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Nonlinear stationary magnetoconvection in a rotating fluid

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We investigate finite-amplitude magnetoconvection in a rotating fluid in the presence of a vertical magnetic field when the axis of rotation is parallel to a vertical magnetic field. We derive a nonlinear, time-dependent, one-dimensional Landau–Ginzburg equation near the onset of stationary convection at supercritical pitchfork bifurcation when

$$T_a \leqslant T_a^* = \frac{[(\pi^2+q_c^2)^2+Q\pi^2]\,[Q(\sigma_2/\sigma_1-1)\,\pi^2-(1+1/\sigma_1)\,(\pi^2+q_c^2)^2]}{Q\,(\pi^2(1/\sigma_1+\sigma_2/\sigma_1)\,(\pi^2+q_c^2)}$$

and a nonlinear time-dependent second-order ordinary differential equation when $T_a = T_a^*$ (from below). $T_a = T_a^*$ corresponds to codimension-two bifurcation (or secondary bifurcation), where the threshold for stationary convection at the pitchfork bifurcation coincides with the threshold for oscillatory convection at the Hopf bifurcation. We obtain steady-state solutions of the one-dimensional Landau–Ginzburg equation, and discuss the solution of the nonlinear time-dependent second-order ordinary differential equation.

1. Introduction

Magnetoconvection, thermohaline convection and convection in binary liquids are examples of double-diffusive systems. In magnetoconvection, the temperature and magnetic field provide the two diffusivities. Overstability is a characteristic feature of double-diffusive convection. This means that for magnetoconvection the principle of exchange of stabilities is not valid, and the first instability can be either stationary or oscillatory. Both double-diffusive convection and convection in a rotating fluid are capable of exhibiting stationary convection at a pitchfork bifurcation, oscillatory convection at a Hopf bifurcation and stationary convection at a Takens–Bogdanov bifurcation (a bifurcation of codimension two or secondary bifurcation). The pitchfork and Hopf bifurcations correspond to primary bifurcations (bifurcations of codimension-one). The rotating double-diffusive system is consequently expected to show a line of codimension-two points that accumulate at a codimension-three point (a tertiary bifurcation).

In this paper we shall consider the problem of nonlinear stationary

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magnetoconvection in a rotating fluid at a supercritical pitchfork bifurcation. We restrict attention to an electrically conducting fluid in an external magnetic field $\mathbf{H}_0 = H_0 \hat{\mathbf{e}}_z$ (which is kept rotating at a constant angular velocity $\mathbf{\Omega} = \Omega \hat{\mathbf{e}}_z$ about the z axis) lying between two horizontal boundaries, which are dynamically free and conductive. The advantage of this choice is that all the equations and boundary conditions may be satisfied by products of sine and cosine functions. In a dimensionless formulation, five parameters are required for the description of the motion: Rayleigh number $R = g\alpha\Delta T d^3/\nu\kappa$, thermal Prandtl number $\sigma_1 = \nu/\kappa$, magnetic Prandtl number $\sigma_2 = \nu/\eta$, Taylor number $T_a = 4\Omega^2 d^2/\nu^2$ and Chandrasekhar number $Q = \mu_m H_0^2 d^2/4\pi\rho_0 \nu\eta$. Here g is the acceleration due to gravity, ΔT is the temperature difference between bottom and top, ν is viscosity, η is magnetic diffusivity (or electrical resistivity), κ is the thermal diffusivity of the incompressible fluid and d is the depth of the layer (or the distance between two horizontal boundaries).

In Sec. 2, we write down the basic equations. Since the bifurcation is a continuous one, only a slow modulation of the convective roll pattern is allowed by the fluid equations near onset. In Sec. 3, we revisit the linear stability analysis discussed by Chandrasekhar (1961) and obtain the pitchfork and Hopf bifurcations, which are bifurcation points of codimension one, and the Takens–Bogdanov bifurcation, corresponding to bifurcation points of codimension two, which accumulate at a codimension-three point. In fact (Cross and Hohenberg 1993), it is possible to tune the system parameters so that a number of eigenvalues, not related by symmetry, become unstable together. These are known as bifurcations of codimension n if n system parameters must be tuned: $n = 3$ in a rotating double-diffusive system on a triple-diffusive system like thermohaline magnetoconvection (in which temperature, saline concentration and magnetic field act as three diffusivities), and $n = 4$ for a rotating thermohaline magnetoconvection system. Now the centre manifold is higher-dimensional, and the normal-form equations must be developed as polynomials in the amplitudes of all the marginal modes. The behaviour as the system parameters are varied slightly away from the codimension-n point can be investigated using the normal forms (universal unfolding). The full behaviour is often complicated, depending on various signs and ratios of coefficients in the normal forms.

The time evolution of a general pattern is developed in Sec. 4 in a region where $T_a < T_a^*$ by means of multiple-scale perturbation theory. In Sec. 4, we derive a two-dimensional nonlinear time-dependent Laundau–Ginzburg equations in a complex amplitude A, with real coefficients near a supercritical pitchfork bifurcation. The phase of the complex amplitude A describes changes in the position and direction of rolls, and its magnitude modulates the intensity of the convective motion. One important result from the two-dimensional nonlinear time-dependent Landau–Ginzburg equation is the presence of defects (topological singularities) in the rolls. The one-dimensional nonlinear time-dependent Landau–Ginzburg equation in which we are interested in this paper cannot describe defects, but, following Cross and Hohenberg (1993), one can study the dynamics of defects of a two-dimensional nonlinear time-dependent Landau–Ginzburg equation. Following Segel (1969), we consider in Sec. 4 the effects of sidewalls on magnetoconvection in a rotating fluid by considering a system of finite lateral width l.

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In Sec. 5, we consider a region where $T_a \to T_a^*$ from below, and study the onset of stationary convection by deriving a nonlinear second-order time-dependent ordinary differential equation and discuss its solution.

