Robust geometric sizing of a small flying wing planform based on evolutionary algorithms

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ABSTRACT

In this paper a geometric sizing method for a small electric powered flying wing is proposed. The geometric sizing method aims to reduce the effects of variations in the power plant characteristics on endurance. This results in a single-objective design optimisation problem where the sensitivity to power plant characteristics of the endurance equation is minimised, constrained to Reynolds number, wing load, wing taper ratio, aircraft size and wing sweep angle. As a result, geometric characteristics of the flying wing such as span, tip chord and root chord are obtained. Flying wing aerodynamic characteristics are obtained by means of an inviscid fluid flow analysis program of the type low-order panel methods, known as CMARC. The optimisation problem involves a non convex function so that it is necessary to rely on heuristic programming methods. In particular an Evolutionary Algorithm based on differential evolution is considered.

NOMENCLATURE

b	wing span
b_W	winglet span
c_M	mean aerodynamic chord

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c_R	root chord
c_T	tip chord
C_{TW}	winglet tip chord
C_B	battery capacity
C_D	drag coefficient
C_L	lift coefficient
C_P	propeller power coefficient
C_T	propeller thrust coefficient
d	propeller diameter
D	drag force
E	endurance
I_B	battery current
J	advance ratio
l_w	diagonal distance proportional to small flying wing size
L	lift force
P	propeller power
P_e	electric power
Re	Reynolds number
S_w	wing area
S	sensitivity to power plant characteristics
T	propeller thrust
V	cruise airspeed
V_B	battery voltage
W	aircraft total weight
W_{eq}	weight of all essential components to flight
W_L	wing load
W_m	aircraft empty weight
W_p	payload
η_m	motor efficiency
η_p	propeller efficiency
λ	taper ratio
μ	air viscosity
ρ	air density
ρ_e	estimated weight per area unit of a wing frame constructed of balsa wood
$\varphi_{0\cdot25}$	sweep angle at the quarter chord
φ_{FW}	winglet leading sweep angle
ω	propeller rotational speed

1.0 INTRODUCTION

Small unmanned aerial vehicles have proven their merits in many civilian and military applications and have steadily gained a growing interest and investment all over the world. Thus, most of the world's major aerospace companies, hundreds of component manufacturers and small companies are hoping to grab a piece on the rapidly expanding market of small unmanned aircraft. In this highly competitive market, small unmanned aircraft needs to meet high standards concerning performance. As a consequence, any synergism that may exist among various subsystems must have to be fully investigated while taking into

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account a wide range of uncertainties for instance, operational conditions and subsystems performance. This state of affairs makes small unmanned aircraft excellent candidates for robust design. Robust design was introduced about half a century ago by Genichi Taguchi as a response to classical design engineering based on experience and extremely costly experiments performed on a black box[1].

Robust design paradigm is based on two fundamental concepts: the measure of the design's sensitivity to changes on environment conditions; and the measure of the loss of reliability due to an unsuitable design. The poor quality designs lie in the large sensitivity of these to variations in the manufacturing process and the environment conditions, and thus high quality designs should minimise such a sensitivity. In this paper, we resort to a recently introduced model-based framework of robust design[7]. In this framework, the robust design problem is changed to the minimisation problem of the covariances of variations in performance functions upon variations in the design environment parameters (a measure of the design sensitivity). In order to compute such a measure of sensitivity, performance needs to be modeled as function, in the mathematical sense.

Design methodologies that aim to fully integrate aircraft subsystems have been applied to obtain non conventional aircraft designs that meet highly performance requirements. In particular, the literature mention three articles: an article where a multidisciplinary design optimisation methodology using a genetic algorithm is applied to integrate subsystems of a micro air vehicle for maximum endurance[4], another where an optimisation method is applied to integrate an hybrid propulsion system with the power required, at the endurance speed, playing the role of the objective function and the optimal results minimise the battery pack increasing the payload[5], thirdly an article where an exhaustive search within the design space is performed to identify optimal design configurations of a proton exchange membrane fuel cell unmanned aerial vehicle[6].

