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Is kinematic geometry an internalized regularity?

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Abstract: A general framework for the explanation of perceptual phenomena as internalizations of external regularities was developed by R. N. Shepard. A particular example of this framework is his account of perceived curvilinear apparent motions. This paper contains a brief summary of the relevant psychophysical data, some basic kinematical considerations and examples, and several criticisms of Shepard's account. The criticisms concern the feasibility of internalization of critical motion types, the roles of simplicity and uniqueness, the contrast between classical physics and kinematic geometry, the import of perceived path curvilinearity, and the relation of perceptual and scientific knowledge.

Keywords: internalization of regularities; kinematic geometry; simplicity

1. Internalization of external regularities as a perceptual explanatory strategy

It is beyond doubt that millions of years of evolutionary processes have profoundly shaped the visual system. However, it is also beyond doubt that evolution is generally neglected by perception theorists, and that detailed accounts of its role are rather uncommon. A rare and welcome exception is the framework developed by Shepard (1984). In a nutshell, he proposes that some perceptual competencies are based on internalizations of external regularities. These regularities are characteristics of the physical environment that were more or less invariant during the evolutionary history of a species or of its ancestors. Through mechanisms of evolutionary adaptation, some outward physical facts were transformed into inward biological constraints. The organisms, having been shaped by the world, in this sense reflect its structure. In consequence, their attempts to recover that structure in the process of perception are based not only on the actual environmental circumstances or the individually acquired knowledge, but also on built-in interiorizations of exterior features.

As an example of an internalized behavioral regularity, Shepard (1984) discusses the relation of the external day-night cycle and the internal sleep-wake cycle. Why are diurnal animals active by day and resting by night? At first sight, it may appear plausible that this activity sequence is directly controlled by the environmental temporal variation of the amount of light, and that no contribution from the organism is necessary to maintain the behavioral cycle, except to be sensitive to the illumination cycle. However, experiments in which animals are kept for extended amounts of time under artificial, completely homogeneous light conditions show that in such circumstances the behavioral cycle continues with relatively small deviations, although eventually they do add up. Thus, although influenced from without, the cycle is also guided from within.

Internalization of external regularities is an intriguing and important explanatory framework for perceptual processes. However, in order to convert such a general notion into particular theories, concrete examples are needed in which the explanatory power of the approach is tested in actual applications. To date, only few cases of such explanations are available. As an example, consider a possible account of the "rigidity principle," the tendency to perceive rigid structure in geometrically ambiguous structure-from-motion displays (Johansson 1964; Ullmann 1979). One way to explain the origin of this principle is to propose that overexposure to rigid body motions in the environment somehow has induced the perceptual apparatus, over the course of evolution, to prefer the rigid interpretation over the infinity of geometrically equally appropriate nonrigid interpretations of stimuli. Another example is the "light-from-above principle," the tendency to favor those three-dimensional (3-D) interpretations of some geometrically ambiguous shaded displays which are in accord with the assumption that the represented scene is illuminated from above rather than from below (Metzger 1975; Ramachandran 1988a). That tendency may be based on the fact that the sun, the primary source of light during the evolution, was invariably located above the illuminated terrestrial scenes. Still another example is Shepard's (1992; 1994) proposal that the three-dimensionality of perceived chromatic surface color is based on the essential three-dimensionality of the color of daylight.

In this paper, I will discuss the perhaps most articulated account of this type: Shepard's theory of internalization of kinematic geometry. In the next section I will describe the phenomena that this theory attempts to explain. In the third

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section some relevant basic kinematic facts will be recounted. In the fourth section the account of these phenomena by Shepard and coworkers will be presented. In section five I will present an examination of this account. My conclusions will be largely negative, that is, I will argue that although the notion of internalization of external regularities certainly has great merit and interest, this particular implementation of that general framework is faced with a number of conceptual and empirical problems.

2. Curvilinear apparent motion

In classical apparent motion displays, a figure is presented first in one spatial position and then in another. Under appropriate conditions, the two static presentations induce the impression that the figure has moved from one position to the other. The characteristics of the motion percept depend on several stimulus parameters. For the present purpose, *orientational* aspects of the stimuli are the most relevant ones. In many apparent motion studies the motion inducing figures take the form of circular disks or rings, that is, they have no intrinsic orientation. Other studies use oriented figures, but with the same orientation in the two positions, such as a vertical rectangle presented in two horizontally displaced positions. In such circumstances, one usually sees the figure move on a rectilinear path between the two positions. But what is the perceived shape of the path if the figures are presented in different orientations in the two positions? Wertheimer (1912) noted that if a line is first exposed in the vertical and then in the horizontal orientation, it is perceived to *rotate* between the two positions. More recently, several studies investigated the dependence of the shape of the perceived path on various stimulus conditions (Bundessen et al. 1983; Farrell 1983; Foster 1975b; Hecht & Proffitt 1991; McBeath & Shepard 1989; Mori 1982; Proffitt et al. 1988; Shepard 1984). Such studies involve apparent motions perceived in both 2-D and 3-D.

2.1. 2-D motions

Figure 1 depicts some examples of stimuli used and variables manipulated in these studies. In each of the four depicted cases, an elongated rectangle is presented in different orientations in two positions. The variables manipulated in the examples are distance, orientation difference, and symmetry. For example, the difference between Figures 1a and 1b is the distance between the centers of the rectangles. In 1c the distance is the same as in 1a, but the relative angle between the two orientations of the major axes of the rectangles is larger. In 1d the distance is the same as in 1a and 1c, and the relative orientation angle is the same as in 1a and 1b; however, in contrast to the other figures, the display is not symmetrical.

A general finding of studies of this type is that the perceived path of apparent motion is not rectilinear but *curvilinear.* The shape of the path is often assessed by having subjects indicate the perceived position of the figure (by means of a probe stimulus or an appropriate gap through which the stimulus would fit) in some intermediate location between the two displayed positions. Another general finding is that for stimuli such as Figure 1a, that is, symmetrical displays with relatively small distance and orientational difference, the perceived path has an approximately circular shape. However, with increasing distance, orientational difference, and with departures from symmetry, the perceived path, while remaining curvilinear, generally exhibits increasingly less curvature, that is, approaches rectilinear shape (McBeath & Shepard 1989; Proffitt et al. 1988).