In Sec. 6, we give our conclusions.

2. Basic equations

Let us consider an electrically and thermally conducting fluid between two horizontal planes with adverse temperature gradients, which is kept rotating at a constant angular velocity $\mathbf{\Omega}$. We assume that the external magnetic field $\mathbf{H}_0$ is in the z direction and d is the vertical distance between the two planes (d is the depth of the layer). These planes bound an incompressible, electrically and thermally conducting fluid of density ρ_0, viscosity μ and electrical resistivity η. μ_m is the magnetic permeability, g is the acceleration due to gravity, β is the adverse temperature gradient, α and κ are respectively the coefficients of thermal expansion and thermal diffusivity of the fluid. We use a Cartesian system of coordinates whose dimensionless vertical coordinate z and dimensionless horizontal coordinates are scaled with d. The velocity vector $\mathbf{v} = (u, v, w)$, temperature θ, time t, pressure p and magnetic field vector $\mathbf{H} = (H_x, H_y, H_z)$ are non-dimensionalized by scales κ/d, βd, d^2/κ, $\mu\kappa/d^2$ and $\kappa H_0/\eta$.

In the Boussinesq approximation, the equations that describe the motion of a rotating and electrically conducting fluid in an external magnetic field are (Chandrasekhar 1961)

$$\boldsymbol{\nabla}\cdot\mathbf{v} = 0, \qquad \boldsymbol{\nabla}\cdot\mathbf{H} = 0, \tag{1}$$

$$\sigma_1^{-1}\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\right)\mathbf{v} - Q\sigma_2\sigma_1^{-1}(\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{H} - Q\frac{\partial\mathbf{H}}{\partial z}$$
$$= -\boldsymbol{\nabla}(p + \tfrac{1}{2}Q\sigma_2\sigma_1^{-1}|\mathbf{H}|^2 + QH_z - \tfrac{1}{8}T_a\sigma_1|\hat{\mathbf{\Omega}}\times\mathbf{r}|^2) + Re\hat{\mathbf{e}}_z + \nabla^2\mathbf{v} + T_a^{1/2}(\mathbf{v}\times\hat{\mathbf{\Omega}}), \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\right)\theta = w + \nabla^2\theta, \tag{3}$$

$$\sigma_2\sigma_1^{-1}\frac{\partial\mathbf{H}}{\partial t} - \boldsymbol{\nabla}\times(\mathbf{v}\times\hat{\mathbf{e}}_z) - \sigma_2\sigma_1^{-1}\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{H}) = \nabla^2\mathbf{H}, \tag{4}$$

where $\hat{\mathbf{\Omega}}$ is the unit vector along the axis of rotation. The Rayleigh number $R = \beta g\alpha d^4/\kappa\nu$, the thermal Prandtl number $\sigma_1 = \nu/\kappa$, the magnetic Prandtl number $\sigma_2 = \nu/\eta$, the Taylor number $T_a = 4\Omega^2 d^4/\nu^2$ and the Chandrasekhar number $Q = \mu_m H_0^2 d^2/4\pi\rho_0\nu\eta$ are non-dimensional numbers. In this paper, we consider the case where the axis of rotation coincides with the vertical ($\hat{\mathbf{\Omega}} = \hat{\mathbf{e}}_z$). The nature of magnetoconvection in rotating fluids depends crucially on the ratio $\sigma_2/\sigma_1 = \kappa/\eta$ of the thermal diffusivity κ to the magnetic diffusivity η. When $\sigma_2/\sigma_1 \ll 1$, as it is in laboratory experiments or in the Earth's core, convection arises first as steady motion, and the main effect of the field is to reduce heat transport. In astrophysical contexts, σ_2/σ_1 is usually very large (owing to radiative heat transport).

It is convenient to reduce (1)–(4) to a single equation as follows.

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The curl of (2) gives

$$\left(\frac{1}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\boldsymbol{\omega}=\frac{1}{\sigma_1}\boldsymbol{\nabla}\times(\mathbf{v}\times\boldsymbol{\omega})-\frac{Q\sigma_2}{\sigma_1}\boldsymbol{\nabla}\times(\mathbf{H}\times\mathbf{J})$$
$$+Q\frac{\partial\mathbf{J}}{\partial z}+R(\boldsymbol{\nabla}\theta\times\hat{\mathbf{e}}_z)+T_a^{1/2}\frac{\partial\mathbf{v}}{\partial z},\quad(5)$$

where the vorticity $\boldsymbol{\omega}=\boldsymbol{\nabla}\times\mathbf{v}$ and the current $\mathbf{J}=\boldsymbol{\nabla}\times\mathbf{H}$. The curl of (5) in turn gives, after use of (1),

$$\left(\frac{1}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2\mathbf{v}-Q\frac{\partial}{\partial z}(\nabla^2\mathbf{H})-R\left(\nabla^2\theta\hat{\mathbf{e}}_z-\boldsymbol{\nabla}\frac{\partial\theta}{\partial z}\right)+T_a^{1/2}\frac{\partial\boldsymbol{\omega}}{\partial z}$$
$$=\frac{1}{\sigma_1}\boldsymbol{\nabla}\times[\boldsymbol{\nabla}\times(\boldsymbol{\omega}\times\mathbf{v})]-\frac{Q\sigma_2}{\sigma_1}\boldsymbol{\nabla}\times[\boldsymbol{\nabla}\times(\mathbf{J}\times\mathbf{H})].\quad(6)$$

Taking the curl of (4), we get

$$\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\mathbf{J}-\frac{\partial\boldsymbol{\omega}}{\partial z}=\frac{\sigma_2}{\sigma_1}\nabla\times[(\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{v}-(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{H}].\quad(7)$$

