulary $\mathcal{V} = \{P_i, F_j, c_k : i \in I, j \in J, k \in K\}$ of distinct symbols, where each $c_k$ is an object symbol and each $P_i$ and $F_j$ is a relation symbol or, respectively, function symbol of a specific arity and type. Formal model theory is used for the definition of logical entailment (Lutz 2012, 83). What distinguishes the RV from syntactic approaches in general is the assumed bipartition of the nonlogical vocabulary $\mathcal{V}$ of a theory into a set $\mathcal{O}$ of obser-

https://doi.org/10.1086/677886 Published online by Cambridge University Press

vation terms and a set $\mathcal{T}$ of theoretical terms.[1] This bipartition leads to a natural tripartition of the set of sentences into $\mathcal{O}$-sentences, $\mathcal{T}$-sentences, and correspondence rules, the latter of which are sentences containing both $\mathcal{O}$- and $\mathcal{T}$-terms (Carnap 1966, chap. 24).

**Definition 1.**—An *RV-theory* $\Theta$ is the union of a set $\Pi$ of higher-order $\mathcal{T}$-sentences (the *postulates of the theory*) and a set $\Gamma$ of higher-order $\mathcal{V}$-sentences (the *correspondence rules*).

Sentences in the extended observation language can be of any order. This is, for example, assumed by Carnap's explication of a theory's synthetic component as its Ramsey sentence (Carnap 1963, sec. 24.D).

**Definition 2.**—*Observations* are given by a set of higher-order $\mathcal{O}$-sentences.

The moniker 'observations' is of course somewhat misleading, but it will be convenient in what follows. What is important is that sets of $\mathcal{O}$-sentences are, in a sense, completely interpreted (cf. Carnap 1963, sec. 24.A).

In contradistinction to the RV, the CE-formalism relies on model theory.[2] Van Fraassen (1980, 64) describes it as follows: "To present a theory is to specify a family of structures, its *models*; and second, to specify certain parts of those models (the *empirical substructures*) as candidates for the direct representation of observable phenomena." Furthermore the models of the theory "are describable only up to structural isomorphism" (van Fraassen 2008, 238; cf. 2002, 22). More formally, this can be phrased as follows:

**Definition 3.**—A *CE-theory* $\langle \{\mathfrak{T}_n\}_{n\in N}, \{\mathbf{E}_n\}_{n\in N}\rangle$ contains a family of structures (the *models of the theory*), and for each structure $\mathfrak{T}_n = \langle T_n, P_i^{\mathfrak{T}_n}, F_j^{\mathfrak{T}_n}, c_k^{\mathfrak{T}_n}\rangle_{i\in I_n, j\in J_n, k\in K_n}$ a set $\mathbf{E}_n$ of *empirical substructures*, such that for each $\mathfrak{E} \in \mathbf{E}_n$, $\mathfrak{E} \subseteq \mathfrak{T}_n$.[3] With each model, a theory also contains every isomorphic structure and its corresponding empirical substructures.[4]

1. Following standard terminology in the philosophy of science (but unfortunately not in logic), I use 'term' synonymously with 'nonlogical symbol'.

2. Note that set theory (which is needed to describe classes of models) and higher-order logic can capture mathematics equally well (Lutz, forthcoming, sec. 5).

3. The notation for $\mathfrak{T}_n$ is close to one often used in the philosophy of science, although it is somewhat infelicitous. The precise notation would be $\mathfrak{T}_n = \langle T_n, \{\langle P_i, P_i^{\mathfrak{T}_n}\rangle : i \in I\}$, etc.$\rangle$

4. To be precise: If $f : T_m \to T_n$ is an isomorphism between $\mathfrak{T}_m$ and $\mathfrak{T}_n$, then the set $\mathbf{E}_n$ of empirical substructures that corresponds to $\mathbf{E}_m$ contains all and only those structures $\mathfrak{E}$ for which there is an $\mathfrak{E}' \in \mathbf{E}_m$ such that $f$ is an isomorphism between $\mathfrak{E}'$ and $\mathfrak{E}$.

https://doi.org/10.1086/677886 Published online by Cambridge University Press

Here and in what follows, $T_n := |\mathfrak{T}_n|$. As a shorthand, define furthermore $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle := \langle \{\mathfrak{T}_n\}_{n \in N}, \{\mathbf{E}_n\}_{n \in N} \rangle$. Definition 3 assumes a standard formalization of the notion of a structure (Chang and Keisler 1990, 19–20; Hodges 1993, 2).Van Fraassen (1989, 366 n. 4) objects to structures being "yolked to a particular syntax," as in "many standard logic texts," which suggests that he would prefer a definition phrased in the formalism of Bell and Machover (1977, 164), who use an index set instead of a vocabulary. This difference is little more than notational, however, since in their definitions of further model theoretic concepts, the index set plays the role of the vocabulary. For examples relevant in what follows, compare the definitions of reduct (or "reduction"), isomorphism, and substructure by Bell and Machover (1977, 164–65) and by Chang and Keisler (1990, 20–23), respectively (cf. Lutz, forthcoming, sec. 3). A reader preferring Bell and Machover's notation will have no problem adapting the following discussion.

