A note on Ferromagnetism. By Mr R. H. FOWLER, F.R.S., Trinity College, and F. C. POWELL, B.A., Gonville and Caius College.

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1. *Introduction.* The purpose of this note is to amplify the discussion and conclusions of a recent paper by one of us*. It has recently been shown by Webster†, drawing largely from the work of Weiss and Forrer‡, that in single crystals of both iron and nickel there is a family of natural directions of *micromagnetization*§ defined by the observed directions of easy *holomagnetization*§. The evidence presented by Webster is completely convincing, and we therefore may refine our picture of micromagnetization by asserting that the micro-elements are all (holo)magnetized in one or other of the directions of easy holomagnetization. These directions are the family of six directions $(\pm 1, 0, 0)$ $(0, \pm 1, 0)$ $(0, 0, \pm 1)$ for iron and the family of eight directions $(\pm 1, \pm 1, \pm 1)$ for nickel. We wish to comment on Webster's theory later in connection with the properties of cobalt.

The more recent paper by Powell, to which this is an addendum, presents a study of the deviation between the direction of holomagnetization and the direction of the effective magnetizing field in single crystals, cubic (iron, nickel) and non-cubic (cobalt). It is there shown, we submit with some convincingness, that the deviation is due to the well-known (l, s) interaction between the spins of the pivoting magnetic electrons and the non-pivoting orbital momentum of the atoms to which they belong. These non-pivoting axes must therefore be assumed to be arranged in fixed directions determined by the crystal lattice and possessing its full symmetry, in particular always possessing a zero resultant. The sign of the deviation effect is accounted for if these axes are distributed with cubic symmetry among the directions $(\pm 1, 0, 0)$, $(0, \pm 1, 0)$ and $(0, 0, \pm 1)$ in nickel and either $(\pm 1, \pm 1, 0)$, $(\pm 1, 0, \pm 1)$, $(0, \pm 1, \pm 1)$ or $(\pm 1, \pm 1, \pm 1)$ in iron. No other directions are at all probable. The magnitude of the deviation effect is consistent with the known magnitude of the (l, s) interaction in these atoms.

* Powell, *Proc. Roy. Soc.* A, vol. 130, p. 167 (1930).

† Webster, *Proc. Phys. Soc. London*, vol. 42, p. 431 (1930).

‡ Weiss and Forrer, *Ann. de Physique*, vol. 5, p. 153 (1926).

§ It seems to us convenient to standardize some such terms to discriminate between the natural state of zero bulk magnetization of a ferromagnetic below the Curie point when micro-elements must still be magnetized to saturation (*micromagnetized state*), and the state of magnetization in bulk when the magnetism of the micro-elements has been aligned (*holomagnetized state*).

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It is possible to go further and deduce from this theory that *if the axes of the atomic angular momenta are arranged as required by this theory of the deviation effect, then the direction of easiest magnetization will be the* (1, 0, 0) *and similar directions in iron and the* (1, 1, 1) *and similar directions in nickel*, in agreement with the facts. We can show further that the (l, s) interaction in the crystal deduced from the deviation effect yields energy differences between the states of holomagnetization along the easiest and other directions, of the right order to account for the observed extra work of magnetization required.

This connection between the deviation effect and the energy differences between various directions of holomagnetization has already been mentioned for iron by Akulov* in connection with his theory of magnetic quadrupole energy. We may therefore properly refer to the connection as Akulov's result. A similar result is true in non-cubic crystals, as we shall see later here for cobalt. In order to incorporate Akulov's result into the present theory it is strictly only necessary to show that this theory leads to a magnetic energy term of the right form. This has not yet been done and we do it here. But it has seemed to us worth while to say rather more about these interesting connections than the bare minimum necessary to incorporate Akulov's result, for it is likely that there is a connection between the observed directions of easy magnetization and the type of the crystal lattice, which is face-centred for nickel, body-centred for magnetic iron (and hexagonal close-packing for cobalt).