In this paper a geometric sizing method for a small flying wing is proposed. We consider that the flying wing is designed to perform a civil reconnaissance mission, where electric power is preferred over liquid fuel power due to lower noise levels. As the powerplant subsystem is the main current consumer and low endurance is one of the principal disadvantages of electric powered flight, it would be interesting to develop a geometric sizing method that aims to reduce the effects of variations in the electric power plant characteristics on endurance.

Formulating the development of the geometric sizing method in terms of the robust design approach results in a single-point design optimisation problem where the sensitivity to power plant characteristics of the endurance equation is minimised, constrained to Reynolds number, wing load, wing taper ratio, aircraft size and wing sweep angle. As a result, geometric characteristics of the flying wing such as span, tip chord and root chord are obtained. Flying wing aerodynamic characteristics are obtained by an inviscid fluid flow analysis program of the type low-order panel methods, known as CMARC[3]. The optimisation problem is addressed using heuristic programming methods. In particular an evolutionary algorithm, based on differential evolution, is applied.

The paper is organised as follows. In Section 2 we present the mission profile. Section 3 is devoted to the formulation of the robust design problem. We introduce the design performance function, the design variables, the design environment parameters and the constraint functions. Section 4 is committed to defining and solving the nonlinear optimisation problem. Finally, we wrap up the paper with some concluding remarks in Section 5.

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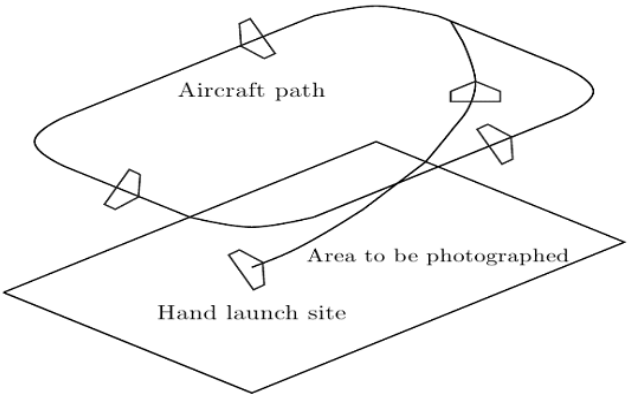


Figure 1. Mission profile.

2.0 MISSION REQUIREMENTS

The mission profile and the available technology usually dictate the size and shape of the vehicle through payload requirements, however in this paper we take a different approach to aircraft sizing by considering a robust design algorithm.

We consider a small flying wing for a civil reconnaissance mission. The mission requires the vehicle to fly 15m off the ground and traverse an area of approximate 2km^2 in about ten minutes, as depicted in Fig. 1. Given the short take off and landing phases, the geometric sizing method considers a performance function valid for cruise. Moreover, it is assumed that the flying wing should be transported in a small suitcase. The transportation constraint and the number of geometric variables to define the design were the motivation to consider a flying wing over a normal aircraft configuration.

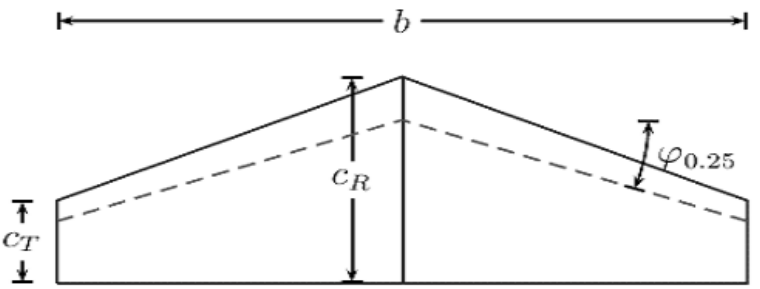


Figure 2. Flying wing planform geometry.