Strictly distinguishing between observable events (phenomena) and unobservable events, van Fraassen (1980, 64) suggests representing phenomena by structures as well: "The structures which can be described in experimental and measurement reports we can call *appearances*."[5] This suggests

> **Definition 4.**—*Appearances* are given by a set $\mathbf{A}$ of structures. A structure $\mathfrak{A} \in \mathbf{A}$ is an appearance.

In early works van Fraassen (1970, sec. 3) relies on "elementary statements" and a "satisfaction function" to give the relation between theory and observations, but later van Fraassen (1980, 64) defines a theory to be "empirically adequate if it has some model such that all appearances are isomorphic to empirical substructures of that model" (cf. van Fraassen 1991, 12).

> **Definition 5.**—A CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ is *empirically adequate* for appearances $\mathbf{A}$ if and only if (iff) there is some $n \in N$ such that for every $\mathfrak{A} \in \mathbf{A}$, there is an $\mathfrak{E} \in \mathbf{E}_n$ with $\mathfrak{E} \cong \mathfrak{A}$.

Definition 5 defines the empirical adequacy of a theory relative to a set of appearances. In contradistinction, empirical adequacy simpliciter is defined as empirical adequacy for the set of all appearances (Monton and Mohler 2008, sec. 1.5).

5. Recently, van Fraassen (2008, 286) has clarified that the phenomena are objective parts of the world, while the appearances are theory-dependent representations of the phenomena. Van Fraassen's formalism describes the relation between theory and appearances, not theory and phenomena.

https://doi.org/10.1086/677886 Published online by Cambridge University Press

Definition 5 assumes the standard notion of isomorphism (e.g., Hodges 1993). Turney (1990, 430) notes that his definitions are "loosely based" on Bell and Machover (1977, 161–68), but he defines the isomorphism of two relational structures as follows (430–31; my terminology):

> **Definition 6.**—Two relational structures $\mathfrak{S}$ and $\mathfrak{S}'$ are *idiosyncratically isomorphic* iff $\mathfrak{S} = \langle S, P_1, P_2, \ldots, P_k\rangle$ and $\mathfrak{S}' = \langle S', P_1', P_2', \ldots, P_k'\rangle$ with $n_i$-ary relations $P_i, P_i'$ and there are bijections $f : S \to S'$ and $g : \{P_1, \ldots, P_k\} \to \{P_1', \ldots, P_k'\}$ such that $\langle u_1, \ldots, u_{n_i}\rangle \in P_i \Leftrightarrow \langle f(u_1), \ldots, f(u_{n_i})\rangle \in g(P_i)$.

In contradistinction, Bell and Machover (1977, 165) effectively assume that the bijection $g$ is determined to map $P_i$ to $P_i'$ for all $1 \le i \le k$ (which leads, up to notational change, to the definition given by Hodges [1993]). Clearly, then, more structures are idiosyncratically isomorphic than isomorphic to each other—too many more, as one can see from the following example loosely inspired by Halvorson (2012, 192). Let the CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ contain all $\mathfrak{T}_n$ and $\mathbf{E}_n$ that fulfill the following conditions: $\mathfrak{T}_n$ is a model of $\Theta = \{\exists^{\ge k} x(x = x)\} \cup \{\forall x(P_k x \to P_i x) : 1 \le i < k\}$ with a set of objects $\{a_i : 1 \le i \le k+1\} \subseteq T_n$ with $a_i \ne a_j$ for $i \ne j$, and $\mathbf{E}_n$ contains the substructures $\mathfrak{E}_n^i$ of $\mathfrak{T}_n$ with domains $E_n^i = \{a_i\}$. CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ makes the substantial claim about the appearances that everything with property $P_k$ also has all properties $P_1, \ldots, P_{k-1}$. But in Turney's definition, $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ is empirically adequate for any class $\mathbf{A}$ of appearances with singleton domains and properties $P_1, \ldots, P_k$.

