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Generated designs: Structure and composition

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Abstract

Generated designs are formed through rules that change, and structures that interpret, descriptions. The structure of design descriptions is examined with particular reference to generative shape descriptions acting as the infrastructure for attributes. Structure is represented by relations among the parts of descriptions. In particular, a closure structure allows relations to be described by the parts themselves. Design processes use descriptions in combination. It is shown how to combine separate descriptions in parallel and sequentially. Rules change descriptions. It is shown how to aggregate local changes consistently across a design and to use rules with structure on their component shapes to highlight selected emergent features. Further, multiple attribute descriptions, including shape and associated properties, are constructed. Their structures are represented algebraically.

Keywords: Description; Relation; Closure; Shape; Attributes

1. INTRODUCTION

The processes of design can be represented in terms of the relations and interactions among tasks and subtasks (Eppinger et al., 1994). Each stage in the process may use a different mix of descriptions. Design descriptions grow and change through the processes of design. Descriptions are used in diverse ways. Geometric descriptions of shape may be of object shape or intermediate shape elements used for analysis, such as the meshing models in finite element analysis. Property or attribute descriptions may deal with such disparate elements as cost, material properties, or associated manufacturing processes. A general view of design descriptions is provided by Stiny (1981). The relations between process stages are constrained and facilitated by the descriptions. This paper examines the structure of design descriptions with particular reference to shapes and how the structures interact with the generative rule systems that are used to create and change designs. The paper starts with design descriptions based on shape descriptions. Later composite descriptions, including shape and physical or functional attributes, are developed.

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2. DESCRIPTIONS AND DESIGN PROCESS

The structure of design descriptions will be represented in terms of the relationships among the parts of a design, which are selected for the description. The idea that a description can be expressed as a selection of parts within well-defined algebras of designs is developed by Stiny (1991). From these insights arise the ideas that the structure of these descriptions can be characterized in terms of topological properties (Stiny, 1994). Changes to structure through generation or interpretation can be made consistent through continuity conditions.

Design descriptions develop through the design process. Coordination of activities in the organization of product development needs to take account of the contingent and iterative natures of design. Model-based methods (Eppinger et al., 1994) are effective in addressing the process view of product development. However, processes require an infrastructure of descriptions. Indeed, we could represent processes by changes made to descriptions. This duality of product and process is encapsulated in generative and grammatical paradigms of designs and design processes (Brown, 1997; Stiny, 1989).

Design descriptions are created and changed in many ways during the processes of design. The generative methods of grammar provide a model for creation and change in design descriptions. This paper concentrates on the types of struc-

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tures that descriptions display, and the freedoms and constraints on these structures during the processes of design. These positions, with the structural on one hand and the generative on the other, are complementary. The perceptual and generative coexist in both the cognition and the creation of designs (Stiny & Gips, 1978).

In a design process a design change may have corresponding changes in component feature descriptions and assembly descriptions. Sometimes a single design transformation corresponds to many component or assembly changes and, in other cases, whole sequences of design transformations correspond to single component or assembly transformations. We observe a structure of related changes that are made to the different descriptions. However, aggregates of these related descriptions are also descriptions. It will be shown how the structure of these combined descriptions is derived from that of the constituents.

Design descriptions necessarily possess structure to organize the information they contain through data structures and standards for product models. For example, geometric models relate parts of the geometry of a design. They may have boundaries with incidence relations or constructive compounds of primitives. Both types of information and structure may be held simultaneously to facilitate the different manipulative and generative purposes of the designer. More complex structures arise from attributes associated with parts of the design.

The structure of design descriptions, and processes that use them, have been approached directly by Rosen and Peters (1996). Particular attention is given to the geometric and shape descriptions in computer aided design, which make extensive use of topological ideas in constructing and manipulating designs. Peters, Rosen, and Shapiro (1994) examine the relations among types of descriptions (e.g., geometric models and manufacturing process representations) in terms of maps between the spaces of descriptions. Each type of description is represented by a metric space. A map between the metric spaces relates changes in one description to changes in the other. A particular role is identified for continuous maps between design and manufacturing descriptions that ensure that changes in one description correspond to comparable changes in the other. Related research by Sarma and Wright (1997) reinforces this approach with reference to incremental design where movement through a space of feasible designs is "dependent on the structure of the feasible space."

Rosen and Peters (1996) emphasize the importance of the interplay between algebra and topology in design descriptions. Stiny (1991, 1994) also develops the algebraic structure, within which design descriptions are placed, as well as the topological structure of individual design descriptions. We note here that certain algebras give rise naturally to associated topologies. Thus, for example, in shape descriptions, the Boolean algebra of subshapes of a shape has an associated topology (Earl, 1997; Koppelberg, 1989). Specific structure relative to this topology arises from the selection of parts of the shape. This selection of parts also provides boundaries and other explicit indications of structure (Earl, 1997). Structure is manifest when parts are selected. The residual or background structure, the Boolean algebra of subshapes, provides no differentiation among parts.