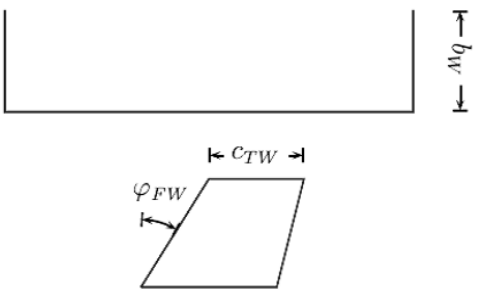


Figure 3. Winglet geometry.

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Given the mission requirements, we need to determine in a systematic way the basic geometric parameters of a small flying wing. In particular, span b, tip chord c_T, root chord c_R and sweep angle $\varphi_{0\cdot25}$, see Fig. 2. As byproducts we obtain payload W_p and cruise airspeed V. The flying wing is equipped with winglets whose size is parameterised in terms of the design variables or kept constant, see Fig. 3.

3.0 ROBUST DESIGN

In recent years, design in engineering is becoming increasingly based on mathematical models. Mathematical models involve multiple quantities, some of which are decided by designers in order to meet performance specifications, under given operation conditions, but some of which, influencing the performance, cannot be completely controlled by designers for spot operating conditions. The aim of robust design is to conceive products whose performance remains between specifications in the presence of variations in operating conditions. In setting up the framework for robust design[7] it is necessary to classify the quantities at play in the design task.

Design variables are quantities to be assigned by the designer. They should meet the performance requirements under given conditions. n design variables are written in the form of vector $x \in \mathbb{R}^n$, that is;

$$x = [x_1 \ x_2 \ \cdots \ x_n]^\mathsf{T} \qquad \ldots (1)$$

Design-environment parameters are variables that define the conditions of the environment under which the designed object is expected to operate. The designer does not have complete control over these parameters which cannot be accurately defined, but approximate or random values can be assigned for them. Design-environment parameters are written in the form of vector $p \in \mathbb{R}^v$, that is,

$$p = [p_1 \ p_2 \ \cdots \ p_v]^\mathsf{T} \qquad \ldots (2)$$

Performance functions define the performance of the design in terms of the design variables and the design-environment variables. The performance functions are written in the form of vector $f \in \mathbb{R}^m$ as following;

$$f = [f_1(x,p) \ \ f_2(x,p) \ \ \cdots \ \ f_m(x,p)]^\mathsf{T} \qquad \ldots (3)$$

It is interesting to observe that n, m and v need not to be related in any way. However, if $m = n$ vector f constitutes a determined set of equations in x which means that the design task is too constrained to allow for robust design. Here, we have $n = 5$, $v = 2$ and $m = 1$.

If p_0 indicates the nominal operating conditions and assuming that Equation (3) is differentiable with respect to p, we have;

$$f(x, p_o + \Delta p) = f(x, p_o) + \left.\frac{\partial f}{\partial p}\right|_{p=p_o} \Delta p + \textit{High Order Terms} \qquad \ldots (4)$$

Note that by neglecting the high order terms and defining

$$\Delta f = f(x, p_0 + \Delta_p) - f(x, p_0), \quad F = \left.\frac{\partial f}{\partial p}\right|_{p=p_o}$$

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Equation (4) can be expressed as follows;

$$\Delta f = F\Delta p \qquad \ldots (5)$$

The Jacobian matrix of f with respect to p measures the sensitivity of the design performance to variations in the design-environment parameters. Hence, the robust design problem can be formulated as[7].

$$\arg \min_{x} \; S(x, p_0), \quad S(x,p_0) = F\,F^{\mathsf{T}} \qquad \ldots (6)$$

In this paper we consider the endurance equation for a small aircraft powered by electric energy as the performance function. The endurance equation developed in Ref. 8, see Appendix A, can be written as follows;

$$f(x,p) = E = \eta_m \frac{\frac{d}{\rho} C_T^{\frac{3}{2}}}{C_P} V_B C_B \left[\frac{V^2 b(c_R + c_T)}{4} C_D \right]^{-\frac{3}{2}} \qquad \ldots (7)$$