The arguments used are semi-classical. The properties of the more elaborate model here contemplated have not yet been worked out quantum-mechanically, as Heisenberg has worked out those of his simpler one. But there is some reason† to expect that such a semi-classical treatment should be at least a fair approximation.

2. *The magnetic energy term of the* (l, s) *interaction theory.* The present theory leads to an expression for the magnetic energy of the crystal, E_{mag},

$$E_{\mathrm{mag}} = \frac{-zJ_0}{2n}\left(\frac{yn}{p}\right)^2 (\Sigma_a \boldsymbol{\zeta}_a)^2 - 2\mu_B\left(\frac{yn}{p}\right)\Sigma_a(\mathbf{H} + \mathbf{G}_a \cdot \boldsymbol{\zeta}_a). \quad \ldots(1)$$

Constant terms have been omitted. The mutual magnetic energy of the pivoting magnetic electrons themselves *in their own field* depends on the saturation, but not on the resultant orientation in a cubic crystal. It has therefore properly been relegated to the omitted constant, for the variation of the degree of saturation in our problems is trivial. For the same reason the contribution $\frac{4}{3}\pi\mathbf{I}$ of the holo-

* Akulov, *Zeit. f. Physik*, vol. 57, p. 249 (1929).
† Powell, *loc. cit.*

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magnetization to $\mathbf{H}$ in (1) is also omitted and $\mathbf{H}$ may be taken to be the effective magnetizing field, that is the external field corrected for the demagnetizing effect. The notation is mainly Heisenberg's: $2n$ is the number of atoms in the crystal, each providing y electrons pivoting as a unit; z is the number of nearest neighbours of any atom; J_0 the exchange interaction integral and μ_B Bohr's magneton; the atoms are divided into p equal groups enumerated by the suffix a; for the atoms of group a the effect of the (l, s) interaction is taken to be equivalent to a fixed magnetic field $\mathbf{G}_a$ acting on the pivoting electrons of that atom, in addition to $\mathbf{H}$, which acts on all groups alike; $|\mathbf{G}_a| (= G)$ is independent of a, but the direction of $\mathbf{G}_a$ varies from group to group; $\boldsymbol{\zeta}_a$ is the fraction of ideal saturation achieved by the electrons of group a. We consider only holomagnetized crystals or equivalent micro-elements.

The theory proceeds (*loc. cit.*) by minimizing E_{mag} subject to

$$(\boldsymbol{\zeta}_a)^2 \leqslant 1 \qquad (all\ a) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

for given $\mathbf{H}$ and $\mathbf{G}_a$, thus determining the equilibrium state. Writing

$$N = \frac{yzJ_0}{2\mu_B}, \quad \boldsymbol{\zeta} = \frac{1}{p}(\Sigma_a \boldsymbol{\zeta}_a), \quad \ldots\ldots\ldots\ldots\ldots (2)'$$

the minimizing $\boldsymbol{\zeta}_a$ satisfy

$$\boldsymbol{\zeta}_a = \frac{N\boldsymbol{\zeta} + \mathbf{H} + \mathbf{G}_a}{|N\boldsymbol{\zeta} + \mathbf{H} + \mathbf{G}_a|}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equation (2)′ evaluated by (3) suffices to study the deviation effects. We have however still to calculate the minimum value of E_{mag}. On substitution of (3) into (1) we find easily

$$-\left(\frac{p}{yn\mu_B}\right) E_{\mathrm{mag}} = \Sigma_a \frac{(N\boldsymbol{\zeta} + 2\mathbf{H} + 2\mathbf{G}_a \,.\, N\boldsymbol{\zeta} + \mathbf{H} + \mathbf{G}_a)}{|N\boldsymbol{\zeta} + \mathbf{H} + \mathbf{G}_a|}. \quad \ldots\ldots (4)$$