3. PARTS AND RELATIONS

Parts or selected subshapes are used for geometrical descriptions of designs. These parts may be primitive shape elements, maximal elements or more general subshapes, including parts with their own structure of subparts. Selected parts are used in the description of feature-based design (Shah, 1991), where designs are interpreted or generated in terms of features. Different feature descriptions are used for different views of the product and feature mappings are used for transformation between descriptions.

Descriptions not only identify parts, but also define their structure by relations between parts. We show how parts can be used to define relations and how this implies that the parts have a particular structure, namely, a closure structure. We demonstrate how these closure structures arise for shapes.

If a shape description is a selection of subshapes, then these should contain information about their relations. This is usually accomplished in geometric models by the use of coordinate systems that locate each of the parts relative to the other parts. However, spatial relations in shapes are described by the component shapes and their composition. To include all information on spatial relations among parts, the sums of parts should also be parts. Further, a unique specification of the way that the parts are configured in a spatial relation also requires that the intersection (or product) of parts is a recognizable part. In addition, descriptions of shapes are assumed finite, that is, there are a finite number of parts.

Thus, for a selection of parts of a shape to contain information about their spatial relations, it is necessary that the sums and products of parts are also parts. Further, the description must extend across the whole of the original shape, giving the property that the sum of parts is the original shape. Therefore, each shape is a part of itself; the part relation is reflexive.

The conditions that the spatial relations between parts are recoverable from the parts yield a closure relation (Ore, 1943*a*, 1943*b*) on the parts. A consequence of this closure relation is that for any subshape of the original shape there is a unique minimal part containing the subshape. This can be defined as the closure of the subshape.

The closure structure and associated closed subshapes (the selected parts) can be viewed as a topology through the standard closure definitions of topology (Kuratowski, 1972). Consider the connection between this topology and the underlying description of a shape as composed of all possible subshapes. The associated topology (Earl, 1997) reflects the lack of structure in this complete description of parts, pre-

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senting a highly disconnected space without boundaries. Thus, the presence of boundaries (Earl, 1997) indicates structure. Boundaries are used in design either as a means to induce structure (as in the case of interpreting geometric models or computer vision images) or for expressing a structure of parts already present in the design description.

4. CLOSURE STRUCTURE

We use the example of shapes and subshapes to illustrate the formal properties of closure structures. For a shape, S, let selected subshapes be denoted $C(S)$ and an associated closure relation $c(s)$ on the subshapes of s of S, which maps a general subshape s to the minimal shape in $C(S)$, which contains s.

The subshapes in $C(S)$ are a closed set of subshapes. Any finite set of subshapes $\{s_i\}$ in $C(S)$ has the properties:

1. The union (or sum) of shapes $\sum s_i \in C(S)$;
2. The intersection (or product) of shapes $\prod s_i \in C(S)$;
3. $S \in C(S)$; and
4. $0 \in C(S)$, where 0 is the empty shape.

The definition of closure $c(s)$ of a subshape s implies that for any finite set of subshapes $\{s_i\}$ in $C(S)$:

1. $cc(s) = c(s)$ for any subshape s in S;
2. $c(\sum s_i) \supseteq \sum c(s_i)$;
3. $c(\prod s_i) \subseteq \prod c(s_i)$; and
4. $c(0) = 0$, where 0 is the empty shape.

The closure relation also satisfies:

5. $c(s) \supseteq s$ for any subshape s in S; and
6. $s_i \geq s_j$ implies $c(s_i) \geq c(s_j)$.

The subshapes $C(S)$ are precisely those subshapes s of S with $c(s) = s$, namely, the subshapes closed under the closure relation.

These definitions are standard from the mathematical literature (Ore, 1943*b*; Kuratowski, 1972) except to note two things. First, they are not applied to point sets but to subshapes. Second, in applying them to shapes only finite sets of subshapes are valid descriptions. These two provisos serve to emphasize that the descriptions created for designs are not mathematical abstractions. The properties above are derived naturally from the design requirements on descriptions, namely, that the selected parts are expressed in terms of relations. The core role of spatial relations in all aspects of design, manufacture, and assembly makes this underlying requirement a useful and potentially powerful starting point for examining the structure of design descriptions.

Closure structures, as described above, are not the most general and it is not the intention here to examine the nuances of these structures, which are presented in the literature. Indeed, because these mostly deal with point-set representations rather than the more structural representations we are considering here, they are not directly applicable to shapes whose underlying structure is of subshapes and the part relation.

The properties presented above for shape descriptions are the model for general design descriptions in terms of parts. For mechanical designs, shape descriptions play an essential role. Engineering properties are associated with shape and relationships between the spatial elements. These generalized descriptions, which are in terms of designs and parts, follow the same pattern as shape descriptions. The theory developed for shape descriptions can readily be generalized. A design may have many descriptions each with a closure structure. We now examine the space of descriptions and some algebraic properties of this space. These allow descriptions to be compared and new ones constructed by composition.