Now taking the scalar product of (5), (6), (4) and (7) with $\mathbf{e}_z$, we get

$$\left(\frac{1}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\xi-Q\frac{\partial J_z}{\partial z}-T_a^{1/2}\frac{\partial w}{\partial z}=-\frac{1}{\sigma_1}(\mathbf{v}\cdot\boldsymbol{\nabla})\xi+\frac{Q\sigma_2}{\sigma_1}(\mathbf{H}\cdot\boldsymbol{\nabla})J_z,\quad(8a)$$

$$\left(\frac{1}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2 w-Q\frac{\partial}{\partial z}(\nabla^2 H_z)-R\nabla_h^2\theta+T_a^{1/2}\frac{\partial\xi}{\partial z}$$
$$=\hat{\mathbf{e}}_z\cdot\left[\frac{1}{\sigma_1}\boldsymbol{\nabla}\times\left((\mathbf{v}\cdot\boldsymbol{\nabla})\boldsymbol{\omega}-\frac{Q\sigma_2}{\sigma_1}\boldsymbol{\nabla}\times[(\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{J}]\right)\right],\quad(8b)$$

$$\sigma_2\sigma_1^{-1}\frac{\partial H_z}{\partial t}-\frac{\partial w}{\partial z}-\nabla^2 H_z=\hat{\mathbf{e}}_z\cdot[\boldsymbol{\nabla}\times(\mathbf{v}\times\hat{\mathbf{e}}_z)],\quad(8c)$$

$$\left(\sigma_2\sigma_1^{-1}\frac{\partial}{\partial t}-\nabla^2\right)J_z-\frac{\partial\xi}{\partial z}=\frac{\sigma_2}{\sigma_1}\hat{\mathbf{e}}_z\{\boldsymbol{\nabla}\times[(\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{v}-(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{H}]\},\quad(8d)$$

where

$$\boldsymbol{\nabla}\times[(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v}]=-\boldsymbol{\nabla}\times(\mathbf{v}\times\boldsymbol{\omega})=(\mathbf{v}\cdot\boldsymbol{\nabla})\boldsymbol{\omega}-(\boldsymbol{\omega}\cdot\boldsymbol{\nabla})\mathbf{v},$$
$$\boldsymbol{\nabla}\times[\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{H}]=-\boldsymbol{\nabla}\times(\mathbf{H}\times\mathbf{J})=(\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{J}-(\mathbf{J}\cdot\boldsymbol{\nabla})\mathbf{H},$$

and ξ and J_z are the z components of the vorticity $\boldsymbol{\omega}$ and current $\mathbf{J}$. Eliminating θ, H_z, ξ and J_z from (8*a–d*) and (3), we get

$$Lw=S,\quad(9)$$

where the linear operator L is given by

$$L=\left[\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{1}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)-Q\frac{\partial^2}{\partial z^2}\right]\left[\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{1}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2\right.$$
$$\left.-Q\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2\frac{\partial^2}{\partial z^2}-R\nabla_h^2\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\right]+T_a\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)^2\left(\frac{\partial}{\partial t}-\nabla^2\right)\frac{\partial^2}{\partial z^2}.\quad(10)$$

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and the nonlinear term S is given by

$$S = \left[\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{1}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)-Q\frac{\partial^2}{\partial z^2}\right]\left[-R\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla_h^2(\mathbf{v}\cdot\boldsymbol{\nabla})\,\theta\right.$$
$$+\frac{Q\sigma_2}{\sigma_1}\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2\frac{\partial}{\partial z}[(\mathbf{H}\cdot\boldsymbol{\nabla})\,w-(\mathbf{v}\cdot\boldsymbol{\nabla})\,H_z]$$
$$\left.+\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)\hat{\mathbf{e}}_z\cdot\left(\frac{1}{\sigma_1}\boldsymbol{\nabla}\times[(\mathbf{v}\cdot\boldsymbol{\nabla})\,\boldsymbol{\omega}]-\frac{Q\sigma_2}{\sigma_1}\boldsymbol{\nabla}\times[(\mathbf{H}\cdot\boldsymbol{\nabla})\,\mathbf{J}]\right)\right]$$
$$-\frac{Q\sigma_2}{\sigma_1}T_a^{1/2}\frac{\partial^2}{\partial z^2}\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)\hat{\mathbf{e}}_z\cdot\{\boldsymbol{\nabla}\times[(\mathbf{H}\cdot\boldsymbol{\nabla})\,\mathbf{v}-(\mathbf{v}\cdot\boldsymbol{\nabla})\,\mathbf{H}]\}$$
$$+\frac{T_a^{1/2}}{\sigma_1}\frac{\partial}{\partial z}\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)^2\left(\frac{\partial}{\partial t}-\nabla^2\right)(\mathbf{v}\cdot\boldsymbol{\nabla})\,\xi$$
$$-\frac{Q\sigma_2}{\sigma_1}T_a^{1/2}\frac{\partial}{\partial z}\left(\frac{\sigma_2}{\sigma_1}\frac{\partial}{\partial t}-\nabla^2\right)^2\left(\frac{\partial}{\partial t}-\nabla^2\right)[(\mathbf{H}\cdot\boldsymbol{\nabla})\,w-(\mathbf{v}\cdot\boldsymbol{\nabla})\,H_z]. \tag{11}$$

It should be noted that the variables θ, H_x, etc. have been eliminated only from the linear parts of (3), and (8*a–d*); hence the left-hand side of (9) contains the linear operator $\mathscr{L}$ operating on w while the right-hand side is the nonlinear part and contains the variables w, θ, H_x, etc.