We may now assume that both $H (= |\mathbf{H}|)$ and G are small compared with $N|\boldsymbol{\zeta}|$, and expand (4) in powers of $|\mathbf{H} + \mathbf{G}_a|/N|\boldsymbol{\zeta}|$ up to and including fourth order terms, obtaining

$$-\left(\frac{p}{yn\mu_B}\right) E_{\mathrm{mag}} = \Sigma_a \left[N|\boldsymbol{\zeta}| + 2|\mathbf{H} + \mathbf{G}_a|\mu + \frac{|\mathbf{H} + \mathbf{G}_a|^2}{N|\boldsymbol{\zeta}|} \tfrac{3}{2}(1 - \mu^2) \right.$$
$$\left. + \frac{|\mathbf{H} + \mathbf{G}_a|^3}{N^2|\boldsymbol{\zeta}|^2} 2(\mu^3 - \mu) + \frac{|\mathbf{H} + \mathbf{G}_a|^4}{N^3|\boldsymbol{\zeta}|^3} \left\{ \frac{-25\mu^4 + 30\mu^2 - 5}{8} \right\} \right]. \quad \ldots\ldots (5)$$

In this expression $\mu = \cos\{\boldsymbol{\zeta}\hat{(}\mathbf{H} + \mathbf{G}_a)\}$. We next observe that (5) is built up of the two expressions $|\mathbf{H} + \mathbf{G}_a|\mu$ and $|\mathbf{H} + \mathbf{G}_a|^2$. These are respectively equal to

$$H\cos(\hat{\mathbf{H}\boldsymbol{\zeta}}) + G\cos(\hat{\boldsymbol{\zeta}\mathbf{G}}_a), \quad H^2 + G^2 + 2HG\cos(\hat{\mathbf{H}\mathbf{G}}_a).$$

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Inserting these values into (5) we obtain the more useful form

$$-\left(\frac{p}{yn\mu_B}\right)E_{\text{mag}} = \Sigma_a \Big[N|\boldsymbol{\zeta}| + 2\{H\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) + G\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)\}$$
$$+\frac{3}{2}\frac{1}{N|\boldsymbol{\zeta}|}\{H^2\sin^2(\hat{\mathbf{H}}\boldsymbol{\zeta}) + G^2\sin^2(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)$$
$$+ 2HG[\cos(\hat{\mathbf{H}}\mathbf{G}_a) - \cos(\hat{\mathbf{H}}\boldsymbol{\zeta})\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)]\}$$
$$-2\frac{1}{N^2|\boldsymbol{\zeta}|^2}\{H\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) + G\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)\}\{H^2\sin^2(\hat{\mathbf{H}}\boldsymbol{\zeta})$$
$$+ G^2\sin^2(\hat{\boldsymbol{\zeta}}\mathbf{G}_a) + 2HG[\cos(\hat{\mathbf{H}}\mathbf{G}_a) - \cos(\hat{\mathbf{H}}\boldsymbol{\zeta})\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)]\}$$
$$+\frac{1}{8}\frac{1}{N^3|\boldsymbol{\zeta}|^3}\{-25[H\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) + G\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)]^4$$
$$+30[H\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) + G\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)]^2[H^2 + G^2 + 2HG\cos(\hat{\mathbf{H}}\mathbf{G}_a)]$$
$$-5[H^2 + G^2 + 2HG\cos(\hat{\mathbf{H}}\mathbf{G}_a)]^2\}\Big]. \quad \ldots\ldots\ldots\ldots(6)$$

We have now to carry out the summation, on the basis that if (X, Y, Z) are the direction cosines of any $\mathbf{G}_a$, then other equal groups occur for each of the 48 sets of direction cosines $(\pm X, \pm Y, \pm Z)$, X, Y, Z permuted. Sums of odd powers of $\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a)$ or $\cos(\hat{\mathbf{H}}\mathbf{G}_a)$ vanish. There remain

$$\Sigma_a \cos^2(\hat{\boldsymbol{\zeta}}\mathbf{G}_a) = \Sigma_a \cos^2(\hat{\mathbf{H}}\mathbf{G}_a) = \tfrac{1}{3}p,$$
$$\Sigma_a \cos(\hat{\mathbf{H}}\mathbf{G}_a)\cos(\hat{\boldsymbol{\zeta}}\mathbf{G}_a) = \tfrac{1}{3}p\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}),$$
$$\Sigma_a \cos^4(\hat{\boldsymbol{\zeta}}\mathbf{G}_a) = \tfrac{1}{3}p\Omega,$$

where

$$\Omega = \tfrac{5}{2}(l^4 + m^4 + n^4)(X^4 + Y^4 + Z^4) - \tfrac{3}{2}(l^4 + m^4 + n^4 + X^4 + Y^4 + Z^4) + \tfrac{3}{2}.$$