5. COMBINATIONS OF DESCRIPTIONS

A design description may contain several views or be expressed with respect to multiple types of properties or attributes. Typically, a design process uses many descriptions. For example, a geometric solid model may be augmented by a finite element geometric model for structural analysis, by component feature models for manufacture, and by subassembly models for functional and behavior representations. It is therefore important to consider how descriptions are combined. Again, we use the specific example of closure structures on shapes.

Suppose $C_1(S)$ and $C_2(S)$ are two closed sets of subshapes describing a shape S. Corresponding closure relations c_1 and c_2 have closures of a subshape s given by $c_1(s)$ and $c_2(s)$ in the two relations. A partial order can be defined on the closure relations with c_1 containing c_2, or $c_1 \geq c_2$, if $c_1(s) \geq c_2(s)$ for all subshapes s in S.

If a closure relation c_1 contains c_2, that is, $c_1 \geq c_2$, then all subshapes that are closed under c_1 are also closed under c_2. This means that $C_1(S) \subseteq C_2(S)$. The $C_2(S)$ description is finer and possesses a greater resolution than the description $C_1(S)$. Alternatively this can be expressed by saying that the $C_2(S)$ description includes all the structure of the $C_1(S)$ description. This relation between descriptions captures the idea of hierarchy of descriptions. However, it is a partial order rather than a strict hierarchy, and descriptions can overlap in more complex relations. In these cases, the descriptions may be taken together as a composite design description. We show two combinations corresponding to the "union" and "intersection" of descriptions.

For descriptions $C_1(S)$ and $C_2(S)$ each consisting of closed sets of subshapes, it is possible to generate the combined description $C_1(S) + C_2(S)$ consisting of the subshapes that are in $C_1(S)$ or $C_2(S)$ and in finite sums or products. Figure 1 shows the parts in two simple descriptions of a shape S and illustrations of parts in the combined description. The

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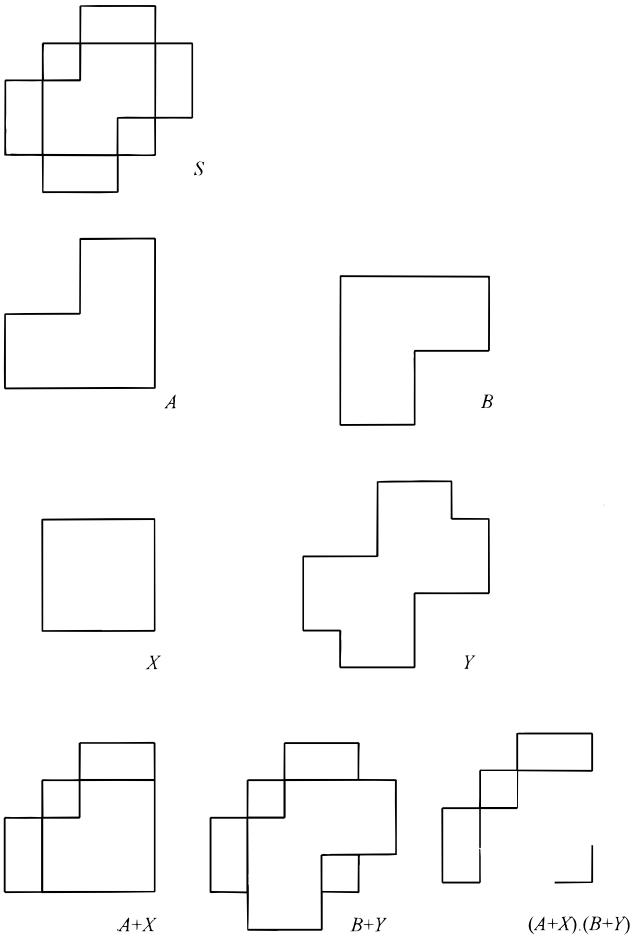


Fig. 1. Shape S composed of line segments. Two descriptions are shown based on selected parts $C_1(S) = \{S, A, B, 0\}$ and $C_2(S) = \{S, X, Y, 0\}$. Three parts of the combined description $C_1(S) + C_2(S)$ are illustrated: $(A + X)$, $(B + X)$, and $(A + X)\cdot(B + Y)$. There are several other parts in the combined description.

shape is composed of line segments and does not depict areas or regions. Another combined description is $C_1(S)\cdot C_2(S)$, defined by subshapes in $C_1(S)$ and $C_2(S)$. These definitions of combinations of descriptions correspond to moving up and down the partial order on closure relations finding the smallest closure relation larger than c_1 and c_2 (denoted $c_1 \vee c_2$ in the partial order) and the largest closure relation smaller than c_1 and c_2 (denoted $c_1 \wedge c_2$ in the partial order). The correspondence of $C_1(S) + C_2(S)$ to $c_1 \wedge c_2$ and $C_1(S)\cdot C_2(S)$ to $c_1 \vee c_2$ is thereby established.

The description $C_1(S) + C_2(S)$ is a finer description than either $C_1(S)$ or $C_2(S)$ because it contains all the divisions in each description as well as the combinations across these descriptions. The description $C_1(S)\cdot C_2(S)$ is a coarse description, containing only those divisions that are shared across the two descriptions.