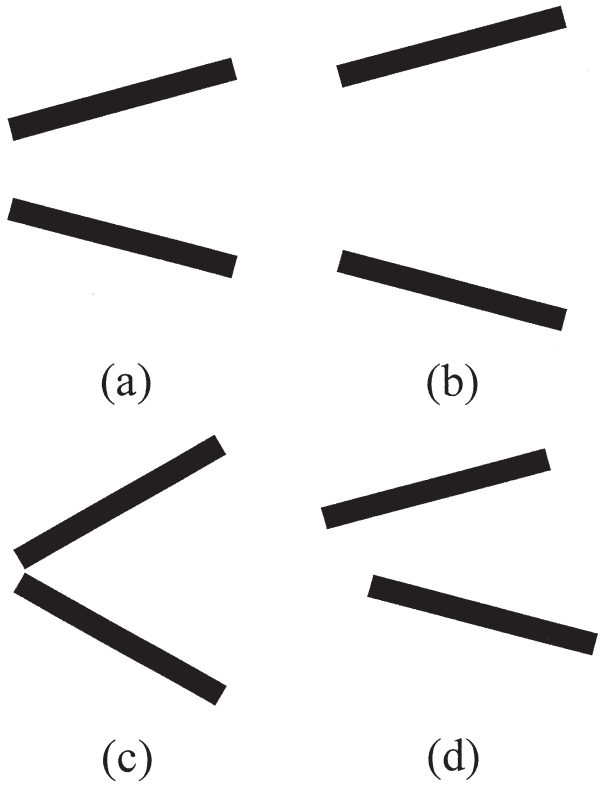


Figure 1. Examples of stimuli in 2-D planar curvilinear apparent motion studies, involving manipulations of distance, angle, and symmetry. See text for details.

2.2. 3-D motions

Apparent motion can also be perceived in depth. In Shepard's (1984) informally described study, the stimuli were randomized polygons presented in various orientations and sizes. The perceived motions were predominantly in depth and their motion was generally screw-like (to be described below). No use of probe stimuli was reported, so that no detailed conclusions about the shapes of the paths are available. In a study by Hecht and Proffitt (1991) the stimuli were drawings of domino-like figures, displayed in a similar fashion as in Figure 1, but depicted in different 3-D orientations. Such stimuli induced apparent motion in depth, whose path was assessed with probe stimuli. The results were similar to the studies of apparent planar motion: while in some favorable conditions the perceived path was approximately circular, in many others it fell short of 3-D circularity.

3. Basic kinematics

In Shepard's approach, the perceived curvilinearity in apparent motion is explained as due to internalizations of features of real motion. The study of the geometry of motion belongs to the province of *kinematics,* a branch of mechanics which is basic for physics and astronomy and has many practical ap-

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plications in engineering and robotics (Bottema & Roth 1979; Mićunovic & Kojić 1988). Thus, in order to analyze both the phenomena as well their explanations in more detail, some relevant elementary kinematic notions and facts will be recounted here. Kinematics is a large and thoroughly mathematical subject. I will only use verbal descriptions in the main text, but will illustrate them geometrically. The stress will be on 2-D motions, but 3-D motions will also be discussed.

3.1. 2-D motions

The issue of main interest here is the following. Let two positions of a rigid planar figure be given, denoted as I (initial) and II (final). A sequential presentation of the figure in the two positions would, under conducive conditions, induce a percept of apparent motion. But how might the figure have *really* moved in the plane from position I to position II? I will call any such transformation a "transporting motion." This section contains a survey and classification of various types of transporting motions.

Clearly, there is an infinity of possible transporting motions between any two positions. Figure 2 depicts geometric sketches of four examples of motions relevant for this discussion; Figure 3 contains additional examples in a somewhat different format, with paths calculated according to appropriate formulas, described in the Appendix. In Figure 2 positions I and II are represented as shaded forms. Two arbitrarily selected points of the figure, A and B, are indicated in their initial locations, AI and BI, and their final locations, AII and BII; point pairs such as AI and AII, and BI and BII, are referred to as "homologous" points. Each case contains, as outlined forms, several snapshots, that is, intermediate positions during the motion of the figure. The trajectory of point A is depicted as a dashed line.

In case 2a the transporting motion consists of a sequence of two phases, the translational phase and the rotational

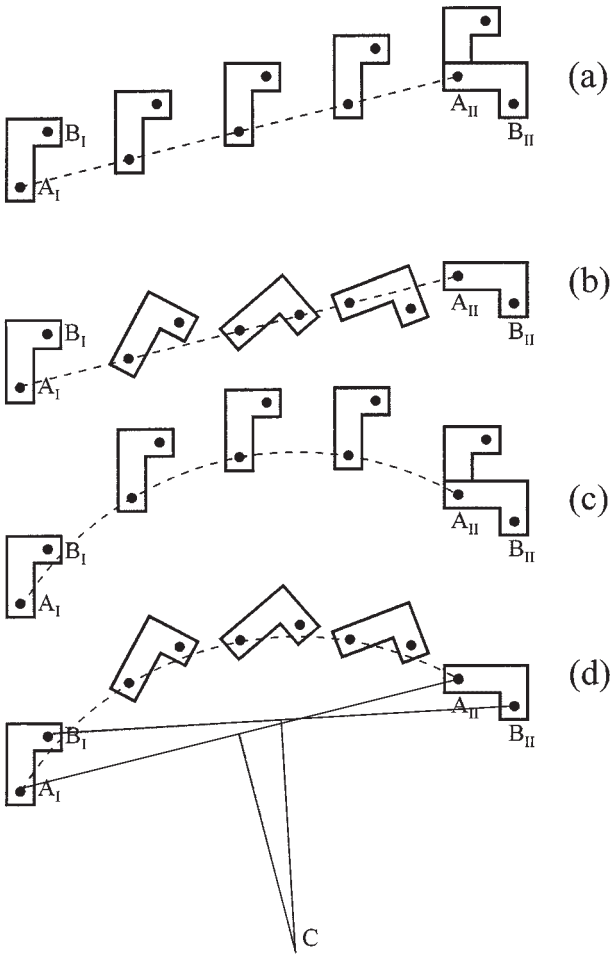


Figure 2. Examples of planar transporting motions. (a) Rectilinear translation followed by rotation about A. (b) Rectilinear translation concomitant with rotation about A. (c) Circular translation followed by rotation about A. (d) This motion can be kinematically defined in two ways. The first possibility is circular translation concomitant with rotation about (a). The second possibility is pure rotation about C. The construction of point C is explained in the text.