Here (l, m, n), (X, Y, Z) are the direction cosines of $\boldsymbol{\zeta}$ and a $\mathbf{G}_a$, and it is assumed that

$$X^4 + Y^4 + Z^4,$$

and therefore also

$$X^2Y^2 + Y^2Z^2 + Z^2X^2,$$

have the same value for all a. On using these results and dividing through by p, we obtain

$$-\left(\frac{1}{yn\mu_B}\right)E_{\text{mag}} = N|\boldsymbol{\zeta}| + 2H\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) + \frac{3}{2}\frac{1}{N|\boldsymbol{\zeta}|}\{H^2\sin^2(\hat{\mathbf{H}}\boldsymbol{\zeta}) + \tfrac{2}{3}G^2\}$$
$$-\frac{2}{N^2|\boldsymbol{\zeta}|^2}H\cos(\hat{\mathbf{H}}\boldsymbol{\zeta})\{H^2\sin^2(\hat{\mathbf{H}}\boldsymbol{\zeta}) + \tfrac{2}{3}G^2\}$$
$$+\frac{1}{8}\frac{1}{N^3|\boldsymbol{\zeta}|^3}\{-25[H^4\cos^4(\hat{\mathbf{H}}\boldsymbol{\zeta}) + 2H^2G^2\cos^2(\hat{\mathbf{H}}\boldsymbol{\zeta}) + \tfrac{1}{3}G^4\Omega]$$
$$+30(H^2 + G^2)[H^2\cos^2(\hat{\mathbf{H}}\boldsymbol{\zeta}) + \tfrac{1}{3}G^2] + 40H^2G^2\cos^2(\hat{\mathbf{H}}\boldsymbol{\zeta})$$
$$-5(H^2 + G^2)^2 - \tfrac{20}{3}H^2G^2\}. \quad \ldots\ldots\ldots\ldots(7)$$

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For present purposes we may pick out from this formula the leading terms on the assumption that

$$N|\boldsymbol{\zeta}| \gg G \gg H.$$

It then reduces to

$$E_{\text{mag}} = -yn\mu_B \Big[N|\boldsymbol{\zeta}| \Big(1 + \frac{G^2}{N^2|\boldsymbol{\zeta}|^2}\Big) + 2H\Big(1 - \frac{2}{3}\frac{G^2}{N^2|\boldsymbol{\zeta}|^2}\Big)\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) - \frac{5}{48}\frac{G^4}{N^3|\boldsymbol{\zeta}|^3}\{25(l^4+m^4+n^4)(X^4+Y^4+Z^4) - 15(l^4+m^4+n^4+X^4+Y^4+Z^4)+9\}\Big]. \quad \ldots\ldots(8)$$

Into this formula we must substitute the equilibrium value of $|\boldsymbol{\zeta}|$, determined from its definition and equations (3). The calculation is similar to the foregoing and need not be given. The result is

$$|\boldsymbol{\zeta}| = 1 - \frac{1}{3}\frac{G^2}{N^2} + \frac{2}{3}\frac{G^2H}{N^3}\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) + \frac{G^4}{16N^4}[25(l^4+m^4+n^4) \times (X^4+Y^4+Z^4) - 15(l^4+m^4+n^4+X^4+Y^4+Z^4) + \tfrac{42}{5}]. \quad \ldots(9)$$