> *Proof.*—Take any appearance $\mathfrak{A} \in \mathbf{A}$. CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ contains the model $\mathfrak{T}_n$ with $T_n = \mathbb{N}$ and $P_i^{\mathfrak{T}_n} = \{i+1, \ldots, k+1\}$ for $1 \le i \le k$, and empirical substructures $\mathbf{E}_n = \{\mathfrak{E}_n^i : 1 \le i \le k+1\}$ with $E_n^i = \{i\}$ and $i \in P_j^{\mathfrak{E}_n^i}$ iff $i > j$. In $\mathfrak{A}$, the element $a \in A$ has some number $l$ of distinct properties: none for $l = 0$ and properties $P_{i_1}, \ldots, P_{i_l}$ for $1 \le l \le k$. For each such $l$, $\mathfrak{A}$ and $\mathfrak{E}_n^{l+1}$ are idiosyncratically isomorphic because $f_l : A \to E_n^{l+1}$, $a \mapsto l+1$ is trivially a bijection, so that $\mathfrak{A}$, $\mathfrak{E}_n^{l+1}$, $f_l$, and $g_l$ fulfill definition 6 when $g_l$ is chosen as follows. For $l = 0$, $g_l = \mathrm{id}$, and for $l > 0$, $g_l$ is any bijection from $\{P_1^{\mathfrak{A}}, \ldots, P_k^{\mathfrak{A}}\}$ to $\{P_1^{\mathfrak{E}_n^{l+1}}, \ldots, P_k^{\mathfrak{E}_n^{l+1}}\}$ with $g_l : P_{i_j}^{\mathfrak{A}} \mapsto P_j^{\mathfrak{E}_n^{l+1}}$ for $1 \le j \le l$. Hence every appearance in $\mathbf{A}$ is idiosyncratically isomorphic to an empirical substructure of the same model $\mathfrak{T}_n$, and therefore $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ would be empirically adequate for $\mathbf{A}$ under Turney's definition. QED

It is for this reason that Turney's definition is inadequate.

**3. Syntactic Empirical Adequacy.** I now want to provide a syntactic description of the notion of a CE-theory and of empirical adequacy. In a CE-

https://doi.org/10.1086/677886 Published online by Cambridge University Press

theory, each element $\mathfrak{E} \in \mathbf{E}_n$ is uniquely determined as the restriction $\mathfrak{T}_n \mid E$ of $\mathfrak{T}_n$ to $\mathfrak{E}$'s domain $E$. Thus for each $\mathfrak{T}_n$, $\mathbf{E}_n$ is determined by the set $\{E : \mathfrak{E} \in \mathbf{E}_n\}$, and each CE-theory is completely determined by its corresponding family, defined as follows:

> **Definition 7.**—Let $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ be a CE-theory. Then its *corresponding family* of structures is given by $\{\mathfrak{T}_n^*\}_{n \in N}$ with $\mathfrak{T}_n^* = \langle T_n, P_i^{\mathfrak{T}_n}, F_j^{\mathfrak{T}_n}, c_k^{\mathfrak{T}_n}, S^{\mathfrak{T}_n^*} \rangle_{i \in I, j \in J, k \in K}$, where $S^{\mathfrak{T}_n^*} = \{E : \mathfrak{E} \in \mathbf{E}_n\}$.

Thus, $\{E : \mathfrak{E} \in \mathbf{E}_n\}$ is the extension of a predicate $S$ of higher order, and for any predicate $P$, $SP$ is the claim that $P$ has an empirical extension. If $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ is a CE-theory, its corresponding family can be described up to isomorphism by a set $\Theta$ of higher-order sentences or a class $\{\Theta_m : m \in M\}$ thereof, where $\{\mathfrak{T}_n^*\}_{n \in N}$ is the union of the models of each set $\Theta_m$ (see Lutz, forthcoming, sec. 2).[6] For a class to be necessary, however, $M$ or at least one $\Theta_m$ has to be infinite, for otherwise the corresponding family would be the class of models of the single sentence $\vee_{m \in M} \wedge \Theta_m$. I am not aware of any scientific theory that would require an infinite $M$ or $\Theta_m$. For instance, the typical mathematical structures needed in physics can be defined by a single higher-order sentence (Enderton 2009, secs. 2, 4). Nonetheless, I will first discuss CE-theories whose corresponding family can be described by a single set of sentences and then discuss the general case.

Each appearance $\mathfrak{A} \in \mathbf{A}$ can be described up to isomorphism by a single set of sentences $\Phi_{\mathfrak{A}}$. Of course, each $\mathfrak{A}$ may have its own distinct domain $A$, so that the sentences in $\Phi_{\mathfrak{A}}$ must be taken as relativized to $A$. Using the symbol '$E_{\mathfrak{A}}$' as name for $A$, this leads to a set $\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})}$ of relativized sentences.[7] The set of all appearances can then be described as the set of sentences $\{\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})} : \mathfrak{A} \in \mathbf{A}\} \cup \{SE_{\mathfrak{A}} : \mathfrak{A} \in \mathbf{A}\}$ containing the relativized descriptions of each appearance and, for each appearance's domain $E_{\mathfrak{A}}$, the higher-order sentence $SE_{\mathfrak{A}}$ stating that $E_{\mathfrak{A}}$ has an empirical extension. Empirical adequacy then becomes semantic consistency:

> **Claim 1.**—Let $\{\mathfrak{T}_n^*\}_{n \in N}$ be the family corresponding to CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$, and let $\Theta$ describe $\{\mathfrak{T}_n^*\}_{n \in N}$ up to isomorphism. Let each appearance $\mathfrak{A} \in \mathbf{A}$ be described up to isomorphism by $\Phi_{\mathfrak{A}}$. Then $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ is empirically adequate for $\mathbf{A}$ iff

6. If the CE-theory is formalized in the notation of Bell and Machover, the index set can be used as a vocabulary (see Lutz, forthcoming, sec. 3).

7. The relativization $\sigma^{(E_{\mathfrak{A}})}$ of a sentence $\sigma$ consists of the restriction of all quantifiers in $\sigma$ to $E_{\mathfrak{A}}$ (see Hodges 1993, theorem 5.1.1). For a set $\Sigma$ of formulas, $\Sigma^{(E_{\mathfrak{A}})}$ is the set of the relativizations of the elements of $\Sigma$. Turney (1990, definition 13) calls relativizations 'qualifications'.

https://doi.org/10.1086/677886 Published online by Cambridge University Press

$$\Theta \cup \{\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})} : \mathfrak{A} \in \mathbf{A}\} \cup \{SE_{\mathfrak{A}} : \mathfrak{A} \in \mathbf{A}\}. \quad (1)$$

is semantically consistent.

*Proof*.—The set (1) is semantically consistent iff it has a model. This is the case iff there is some model $\mathfrak{T}_n^*$ of $\Theta$ such that for each $\mathfrak{A} \in \mathbf{A}$, $\mathfrak{T}_n^* \vDash \{SE_{\mathfrak{A}}\} \cup \Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})}$. Because $S^{\mathfrak{T}_n^*} = \{E : \mathfrak{E} \in \mathbf{E}_n\}$, this holds iff $E_{\mathfrak{A}}$ is interpreted by the domain $E$ of some $\mathfrak{E} \in \mathbf{E}_n$ such that $\mathfrak{T}_n^*[E_{\mathfrak{A}}/E] \vDash \Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})}$. Since $\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})}$ does not contain the term $S$, this is the case iff $\mathfrak{T}_n[E_{\mathfrak{A}}/E] \vDash \Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})}$. Induction on the complexity of formulas shows that this holds iff $\mathfrak{E} = \mathfrak{T}_n|E \vDash \Phi_{\mathfrak{A}}$ and thus, by assumption, iff $\mathfrak{E}$ is isomorphic to $\mathfrak{A}$.[8] Therefore, the set (1) is semantically consistent iff there is some $n \in N$ such that for each $\mathfrak{A} \in \mathbf{A}$, there is an $\mathfrak{E} \in \mathbf{E}_n$ with $\mathfrak{E} \cong \mathfrak{A}$. QED

If a CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ can be described by a class $\{\Theta_m : m \in M\}$ of sets of sentences, claim 1 entails that $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ is empirically adequate iff $\Theta_m \cup \{\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})} : \mathfrak{A} \in \mathbf{A}\} \cup \{SE_{\mathfrak{A}} : \mathfrak{A} \in \mathbf{A}\}$ is semantically consistent for at least one $m \in M$. The use of semantic consistency is not forbidden for syntactic approaches since their core assumption is not that semantic concepts cannot be used at all but rather that the description of theories has to be syntactical (cf. Lutz, forthcoming, sec. 5).

The syntactic description of empirical adequacy that is given in claim 1 differs from the one given by Turney (1990, 446–47) in a number of respects. For example, Turney assumes that a CE-theory is an elementary class that contains only relational structures. The former arguably puts him at odds with van Fraassen (1989, 211), while the latter requires elaborate reformulations of most scientific theories, which typically contain constants and functions. Most important for what follows, Turney provides a syntactic reformulation of each concept of definition 5 but keeps its overall structure intact. Specifically, he does not provide a single set of sentences that captures both the family of models and the family of sets of empirical substructures of a CE-theory. For this reason, the possibility of capturing empirical adequacy in the RV is not obvious in his syntactic description.