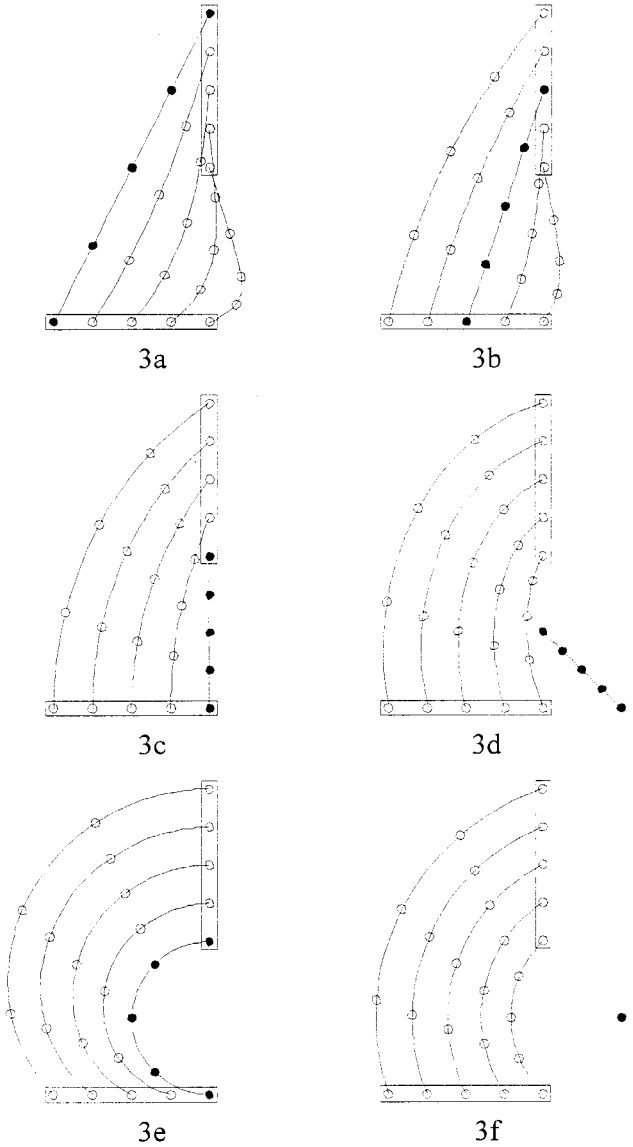


Figure 3. Examples of planar transporting motions. The black disks denote the positions of the pole. The first four figures depict rectilinear translation + rotation. (a) Leftmost point on the figure is the pole. (b) Middle point on the figure is the pole. (c) Rightmost point on the figure is the pole. (d) Pole does not belong to the figure, but is rigidly connected to it. (e) Circular translation + rotation. Rightmost point on the figure is the pole. (f) Pure rotation. The stationary pole is denoted by the black disk.

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phase. The first phase involves *rectilinear* translation, a type of motion in which the trajectories of all points of the figure are parallel straight lines of equal length. In the last position of the first phase point A has reached its final location AII, but point B has not. In the second phase, the figure rotates about A, through an angle that moves point B and all other points of the figure into their final positions. Point A is the "pole" or center of rotation. The trajectories of all points of the figure in the second phase are concentric circular segments, whose radii are given by the distances of the points from the pole A.

Several variants of this type of transporting motion can be implemented. For example, the two phases might be interchanged, such that the figure first performs the rotatory motion about AI and then the appropriate translation. Or, as represented in case 2b, the two phases might completely temporally overlap: as it translates, the figure also rotates about A, and does it in such a manner that point B arrives at location B_{II} at just the moment that point A arrives at location A_{II}. Note that in this motion point A, the pole, travels along a rectilinear trajectory, while point B and all other points rotate about A. Thus, their trajectories are circular *relative* to the pole; relative to the stationary background the motions also have a translatory component, so that their trajectories are more complicated curves, belonging to the family of cycloids.

The choice of point A as the pole in cases 2a and 2b is arbitrary. For example, in order to perform the motion between positions I and II, point B might also serve as the pole, and would move rectilinearly from B_I to B_{II}, while all the other points, including point A, would rotate about it. Different choices of the pole induce different trajectories of the points, but the total angle of rotation must be identical.

The effect of variation of the position of the pole is illustrated in Figure 3. This figure depicts several ways in which a rectangular object might move from a horizontal position I to a vertical position II. Five points of the figure and their trajectories are indicated. The initial and final positions of the points, as well as three intermediate snapshots, are depicted as small circles. The *filled* circles indicate the positions of the point that serves as the pole. In Figure 3a the pole is the leftmost point of the figure in the horizontal position, in Figure 3b the pole is the middle point, and in Figure 3c it is the rightmost point. Note that in these figures the pole moves on a rectilinear trajectory, different in each case, while the trajectories of the other points are curved, and are also different in different cases. In Figure 3d the translating pole does not belong to the moving figure, but is assumed to be rigidly connected to it. In this case the trajectories of all points of the figure are curvilinear. Rotation of the figure about the pole is 90° clockwise in each case.

In all these examples the translational component of the transporting motion is rectilinear. However, there is also a *curvilinear* variant of translational motion. As in rectilinear translation, in this type of motion the trajectories of all points are congruent (they have equal shape, size, and orientation, and thus are completely superimposable), but they are curvilinear. An example of such a motion, involving *circular* translation, is presented as the first phase of motion depicted in case 2c; in this phase all points move on congruent circular arcs. The second phase of the motion is a rotation about pole A, as in case 2a. Note that although trajectories of individual points are circular in both circular translation and rotation, these two types of motion are by no means identical. In rotation, all points rotate about a common center, their trajectories are concentric circular arcs whose lengths are different for different radii, and the figure changes its orientation during the motion. In contrast, in circular translation each point has a different center of rotation, but the radii are all equal, the trajectories of all points are congruent circular arcs, and the figure retains its orientation during the motion.

Case 2d presents a temporal overlap of translation and rotation, analogous to case 2b, except that in this case translation is circular: as it translates along circular arcs, the figure also rotates about point A as the pole. The role of the straight lines and point C in this figure will be discussed below. Another example of this type of motion is presented in Figure 3e, with the rightmost point as the pole.

A common feature of examples presented so far is that they are all cases of various combinations of both translations (rectilinear or circular) and rotations. Indeed, all planar motions can be represented as such combinations, with pure translations and pure rotations as special cases. Now, it may appear at first sight that when the two positions of a figure have different orientations, *both* translation and rotation would generally be necessary to transport the figure from one position to the other. Interestingly, this is not the case: it can be shown that, given two positions of arbitrarily different orientations, a specific *pure rotation* always suffices as a transporting motion, without the need of a translational component. This fact is illustrated by case 2d. Note that this case was used above to illustrate a combination of circular translation and rotation. However, this particular motion can also be instantiated by a pure rotation, but about a different, stationary center. In order to find this center a simple geometrical construction, indicated in Figure 2d, may generally be used; for exceptions, see below. It involves constructing the perpendicular bisectors of lines A_IA_{II} and B_IB_{II} (which join the homologous points), and finding the intersection C of these bisectors. An elementary geometric argument proves that triangles CA_IB_I and $CA_{II}B_{II}$ are congruent, from which it can be deduced that the figure as a whole can indeed be transported from position I into position II by a rotation about C. Figure 3f presents another example of a pure rotational motion about a static pole, depicted as a filled circle. Note that trajectories of all points are circular arcs whose common center is the pole. As in case 2d, this particular motion can also be instantiated as a combination of translation and rotation about a moving pole, as is shown in the Appendix.