The structure of possible descriptions is a partial order. This leads us to view the descriptions as part of a lattice (Birkhoff, 1967) of descriptions rather than a hierarchy. Although strict hierarchies are not precluded and are used in many design cases, there is the possibility that some relations between descriptions and thus descriptions themselves might be missed or hidden. Another way of viewing the combined description $C_1(S) + C_2(S)$ is that the corresponding closure $c_1 \wedge c_2$ of a subshape s in S is the intersection of the closures of the subshape in c_1 and c_2, that is, $(c_1 \wedge c_2)(s) = c_1(S)\cdot c_2(S)$.

Combinations of descriptions can be extended to cover the case in which the parts of one description are placed side by side with the parts of another description. This Cartesian product of descriptions corresponds to multiple descriptions being used simultaneously. There is another type of product combining descriptions that rather than putting the two descriptions together in parallel applies them sequentially. Thus, a description is examined from the perspective of another description. It will be unlikely that this modulated view will satisfy all the properties of the original description, but there are simple conditions on the two descriptions that lead to the product also being a description.

The "sequential" product can be defined in terms of closure relations c_1 and c_2 on shape S as $c_1(c_2(s))$ for subshapes s in S and denoted by a product notation $c_1 \times c_2(s)$. If $c_1 \geq c_2$, then $c_1 \times c_2 = c_1$, that is, the coarsest description dominates in the product. Further, in this case, the product is a description. In line with these observations the join of two descriptions $c_1 \vee c_2$ (which is the smallest description containing c_1 and c_2) will satisfy $c_1 \vee c_2(s) \supseteq c_1 \times c_2(s)$ for all subshapes s in S. In fact $c_1 \vee c_2$ is the smallest closure relation for which these inclusions hold. Thus, although $c_1 \times c_2$ may not be a closure, it generally represents a coarser description than either c_1 or c_2. This is to be expected as the product description effectively looks at the divisions created by c_1 through the perspective of the description c_2. The result cannot be a finer subdivision than either c_1 or c_2. An illustration is shown in Figure 2 where the shapes are plane regions or areas. The boundary lines indicate the extent of the areas but are not part of the shapes. By taking different structures on the two parts S_1 and S_2, different closure structures on the whole shape are constructed. Figure 2(ii) has parts in S_1 indicated by the three shaded squares and the whole of subshape S_2. Figure 2(iii) has parts in S_2 indicated by the three shaded squares and the whole of subshape S_1. Some of the parts composing the two different descriptions $C_1(S)$ and $C_2(S)$ are also indicated. The sequential product description is shown in Figure 2(iv) with some of its parts shown shaded. Figure 2(iv) demonstrates how the sequential product has "coarser" parts. Further, note that this sequential product does not produce a closure relation and is thus not a proper description.

Returning to the question of finding the conditions under which $c_1 \times c_2$ satisfies all the conditions of a description, the previous result suggests that $c_1 \times c_2$ can only be a description if $c_1 \vee c_2 = c_1 \times c_2$. This is equivalent to requiring that when the closure c_1 acts on the description c_2 it does not produce any additional divisions. There is a further sym-

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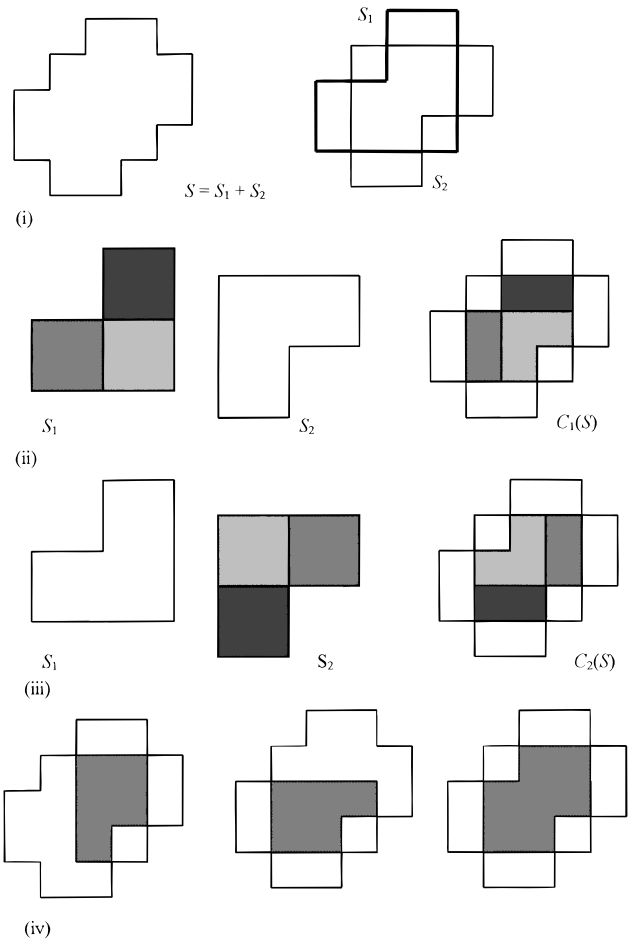


Fig. 2. (i) Shape S composed of a connected area defined by boundary shown. The shape is considered as composed of two areas S_1 and S_2, which are further subdivided. (ii) Area parts of shape S as basis for description $C_1(S)$ with some closed parts of the description shown shaded. (iii) Parts of shape S as basis for description $C_2(S)$ with some closed parts of the description shown shaded. (iv) Some parts of the description formed by the product $c_1 \times c_2$ of corresponding closures are shown by the shaded areas.

metry that $c_1 \times c_2 = c_2 \times c_1$ if and only if both products are closure relations.