In the fourth order term here (l, m, n) represents strictly the direction of $N\boldsymbol{\zeta} + \mathbf{H}$, but in this term this direction may be identified with that of $N\boldsymbol{\zeta}$. On using this result we find that

$$E_{\text{mag}} = -yn\mu_B \Big[N\Big(1 + \frac{2}{3}\frac{G^2}{N^2}\Big) + 2H\Big(1 - \frac{1}{3}\frac{G^2}{N^2}\Big)\cos(\hat{\mathbf{H}}\boldsymbol{\zeta}) - \frac{1}{72}\frac{G^4}{N^3}\{75(l^4+m^4+n^4)(X^4+Y^4+Z^4) - 45(l^4+m^4+n^4+X^4+Y^4+Z^4)+19\}\Big]. \quad \ldots\ldots(10)$$

This is the final expression required. The first term is the energy of the magnetization in the "Weiss" field. The second term is its energy in the effective magnetizing field $\mathbf{H}$ corrected for loss of saturation, and depends only trivially on (l, m, n). The third term fixes the variation of the magnetic energy with the direction of magnetization (l, m, n), the important part being

$$yn\mu_B \cdot \frac{5}{24}\frac{G^4}{N^3}\{5(X^4+Y^4+Z^4) - 3\}(l^4+m^4+n^4). \quad \ldots(11)$$

An equivalent expression for the variable energy is

$$-yn\mu_B \cdot \frac{5}{12}\frac{G^4}{N^3}\{5(X^4+Y^4+Z^4) - 3\}(l^2m^2+m^2n^2+n^2l^2). \quad \ldots(12)$$

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3. *Applications to iron and nickel.* The form of (12) shows at once that the theory proposed incorporates Akulov's theorem, but the synthesis of experimental facts thereby achieved seems to us to merit further comment from the point of view of the present theory. It has been shown (*loc. cit.*) that for nickel, most probably $(X, Y, Z) = (1, 0, 0)$. The variable part of the energy then becomes

$$+ 2yn\mu_B \cdot \frac{5}{24} \frac{G^4}{N^3} (l^4 + m^4 + n^4),$$

and must be as small as possible for the direction of easiest magnetization, which is therefore $(1, 1, 1)$, $l = m = n = \frac{1}{\sqrt{3}}$, in agreement with the facts. The energy difference when magnetized in this direction and in the $(1, 0, 0)$ direction, say, is

$$2yn\mu_B \left[\frac{5}{36} \frac{G^4}{N^3}\right]. \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

For iron probably $(X, Y, Z) = (1, 1, 1)$. Powell's discussion allows also $(1, 1, 0)$, but we shall see later that this is unlikely. Then the variable energy is

$$- 2yn\mu_B \cdot \frac{5}{36} \frac{G^4}{N^3} (l^4 + m^4 + n^4),$$

and must be as small as possible algebraically for easy magnetization, so that $(l, m, n) = (1, 0, 0)$, again in agreement with the facts*. The energy difference between magnetization in this direction and in $(1, 1, 1)$, say, is

$$2yn\mu_B \left[\frac{5}{54} \frac{G^4}{N^3}\right]. \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

4. *Connection of the values of* (X, Y, Z) *with the nature of the crystal lattice.* The nature of the crystal lattice, face-centred for nickel, body-centred for iron, is presumably determined by atomic interchange forces which are large compared with the magnetic forces here in contemplation. We do not know why the lattices are different for the two metals, but given that they are different we may perhaps guess that the (X, Y, Z) of the atomic axes must select the values similar to $(1, 1, 1)$ for a body-centred lattice and those similar to $(1, 0, 0)$ or $(1, 1, 0)$ for a face-centred one. For there are eight neighbours to each atom in the body-centred lattice, and we can arrange a repeating pattern out of the eight $(1, 1, 1)$ directions in which each atom is surrounded by one atom of each group but not out of the $6\,(1, 0, 0)$'s or the $12\,(1, 1, 0)$'s. In the face-centred lattice there are twelve neighbours and we can arrange

* (l, m, n) must equal $(1, 0, 0)$ also if $(X, Y, Z) = (1, 1, 0)$.