**4. Empirical Adequacy in a Bipartitioned Vocabulary.** Given that the empirical adequacy of a theory can be phrased syntactically in a single vocabulary, capturing empirical adequacy in the bipartitioned vocabulary of the RV is very simple. One can simply treat the original vocabulary as theoretical ($\mathcal{T}$) and introduce a second observational term for each theoretical

8. For a quick plausibility consideration, note that $\mathfrak{T}_n[E_{\mathfrak{A}}/E] \vDash \Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})} \Leftrightarrow \mathfrak{T}_n[E_{\mathfrak{A}}/E]|E \vDash \Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})} \Leftrightarrow \mathfrak{T}_n[E_{\mathfrak{A}}/E]|E \vDash \Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})} \wedge \forall x E_{\mathfrak{A}} x \Leftrightarrow \mathfrak{E} = \mathfrak{T}_n|E \vDash \Phi_{\mathfrak{A}}$.

https://doi.org/10.1086/677886 Published online by Cambridge University Press

one. Thus, for each $P_i$ one can introduce an observational predicate symbol $P_i^o$, and analogously for function and constant symbols, leading to an observational vocabulary $\mathcal{O}$. Let $\Phi^o$ be the set of sentences that results from replacing each nonlogical constant in the set $\Phi$ of sentences by its observational counterpart. Then the empirical adequacy of a CE-theory can be expressed as the compatibility of an RV-theory with the observations:

**Corollary 1.**—Let $\{\mathfrak{T}_n^*\}_{n\in N}$ be the family corresponding to CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$, and let $\Pi$ describe $\{\mathfrak{T}_n^*\}_{n\in N}$ up to isomorphism. Let each appearance $\mathfrak{A}$ be described up to isomorphism by $\Phi_{\mathfrak{A}}$. Then $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ is empirically adequate for $\mathbf{A}$ iff

$$\Pi \cup \Gamma \cup \{(\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})})^o : \mathfrak{A} \in \mathbf{A}\} \cup \{S^o E_{\mathfrak{A}}^o : \mathfrak{A} \in \mathbf{A}\}. \tag{2}$$

is semantically consistent, where

$$\Gamma = \{P_i^o = P_i\}_{i\in I} \cup \{S^o = S\} \cup \{F_j^o = F_j\}_{j\in J} \cup \{c_k^o = c_k\}_{k\in K}.$$

*Proof.*—The reducts of the models of $\Pi \cup \Gamma$ to $\mathcal{O}$ are the models of $\Pi$ with their symbols renamed as described above. Hence the corollary follows from claim 1 by a simple renaming of symbols. QED

Corollary 1 establishes that empirical adequacy is nothing but the compatibility of an RV-theory consisting of theoretical postulates $\Pi$ and correspondence rules $\Gamma$ (definition 1) with observation sentences (definition 2). Like claim 1, corollary 1 applies only to CE-theories whose corresponding family can be described by a single set of higher-order sentences. Again with the caveat that few if any theories will require a description of their models by a class $\{\Pi_m : m \in M\}$ of sets of sentences, corollary 1 can be generalized to this case: the CE-theory is then empirically adequate iff the set $\Pi_m \cup \Gamma \cup \{(\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})})^o : \mathfrak{A} \in \mathbf{A}\} \cup \{S^o E_{\mathfrak{A}}^o : \mathfrak{A} \in \mathbf{A}\}$ is semantically consistent for at least one $m \in M$. The use of semantic consistency is not incompatible with the RV since it is essentially the same as the use of semantic entailment.

Corollary 1 is somewhat unsatisfying in that there is no difference between theoretical and observational concepts except their names. This is not a problem of the RV itself but rather inherited from CE—which will become important in section 5. Nonetheless, there are some ways to provide the correspondence rules with a somewhat more elaborate role, even within the confines of CE. In keeping with Carnap's hope that all observational terms can be defined in theoretical terms and his almost universally accepted conjecture that the converse is not generally possible (Carnap 1966, 234), one can restrict the identification of theoretical with observational terms to observable objects. For the relation symbols, the correspondence rules then have the form

https://doi.org/10.1086/677886 Published online by Cambridge University Press

$$\{\forall \bar{x}[P_i^o \bar{x} \leftrightarrow \exists Y(SY \wedge Yx_1 \wedge \ \ldots \ \wedge Yx_{m_i}) \wedge P_i \bar{x}] : i \in I\} \cup \{S^o = S\}, \quad (3)$$

where $m_i$ is the arity of $P_i$ and $\bar{x}$ stands for a string of $m_i$ distinct variables.[9] Things are not as straightforward for functions since functions have to be defined over the whole domain. One could thus substitute an $n_j + 1$-ary observational relation $Q_j^o$ for each $n_j$-ary theoretical function $F_j$ with

$$\forall \bar{x}[Q_j^o x_1 \ldots x_{n_j+1} \leftrightarrow \exists Y(SY \wedge Yx_1 \wedge \ \ldots \ \wedge Yx_{n_j}) \wedge F_j x_1 \ldots x_{n_j} = x_{n_j+1}], \quad (4)$$

which would also mean that the descriptions $(\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})})^o$ of the appearances have to be changed accordingly. It is therefore more convenient to disappoint Carnap's hope for explicit definitions and rely for each $F_j$ on the conditional definition

$$\forall \bar{x}[\exists Y(SY \wedge Yx_1 \wedge \ \ldots \ \wedge Yx_{n_j}) \rightarrow (F_j^o \bar{x} = F_j \bar{x})]. \quad (5)$$

However, since the value of the observational functions is irrelevant outside the domains of the empirical substructures, the conditional definitions could be completed by conditionally defining that outside of the empirical subdomains, the observational functions all have the same arbitrarily chosen value.