When two differently oriented positions of a figure are given, any point in the plane can be used as the pole about which the figure can be rotated from position I into a position in which it has the same *orientation* as in position II. But in almost all cases this position will be different than position II, so that an additional, translatory motion (which is orientation preserving) is necessary to complete the transporting motion. However, there is a unique point in the plane, determined by the geometric construction presented in Figure 2d, such that when the figure is rotated about it by the required angle, it is already transported to position II, and no additional translation is necessary.

The motion from I to II is not uniquely determined by the center of rotation. The example depicted in Figure 2d involves clockwise rotation, but counter-clockwise rotation

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could also be used. Furthermore, in moving from I in either direction, the figure might have performed one or more additional full 360° rotations before coming to a stop at II. However, if one always opts for the shortest route, then there exists only a single appropriate pure rotation, except if it involves 180°, when two routes have equal lengths. Ignoring this last detail, it can be said that in this sense pure rotation is a *unique* transporting motion. In contrast, there are infinitely many *different* translation + rotation combinations, though each unique in itself, that will get the figure from I to II.

In all discussed examples positions I and II had different orientations. When they have the same orientation, a *pure translation* suffices to transport the figure from I to II, and the rotation component is not needed. In such cases the above intersection-of-perpendicular-bisectors construction fails: lines A_IA_{II} and B_IB_{II} are parallel, so that their perpendicular bisectors are also parallel and do not intersect, and thus no center of rotation can be constructed. This suggests that in such cases the figure cannot be transported from I to II by a pure rotation. Nevertheless, one can assert that even pure translation is in fact a rotation, but around an *infinitely* distant pole. If this is accepted, then pure rotation, just as translation + rotation, is a truly general type of transporting motion, in that it can be used to transport a figure between any two positions in the plane, regardless of their orientations.

There is another class of cases in which the above general geometric procedure fails, and that is when the two perpendicular bisectors coincide, and thus have no unique intersection point. Such a situation arises in symmetrical cases, such as those depicted in Figure 1a, 1b, and 1c. In these special cases, which are most often used in psychophysical experiments, the center of rotation can be found as the intersection of lines A_IB_I and $A_{II}B_{II}$, a procedure which does not work in the general case.

In all examples presented so far, the translational component was either zero (as in pure rotation), or it had a constant direction (as in rectilinear translation), or a constantly but uniformly changing direction (as in circular translation). However, in general the direction of translation may change from point to point, so that the figure may translate along an arbitrarily curved line, while rotating concomitantly. Such motions can always be represented as pure rotations, but around a mobile pole.

The specifications of motions in the above examples are incomplete, because to define the motion of a figure it is not enough to determine the trajectories of its points. One also needs to specify the temporal manner of motion along the trajectories. The two basic possibilities are uniform and nonuniform motion. In uniform translations equal distances are covered in equal times, whereas in uniform rotations equal angles are covered in equal times. In contrast, in nonuniform motions the velocities of points may change over time. Thus, a general transporting motion from I to II may involve arbitrary accelerations, decelerations, and direction reversals.

3.3. 3-D motions

Kinematics in 3-D is in many ways analogous to the 2-D case, but it is also much more complex, both mathematically and visually. I will only discuss some basic facts which are relevant for the present purpose. The same main question as in the case of 2-D motions can be posed. We are given two arbitrary spatial positions of a rigid body, denoted as I and II. How might the body have moved from the initial to the final position?

As in the plane, the transporting motions in 3-D can be performed in an infinity of ways by combinations of various types of translations and rotations. Translation in 3-D is defined in the same way as in 2-D, except that the trajectories are spatial curves; rectilinear translation is specified by a constant 3-D direction. Rotation in 3-D is not defined as in 2-D with respect to a point (rotation center), but with respect to a line (rotation axis). The circular trajectories of the rotating points are not all concentric (as in the 2-D case), but they are co-axial, as their centers all belong to the rotation axis, with the planes of all trajectories being perpendicular to the axis. Similarly to the 2-D case, the temporal order of translation and rotation is arbitrary: the translation could be performed first and then the rotation; translation and rotation might be concomitant; and so on. In general, the translation direction may vary arbitrarily during the motion, and the rotation axis may change its location and orientation.

As in the planar case, there are many different translation + rotation combinations that can transport a figure from I to II. However, in contrast to the 2-D case, pure rotation in 3-D does not qualify as a general, unique type of transporting motion: given two positions of a rigid body, although in some cases it can be moved from one position into the other by a pure rotation, this is not true in general. Still, there is a particular type of a rectilinear translation + rotation combination which has a uniqueness status similar to pure rotation in 2-D. This specific combination is called "helical" or "twisting" or "screw" motion. Its specificity is that, whereas in general rectilinear translation + rotation combinations the rotation axis has a different orientation from the direction of translation, in helical motions these two orientations are equal, and thus the rotation axis is parallel to the direction of translation. It can be shown that, given two arbitrary positions of a rigid body, it can always be transported from the first position into the second position by an (almost) unique helical motion; this kinematic fact is often referred to as *Chasles's theorem.* I say "almost" unique, because the same considerations about clockwise and counter-clockwise directions and multiple turns apply as in the case of pure rotations in 2-D. The geometric construction and analytical expression of this unique transporting motion is much more complicated than for pure rotation in the 2-D case.

4. Shepard's account

How can the kinematic facts on possible transporting motions reviewed in section 3 help explain the empirical data on apparent motion paths described in section 2? One answer is: by applying the notion of internalization of external constraints sketched in section 1 (Carlton & Shepard 1990a; 1990b; McBeath & Shepard 1989; Shepard 1984; 1994). The general idea is that our perceptual systems have internalized some relevant regularities of real external motions, which then determine the motion impressions induced by apparent motion stimuli. In the following, the crucial arguments are presented using citations from the relevant papers:

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> The paths that are psychologically favored . . . provide information about our internalized principles concerning the ways in which objects transform in the world. (Carlton & Shepard 1990a, p. 133)

What are these internalized principles of external motions? One possibility is as follows:

> On the basis of the assumption that we have internalized the regularities that have prevailed in the terrestrial environment throughout biological evolution, one might at first suppose that our internalized rules for the motion of rigid bodies in space would correspond to the laws of motion of classical, Newtonian mechanics. (Carlton & Shepard 1990a, p. 142)

How do the laws of classical mechanics govern object motions?

> Any continuous rigid motion would be consistent with the laws of motion in the presence of arbitrarily changing forces. In the absence of external forces (such as those of friction, air resistance, and gravity), however, the center of mass of the object must, according to classical physics, traverse a straight line at a constant velocity between the corresponding centers of the object as it appears in the two given images. (Carlton & Shepard 1990a, p. 142)

Furthermore, the appearance of apparent motion stimuli suggests an additional component:

> Because the two views also differ by a rotation, such a motion would have to be accompanied by an additional, apparent rotational transformation. (Shepard 1984, p. 425)

An example of such a type of motion, in 2-D, is presented in this paper as Figure 3b.