3. Linear stability

Following Chandrasekhar (1961), we perform a linear stability analysis of the problem by substituting

$$w = W(z)\,e^{i(q_x x+q_y y)}\,e^{pd} \tag{12}$$

into the linearized version of (9), namely

$$Lw = 0,$$

and obtaining an equation

$$\left\{(D^2-q^2-p)\,(D^2-q^2)\left[\left(D^2-q^2-\frac{\sigma_2}{\sigma_1}p\right)\left(D^2-q^2-\frac{1}{\sigma_1}p\right)-QD^2\right]^2\right.$$
$$\left.+T_a D^2(D^2-q^2-p)\left(D^2-q^2-\frac{\sigma_2}{\sigma_1}p\right)^2\right\}W$$
$$=-Ra^2\left(D^2-q^2-\frac{\sigma_2}{\sigma_1}p\right)\left[\left(D^2-q^2-\frac{\sigma_2}{\sigma_1}p\right)\left(D^2-q^2-\frac{1}{\sigma_1}p\right)-QD^2\right]W. \tag{13}$$

In this section, we consider only idealized boundary conditions. Substituting $W = \sin\pi z$ in equation (13), we get a fifth-degree polynomial in p of the form

$$a_5 p^5+a_4 p^4+a_3 p^3+a_2 p^2+a_1 p+a_0 = 0, \tag{14}$$

where $a_0, a_1, a_2, a_3, a_4, a_5$ are real constants and depend on σ_1, σ_2, q, Q and R.

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Setting $p = 0$ and $R = R_s$ (the value of R for the onset of stationary convection) in (14), we get

$$R_s = \frac{(\pi^2+q^2)^3}{q^2} + \frac{Q\pi^2(\pi^2+q^2)}{q^2} + \frac{T_a\pi^2(\pi^2+q^2)}{q^2} - \frac{QT_a\pi^4(\pi^2+q^2)}{q^2[(\pi^2+q^2)^2+Q\pi^2]}. \quad (15)$$

For $Q = 0$, $T_a \neq 0$ and for $Q \neq 0$, $T_a = 0$ we can see from (15) that rotation or a magnetic field inhibits the onset of convective instability. However, when both are present ($Q \neq 0$, $T_a \neq 0$), instead of acting together, they will have conflicting tendancies. First, they will reduce the theshold for onset of convective instability, which will attain some minimum at $q = q_{sc}$, and then they will increase the threshold for the onset of convective instability. The critical wavenumber $q = q_{sc}$ is obtained by finding the minimum of $R_s(q)$, and is given by (Das and Bhattacharjee 1991)

$$0 = \frac{3(\pi^2+q_{sc}^2)^2}{q_{sc}^2} - \frac{(\pi^2+q_{sc}^2)^3}{q_{sc}^4} - \frac{Q\pi^2}{q_{sc}^4} - \frac{Ta\pi^2}{q_{sc}^4} + \frac{QT_a\pi^4}{q_{sc}^4}\frac{2q_{sc}^2(\pi^2+q_{sc}^2)+Q\pi^2+(\pi^2+q_{sc}^2)^2}{[(\pi^2+q_{sc}^2)^2+Q\pi^2]^2}. \quad (16)$$

The critical number R_{sc} is obtained by setting $q = q_{sc}$ in (15).

Setting $p = i\omega$ and $R = R_o$ (the value of R for the onset of oscillatory convection) in (14) and equating real and imaginary parts, we get

$$a_4\omega^4 - a_2\omega^2 + a_0 = 0, \quad (17a)$$

$$a_5\omega^5 - a_3\omega^3 + a_1\omega = 0. \quad (17b)$$

For marginal stability, $p = i\omega$, where ω is real. We can have marginal stability if either

(a) $\omega = 0$ and $a_0 = 0$ or
(b) $\omega^2 = [a_3 \pm (a_3^2 - 4a_1a_5)^{1/2}]/2a_5$,

with

$$a_3^2 > 4a_1a_5, \quad (18a)$$

$$a_4a_3^2 - 2a_1a_4a_3 - a_2a_3a_5 + 2a_0a_5^2 \pm (a_4a_3 + a_2a_5)(a_3^2 - 4a_1a_5)^{1/2} = 0. \quad (18b)$$

These are the conditions for primary bifurcations, namely (a) a pitchfork bifurcation corresponding to stationary convection, or (b) a Hopf bifurcation corresponding to oscillatory convection.

At the Takens–Bogdanov bifurcation (which is a secondary bifurcation or a bifurcation of codimension two), we have for idealized free–free boundary conditions

$$a_0 = a_1 = 0,$$

$$R_{sc} = R_{oc} = R_c, \quad q_{oc} = q_{sc} = q_c,$$

but for realistic rigid–rigid boundary conditions

$$a_0 = a_1 = 0,$$

$$R_{sc}(q_{sc}) = R_{oc}(q_{oc}), \quad q_{sc} \neq q_{oc}.$$

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$a_0 = a_1 = 0$ corresponds to

$$T_a = T_a^* = \frac{[(\pi^2+q^2)^2+Q\pi^2]^2\,[Q\pi^2(\sigma_2/\sigma_1-1)-(1+1/\sigma_1)\,(\pi^2+q^2)^2]}{Q\pi^4(1/\sigma_1+\sigma_2/\sigma_1)\,(\pi^2+q^2)}. \quad (19)$$