For reasons that will be discussed in section 5, observational constant symbols still have to be identified with their theoretical counterparts, which leads to

> **Corollary 2.**—In corollary 1, $\Gamma$ can also be the union of sets (3), (5), and $\{c_k^o = c_k : k \in K\}$.
>
> *Proof.*—Since the modification does not change the extensions of the observational terms within any set that is in the extension of $S$, the corollary follows immediately from corollary 1. QED

This, then, shows that van Fraassen's and Turney's conjecture about the impossibility of capturing empirical adequacy in the RV is incorrect. The use of embeddings is thus not superior to the use of correspondence rules when connecting theory and observations. In fact, the CE-formalism is but a special case of the RV-formalism, with specific restrictions on the form of observation sentences, correspondence rules $\Gamma$, and postulates $\Pi$. In the next section, I will argue that because of these restrictions, the CE-formalism is actually inferior to the RV-formalism.

9. To the extent that his terse sketch allows a comparison, Friedman (1982, 276–77) seems to have this kind of correspondence rule in mind.

https://doi.org/10.1086/677886 Published online by Cambridge University Press

**5. Beyond Empirical Substructures.** Definition 3 puts enormous constraints on the formalization of theories. For it follows from the definition of a substructure that every constant of a model $\mathfrak{T}_n$ of a CE-theory has to be in the domain $E$ of each of its empirical substructures $\mathfrak{E} \in \mathbf{E}_n$. Furthermore, every function of the structure $\mathfrak{E} \in \mathbf{E}_n$ must map all (tuples of) elements of $E$ to elements of $E$ (Hodges 1993, lemma 1.2.2). For a CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ to be possibly empirically adequate, it must be possible that every appearance $\mathfrak{A}$ is isomorphic to some empirical substructure of some $\mathfrak{T}_n$. And since $\{\mathfrak{T}_n\}_{n \in N}$ is closed under isomorphism, according to $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ every empirical substructure of every $\mathfrak{T}_n$ could be an appearance (Hodges 1993, ex. 1.2.4b). Therefore, according to $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$, every constant of $\mathfrak{T}_n$ represents something observable (since it represents an aspect of a phenomenon) and all functions of $\mathfrak{T}_n$ map (tuples of) elements of $T_n$ that partake in the representation of phenomena only to other such elements of $T_n$. It is clear that this is not the case for actual theories about, say, elementary particles.

One may object to these criticisms (and definitions 3–5) on the grounds that the terms 'embedding', 'substructure', and 'isomorphism' are not meant literally. However, the terms are well defined within but not outside of model theory, where in general they also do not occur together. And the objection makes van Fraassen use these terms in different, undefined ways without pointing this out and subsequently helping himself to results from model theory without any justification. For example, van Fraassen (1980, 43) discusses cases in which "every model of $T_1$ can be embedded in (identified with a substructure of) a model of $T_2$." The parenthetical equivalence claim relies on the model theoretic definition of 'embedding' and 'substructure', the closure of the class of models of a CE-theory under isomorphism, and the equivalence of embedding and the substructure relation for isomorphically closed classes of structures (Hodges 1993, ex. 1.2.4b). If the terms were not meant in the model theoretic sense, there would be no justification for this equivalence claim.

The problems resulting from van Fraassen's reliance on substructures can easily be solved in the RV because RV-theories can contain nonobservational terms, which themselves do not have to be connected directly to observations. Thus even without loosening the restrictions on the observation sentences inherited from CE, the RV allows for connections between arbitrarily complex theoretical concepts, named by $\mathcal{T}$-terms, and simple observational concepts, named by $\mathcal{O}$-terms: postulates $\Pi$ can introduce $\mathcal{T}$-terms and correspondence rules $\Gamma$ can go beyond simple identification or identification in a restricted domain. Hence in an RV-theory, there can be constant symbols in $\mathcal{T}$ that name unobservable events and function symbols in $\mathcal{T}$ that name functions from observable to unobservable events. The empirical adequacy of such a more general RV-theory $\Pi \cup \Gamma$ can still be defined

https://doi.org/10.1086/677886 Published online by Cambridge University Press

as the semantic consistency of the set (2) of sentences. This solution in the RV no doubt has a counterpart in model theory, but it would in all likelihood either require a significant reformulation of most scientific theories before they can be analyzed in the CE-formalism or involve a generalization of the CE-formalism.[10] In the latter case the formalism would not be the one criticized here and conjectured by van Fraassen to be superior to the RV-formalism.