This endurance equation is expressed in terms of electric power plant characteristics η_m, C_T, C_p, d, C_B and V_B, aircraft drag coefficient C_D, and geometric characteristics c_R, c_T and b. In the following, it is assumed that the battery capacity and the battery voltage remain constant. In order to evaluate the performance function[7] it is necessary to compute the aircraft polar to obtain the C_D. Assuming an ideal cruise condition we have;

$$L = W,\; T = D$$

and then

$$C_L = \frac{4(W_m + W_p)}{\rho V^2 b(c_R + c_T)}$$

Finally, computing the C_L, from the drag polar, the C_D can be obtained.
Hence, in terms of the robust design framework one has

$$f(x,p) = E(c_R, c_T, b, W_p, V, C_T, C_p)$$

$$[\, x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \,]^{\mathsf{T}} \;\; = \;\; [\, c_R \;\; W_p \;\; b \;\; V \;\; c_T \,]^{\mathsf{T}}$$

$$[\, p_1 \; p_2 \,]^{\mathsf{T}} = [\, C_T \;\; C_p \,]^{\mathsf{T}}$$

Note that the resulting robust design problem involves just a single objective design[9] due to the fact that the problem only refers to the cruise condition.

In order to define a baseline planform the following set of constraint functions is considered.

- *Taper Ratio*. The taper ratio is the ratio between root and tip chords, that is, $\lambda = c_T/c_R$ (as defined in Ref. 10), and we consider;

$$0{\cdot}375 \le \lambda \le 1 \qquad \ldots (8)$$

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- *Sweep Angle*. The sweep angle at the quarter chord line for a wing with trailing edge as shown in Fig. 2 is given by;

$$\text{Tan}\,(\varphi_{0\cdot 25}) = \frac{3}{2}\left(\frac{c_R - c_T}{b}\right)$$

The main aerodynamical effect of the sweep angle is to shift the line of action of the lift force modifying the longitudinal dynamic characteristics of the flying wing. Given that in this paper, the dynamic characteristics of the flying wing are not addressed, we select the sweep angle domain in such a way that the resulting flying wing preserves the geometry portrayed in Fig. 2. We select;

$$10^{\circ} \leq \arctan\left[\frac{3}{2}\left(\frac{c_R - c_T}{b}\right)\right] \leq 45^{\circ} \quad \ldots (9)$$

- *Wing load*. We consider an empirical rule from radio controlled model aircraft builders, concerning wing load. This rule states that if W_L satisfies

$$17\cdot 9523\text{N/m}^2 \leq W_L \leq 29\cdot 9205\text{N/m}^2 \quad \ldots (10)$$

then the resulting aircraft will have good characteristics to be flown using radio control[11].

- *Cruise airspeed*. Flying at low Reynolds number present a major challenge in terms of adequate lift generation as well as stability and control requirements. Thus, we propose a cruise airspeed such that the Reynolds number is inside the following range[12]

$$8\cdot 0E04 \leq \frac{\rho V c_M}{\mu} \leq 18\cdot 0E04 \quad \ldots (11)$$

- *Aircraft size*. As a measure of the aircraft size l_w we consider the diagonal of the equivalent rectangular wing plan form with side lengths b and $_{CR}$, that is, $l_w = \sqrt{c_R^2 + b^2}$ Concerning the aircraft size we impose;

$$0\cdot 8\text{m} \leq l_w \leq 1\cdot 2\text{m} \quad \ldots (12)$$

It is important to underline that operational autonomous aerial vehicles that perform useful work have considerably higher wing loadings and aircraft speeds than the values proposed in this work.