At $T_a = T_a^*$, (14) has $\omega = 0$ as a double zero, and we find relaxation oscillations for a suitable choice of parameters. Near the secondary bifurcation, the conductive state becomes unstable against both stationary and oscillatory modes at pitchfork and Hopf bifurcations (which are primary bifurcations), i.e. the real parts of two eigenvalues pass through zero nearly simultaneously. This violates the assumption made for deriving the nonlinear Landau–Ginzburg equation in Sec. 4. T_a^* given by (19) is physically meaningful only if T_a^* is positive, and this implies that

$$Q > Q_c = \frac{(\pi^2+q_c^2)^2\,(1+1/\sigma_1)}{\pi^2(\sigma_2/\sigma_1-1)}, \quad (20a)$$

$$\frac{\sigma_2}{\sigma_1} > 1. \quad (20b)$$

In laboratory experiments and in planetary cores, $\sigma_2/\sigma_1 \ll 1$, and T_a^* is always negative. Thus only primary bifurcations exist in laboratory experiments and in planetary cores. For $a_3^2 = 4a_1 a_5$, the system described by (14) has simultneous Hopf and pitchfork bifurcations, and quasiperiodic solutions can be obtained in the nonlinear regime (Arneodo *et al.* 1980). For $a_0 = a_1 = a_2 = 0$, by eliminating R and Q, we get $T_a = T_a^{**}$, where $\omega = 0$ corresponds to a triple zero and we have a bifurcation of codimension three corresponding to tertiary bifurcation. $T_a = T_a^{**}$ can be obtained by eliminating Q from

$$T_a = \frac{[(\pi^2+a^2)^2+Q\pi^2]^2\,[Q\pi^2(\sigma_2/\sigma_1-1)-(1+1/\sigma_1)\,(\pi^2+q^2)^2]}{Q\pi^4(\pi^2+q^2)\,(1/\sigma_1+\sigma_2/\sigma_1)}, \quad (19a)$$

$$T_a = (\pi^2+q^2)\left(\frac{\sigma_2^2}{\sigma_1^2}-\frac{2}{\sigma_1}-\frac{2\sigma_2}{\sigma_1}\right)[(\pi^2+q^2)^2+Q\pi^2]^2$$

$$-\frac{(\pi^2+q^2)^3\,[(\pi^2+q^2)^2+Q\pi^2]^2}{(\sigma_2/\sigma_1)\,\pi^2[(\pi^2+q^2)^2\,(2-2/\sigma_1)+(Q\pi^2/\sigma_1)\,(2+\sigma_2/\sigma_1)]}. \quad (19b)$$

Equation (19) gives a line of codimension-two points that accumulate at a codimension-three point obtained by eliminating Q from (19a, b).

4. Derivation of time-dependent nonlinear Landau–Ginzburg equation

The existence of a threshold (critical value of the Rayleigh number, namely $R = R_{sc}$) and the cellular structure (critical wavenuymber $q = q_{sc}$) for a fixed Taylor number T_a and a fixed Chandrasekhar number Q are the main characteristics of stationary magnetoconvection in a rotating fluid. In this section, we treat the region near the onset of stationary convection, i.e. we

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describe what happens as the Rayleigh number is slightly above its value at the onset of stationary convection ($R > R_{sc}$) by introducing ϵ as

$$\epsilon^2 = \frac{R - R_{sc}}{R_{sc}} \ll 1.$$

To simplify the problem, we assume the formation of cylindrical rolls. The axis of a cylindrical roll is taken to be the y axis, so that the y dependence goes from (9). The z dependence is contained entirely in the sine and cosine functions, which ensures that the idealized free–free boundary conditions are satisfied. For values of the control parameter R close to the threshold values R_{sc} ($\epsilon^2 \ll 1$), the structure of the short length scales will be insensitive to ϵ, but a slow modulation in space and time is possible by making use of the band of unstable solutions, and the linear growth is likely to saturate owing to nonlinear effects. This behaviour can be analysed by writing solutions of (1)–(4) in power series as

$$f = \epsilon f_0 + \epsilon^2 f_1 + \epsilon^3 f_2 + \dots, \tag{21}$$

where $f = (u, v, w, \theta, H_x, H_y, H_z)$ and we assume

$$w_0(x, y, z, t) = A(X, Y, T)\, e^{iq_{sc}x} \sin \pi z + \text{c.c.}, \tag{22}$$

where u_0, v_0, θ_0, H_{x0}, H_{y0} and H_{z0} are obtained from (22) and the linearized versions of (1)–(4). Here c.c. stands for the complex conjugate and $e^{iq_{sc}x} \sin \pi z$ is the critical mode for the linear problem at $R = R$ and $q = q_{sc}$. The amplitude $A(X, Y, T)$ is allowed to depend on the slow variables $X = \epsilon x$, $Y = \epsilon^{1/2} y$ and $T = \epsilon^2 t$. The differential operators can be expressed as

$$\left.\begin{aligned} \frac{\partial}{\partial x} &\to \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial X}, \qquad \frac{\partial}{\partial y} \to \epsilon^{1/2} \frac{\partial}{\partial Y}, \\ \frac{\partial}{\partial z} &\to \frac{\partial}{\partial z}, \qquad \frac{\partial}{\partial t} \to \epsilon^2 \frac{\partial}{\partial T}. \end{aligned}\right\} \tag{23}$$

With (21) and (23), the nonlinear equations (1)–(4) are transformed into a set of linear inhomogeneous equations. The solvability condition for the latter yield the amplitude equation. Using (21)–(23), (10) can be written as

$$L = L_0 + \epsilon L_1 + \epsilon^2 L_2 + \dots,$$

and one can see from (11) that the nonlinear contribution starts from the term of order ϵ^3. Substituting (21)–(23) into (9), we get, to the lowest order in ϵ (namely the coefficient of ϵ),

$$L_0 w_0 = 0,$$

which implies that

$$R_{sc} = \frac{\pi^2 + q_{sc}^2}{q_{sc}^2} + \frac{Q\pi^2(\pi^2 + q_{sc}^2)}{q_{sc}^2} + \frac{T_a \pi^2 (\pi^2 + q_{sc}^2)}{q_{sc}^2} - \frac{QT_a \pi^2 (\pi^2 + q_{sc}^2)}{q_{sc}^2} [(\pi^2 + q_{sc}^2)^2 + Q\pi^2].$$