**6. Beyond Consistency.** Corollary 1 allows a direct comparison of the RV and CE. Its relevance is demonstrated by a previous comparison given by Muller (2010, 90), who, in line with Carnap (1966, 240–41), states that one aim of scientific theories in the RV is *observational adequacy* ("ObsAdeq"), for which a theory must entail the observations made so far. He points out three differences to empirical adequacy ("EmpAdeq"): "(i) ObsAdeq relies on the distinction between theoretical and observational concepts whereas EmpAdeq does not . . . ; (ii) ObsAdeq depends on historical time whereas EmpAdeq is timeless (quantifies universally over historical times); (iii) EmpAdeq requires for its definition a conception of a scientific theory that . . . cannot be defined in the [RV]" (Muller 2010, 90 n. 8). Corollary 1 shows that Muller's claim iii, which is again but a conjecture, is false. It also shows that difference i is easily overcome by introducing an observational vocabulary and identity statements as correspondence rules. The discussion in section 5 furthermore shows that this difference speaks in favor of the RV. Difference ii is not one of principle but at best one of primacy of definition: one could define 'preliminary empirical adequacy' as 'empirical adequacy given the appearances **A** observed so far' and define 'eternal observational adequacy' as 'observational adequacy at all times'.

Muller misses the real difference, which is obvious in the syntactic paraphrase: a theory is observationally adequate if it entails all observations, while it is empirically adequate if it is not incompatible with them. Compared to observational adequacy, then, empirical significance is cheap: a tautology is compatible with any set of sentences describing the appearances, and thus its set of models is empirically adequate. This is obvious given claim 1, and it can also be translated into the CE-formalism: the CE-theory corresponding to a tautology contains the class of all structures and for each structure the set of all its substructures as empirical substructures. This CE-theory is empirically adequate given any appearances **A**. Thus claim 1 throws doubt on van Fraassen's claim that science "aims to give us theories that are empirically adequate" (van Fraassen 1980, 12), at least if empirical adequacy is meant to be the only aim of science, or even a particularly interesting one.

10. Indeed, I think substituting relativized reducts for substructures in definition 5 suffices for avoiding the objections presented here.

https://doi.org/10.1086/677886 Published online by Cambridge University Press

This criticism has already been put forth by Worrall (1984, sec. 3) and Percival (2007, 87), who suggests as one of three possible replacements for van Fraassen's account of the aim of science that for each context $c$, science aims for theories that are empirically adequate and "fully empirically informative with respect to all the phenomena that are relevant at $c$."[11] Unfortunately, Percival does not provide an explication of the latter notion in the CE-formalism but rather stipulates that a theory "is 'fully empirically informative' with respect to certain phenomena iff it expresses all true propositions that are solely about those phenomena" (n. 44). In his reply, van Fraassen (2007, sec. 1.1) acknowledges the need for informative theories but does not give an explication either.

To arrive at a precise notion of full empirical informativeness, one can look to the RV. An RV-theory $\Theta$ expresses all and only true propositions about some appearances $\mathbf{A}$ iff $\Theta \models \{\Phi_{\mathfrak{A}}^{(E_{\mathfrak{A}})} : \mathfrak{A} \in \mathbf{A}\} \cup \{SE_{\mathfrak{A}} : \mathfrak{A} \in \mathbf{A}\}$. For a CE-theory $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$, then, every model $\mathfrak{T}_n^*$ of the corresponding family described by $\Theta$ is also a model of the observations. It is not very difficult to see that this is the case iff every empirical substructure $\mathfrak{E} \in \mathbf{E}_n$ of every structure in $\langle \mathfrak{T}_n, \mathbf{E}_n \rangle$ is isomorphic to an appearance $\mathfrak{A} \in \mathbf{A}$ and, since every $\mathfrak{A} \in \mathbf{A}$ is described up to isomorphism, vice versa. Since this condition already entails empirical adequacy, it formalizes the aim of science as suggested by Percival if context $c$ is taken to determine the appearances $\mathbf{A}$.

**7. Conclusion.** The preceding discussion has brought out a number of points. First, the RV can describe scientific theories and their relation to the observations if CE can, while the opposite is not obviously the case. Second, the CE-formalism can and should be extended in at least two respects. It should allow for concepts in theories that do not also occur in the appearances, and it should include the explication of 'full empirical informativeness' among its formal desiderata for a CE-theory. Finally, and as a suggestive corollary to this and previous discussions, the philosophy of science would be better off if syntactic and semantic methods in general and the RV and CE in particular were not seen as in opposition but, correctly, as means of mutual support and improvement.