4.0 ROBUST DESIGN OPTIMISATION PROBLEM

The single-point robust design optimisation problem is formulated as the minimisation of the sensitivity function[6] with respect to the design-environment parameters vector p, subject to the constraint functions (8)-(12), that is;

$$\textit{Minimise} \quad S(x, p_0) = \left(\frac{\partial E}{\partial p_1}\right)^2\Bigg|_{p=p_o} + \left(\frac{\partial E}{\partial p_2}\right)^2\Bigg|_{p=p_o}$$

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$$
\textit{subject to}\begin{cases}
g_1(x)=\dfrac{x_5}{x_1}-1\le 0 & g_2(x)=-\dfrac{x_5}{x_1}+0\cdot 375\le 0\\
g_3(x)=\dfrac{3}{2}\dfrac{x_1-x_5}{x_3}-1\le 0 & g_4(x)=-\dfrac{3}{2}\dfrac{x_1-x_5}{x_3}+0\cdot 1763\le 0\\
g_5(x)=\dfrac{2(x_2+W_m)}{(x_1+x_5)x_3}-3\cdot 05\le 0 & g_6(x)=-\dfrac{2(x_2+W_m)}{(x_1+x_5)x_3}+1\cdot 83\le 0\\
g_7(x)=\dfrac{\rho}{\mu}c_M x_4-18\cdot 0E+04\le 0 & g_8(x)=-\dfrac{\rho}{\mu}c_M x_4+8\cdot 0E+04\le 0\\
g_9(x)=\sqrt{x_1^2+x_3^2}-1\cdot 2\le 0 & g_{10}(x)=-\sqrt{x_1^2+x_3^2}+0\cdot 8\le 0
\end{cases}
$$

where the mean aerodynamic chord is expressed as;

$$c_M=\frac{2}{3}\frac{c_R^2+c_Rc_T+c_T^2}{c_R+c_T}=\frac{2}{3}\frac{x_1^2+x_1x_5+x_5^2}{x_1+x_5}$$

4.1 Evolutionary Algorithm

In this work we use an Evolutionary Algorithm (EA) based on Differential Evolution, Equation (14). This is a population-based direct global search technique, and uses a population P of size N,

$$P=\{x_{(1,G)}, x_{(2,G)}, \cdots, x_{(N,G)}\}$$

where each individual $x_{(i,G)}$ is represented by a real-number vector with dimension n For each individual $x_{(i,G)}=\{x_1, x_2 ..., x_n\}$ in generation G, a trial vector is created applying mutation and crossover operators.

4.1.1 Mutation

$$v_{(i,G)}=x_{(r1,G)}+F\,[x_{(r2,G)}-x_{(r3,G)}]$$

where $r_1, r_2, r_3, \in [1,2,3, \cdots, N]$ are, randomly generated and, mutually independent integers among them and from the reference vector index $x_{(i,G)}$. $F\in[0, 2]$ is a constant factor controlling the amplification of the difference $x_{(r2,G)}-x_{(r3,G)}$.

4.1.2 Crossover

With the aim of improving diversity in the generation of parameter vectors, trial vectors are generated by applying a crossover operator between mutant and reference vectors as follows:

$$
\begin{aligned}
u_{(ji,G+1)}&=v_{(ji,G+1)} \quad \textit{if } (randb(j)\le CR) \text{ or } j=rnbr(i)\\
u_{(ji,G+1)}&=x_{(ji,G)} \quad \textit{if } (randb(j)>CR) \text{ and } j\ne rnbr(i)
\end{aligned}
\quad , \; j=1,2, \cdots, n
$$

In the above equation, randb(j) is the j-th evaluation of a randomly generated number created

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in the set [0, 1]. $CR \in [0, 1]$ is a constant. Both, F and CR are user-defined parameters. rnbr(i) is a random integer in the set $[1,2, \ ,n]$, assuring that $u_{(i,G+1)}$ receives at least one parameter from $v_{(ji,G+1)}$.