To the next order in ϵ (namely the coefficient of ϵ^2), we get

$$L_1 w_0 + L_0 w_1 = 0.$$

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$L_1 w_0 = 0$ implies that $(\partial R_s/\partial q)_{q=s_{sc}=0}$, and hence $w_1 = 0$. Using the equation of continuity, we have $u_1 = 0$. Similarly, $v_1\, \theta_1\, H_{x1}, H_{y1}$ and H_{z1} can be obtained as

$$\left.\begin{aligned} v_1 &= -\frac{i\pi^2 T a^{1/2}}{4\sigma_1 q_{sc}^3(\pi^2+q_{sc}^2)}(A^2 e^{2iq_{sc}x} - \text{c.c.}),\\ \theta_1 &= \frac{R_{sc}}{2\pi(\pi^2+q_{sc}^2)}|A|^2 \sin^2 \pi z,\\ H_{x1} &= H_{y1} = 0,\\ H_{z1} &= \frac{\sigma_2}{\sigma_1(\pi^2+q_{sc}^2)}|A|^2 \sin^2 \pi z. \end{aligned}\right\} \tag{24}$$

To the next order in ϵ (namely the coefficient of ϵ^3), we get

$$L_0 w_2 = w_1 - L_2 w_0. \tag{25}$$

In order that (25) be solvable when $L_0 w_0 = 0$, one requires that its right-hand side be orthogonal to w_0, which is ensured if the coefficient of $\sin \pi z$ in $w_1 - L_2 w_0$ is zero. This implies that

$$\lambda_0 \frac{\partial A}{\partial T} - \lambda_1 \left(\frac{\partial}{\partial X} - \frac{i}{2q_{sc}}\frac{\partial^2}{\partial Y^2}\right)^2 A - \lambda_2 A + \lambda_3 |A|^2 A = 0, \tag{26}$$

where λ_0, λ_1, λ_2 and λ_3 are real coefficients in (26). Equation (26) is a time-dependent, nonlinear, two-dimensional Landau–Ginzburg equation describing magnetoconvection in a rotating fluid. When there is no rotation, λ_0, λ_1, λ_2 and λ_3 are always positive. Equation (26) describes the variations of the pattern on the slow time scale $\epsilon^2 t$, the slow spatial scale ϵx perpendicular to the rolls, and the slow spatial scale $\epsilon^{1/2} y$ parallel to the rolls. If the system is not invariant under rotations in the plane then the amplitude equation takes the form

$$\lambda_0 \frac{\partial A}{\partial T} - \lambda_1 \left(\frac{\partial^2 A}{\partial X^2} + \frac{\partial^2 A}{\partial Y^2}\right) - \lambda_2 A + \lambda_3 |A|^2 A = 0, \tag{27}$$

instead of (26), with different slow spatial scales in ϵx and ϵy in the x and y directions. This situation occurs in electrohydrodynamic convection in nematic liquid crystals (Bodenschatz *et al.* 1988), where the nematic director provides the microscopic anisotropy. Equation (27) also occurs in Taylor–Coutte flow, where axial and azimuthal directions are not equivalent. We thus have the somewhat paradoxical situation that the two-dimensional, nonlinear, time-dependent Landau–Ginzburg equation (26) for isotropic system is anisotropic, whereas for the anisotropic system of electrodynamic convection in nematic liquid crystals it takes the isotropic form of the two-dimensional, nonlinear, time-dependent Landau–Ginzburg equation (27). The reason for this is that the roll pattern breaks the rotational symmetry in the isotropic system so that transverse and longitudinal variations are qualitatively different, whereas in the anisotropic system they can be made the same by a simple scale change. These equations (26) and (27) correspond to reduced versions of the complete dynamics, which contain all the symmetries of the problem. They may be solved more easily and describe the dynamics of the system on long space–time scales close to the bifurcation point.

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One important limitation of the amplitude equation (26) is that it only describes situations in which the rolls are everywhere almost normal to a particular direction, labelled the x direction, and the roll orientation varies by an angle $U(\epsilon^{1/2})$. In the problem of rotating magnetoconvection, λ_0 is positive only if

$$T_a < T_a^* = \frac{[(\pi^2+q_c^2)^2+Q\pi^2]^2\,[Q\pi^2(\sigma_2/\sigma_1-1)-(1+1/\sigma_1)\,(\pi^2+q_c^2)^2]}{Q\pi^4(1/\sigma_1+\sigma_2/\sigma_1)\,(\pi^2+q_c^2)}.$$

$\lambda_0 = 0$ at $T_a = T_a^*$, and our scaling used in deriving (26) breaks down. In the next section, by using a new scaling for time, we derive at $T_a = T_a^*$ (from below) a nonlinear, second-order ordinary differential equation. λ_2 is always positive. λ_3 is positive only if

$$(\pi^2+q^2sc)^3+Q\pi^2(\pi^2+q_{sc}^2)+\pi^2T_a > \frac{\sigma_2}{\sigma_1},\ \frac{\pi^4}{q_{sc}^2}Q(\pi^2+q_{sc}^2)+\frac{\pi^4}{q_{sc}^2}\frac{1}{\sigma_1}T_a^{1/2}. \tag{28}$$

In this and the next section we assume that (23) is satisfied, so that λ_3 is always positive. λ_1 is positive for $0 < T_a < T_{ac}$ and negative for $T_a > T_{ac}$. In this section, we neglect the value of $T_a = T_{ac}$, which will make λ_1 zero. Dropping the time dependence and the y dependence from (26), we get

$$\frac{d^2A}{dX^2}+\frac{\lambda_2}{\lambda_1}\left(1-\frac{\lambda_3}{\lambda_2}|A|^2\right)A = 0. \tag{29}$$

For $\lambda_1 > 0$, the solution of (29) is given by

$$A(X) = A_0\tanh\left(\frac{X}{\Lambda}\right), \tag{30}$$

where

$$A_0 = \left(\frac{\lambda_2}{\lambda_3}\right)^{1/2},\qquad \Lambda = \left(\frac{2\lambda_1}{\lambda_2}\right)^{1/2}. \tag{31}$$