## REFERENCES

Bell, J. L., and M. Machover. 1977. *A Course in Mathematical Logic*. Amsterdam: North-Holland.

Carnap, R. 1963. "Replies and Systematic Expositions." In *The Philosophy of Rudolf Carnap*, ed. P. A. Schilpp, 859–1016. Chicago: Open Court.

———. 1966. *Philosophical Foundations of Physics: An Introduction to the Philosophy of Science*. New York: Basic.

Carus, A. W. 2007. *Carnap and Twentieth-Century Thought: Explication as Enlightenment*. Cambridge: Cambridge University Press.

11. The other two possible replacements are outside the scope of this discussion.

https://doi.org/10.1086/677886 Published online by Cambridge University Press

Chang, C. C., and H. J. Keisler. 1990. *Model Theory*. 3rd ed. Amsterdam: North-Holland.
Enderton, H. B. 2009. "Second-Order and Higher-Order Logic." In *Stanford Encyclopedia of Philosophy*, ed. E. N. Zalta. Stanford, CA: Stanford University.
Friedman, M. 1982. Review of *The Scientific Image*, by Bas van Fraassen. *Journal of Philosophy* 79 (5): 274–83.
Halvorson, H. 2012. "What Scientific Theories Could Not Be." *Philosophy of Science* 79 (2): 183–206.
———. 2013. "The Semantic View, If Plausible, Is Syntactic." *Philosophy of Science* 80 (3): 475–78.
Hodges, W. 1993. *Model Theory*. Cambridge: Cambridge University Press.
Justus, J. 2012. "Carnap on Concept Determination: Methodology for Philosophy of Science." *European Journal for Philosophy of Science* 2 (2): 161–79.
Lutz, S. 2009. "Ideal Language Philosophy and Experiments on Intuitions." *Studia Philosophica Estonica* 2 (2): 117–39.
———. 2012. "On a Straw Man in the Philosophy of Science: A Defense of the Received View." *HOPOS: Journal of the International Society for the History of Philosophy of Science* 2 (1): 77–120.
———. Forthcoming. "What's Right with a Syntactic Approach to Theories and Models?" *Erkenntnis*. doi:10.1007/s10670-013-9578-5.
Maher, P. 2007. "Explication Defended." *Studia Logica* 86 (2): 331–41.
Monton, B., and C. Mohler. 2008. "Constructive Empiricism." In *Stanford Encyclopedia of Philosophy*, ed. E. N. Zalta. Stanford, CA: Stanford University.
Mormann, T. 2007. "The Structure of Scientific Theories in Logical Empiricism." In *The Cambridge Companion to Logical Empiricism*, ed. A. Richardson and T. Uebel, 136–62. Cambridge: Cambridge University Press.
Muller, F. A. 2010. "Reflections on the Revolution at Stanford." *Synthese* 183 (1): 87–114.
Percival, P. 2007. "An Empiricist Critique of Constructive Empiricism: The Aim of Science." In *Images of Empiricism: Essays on Science and Stances, with a Reply from Bas C. van Fraassen*, ed. B. Monton, chap. 6, 83–116. Oxford: Oxford University Press.
Richardson, A. 2007. "'That Sort of Everyday Image of Logical Positivism'—Thomas Kuhn and the Decline of Logical Empiricist Philosophy of Science." In *The Cambridge Companion to Logical Empiricism*, ed. A. Richardson and T. Uebel, 346–70. Cambridge: Cambridge University Press.
Turney, P. 1990. "Embeddability, Syntax, and Semantics in Accounts of Scientific Theories." *Journal of Philosophical Logic* 19:429–51.
van Fraassen, B. C. 1970. "On the Extension of Beth's Semantics of Physical Theories." *Philosophy of Science* 37 (3): 325–39.
———. 1980. *The Scientific Image*. Oxford: Clarendon.
———. 1989. *Laws and Symmetry*. Oxford: Clarendon.
———. 1991. *Quantum Mechanics: An Empiricist View*. Oxford: Clarendon.
———. 2002. *The Empirical Stance*. New Haven, CT: Yale University Press.
———. 2007. "From a View of Science to a New Empiricism." In *Images of Empiricism: Essays on Science and Stances, with a Reply from Bas C. van Fraassen*, ed. B. Monton, chap. 15. Oxford: Oxford University Press.
———. 2008. *Scientific Representation: Paradoxes of Perspective*. Oxford: Clarendon.
Worrall, J. 1984. "An Unreal Image." Review of *The Scientific Image*, by Bas van Fraassen. *British Journal for the Philosophy of Science* 35:65–80.

https://doi.org/10.1086/677886 Published online by Cambridge University Press