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HANDBOOK OF MATRICES

H. Lütkepohl

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This book is a collection of matrix results that the author found important for his own work. To make the book reasonably self-contained, the author has also added many elementary results. The book contains definitions and results only and no proofs.

The collection is a very personal one. There is a lot of emphasis on commutation, duplication, and elimination matrices (which, of course, I welcome) and on matrix derivatives (on which more later). In addition to a list of symbols, a dictionary of terms, and an index, there are 11 chapters: (1) definitions, notation, terminology, (2) rules for matrix operations, (3) matrix valued functions of a matrix, (4) trace, determinant, and rank of a matrix, (5) eigenvalues and singular values, (6) matrix decompositions and canonical forms, (7) vectorization operators, (8) vector and matrix norms, (9) properties of special matrices, (10) vector and matrix derivatives, (11) polynomials, power series, and matrices.

One essential feature of a "cookbook" like this is that it is free of errors. Fortunately this appears to be the case. I have checked a number of results, and I have found no errors. The book should therefore be a useful tool for econometricians and time-series statisticians.

Personally, I must admit, I don't like "cookbooks." In my own work (and I expect in many colleagues' work as well) the result that I need is very seldom directly available in the literature. Usually I need to extend a result or combine two results. Without knowing the proof of the existing results, it is very difficult to obtain these extensions. So I prefer books that contain not only results but also proofs and discussion. This is true in particular for (matrix) differential calculus where—apart from basic results (inverse, determinant)—it is the *method* that needs to be learned. A key ingredient in this method is the chain rule. If we wish to employ the chain rule for vector derivatives and matrix derivatives, we have to use the correct method of differentiation, and this is most easily accomplished by the use of differentials (see Magnus and Neudecker, 1988). There is no word about differentials in Lütkepohl's collection and hence he is forced to list many, many derivatives. The correct method, however, can be learned in a few hours, and then most of the results and many more can be derived.

Let me explain by taking one simple example. On page 178, we find the result

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$$\frac{\partial \operatorname{tr}(X'X)}{\partial X} = 2X \tag{1}$$

and many other results of the same type: $X'AX$, XAX', XAX, $AXX'A'$, etc. Using differentials, the derivation goes

$$\begin{aligned} d \operatorname{tr} X'X &= \operatorname{tr} d(X'X) = \operatorname{tr}(dX)'X + \operatorname{tr} X'dX \\ &= 2 \operatorname{tr} X'dX = 2(\operatorname{vec} X)' d \operatorname{vec} X. \end{aligned} \tag{2}$$

The derivative is *not* $2X$ but $2(\operatorname{vec} X)'$, a row vector. The matrix $2X$ does, of course, contain all the partial derivatives, but it is not the derivative, and hence the chain rule cannot be applied. Suppose, for example, that X is a function of a vector α, $X = X(\alpha)$, and we wish to find the derivative of $\operatorname{tr}(X'X)$ with respect to α. Then (1) does not help us, but from (2) we get simply

$$d \operatorname{tr} X'X = 2(\operatorname{vec} X)' d \operatorname{vec} X = 2(\operatorname{vec} X)' \frac{\partial \operatorname{vec} X}{\partial \alpha'} d\alpha, \tag{3}$$

and hence the derivative is now $2(\operatorname{vec} X)' \partial \operatorname{vec} X / \partial \alpha'$. Without the chain rule and differentials, such simple and basic results are very much harder to obtain.

I have one further point of critique. When discussing matrix functions $F(A)$ of some real symmetric $n \times n$ matrix A (e.g., e^A), it is customary to apply this function to the eigenvalues of A. Hence, if $T'AT = \Lambda$ (diagonal) and $T'T = I_n$, then

$$A = T\Lambda T', \qquad e^A = T(e^\Lambda)T', \tag{4}$$

where

$$e^\Lambda = \begin{pmatrix} e^{\lambda_1} & & \\ & \ddots & \\ & & e^{\lambda_n} \end{pmatrix}. \tag{5}$$

It seems to me that this is the most obvious and useful way to extend scalar definitions to matrix definitions. Hence, $A > 0$ would mean that all eigenvalues of A are positive (A positive definite) and *not* that all elements of A are positive. (I realize that "positive" matrices, where all elements are positive, play a role in mathematical economics, but positive definite matrices are far more important.) Nevertheless, the author defines $A > 0$ to mean that all elements of A are positive. Similarly, he defines $|A|_{\text{abs}}$ as the matrix obtained from A by taking absolute values of each element of A separately. Surely a more useful definition would be $|A|_{\text{abs}} = T|\Lambda|_{\text{abs}}T'$?

In spite of these critical points, Lütkepohl's book will be a useful and reliable reference for econometricians. It is well organized, carefully produced, and contains a lot of useful results.

REFERENCE

Magnus, J.R. & H. Neudecker (1988) *Matrix Differential Calculus with Applications in Statistics and Econometrics.* Chichester: Wiley.

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