For $\lambda_1 < 0$, the solution of (29) is given by

$$A(X) = A_0\,\mathrm{sech}\left(\frac{X}{\Lambda}\right), \tag{32}$$

where

$$A_0 = 2\left(\frac{\lambda_2}{\lambda_3}\right)^{1/2},\qquad \Lambda = \left[\frac{2(-\lambda_1)}{\lambda_2}\right]^{1/2}. \tag{33}$$

If we consider a system with finite aspect ratio, and if L is the length of the system, then, following Segel (1969), we can show that

$$R_{sc}(L) = R_{sb}(q_{sb}) = R_{sc}(q_{sc})+\frac{\lambda_1}{\lambda_2}\frac{\pi^2}{L^2}R_{sc}(q_{sc}). \tag{34}$$

Here $R_{sc}(q_{sc})$ is a critical Rayleigh number corresponding to $L = \infty$ (when q_{sb} becomes q_{sc}). $R_s(L) = R_{sb}(q_{sb})$ is the critical Rayleigh number of the system of length L, and $R_{sb}(q)$ has a minimum when $q = q_{sb}$. Segel (1969) assumed that the

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horizontal surfaces are stressfree, but the lateral boundaries in the enclosure are no-slip. This forces the amplitude A to be zero at the lateral boundaries. Thus, for $\lambda_1 > 0$, we fix a lateral boundary at $X = 0$, and, for $\lambda_1 < 0$, we fix lateral boundaries at $X = -\frac{1}{2}L$ and $X = \frac{1}{2}L$. Thus for both $\lambda_1 > 0$ and $\lambda_1 < 0$, the amplitude will increase as one moves away from the lateral wall. Thus $\lambda_1 > 0$ implies that convection sets in when $R_{sb}(q_{sb}) > R_{sc}(q_{sc})$. Thus, for $\lambda_1 > 0$, finite L inhibits the onset of stationary convection. For $\lambda_1 < 0$, convection sets in at a Rayleigh number $R_{sb}(q_{sb})$ lower than $R_{sc}(q_{sc})$. Thus, for $\lambda_1 < 0$, finite L accelerates the onset of stationary convection by reducing its threshold.

5. Derivation of time-dependent nonlinear second-order ODE near onset of stationary convection when $T_a \to T_a^*$ (from below)

As $T_a \to T_a^*$ (from below), the coefficient of $\partial A/\partial T$ in (26), namely λ_0, becomes zero, and the analysis of Sec. 4 breaks down. To rederive an equation valid as $T_a \to T_a^*$ (from below), we assume that the amplitude A is allowed to depend on the slow time variable $T = \epsilon t$. We expand the dependent variables

$$f = (u, v, w, \theta, H_x, H_y, H_z)$$

as

$$f = \epsilon f_0 + \epsilon^2 f_1 + \dots, \tag{21}$$

where w_0 is given by (22) and u_0, v_0, θ_0, H_{x0}, H_{y0} and H_{z0} can be obtained from (22) and the linearized versions of (1)–(4). $A(T)$ is a slowly varying function of time only, and ϵ is given by

$$\frac{R - R_c}{R_c} = \epsilon^2.$$

The differential operator can be expressed as

$$\frac{\partial}{\partial z} \to \frac{\partial}{\partial z}, \qquad \frac{\partial}{\partial x} \to \frac{\partial}{\partial x}, \qquad \frac{\partial}{\partial t} \to \epsilon \frac{\partial}{\partial T}. \tag{35}$$

Using (35), the operator L of (10) can be written as

$$L = L_0 + \epsilon L_1 + \epsilon^2 L_2 + \dots \quad .$$

Substituting (21), (35) into (9), we get, to the lowest order in ϵ,

$$R_c = \frac{(\pi^2 + q_0^2)^3}{q_c^2} + \frac{Q\pi^2(\pi^2 + q_c^2)}{q_c^2} + \frac{Ta^*\pi^2(\pi^2 + q_0^2)}{q_0^2} - \frac{QT_a^*\,\pi^4(\pi^2 + q_0^2)}{q_0^2[(\pi^2 + q_0^2)^2 + Q\pi^2]}, \tag{36}$$

where T_a^* is given by (19). To the next order in ϵ, we have

$$L_1 w_0 = 0,$$

which implies that $\lambda = 0$. To the same order (the coefficient of ϵ^2), we get, as in Sec. 4, the values of u_1, v_1, w_1, θ_1, H_{x1}, H_{y1} and H_{z1}. Substituting these into (9), we get, to the order of ϵ^3,

$$B_0 \frac{d^2A}{dT^2} - \lambda_2 A + \lambda_3 |A|^2 A = 0. \tag{37}$$

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Here β_0 is real and positive, $\lambda_2 > 0$, and we assume that the condition (28) is satisfied, so that $\lambda_3 > 0$. Let

$$A = a(T)\, e^{i\phi},$$

where the phase ϕ is constant. Then (37) reduces to

$$\frac{d^2a}{dT^2} - \beta_1 a + \beta_2 a^3 = 0, \tag{38}$$

where $\beta_1 = \lambda_2/\beta_0 > 0$ and $\beta_2 = \lambda_3/\beta_0 > 0$. The solution of (38) can be obtained in terms of elliptic functions of the first kind.

Linear stabilization of (37) occurs only if $\beta_1 > 0$. For $\beta_1 > 0$, the solution of (37) saturates to an amplitude of the order of $(\lambda_2/\lambda_3)^{1/2}$. Thus the unstable mode undergoes a regime of linear growth, which is limited by nonlinearity when $\beta_2 > 0$. The system then settles into a stable homoclinic orbit with an amplitude modulation of the order of $(\lambda_2/\lambda_3)^{1/2}$.