6. TRANSFORMATIONS OF DESCRIPTIONS

The above discussion has considered how descriptions can be combined and indicates how multiple descriptions can be used simultaneously. If these are considered as descriptions related by maps or functions, there is an immediate connection to transforming descriptions during design processes. For rule-based generative processes, the rules themselves give rise to the maps. As maps between structures, they are characterized by the extent to which they preserve structure. Those that maintain the relations between parts, although the designs may change, are isomorphisms; these changes might occur at detailed design stages where dimensions and tolerances of components are specified, but without any configurational or structural change.

More important and general are the continuous maps that do not lose structure. Any divisions that can be maintained within the framework of the changing agent are retained. The formal details are given in Stiny (1994), where a variety of maps based on rules are examined. Further details on rule-based change are developed by Krishnamurti and Stouffs (1997). Continuity is a condition that keeps all parts in place under the map. We examine the ways that changes between descriptions can be divided among many parallel local changes.

The application of a shape rule to a shape S in a generative system will remove a subshape of S and "replace" it with another shape. The structure of the generated shape need not be prescribed by the rule and ultimately the history of generation to that point. Continuity conditions provide a way of ensuring that structure created at one stage of generation is not discarded, without intervention of rules, at subsequent stages. Stiny (1994) emphasizes that the map between structures at successive stages of generation although based on the action of the rule is not determined by the rule. There are many ways to interpret a rule application as a map between closure structures on shapes. One of Stiny's concerns was to allow "post hoc" structure identification so that it is possible to explore how shape computations corresponding to rule applications might be made continuous. There is freedom to interpret the shapes participating in a rule by assignment of closure structures and freedom to interpret the rules by assignment of mappings between the closure structures.

Sequences of rule applications can be identified as continuous if each of the rule applications is continuous. We look a little more closely at the way that changes to shapes are implemented by shape rules. A rule r: $a \rightarrow b$ is indiscriminate in that it does not distinguish parts in shapes a or b. Consider the case in which a change is made to a shape by two rules, r_1 and r_2, applied in parallel. However, each of these rules is local in the sense that they are only applied to specified parts of the shape rather than the whole shape. This mirrors the case in which design development is conducted on selected parts of a design. The selected parts may overlap or indeed one may be contained in the other, as when components of subassemblies are changed at the same time as the assembly itself.

How are these local changes combined? One way would be to put the two selected parts, S_1 and S_2, together and reform the rule as a single operation. Suppose each of the changes is continuous with respect to the corresponding closure structures on S_1, S_2 and their images S_1', S_2' under the rule application. The shape $S_1 + S_2$ can be given a natural closure structure by adding extra structure in the intersection $S_1 \cdot S_2$ comprising intersections of closed shapes in S_1 and S_2. The intersection thus displays a finer structure. Alternatively, this structure is derived as the sum of two closures on $S_1 + S_2$. One of these is obtained by adding the

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shape S_2 to the closure structure for S_1 together with intersections of S_2 with closed shapes in S_1 to give $C_1(S_1 + S_2)$. The other closure $C_2(S_1 + S_2)$ is constructed symmetrically. The sum of closures $(C_1 + C_2)(S_1 + S_2)$ as defined previously, is finer than each of C_1 and C_2 containing all divisions in C_1 and C_2 as well as a more refined structure of parts in the intersection $S_1 \cdot S_2$. Figure 3(i) shows a shape with two parts S_1 and S_2 identified. The shape and the structures of parts are similar to those considered in Figure 2. The parts need not cover the whole shape, as is the case in this example. Figures 3(ii) and 3(iii) illustrate how closure structures $C(S_1)$ and $C(S_2)$ on two parts S_1 and S_2 of a shape S induce closure structures $C_1(S_1 + S_2)$ and $C_2(S_1 + S_2)$ on $S_1 + S_2$. The shaded parts serve to indicate the closed parts present in the shapes. They are representative with each closure description possessing several other parts. Figure 3(iv) combines the closure structures to give a finer closure structure on $S_1 + S_2$. Again, representative closed parts are indicated by the shaded squares.