However, the data reviewed in section 2 suggest that such a combination of rectilinear translation and rotation is generally not perceived (but see Farrell 1983; Mori 1982). Therefore, a different source of internalized constraints is proposed, provided by the principles of kinematic geometry. Although we are able to experience any given type of *external* motion, when we are presented by discrete displays inducing *apparent* motion, "the default motions that are experienced in the absence of external support are just the ones that reveal, in their most pristine form, the internalized kinematics of the mind and, hence, provide for the possibility of an invariant psychological law" (Shepard 1994, p. 10). What is the form of this internalized kinematics of the mind? In general, what is perceived both in 2-D and in 3-D is " . . . the unique, simplest rigid motion that will carry the one view into the other . . . " (Shepard 1984, p. 426). More specifically, describing Foster's (1975b) data on perceived 2-D motions, Shepard notes that "the motion tended to be experienced over that unique circular path that rigidly carries the one figure into the other by a single rotation about point P [the pole], in the plane" (Shepard 1984, p. 425). This type of motion is depicted here in Figures 2d and 3f. Why is it that the perceptual system prefers pure rotation over rotation + translation? "The pure rotation . . . could be regarded as the simplest motion. . . . Any other motion would require, at least, a rotation through the same angle and, in addition, translation" (Carlton & Shepard 1990a, p. 152). Similar accounts are given in Shepard (1984, p. 425) and Shepard (1994, p. 8).

Analogous considerations apply to 3-D. Describing his data on perceived 3-D motions, Shepard notes that "out of the infinite set of transformational paths through which the one shape could be rigidly moved into congruence with the other, one tends to experience that unique, minimum twisting motion prescribed by kinematic geometry" (Shepard 1984, p. 425); the reference here is to Chasles's theorem. Helical motion tends to be perceived because it is "the geometrically simplest and hence, perhaps, the most quickly and easily computed. Certainly, within a general system suitable for specifying all possible rigid motions, such a motion requires the minimum number of parameters for its complete specification" (Shepard 1994, p. 7).

However, the data reviewed in section 2 indicate that in experiments pure rotation or helical motion is not generally perceived. The reported shapes of the perceived paths in these studies are mostly curvilinear, but their shape is neither purely rectilinear nor purely circular or helical. In order to account for such data, McBeath and Shepard (1989) invoke various spatiotemporal processing limitations of the visual system. On the other hand, Carlton and Shepard (1990a) argue that "our perceptual systems may have internalized still more general geometrical principles under which the physical and geometrical principles so far considered are both subsumable as special cases" (p. 168).

> The relevant principles that have been invariant throughout evolutionary history appear to be of two general types – namely physical and geometrical . . . The principles of kinematic geometry, which are in some ways more general and pervasive, may be more internalized than the principles of classical physics. Perhaps different weighted combinations of the two types of principles may yield the best accounts of data from different individuals or under different conditions. (p. 174)

Similarly, Shepard (1994) argues that internalized knowledge of classical physics "may be contaminated, to a variable degree across individuals and conditions of testing, by a more deeply internalized wisdom about kinematic geometry" (p. 8).

In summary, this account claims that during the course of evolution our perceptual systems have internalized some invariant regularities that govern the motions of external objects between two discrete spatial positions. In apparent motion displays, in which there is no real motion in the stimulus, such internalized constraints guide the percept. The discrepancies of empirical data from predictions may derive from a compromise between principles of classical physics and kinematic geometry.[1]

5. An examination of Shepard's account

I will discuss several issues that I find problematic for this theory. They concern questions of internalization of regularities and principles, the roles of simplicity and uniqueness, the import of perceived curvilinearity, the contrast of kinematic geometry and classical physics, and the relation of perceptual and scientific knowledge.

5.1. Internalization of external regularities

There is an important difference between Shepard's account of apparent motion and other evolutionary grounded explanations noted in the introduction. The difference concerns what it is that is internalized. In the examples described earlier, the presumed internalization concerns an invariant or recurrent feature of the environment, such as the light-dark cycle (in the endogenous rhythms account), the predominance of rigid bodies in the environment (in the rigidity principle account), the direction of the sun with respect to the earth (in the account of the light-from-above

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principle), and the spectral composition of daylight (in the account of the three-dimensionality of perceived chromatic color). In all these cases, the internalization account proposes that a pervasive external regularity is the source of an evolutionary acquired internal mechanism that underlies a behavioral effect. In contrast, as argued below, in Shepard's apparent motion account such a pervasive external regularity is missing.

Consider first the idea of the internalization of classical physics. As Carlton and Shepard (1990a) themselves point out, the type of motion that is assumed to be internalized (uniform rectilinear motion of the center of mass of the body) is derived in classical physics under the assumption of "the absence of external forces (such as those of friction, air resistance, and gravity)." The problem here is that in the terrestrial environment these external forces were never absent. How then could the perceptual system have internalized this type of motion?

To illustrate this, consider a possible "classical physics world," devoid of friction, air resistance, and gravity. Accordingly, all its objects, organic and nonorganic, are perfectly smooth entities floating about in vacuum in a gravitation-free field. All they do between collisions is to translate rectilinearly while rotating concomitantly. One day the psychologists of that world perform perceptual experiments in which they project an object first in one position in space and then in another. Their subjects report that they actually see the body as moving on a rectilinear path from one position to the other. An explanation suggests itself: their perceptual systems have internalized an external regularity of their world. In such a world this would indeed be a plausible explanation. But in our world friction, air resistance, and gravity, as well as other factors are pervasive, and thus the motions of inorganic (not to mention organic) bodies are much more complicated. Thus, it is not clear what regularities the perceptual systems might have extracted during evolution.

The idea of the internalization of the principles of kinematic geometry can be criticized in a similar fashion. Consider a possible "kinematic geometry world," a universe in which the trajectories of most objects are, for some peculiar reason, predominantly circular arcs in 2-D and helical arcs in 3-D. In such a world, these types of motions might indeed have been internalized by their inhabitants. But in our world these particular types of motions do not appear to be typical or representative. Thus, there apparently is no corresponding pervasive external regularity to be internalized. Consequently, tendencies for perceived circular or helical motions can hardly be based upon internalization of invariant environmental features.

A possible reply to such criticism might be to claim that it is not the internalization of external *regularities* that is supposed to be operative in the apparent motion account, but the internalization of certain physical or geometrical *principles.* Such a formulation is indeed clearly indicated by several quotes in section 4. Note, however, that such an interpretation would concede that the apparent motion account is indeed different from other described internalization accounts in this respect. I will deal with this possibility in a later section.