6. Conclusions

We have studied magnetoconvection in a rotating fluid corresponding to a regime $T_a \leqslant T_a^*$. Equations (15) and (16) show that $R_s = R_{sc}$ and $q = q_{sc}$ correspond to minimum of $R_s(q)$, and the onset of stationary convection at the pitchfork bifurcation corresponds to the case of marginal stability. The Takens–Bogdanov bifurcation corresponds to $a_0 = a_1 = 0$, and, by eliminating the Rayleigh number, we get the value of $T_a = T_a^*$ given by (19). Near the onset of stationary convection (for $R_s > R_{sc}$ such that $(R_s - R_{sc})/R_{sc} = \epsilon^2$) and for the amplitude equation for $A(X, Y, T)$ is governed by the nonlinear, time-dependent, two-dimensional Landau–Ginzburg equation (26). For $T_a < T_a^*$, $\lambda_0 > 0$, but λ_1 can be positive, negative or zero. For $T_a < T_a^*$, our derivation of (26) breaks down when $\lambda_1 = 0$, and we get two stationary solutions, namely

$$A(x) = \begin{cases} A_0 \tanh\left(\dfrac{X}{\Lambda}\right) & \text{for } \lambda_1 > 0, \\ A_0 \operatorname{sech}\left(\dfrac{X}{\Lambda}\right) & \text{for } \lambda_1 < 0. \end{cases}$$

If $R_s(L) = R_{sb}(q_{sb})$ is the critical Rayleigh number of a system of finite length L and $R_{sb}(q_{sb})$ has a minimum when $q = q_{sb}$ then $R_{sb}(q_{sb})$ is connected with $R_{sc}(q_{sc})$ by (34). From (34), if λ_1 is positive then finite L inhibits the onset of stationary convection, but if it is negative then finite L accelerates the onset of stationary convection by reducing the threshold for the onset of stationary convection. For $T_a \to T_a^*$ (from below), the amplitude equation is governed by the nonlinear equation (37). The solution of (37) can be obtained as

$$A = a(t)\, e^{i\phi} \qquad (\phi = \text{const}),$$

where $a(T)$ can be written in terms of elliptic integrals of the first kind. For $T_a > T_a^*$, one gets coupled nonlinear Landau–Ginzburg equations with complex coefficients in $A_L(X, Y, T)$ and $A_R(X, Y, T)$. Here $A_R(X, Y, T)$ is the amplitude of the left-travelling wave and $A_R(X, Y, T)$ is the amplitude of the right-travelling wave. These coupled equations describe modulations of the waves on slow space

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and time scales, and correspond to oscillatory convection at the Hopf bifurcation. The special cases of pure travelling waves ($A_L = 0$ or $A_R = 0$) and standing waves ($A_L = A_R$) can be recovered. Matthews and Rucklidge (1993) have considered travelling and standing waves in magnetoconvection by assuming that A_L and A_R depend only on the slow time scale. In this paper, we have not considered the region of oscillatory convection corresponding to $T_a > T_a^*$. For $T_a \to T_a^*$ (from above), one gets a nonlinear second-order ordinary differential equation in the complex amplitude A of the type (Cross and Hohenberg 1993)

$$\frac{d^2A}{dT^2} - (\beta_1 - g_2|A|^2)\frac{dA}{dT} - \beta_2 A - g_1|A|^2 A - \beta_2 g_3\left(A\frac{dA^*}{dT} + A^*\frac{dA}{dT}\right)A = 0. \quad (39)$$

Here $\beta_1 = \beta_2 = 0$ defines the codimension-two point (corresponding to $a_0 = a_1 = 0$). When $\beta_1 = \beta_2 = 0$, Knobloch and Proctor (1985) have analysed (39) for the double-diffusive convection problem. The value of T_a^* given by (19) is positive if $Q > Q_c$ and $\sigma_2/\sigma_1 > 1$. In laboratory experiments and in planetary cores $\sigma_2/\sigma_1 \ll 1$, and hence T_a^* is always negative. Thus only primary bifurcations exist in laboratory experiments and planetary cores.

Equation (19) gives T_a^* as a function of Q, and this corresponds to a line of codimension-two points that accumulate at a codimension-three point obtained by eliminating R and Q from $a_0 = a_1 = a_2 = 0$. If we divide (14) by $a_5 p^2 + a_4 p + a_3$, we get

$$p^3 + M_2 p^2 + M_1 p + M_0 = 0, \quad (40)$$

where M_0, M_1 and M_2 depend on a_0, a_1, a_2, a_3, a_4 and a_5. By eliminating R and Q from $a_0 = a_1 = a_2 = 0$, we get $T_a = T_a^{**}$, where $\omega = 0$ corresponds to a triple zero and we have a bifurcation of codimension three, corresponding to a tertiary bifurcation. Following Arneodo *et al.* (1985), the amplitude equation near a point of codimension three is given by

$$\frac{dA}{dT^3} + \mu_2\frac{d^2A}{dT^2} + \mu_1\frac{dA}{dT} + \mu_0 A = g\left(A, A^*, \frac{dA}{dT}, \frac{dA^*}{dT}, \frac{d^2A}{dT^2}, \frac{d^2A^*}{dT^2}\right), \quad (41)$$

where g is a nonlinear function of A, A^*, dA^*/dT and dA^*/dT. Equation (41) can be written in the form of a Lorentz attractor by writing

$$\left.\begin{aligned} \frac{dA}{dT} &= B(T), \\ \frac{dB}{dT} &= C(T), \\ \frac{dC}{dT} &= g(A, A^*, B, B^*, C, C^*) - \mu_2 C - \mu_1 B - \mu_0. \end{aligned}\right\} \quad (42)$$

Thus we conclude that chaos is possible for magnetoconvection in a rotating fluid near a point of codimension three (for $T_a \to T_a^{**}$.

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