4.1.3 Selection

Once the trial vector is generated, its objective function is evaluated and compared with the reference vector to decide which one pass to next generation ($G = 1$) as follows:

For the unconstrained version of the differential evolution algorithm, comparison between reference and trial vectors is done on the basis of objective function value. i.e. if $f(u_{(i,G+1)}) < f(x_{(i,G)})$, the trial vector $u_{(i,G+1)}$ replaces the reference vector $x_{(i,G)}$, otherwise the reference vector is maintained for the next generation. For the present case, being the problem a constrained one, the following rules are considered when comparing the reference and trial vectors.

1. If both vectors are feasible: The one with the lower objective function value survives to the next generation.
2. If one vector is feasible and the other is not: the feasible vector survives to the next generation.
3. If both vectors are unfeasible: The one with lower constraints violations survives to the next generation.

4.1.4 Pseudocode

```
BEGIN
Initialise Generation  G0
 Evaluate fobj for x(i,G0) in G0, i = 1,2, ... , N
  For G = 1 to Gmax
    Begin
       In each generation do:
       Begin
          For each reference vector x(i,G), create a trial vector v(i,G)
          Evaluate the trial vector v(i,G)
          Based on fobj decide whether x(i,G) or v(i,G) passes to next generation
       End
    End
END
```

4.1.5 Parameters values

Propeller characteristics, in terms of thrust, power and efficiency, were obtained as functions of $J = V/(\omega d)$ using the QPROP software[13]. For an APC 8 × 6 propeller, we obtain the curves shown in Fig. 4. The following parameters were taken as reference values in the optimisation process: $C_T = 0{\cdot}0726227$, $C_P = 0{\cdot}0634168$, which coincide to a maximum propeller efficiency $\eta_p = 0{\cdot}7700827$ for $J = 0{\cdot}67$. Therefore, we have;

$$p_0 = [0{\cdot}0726227 \quad 0{\cdot}0634168]^T$$

We consider a lithium polymer battery with the following characteristics $C_B = 2628$As and $V_B =$

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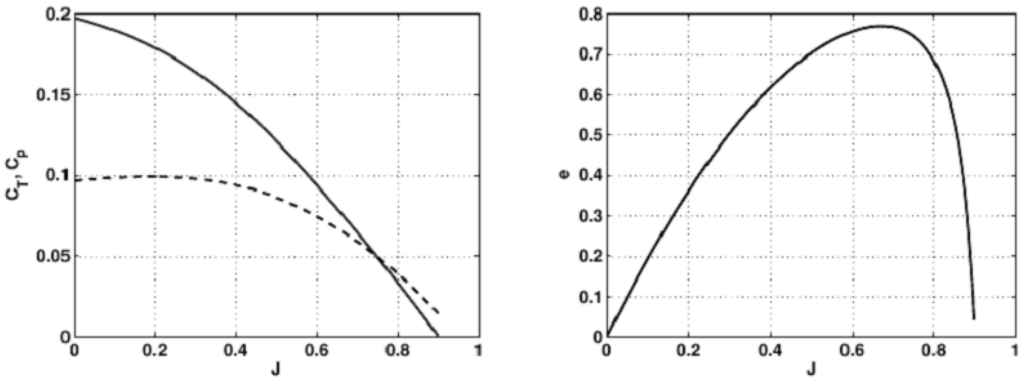


Figure 4. (Left) Thrust coefficient (continuous line), power coefficient (dashed line) (Right) Propeller efficiency.

11·1V. The empty weight was estimated to be $W_m = W_{eq} + \rho_e S$ with W_{eq} 0·1352kg and ρ_e 1·082134kg/m^2. We consider $\eta_m = 0{\cdot}8$. The wing aerofoil is an Eppler66. The winglet geometric parameters are $b_W = 0{\cdot}15b$, $c_{TW} = 0{\cdot}7_{CT}$ and $\varphi_{FW} = 45°$. The winglet aerofoil profile is a NACA0009.