5.2. Simplicity

An examination of Shepard's explanation of curvilinear apparent motion indicates that, in addition to internalization, other notions also play a large or even predominant role. For example, an inspection of the citations in section 4 reveals that the notion of *simplicity* figures prominently in the accounts of the shapes of perceived motions. Note that such an account does not explicitly invoke the idea of internalization of external regularities. For example, it is not claimed that pure rotations in 2-D are perceptually preferred because, say, they are internalized to a larger extent than combinations of rotations and translations. Rather, pure rotations are singled out because they are simpler. Thus Shepard's account is to an important extent based on the concept of simplicity, and not only on the notion of internalization of external regularities. Recall, however, that only in some conditions subjects report circular motions, whereas in others the perceived motion path is generally less curved. Thus, even if the propensity for this type of simplicity is in some way embodied in the perceptual system, it can easily be countered by other factors.

Now, what could be more obvious than the fact that rotation is simpler than rotation *and* translation? However, as it turns out, this matter is more complex, and the notion of comparing the simplicity of different types of motions is in fact itself not very simple or straightforward.

The intuitively compelling claim that pure rotation is simpler than rotation + translation is not only based on the fact that the former motion has one component and the latter has two, but also on the notion that "rotation" in pure rotation is the same thing as in rotation + translation. However, this is not quite the case. The two rotations do have in common the specification of the angle of rotation. However, there is a difference concerning the manner of determination of the center of rotation. In translation + rotation the pole can be chosen freely, so that any point in the plane can serve as the center of rotation. In contrast, as was shown above, in pure rotation the position of the pole must be constructed on the basis of the features of the two positions of the figure, and only a single point (in some cases infinitely distant) can serve as the pole.

There is another difference between the two types of motions, and that is, of course, that in contrast to pure rotation, in rectilinear translation + rotation two coordinates of the 2-D translation direction need also to be determined. However, the advantage of getting rid of translation is offset by the need to establish the two coordinates of the pole in pure rotation.

Furthermore, it is interesting to note that the plotting formulas that I have used to draw Figure 3 (see Appendix), are identical for cases of translation + rotation (3a–3d) and pure rotation (3f). The same program is used to draw all five cases, using different parameter values for each case. The specificity of case 3f is only in that both translation coordinates are zero. However, as zero is just another number, case 3f is just another case of motion, and, at least in such a representational format, it is not qualitatively singled out from other cases and requires the same number of parameters for its specification. In addition, as noted in section 3 and in the Appendix, the motions that were specified as pure rotations (case 2d and case 3f) can be exactly duplicated by specifying them as instances of translation + rotation.

In kinematics textbooks it is shown that general motion in the plane involves three degrees of freedom, two for translation and one for rotation. In contrast, pure rotation requires only a single degree of freedom (for the rotation angle), but only if the center of rotation can be established

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in advance. If that is not the case, two additional pieces of information are required to specify its position in the plane. Thus, given a simplicity metric in terms of the number of degrees of freedom, translation + rotation is as simple (or as complex) as pure rotation, since the free choice of the pole does not use up degrees of freedom. Thus, by this criterion, pure rotation is not singled out from other motions by virtue of its simplicity.

In the preceding paragraphs pure rotation and rotation + translation were compared with respect to the number of pieces of information needed to specify them. As cited in section 4, Shepard (1994) suggests that simpler motions are those that are specified with fewer parameters, and that for that reason they may be calculated more quickly and easily. However, note that different parameters may themselves be computed with different ease, so that comparing just the number of informations might not be a very sensitive measure of simplicity. Therefore, rather than considering just the sheer number of parameters, it may be perceptually more relevant to discuss the potential processes by which the values of these parameters could be established.

When presented with two different spatially offset and temporally sequential positions, I and II, on what basis might the visual system come up with the parameters needed for the specification of a particular transporting motion? The angle of rotation, needed for both pure rotation and translation + rotation, may be established by the difference of orientations of the figures in the two positions. In translation + rotation, the center of rotation may be chosen to reside at some visually salient location of the figure, such as at its endpoints or center; the direction of rectilinear translation may be given by the orientation of the virtual line connecting two homologous points, which themselves may be singled out by the same visual criteria as the pole. Such motions are represented in Figures 3a, 3b, and 3c. Among them, case 3c may be singled out because it involves the shortest path of the pole.

In contrast to translation + rotation, in pure rotation the main problem is to specify the center of rotation. As noted above, the specific procedure for symmetrical displays is relatively simple, but it cannot be generalized to nonsymmetrical displays. The general procedure presented in section 3 is also simple enough to execute, once one has a compass and a ruler at one's disposal. But it is not clear how the visual system may go about to find midpoints of virtual lines connecting points that are not presented simultaneously, then to construct, at those midpoints, lines perpendicular to them (bisectors), and finally to ascertain the location of the intersection of the bisectors, which may be removed at some distance (up to infinity) from the presented figures. Perhaps the visual system could use some other, more easily computable type of construction, but this remains to be shown. Shepard (1984) suggests that the specification of the rotation center might be similar to the way this center is established in so-called Glass patterns; however, these are very different types of displays than apparent motion stimuli, and in the example pattern that is provided (his Fig. 4b), the rotation center is visually quite conspicuous. Thus this particular piece of information appears to be much harder to establish than the others.

In sum, when rotation + translation and pure rotation are compared just with respect to the number of component motions, then pure rotation (one component) appears simpler than rotation + translation (two components). However, when they are compared with respect to the number of degrees of freedom, or the number of parameters needed in a general procedure, then the two types of motions appear to be equally simple. Finally, when they are compared with respect to the easiness of visual computations of the required motion parameters, rotation + translation appears simpler. The least that can be said in conclusion is that simplicity may not be a reliable criterion by which pure rotation can be perceptually singled out from other types of motions in 2-D. A similar argument would apply concerning the presumed simplicity of helical motions in 3-D. In this case, a particular difficulty, analogous to finding the position of the pole in pure rotation in 2-D, is the procedure by which the appropriate axis of rotation can be specified in the general case, because it is complicated analytically and sometimes far from obvious.

5.3. Uniqueness

The notion of uniqueness features prominently in Shepard's account. The quotations cited in section 4 stress the uniqueness of pure rotation in 2-D and of helical motion in 3-D. Thus, even though these motions may not be kinematically simpler than general translation+rotation combinations, there is something that may make them kinematically salient, and that is their uniqueness. Uniqueness is clearly a different notion than simplicity (since unique entities need not be simple nor do simple entities need to be unique), nor does it relate in any obvious way to internalization of external constraints (since unique features need not be internalized nor do internalized features need to be unique). Thus, uniqueness appears to be a third, independent component in Shepard's account, besides simplicity and internalization. According to this explanatory strategy, the perceptually preferred motion paths are those that are kinematically unique.