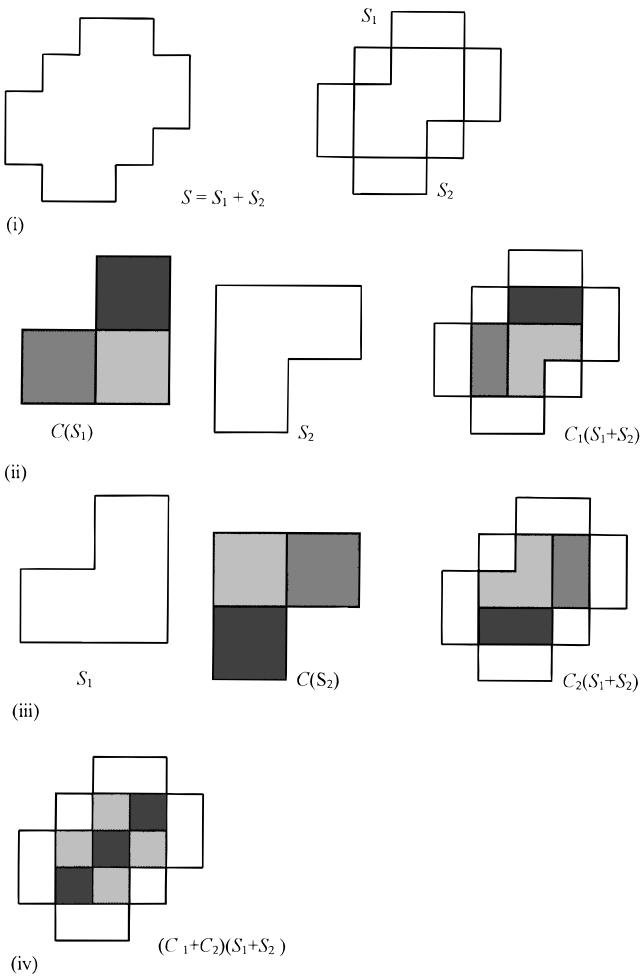


Fig. 3. (i) Shape S with two parts S_1 and S_2. (ii) The part S_1 has a closure structure $C(S_1)$, which induces closure structure $C_1(S_1 + S_2)$ on $S_1 + S_2$. (iii) The part S_2 has closure structure $C(S_2)$, which induces closure structure $C_2(S_1 + S_2)$ on $S_1 + S_2$. (iv) The sum of closure structures $(C_1 + C_2)(S_1 + S_2)$ on $S_1 + S_2$ has finer divisions in the intersection of S_1 and S_2 as shown by the shaded squares.

In the original case, the two rules provided changes that were individually continuous. Suppose that both are made together according to the rule $a_1 + a_2 \rightarrow b_1 + b_2$. With closure $C_1 + C_2$ on $S_1 + S_2$ and a corresponding construction of closure on the result of the rule, the composite rule is continuous according to the interpretation of the rules in terms of maps. As there are many interpretations of rules by maps (Stiny, 1994) we need to assume that the particular form of the maps for the rules r_1 and r_2 is the same. Figure 4(i) shows a shape S with subshapes S_1 and S_2, which each have their own structures. Rules r_1 and r_2 (Fig. 4(ii)) apply to these subshapes with matching shape and structure. The structures of parts in shapes S_1 and S_2 are indicated by shaded areas on the left sides of the rules. The rules replace the shapes S_1 and S_2 with S_1' and S_2'. The shapes S_1' and S_2' have their own structure of parts indicated by the shaded areas on the right side of the rules. The induced structures $C_1(S_1' + S_2')$ and $C_2(S_1' + S_2')$ on $S_1' + S_2'$ are combined to form the structure $(C_1 + C_2)(S_1' + S_2')$ indicated by the shaded areas in Figure 4(iii). A form of the composite rule in which the rules apply in parallel is shown in Figure 4(iv) with structure of the left and right sides of the rule indicated by shaded areas.

Other combinations of rules can be examined. For example, the rule substitutions may not take a_1 and a_2 together, instead taking them separately in the form $S_1 + S_2 \rightarrow ((S_1 - \tau a_1) + (S_2 - \tau a_2)) + (\tau b_1 + \tau b_2)$, where τ is the transformation used in applying the rule. The combined rule now only applies to the two selected parts S_1 and S_2 independently. Then a change in an area of a design, which is shared by the selected parts, under one rule is not always maintained in the combined rule. The other component of the rule may not affect that area of the design. It appears that the combined rule is still continuous. This formal situation corresponds to the case in which different aspects of a design are represented at some stage by the same shape.

However, although merging at one stage in generation they maintain their separate identities to emerge later. For example, an underlying grid or more general modular scheme may coincide with shape features, but in changing the shape features it would be desirable to leave the grid in place or at least allow it to change according to its own rules.

Thus, we see that by adding more structure to a shape, it is possible to combine local changes so that they are made consistently across the design. We now consider how rules themselves can be used to add structure. The freedoms to interpret shapes (through the selection of parts) in the search to give meaning to shape computations allow some constraints to be applied within the rules. These constraints might take the form of distinguishing parts of the shapes in the specification of a shape rule. Thus, a rule r: $a \rightarrow b$ may have prescribed structure on the shapes a and b, which constrain the structure of the shape to which the rule can apply as well as imposing partial structure on the result.