The aerodynamic parameters were computed using the Computational Fluid Dynamics code CMARC. Concerning air parameters, air density and air viscosity, we consider their values at the altitude and average temperature of Mexico City, that is, $\rho = 0{\cdot}9\text{kgm}^{-3}$ and $\mu = 13{\cdot}59E-06\ \text{N s m}^{-2}$.

5.0 RESULTS

The algorithm was allowed to run during 100 generations, and 25 independent runs were executed. The population size was set to 50 individuals, summarising 5,050 solution candidates, each run, being evaluated (including the initial generation) each one took one minute approximately, in a 2·2GHz processor. The parameters for the Differential Evolution optimiser were set to $F = 0{\cdot}5$ and $CR = 0{\cdot}9$. The design variables of the best, median and worst geometric configuration, at generation 100, is summarised in Table 1 while the corresponding sensitivity values are presented in Table 2.

Table 1
Design variables at generation 100

Design variable	c_R	W_p	b	V	C_T
Best	0·3340	0·4672	1·1521	10	0·1978
Worst	0·3350	0·4632	1·1520	10	0·1936
Median	0·3340	0·4644	1·1479	10	0·1976

Table 2
Sensitivity values

Sensitivity	S
Best	6·32901 E09
Worst	6·49631 E09
Median	6·38276 E09

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In Table 3 the constraint values for the best wing are summarised. As it can be verified the constraint values are inside their allowed domain.

Table 3
Constraint values for the best wing

Constraint	λ	$\varphi_{0\cdot25}$	R_e	W_L	l_W
Best wing	0·5922	10·0556°	179,942	18·8548	1·1995

The evolution of sensitivity mean over the 25 runs is shown in Fig. 5, as can be observed it is minimised. The sensitivity values for the worst and the best individuals every ten generations are also shown. The wing polar of the best individual is plotted in Fig. 6. In this figure, along with the corresponding point for the operating condition that minimises the sensitivity S (the result obtained from the optimisation process), two additional important aerodynamic conditions are plotted. The first one corresponds to the operational point that maximises the ratio C_L/C_D. This condition allows the design to maximise the range of the aircraft. The second one corresponds to the point that maximises the ratio C_L^3/C_D^2. This operating point will maximise the endurance of the aircraft. From the polar plot, we can observe that the point minimising the

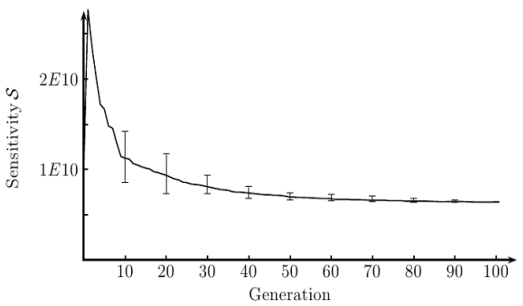


Figure 5. Convergence plot of the sensitivity mean over 25 runs. Vertical lines represent the sensitivity value for the worst (top) and best (bottom)

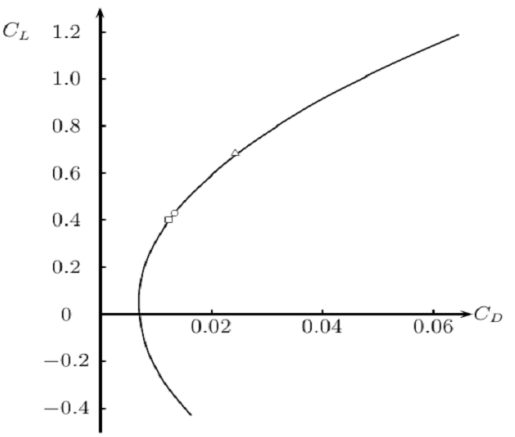


Figure 6. Aircraft polar of the best geometric configuration. ○ operation point to minimise sensitivity. □ operation point to maximise the ratio C_L/C_D. Δ operation point to maximise the ratio C_L^3/C_D^2.