However, the promotion of uniqueness from a kinematical feature into a perceptual principle would face some problems. For example, circular and helical motions are indeed unique in the sense explicated in section 3. But, there are other kinematical criteria in terms of which some other motions might be unique. For example, given two positions of a figure, one way to choose a particular transporting motion among the others could be to look for the motion for which some prominent feature of the figure, such as one of its endpoints, moves over the shortest path. Another candidate could be the motion for which the average length of paths of homologous points is shortest. Still another possibility would be to single out the motion for which these paths are, on average, least curved. Thus, no type of motion is *uniquely* unique: rather, uniqueness is a feature that is relative to some criterion, and the choice of one criterion rather than of another would have to be argued on some separate grounds.

But why would a perceptual system prefer a kinematically unique motion over a kinematically nonunique one in the first place? Such preference might perhaps be plausible if it were to refer to visual uniqueness or salience. However, inspect for a moment the examples of motions in Figure 3. It can be seen that all cases are different and possess some specific features. Figure 3f, kinematically unique since it is a case of pure rotation, does not appear to be especially visually unique (or, for that matter, simple). The physicist might single it out, but why should the perceiver

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do so? Also, recall again that only in some conditions do subjects report approximately circular paths in 2-D and helical paths in 3-D. As in the case of simplicity, uniqueness – if it is embodied – can apparently be easily overridden.

5.4. Kinematic geometry and classical physics contrasted

According to Shepard's analysis, classical physics and kinematic geometry yield different predictions for apparent motion paths: classical physics predicts rectilinear paths, whereas kinematic geometry predicts circular and helical paths. Such an account conveys a portrayal of kinematic geometry and classical physics as two theories that can have different predictions about some aspect of reality, similar to, say, Newtonian and Einsteinian theory providing mutually incompatible accounts of the speed of light or the shape of its trajectory.

However, such a portrayal is not appropriate. Classical physics *adds* to kinematic geometry, it does not contradict it. Physics textbooks often treat motion by first discussing the more mathematical aspects (kinematics), and then introducing the more physical notions (dynamics). Whereas kinematic geometry describes the ways bodies move, classical physics, accepting this description, goes on to inquire about the physical causes of their motions. Thus kinematic geometry involves the concepts of displacement, trajectory, velocity, and so on, and classical physics involves, in addition, the concepts of mass, force, inertia, and so on. For example, kinematic geometry *describes* the shapes and velocities of trajectories of heavenly bodies, whereas classical physics deduces these shapes and velocities, using a richer set of concepts and laws.

With respect to the issue at hand, a rectilinear path is as compatible with kinematic geometry as is a circular path. Kinematic geometry does not "prescribe" a circular or helical path, since it is not in the business of prescribing paths but describing them. It is classical physics that, given some additional assumptions, singles out the rectilinear path among the many possible paths offered by kinematic geometry. Thus, the classical physics prediction is not a rival of the kinematic geometry prediction, and a test of perceived path shape is not a test between kinematic geometry and classical physics.

5.5. The import of path curvilinearity

The main empirical support for Shepard's account is the curvilinearity of perceived paths in apparent motion displays. However, curvilinear paths are not exclusively indicative of circular motions (in 2-D) or helical motions (in 3-D). In fact, they are just as compatible with many translation + rotation combinations. As illustrated by examples in Figure 3, all 2-D motions (except for pure rectilinear translation), involve curvilinear translations for all figure points, except for rectilinearly moving poles. The geometrical center of the figure moves rectilinearly only if it is the center of rotation (Fig. 3b), otherwise it moves curvilinearly. Several cases of translation + rotation combinations (such as Figs. 3c, 3d, and 3e) appear visually relatively similar to pure rotation (Fig. 3f), and it might be difficult to distinguish them on phenomenal grounds, or to decide to which category a given apparent motion trajectory belongs. Furthermore, some empirical studies have provided evidence that perceived paths in some apparent motion displays are not purely circular but are rather translation + rotation combinations (Farrell 1983; Mori 1982). In sum, the curvilinearity of perceived motion paths in 2-D does not provide much support for the idea that a crucial component of their shape is best described by pure rotation. Similarly, there appears to be no specific empirical evidence that perceived paths in 3-D are generally preferentially truly helical (meaning that the translation direction must be parallel to the rotation axis) rather than general translation + rotation combinations (in which this parallelism does not necessarily have to hold).

5.6. Perceptual and scientific knowledge

It was argued above that it is not likely that the visual system has internalized the particular *types* of motions that are central in Shepard's account. However, one could argue that what was internalized instead are some kinematic *principles* of motion, such as Chasles's theorem. The main problem for this idea is to make plausible how and why the visual system would have internalized such principles. They are not "out there" in the same sense as external regularities, which are recordable by physical or biological sensory systems. Rather, these principles involve some mathematical truths concerning the generality and uniqueness of a specific class of potential transporting motions between two arbitrary discrete positions. This type of issue is of great interest to mathematicians and is therefore duly discussed, among other theorems, in kinematic textbooks. But what would impel a biological visual system to pose, let alone solve such a problem? As noted, such motion types appear not to be predominantly encountered in the natural environment. In technical applications they are used in some cases but not in others, and the decision which concrete type of motion is chosen in a particular mechanical device is not governed by mathematical uniqueness but by technical efficiency.

Consider the general idea that, through their contact with the environment, our perceptual systems have extracted certain principles that are strongly analogous to laws, axioms, and theorems articulated by the scientific community. Such a notion is intriguing, but it should be treated with caution, and a more concrete analogy between the operation of the perceptual system and the conduct of scientific inquiry may be grossly inappropriate. For example, it is certainly true that biological organisms have acquired immense experience of geometric features of their environment. But does that mean that their visual systems must embody and apply the basic axioms and theorems one would find listed in geometry textbooks such as, say, that the square on the hypotenuse is equal to the sum of the squares on the kathetes? Or, note that all our life we are exposed to and are ourselves the sources of physical forces of various kinds. But can we conclude from this that our visual system must have extracted the overarching principle that Force is equal to Mass times Acceleration, and that it applies this principle in the interpretation of visual events?

The task of the visual system is, in part, to inform the perceiver about the makeup of the current environment. The task of the scientist is, in part, to extract analytic order out of bewildering complexity. These two tasks are different, and the ways they are accomplished may well also be different. Furthermore, the type of information offered by vision

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may not always be conducive to the sort of problems faced by science. This means that when we theorize about the world we often have to think beyond what we see. The history of the physics of motion provides an example of adverse effects that everyday experiences may have on the development of scientific theories. Aristotelian physics assumed, quite plausibly, that whenever a body is in motion, some force must be at work, and that with the cessation of the force the body must stop. After all, what is more obvious than the fact that if you want to move a rock you must push, and if you stop pushing, it will stop moving. Thus, in Christian cosmology angels had to be recruited to eternally push the planets along crystalline spheres to prevent them from stopping. It took the brilliance of a Galileo to conceive the concept of inertia which claims, against all the evidence of the terrestrial senses, that once in motion a body would forever move on, without the need of any force to keep it going, provided that no forces act on it. To make this plausible, we can imagine a perfectly smooth horizontal surface on which a perfectly smooth ball is set rolling. Then we realize that there is nothing to make it stop. But this is a *thought* experiment. No one has ever *seen* such perfect objects. It took the genius of a Newton to establish that when forces are involved it is not in motions as such but in changes of motions. Thus a real ball on a real surface will eventually stop rolling, not because some force has ceased to act, but because another force, friction, acts against inertia.