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similar surroundings for each atom out of the 6 (1, 0, 0)'s or the 12 (1, 1, 0)'s but not out of the 8 (1, 1, 1)'s. The deviation effect in nickel has already selected the (1, 0, 0) directions as the most probable, ruling out the (1, 1, 0)'s.

These remarks of course are only speculative; they suggest the possibility of further verification by extended calculations of the energy of various systems of magnetic dipoles, which have never yet been undertaken. But it seems fair to conclude that the (l, s) interaction theory not only gives a satisfactory account of the deviation effects and directions of easy magnetization in cubic crystals, but also does so in a way which seems likely to fit into further developments of our knowledge of the properties of crystal lattices.

5. *Cobalt, with remarks on non-cubic ferromagnetic crystals in general.* It is natural to examine whether the (l, s) interaction theory will not also apply with equal success to a simple non-cubic crystal such as cobalt, using the measurements of Kaya*. It has been already shown that it accounts properly for the deviation effects. Equation (5) continues to apply, but need only be used down to second order terms. We have therefore to use

$$-\left(\frac{p}{yn\mu_B}\right) E_{\text{mag}} = \Sigma_a \left\{ N|\boldsymbol{\zeta}| + 2|\mathbf{H}+\mathbf{G}_a|\mu + \frac{3}{2}\frac{|\mathbf{H}+\mathbf{G}_a|^2}{N|\boldsymbol{\zeta}|}(1-\mu^2)\right\}$$

with the same values of $|\mathbf{H}+\mathbf{G}_a|\mu$ and $|\mathbf{H}+\mathbf{G}_a|^2$ as before.

We may now only assume that the (X_a, Y_a, Z_a) of the p groups balance out the atomic orbital momentum so that

$$\Sigma_a X_a = \Sigma_a Y_a = \Sigma_a Z_a = 0,$$

and that the axes are chosen in such a way that also

$$\Sigma_a X_a Y_a = \Sigma_a Y_a Z_a = \Sigma_a Z_a X_a = 0. \quad \ldots\ldots\ldots\ldots (15)$$

Such conditions could always be fulfilled in any crystal. Then

$$\Sigma_a \cos(\hat{\mathbf{G}_a\boldsymbol{\zeta}}) = 0, \quad \Sigma_a \cos(\hat{\mathbf{H}\mathbf{G}_a}) = 0,$$

$$\frac{1}{p}\Sigma_a \cos^2(\hat{\mathbf{G}_a\boldsymbol{\zeta}}) = l^2\bar{X}^2 + m^2\bar{Y}^2 + n^2\bar{Z}^2,$$

where $\bar{X}^2$, etc. are the mean values of X^2, etc. over the various groups. Then, retaining only the most important terms,

$$E_{\text{mag}} = -yn\mu_B\left\{N|\boldsymbol{\zeta}| + 2H\cos(\hat{\mathbf{H}\boldsymbol{\zeta}}) + \frac{3}{2}\frac{G^2}{N|\boldsymbol{\zeta}|}(1-[l^2\bar{X}^2 + m^2\bar{Y}^2 + n^2\bar{Z}^2])\right\}. \quad \ldots\ldots (16)$$

* Kaya, *Science Reports, Tôhoku University* (1928), p. 1157.

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We require also the value of $|\boldsymbol{\zeta}|$, which is obtained in the same way as before and found to be*

$$|\boldsymbol{\zeta}| = 1 - \frac{1}{2}\frac{G^2}{N^2}(1 - \{l^2\bar{X}^2 + m^2\bar{Y}^2 + n^2\bar{Z}^2\}). \quad \ldots\ldots(17)$$

Therefore to the required order

$$E_{\text{mag}} = -yn\mu_B\left\{N + 2H\cos(\mathbf{H}\hat{\boldsymbol{\zeta}}) + \frac{G^2}{N}(1 - [l^2\bar{X}^2 + m^2\bar{Y}^2 + n^2\bar{Z}^2])\right\}. \quad \ldots\ldots(18)$$