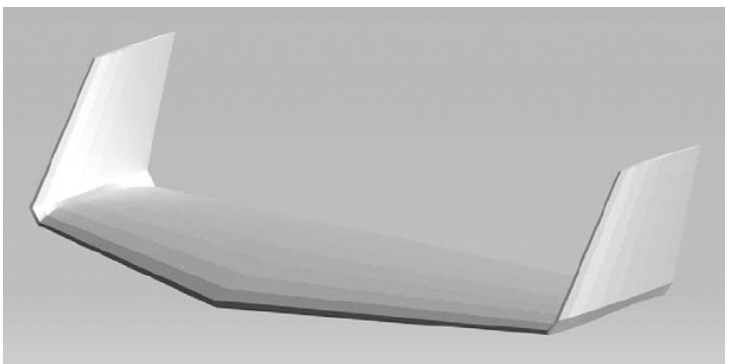


Figure 7. Geometric configuration of the best solution.

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sensitivity S lays between the region of maximum range and maximum Endurance, and it is closer to the point maximising the range. So we can conclude that the design minimises the sensitivity S without excessively degrading the maximum range capability of the aircraft. Finally, the best flying wing geometry is shown in Fig. 7.

6.0 CONCLUSIONS

A model based robust design optimisation technique has been applied to propose a geometric sizing method for a small flying wing powered by electric energy. The proposed method aims to reduce the effect of the power plant characteristics on aircraft endurance. The design variables have been constrained in order to reduce their domain.

The optimisation problem has been solved through an evolutionary algorithm based on differential evolution. After 100 generations the mean of the objective function (sensitivity) is reduced from 1·1043E10 to 6·3290E09 giving an improvement of 47·14%.

The obtained results give to the designer a first approach to define basic geometric design variables of the flying wing, in terms of the robustness with respect to variations in power plant characteristics. Moreover, the proposed method also indicates to the designer some consequences of having robustness, such as the fact that the operational point for robustness does not exactly coincide with the operational point for aerodynamic efficiency.

The proposed method is intended as a starting point for further studies that may provide detailed insights on the complex interactions that show up in an authentic aircraft design context. For instance, by formulating other robust design scenarios modifying the definition of the design variables and design-environment parameters. These new robust design scenarios will enhance the possibilities for the designer.

7.0 ACKNOWLEDGMENTS

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Appendix A

Motor efficiency is defined as

$$\eta_m = P/P_e \qquad \ldots (13)$$

Considering that the propeller thrust and propeller power are defined in Ref (2);

$$T = \rho\omega^2 d^4 C_T$$

$$P = \rho\omega^3 d^5 C_P$$

the propeller power can be expressed as follows[2];

$$P = \frac{1}{\rho^{\frac{1}{2}}} \frac{1}{d} \frac{C_P}{C_T^{3/2}} T^{\frac{3}{2}} \qquad \ldots (14)$$

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on the other hand, the electric power can be expressed as;

$$P_e = I_B V_B \quad \ldots (15)$$

Replacing Equations (14) and (15) into (13) and solving for I_B, we obtain

$$I_B = \frac{C_P T^{\frac{3}{2}}}{\eta_m \rho^{\frac{1}{2}} d V_B C_T^{3/2}} \quad \ldots (16)$$

Endurance for an electric powered plant is defined as

$$E = C_B / I_B \quad \ldots (17)$$

According to Equations (16) and (17) one has

$$E = \frac{\eta_m \rho^{1/2} d V_B C_B C_T^{3/2}}{C_P T^{3/2}} \quad \ldots (18)$$

Now, form the cruise condition, we have

$$T = \frac{1}{2} \rho S_w V^2 C_D \quad \ldots (19)$$

Finally, replacing Equation (18) into (19) the endurance Equation (7) is obtained.

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