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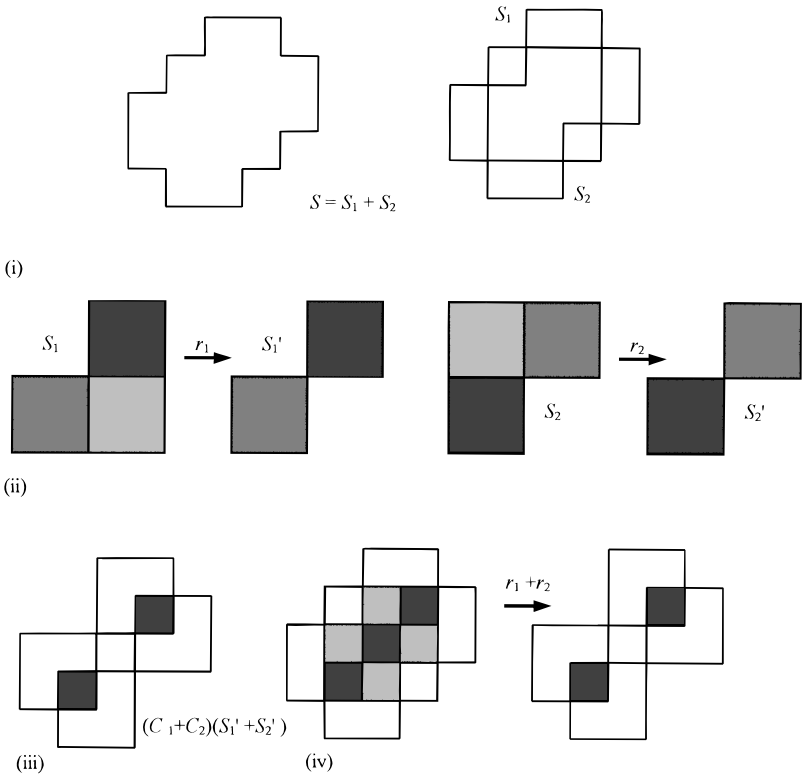


Fig. 4. (i) Shape S with two parts S_1 and S_2. (ii) The part S_1 has a closure structure indicated by the shaded areas on the left side of rule r_1, which induces closure structure $C_1(S_1 + S_2)$ on $S_1 + S_2$. The part S_2 has closure structure indicated by the shaded areas on the left side of rule r_2, which induces closure structure $C_2(S_1 + S_2)$ on $S_1 + S_2$. (iii) The induced closure on $S_1' + S_2'$, $(C_1 + C_2)(S_1' + S_2')$. (iv) The composite rule $r_1 + r_2$.

A rule is discriminating where it can be applied to specific spatial relations among parts rather than the overall shape. Further, the rule will partially determine its interpretative effect in picking out features and spatial relations in the productions using the rule. More concretely, the implementation of design changes to an engineering component picks out a particular configuration of features and replaces it with another related set of features. The way that the result of a rule is interpreted depends on the implicit structure in the rule.

Suppose a rule r: $a \rightarrow b$ has closure structures C_a and C_b on a and b. The application of the rule to a shape S should exhibit a matching of the parts of a with corresponding parts of the shape S. Consider the application of this rule with an interpretation of the parts of the resultant shape provided by a combination of two elements of structure. First, the structure on shape τb, where τ is the application transformation, and second, the residue of the structure on shape a after rule application (that is, the shape $S - \tau a$). There are now two shapes, $S - \tau a$ and τb, each with a closure structure. To combine these we can extend both trivially to the total shape $S - (\tau a - \tau b)$ and then combine the closures by addition to create the finer closure structure.

A particular case is presented by the rule that does not change shape, but changes structure. These rules of the form r: $a \rightarrow a$ allow the change of structure to parts of the shape S. They provide a generative description of the structural interpretation of a shape. It might be argued that generative shape change and shape interpretation be considered separately, but in practice change is commonly effected with intent to produce an intended interpretation, for example, to satisfy engineering requirements. This is not to imply that design actions are determined by intent, but that intent constrains the outcomes of rules. Thus, a design rule might focus on a particular area of a design, changing its shape and structure of parts intentionally, but leave free interpretation elsewhere (perhaps within the constraints of continuity). Emergent parts are possible outside the prescriptions of the structured rule. Thus, for example, an engineering subassembly may be replaced by an alternative. The intent may be to change properties such as cost or performance. The new assembly design as a whole might possess new emergent features such as free space or alignments, which can be used by other rules.

7. MULTIPLE ATTRIBUTE DESCRIPTIONS

A useful feature of shape descriptions is the algebraic systems in which the descriptions lie. These classify descriptions in geometric terms through the dimensions of the shape elements they use and through other properties or attributes attached to parts of the shape. Complex descriptions have components in many algebras and are considered as descriptions in the product algebras (Stiny, 1991). In this paper we indicate how to deal with these multiple descriptions using a simple example of a description with a shape and a property component. The framework for this development com-

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prises the weighted shapes of Stiny (1992) and attribute grammars applied to design by Brown et al. (1995).