It is not hard to find additional examples in which our individual senses are likely to delude our common sense about some aspect of reality, and where scientific progress is achieved only through increasing abstractions from the sensory givens. Such examples indicate that although perceivers and scientists may share the task of extracting regularities, their data, competencies, strategies, and goals are not the same. This makes the task of those perceivers who are also scientists of perceivers all the more difficult.

6. Conclusion

What is the relation between empirical data on curvilinear apparent motion and the particular circular/helical motions described in kinematic geometry? Why is it that stimuli such as those depicted in Figure 1 induce the types of apparent motions described in section 2? According to the argument presented here, it is not because such motion types are internalized, since they are not predominant in the exterior in the first place. It is not because they are kinematically simpler than other motions, since it is questionable that they are kinematically simpler. They *are* kinematically unique in a specific sense, but there are other types of uniqueness as well, and it is not clear why a perceptual system should care about kinematic uniqueness anyway. It is not because principles of kinematic geometry are preferred over principles of classical physics, since these two fields are not in predictive rivalry. It is not because empirical data predominantly indicate such motion types, since they are compatible with many other types of motion as well. Finally, it is not because certain principles of motion are extracted by the visual system from the environment, as it is hard to see how and why they should be internalized.

It should be stressed that these negative conclusions do not apply to the general idea of the internalization of external regularities. It is indeed very plausible that perceptual mechanisms are affected by evolutionary adaptational processes of the species in its contact with the environment. However, the unpacking and empirical testing of this notion is by no means straightforward, and remains as an important issue for further research.

APPENDIX

Motion of a rigid body can be decomposed into translation and rotation. Symbolically, M = T + R. This decomposition can be mathematically represented in several forms, usually involving matrices and/or vectors. Here I will use the coordinate (parametric) form, which is less elegant but is more suited for plotting. It involves separate equations for the two Cartesian coordinates, that is, $M_x = T_x + R_x$, and $M_y = T_y + R_y$.

The rotation components are identical for all six examples presented in Figure 3. They take the form:

$$R_x(t) = P_x + (A_x - P_x)\cos \omega t - (A_y - P_y)\sin \omega t$$

$$R_y(t) = P_y + (A_x - P_x)\sin \omega t + (A_y - P_y)\cos \omega t$$

Here P_x and P_y are the coordinates of the pole (rotation center), and ω is the angular velocity, assumed as constant. These three parameters specify the rotation component. Time is denoted as t, and A_x and A_y are coordinates of points on the moving body (or rigidly attached to it) in the initial (horizontal) position. Five representative points are depicted in the examples. The origin of the Cartesian coordinate system was chosen to coincide with the leftmost point. Thus, in the initial (horizontal) position, I, all points lie on the x-axis, with the x-coordinates being 0, 0.25, 0.50, 0.75, and 1, and the y-coordinates being all zero. In the final (vertical) position, II, all points have 1 as the x-coordinate, and the corresponding y-coordinates are, respectively, 2, 1.75, 1.50, 1.25, and 1.

The translation components for cases 3a–3d and 3f take the form:

$$T_x(t) = D_x\, t$$

$$T_y(t) = D_y\, t$$

Here D_x and D_y give the direction of the translation, which is assumed constant.

Note that for t = 0, $M_x(t) = A_x$, and $M_y(t) = A_y$, that is, in the beginning the points are in their initial positions. As time increases, $M_x(t)$ and $M_y(t)$ change, that is, the points move, and their trajectories are different for different values of P_x, P_y, 4, D_x, and D_y. However, for t = 1 all points end up in final positions which are identical in all six cases. The particular values of these five constants for cases 3a–3d and 3f are given in Table 1.

Table 1. *Parameters for motion equations corresponding to examples depicted in Figure 3*

CASE	P_x	P_y	ω	D_x	D_y
3a	0	0	$-\pi/2$	1	2
3b	0.5	0	$-\pi/2$	0.5	1.5
3c	1	0	$-\pi/2$	0	1
3d	1.5	0	$-\pi/2$	−0.5	0.5
3f	1.5	0.5	$-\pi/2$	0	0

It can be seen that in all cases the extent of rotation is $\pi/2$ clockwise. In cases involving translation+rotation (3a–3d), the rotation pole in the initial position is chosen to lie on the x-axis (P_y=0 in all cases). The coordinates of the direction of translation are given as the differences of the coordinates of the pole in the final and in the initial position. In the case involving pure rotation (3f) the translation coordinates are zero. The coordinates of the pole in this case can be found through the geometric construction de-

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scribed in the text, or by corresponding analytical formulas (Bottema & Roth 1979), given by

$$P_x = [O_x - O_y \cot(\omega/2)]/2$$

$$P_y = [O_x \cot(\omega/2) + O_y]/2$$

Here O_x and O_y are the coordinates of the point to which the origin of the coordinate system is transported by the rotation, and ω is the angular extent of rotation. In the case at hand, $O_x = 1$, $O_y = 2$, and $\omega = -\pi/2$, so that the coordinates of the pole are $P_x = 1.5$, $P_y = 0.5$, as given in Table 1.

In case 3e, the translation is not rectilinear but has the shape of an arc of a circle. Its components take the form:

$$T_x = R \cos(\sigma + \tau t) - R \cos \sigma$$

$$Ty = R \sin(\sigma + \tau t) - R \sin \sigma$$

Here R is the radius of the circular arc, σ is the angle of the starting point of the arc, and τ is its angular extent. In the present case $R = 0.5$, $\sigma = -\pi/2$, and $\tau = -\pi$. The rotation parameters are $P_x = 1$, $P_y = 0$, $\omega = -\pi/2$. Thus the pole is the same as in case 3c, but in this case its path is semicircular.

The motion in case 3f, which was represented above as an instance of pure rotation, can also be defined as a case of circular translation + rotation, with the following parameters. As in case 3c, the coordinates of the rotation pole are $P_x = 1$, $P_y = 0$, and the extent of rotation is $\omega = -\pi/2$. However, the parameters of circular translation in this case are $R = \sqrt{0.5}$, $\sigma = -3\pi/4$, and $\tau = -\pi/2$.

NOTE

1. The scientific fields that Shepard calls "kinematic geometry" and "classical physics" correspond to topics often referred to in mechanics texts as "kinematics" and "dynamics," but I stick with Shepard's labels in this paper.

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