If Z is along the hexagonal axis for cobalt, as from the symmetry it can be, in conformity with (15), then also $\bar{X}^2 = \bar{Y}^2$ and the sign of the deviation effect requires

$$\bar{X}^2 = \bar{Y}^2 > \bar{Z}^2.$$

The important directional terms in E_{mag} are then

$$2yn\mu_B\left(\frac{G^2}{2N}\right)(l^2 + m^2)(\bar{X}^2 - \bar{Z}^2).$$

This is least when $l = m = 0$, $n = 1$, i.e. when the magnetization is along the hexagonal axis in agreement with observation. The magnetization curves of Kaya are reproduced in Fig. 1 below. To magnetize at right angles to the hexagonal axis requires an extra energy

$$2yn\mu_B\left(\frac{G^2}{2N}\right)(\bar{X}^2 - \bar{Z}^2), \quad \ldots\ldots(19)$$

which is of course very large compared with the fourth order energies for iron or nickel.

The contrast between hexagonal cobalt and cubic iron and nickel suggests further reflections. It suggests that one of the primary reasons (perhaps *the* primary reason) for the difficulty of magnetizing ordinary cobalt and other non-cubic materials is the lower symmetry of their micro-crystals which leaves them with second order directional terms of the (l, s) interaction as compared with fourth order terms which are all that remain in cubic crystals.

The difference in the nature of the magnetization curves for cobalt and nickel (or iron) is also significant and provides additional strong support for Webster's theory of these curves. For on his

* There is a misprint in Powell, *loc. cit.*, where the first equation of § 5 should obviously read

$$\zeta_x = \frac{a_x}{a}\left\{1 - \frac{1}{2}\frac{G^2}{N^2} + \frac{3}{2}\frac{G^2}{N^4}(Pa_x^2 + Qa_y^2 + Ra_z^2)\right\} - \frac{G^2}{N^2}\frac{Pa_x}{a} + O(G^4),$$

or what is the equivalent,

$$\zeta_x = \frac{a_x}{a}\left\{1 - \frac{1}{2}\frac{G^2}{N^2}\left(1 - 3\left[P\frac{a_x^2}{a^2} + Q\frac{a_y^2}{a^2} + R\frac{a_z^2}{a^2}\right]\right)\right\} - \frac{G^2}{N^2}P\frac{a_x}{a} + O(G^4).$$

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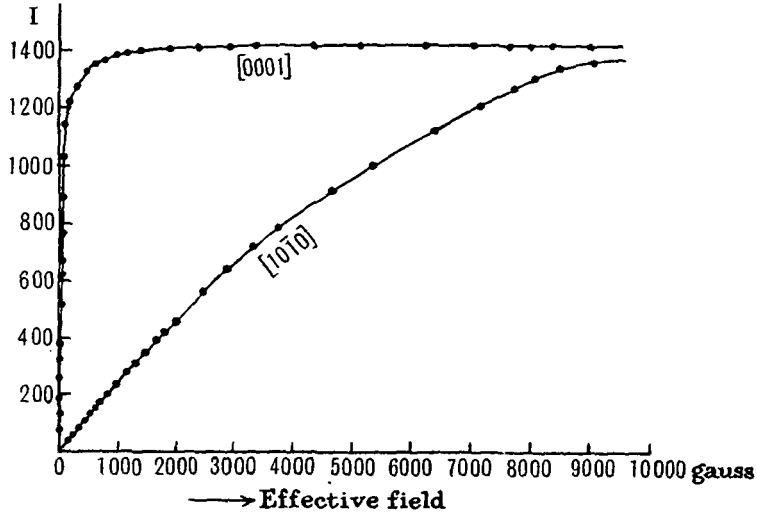


Fig. 1. Magnetization curves for cobalt along (0001) and normal (10$\bar{1}$0) to the hexagonal axis.