Consider a description of shape S with parts $C(S)$. Each part s_i in $C(S)$ has associated with it an attribute value p_i. The description is composed of composite parts (s_i, p_i), where attribute values will need to satisfy internal consistency conditions for "sums" and "products" of composite parts:

$$(s_i\, p_i) + (s_j, p_j) = (s_i + s_j, \min\{p_i, p_j\})$$

$$(s_i, p_i) \cdot (s_j, p_j) = (s_i \cdot s_j, \max\{p_i, p_j\}).$$

These conditions give a weak property to the extensive part $s_i + s_j$ and a strong property to the included part $s_i \cdot s_j$. The limitations of this simple definition are that the property values may not form a strict order and the minimum and maximum of properties cannot be given. If property values are partially ordered, then the minimum and maximum are replaced by greatest lower bound and least upper bound. The richness of the property structure was clearly envisaged by Stiny (1992), although no specific partially ordered structure was investigated there. The resulting design descriptions have a composite structure. One component is the shape structure and the other the property structure. In the definition indicated above, there is a parallel between the structure of the shape descriptions and the associated properties. Using the minimum and maximum in the algebraic operations on the composite parts, we find the following relations of shape and properties: $s_i \leq s_j \Rightarrow p_i \geq p_j$. Note the special case when all shapes in the shape description component have the same property value and the structure collapses to the shape description alone.

The composite description can be decomposed into a series of descriptions corresponding to each property value. This provides an example of several descriptions being used simultaneously. For each property value p_i select parts s_j such that $p_j \leq p_i$, that is, those parts that have property value $\leq p_i$. Let these induced descriptions for property value p_i be denoted $C_i(S)$. For $p_i \leq p_j$, we find that $C_i(S) \leq C_j(S)$ and the corresponding closure relations satisfy $c_i \geq c_j$. Thus, as the property value is set at a value that discriminates less then there are fewer parts and less structure. A related set of descriptions picks out parts that have higher property values. These latter are not descriptions of the whole design in contrast to the earlier descriptions that picked out parts that have lower property values. The descriptions only apply to a subshape identified by the property value and neglects parts with lower values.

8. PRODUCTS OF DESCRIPTIONS

Finally, consider the case in which there are two descriptions $C(S)$ and $C(T)$ of different designs S and T. Combined design descriptions can come from many sources. For example, there is a product description $C(S) \times C(T)$ based on pairs of elements from $C(S)$ and $C(T)$ with appropriate finite-shape sums and products added. The development of such products follows that of product topologies (Kuratowski, 1972) just as the closure structures parallel the development of topology through the closure axioms. A feature of these product descriptions is that they can be applied to the descriptions of designs in the product algebras examined by Stiny (1991). Examples of applications of these product algebras to spatial layouts can be found in Krishnamurti and Earl (1998).

In these product algebra descriptions, designs are described through geometric elements of different dimensions such as points, lines, planes, and volumes, as well as types of attributes with value structures (for example, partial orders). There is a range of descriptions, each attending to particular types of geometric elements or attributes. These different descriptions may be interrelated. For example, in mechanical design, the strength of a feature is related to material attributes and shape. The product allows complex multiple descriptions to be constructed. Projection maps take slices of these multiple descriptions, showing particular geometric elements or properties. Note that the structure built into the product description is more complex than the aggregation of individual structures; the product contains relations among parts across component descriptions, as well as among the elements within the component descriptions.

9. CONCLUSION

Design descriptions are examined as compositions of parts. This view is derived from the capability to include emergence and generation within the computational framework based on such descriptions. Examination of the detailed structure of these descriptions shows the necessity for closure properties. These are deduced from requirements for design descriptions to express relations among parts. For shapes, these are spatial relations. Combinations of descriptions dealing with descriptions of the same type can be constructed in several ways to give other descriptions of that type. The algebraic construction of new descriptions presents many ways to look at a single design. Knowledge of this space of possible descriptions is key to understanding emergent properties and how designs can be changed by applying rules.

Rules change designs, but do not necessarily determine corresponding changes in their structure. These changes in structure can be constrained by conditions such as continuity of change, but in design are generally allowed extensive freedom. The views or interpretations of a design are further constrained by assigning structure to the rule itself. The remaining structure is then fitted around this predefined structure. The implicit structure on a design derived from the rules that generate it can be associated with an intention to effect change in structure or view of the design.

In the latter part of the paper, methods of combination are outlined for descriptions of different types. One instance examines shapes with attached property values and

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another considers a more general product of descriptions over different types.

Designs are complex, multidimensional structures as demonstrated in the breadth of attributes and relations they encompass and the depth of the hierarchy of levels of detail and abstraction in which designs are described. The results presented illustrate some of the formal tools that are useful in constructing design descriptions. Generative systems change designs and descriptions. The generative history of a design provides the constraints and context for a descriptive history of evolving structure and meaning. Descriptions are woven through the generative history of a design; created, composed, and discarded. This algebra of structure and composition is key to analyzing the freedoms and constraints in generative design systems.

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