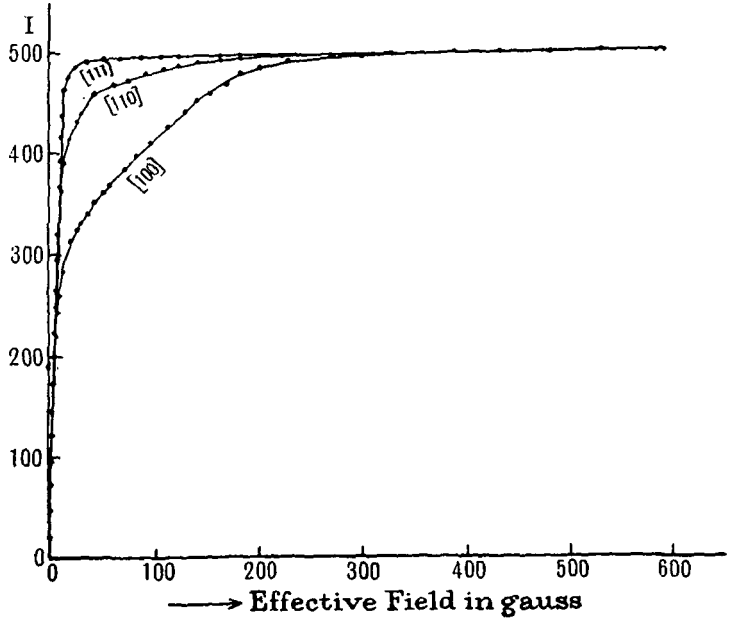


Fig. 2. Magnetization curves for nickel along the principal axes in the (110) plane.

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view a natural unmagnetized single crystal of cobalt will be micro-magnetized in small blocks along the directions of easy magnetization, which are here only two, namely in one or other direction along the hexagonal axis. When we attempt to magnetize a cobalt crystal in some direction at right angles to this axis, there is no occasion for any change from one direction of easy magnetization to another and therefore *no first stage*. The magnetization curve for a hard direction should depart from the magnetization curve for the easy direction *for the smallest magnetizations*. This is exactly what is observed, in striking contrast to the magnetization curves of cubic nickel (see Figs. 1 and 2) or iron.

Finally, one may indulge in a rash prediction. Cobalt changes to a face-centred cubic lattice at higher temperatures while still ferromagnetic. It would be interesting to determine the details of the ferromagnetism above the transition point. The theory suggests that cobalt should change sharply at the transition temperature into a substance with the magnetic characteristics of nickel so far as deviation effects and the direction of easy magnetization (1, 1, 1) are concerned, and that the extra work required to produce saturation in the hard (1, 0, 0) direction should fall to something of the order of $\frac{1}{2}$ per cent. to 1 per cent. of its value in the hexagonal state.

6. *Akulov's comparison for nickel and cobalt.* Akulov has already shown that the energy difference (14) correctly represents the extra work which must be done to magnetize iron in the direction (1, 1, 1) over that necessary in the easy (1, 0, 0); this extra work is measured by the difference in the values of $\int H dI$ taken along the two I-H curves and is therefore measured by the area enclosed between them. We shall, in conclusion, record the similar results for the data and theory for nickel and cobalt.

For nickel, using Powell's values* of G and N, the energy difference (13) is $1{\cdot}8 \times 10^4$ ergs/cm.^{3}, which agrees extremely closely with the energy difference of the (1, 1, 1) and (1, 0, 0) curves in Fig. 2.

For cobalt, using Powell's values, the energy difference (19) is $5{\cdot}5 \times 10^6$ ergs/cm.^{3}, which again agrees closely with the value estimated from the magnetization curves in Fig. 1. The closeness of the agreement in these cases is remarkable, in view of the considerable differences between the saturated magnetization and the theoretical value, which is $2yn\mu_B$ in all cases.

* The values given in Powell, *loc. cit.*, for Fe and Ni are not quite correct for the data used to determine them. The correct values are

Ni $G = 6{\cdot}0 \times 10^5$ gauss

Fe $G = 12{\cdot}1 \times 10^5$,, $(X, Y, Z) = (1, 1, 0)$

Fe $G = 9{\cdot}4 \times 10^5$,, $(X, Y, Z) = (1, 1, 1)$

See a forthcoming discussion of magnetostriction by Powell.

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