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ARTICLES

A PATENTABILITY REQUIREMENT AND INDUSTRY-TARGETED R&D

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I introduce a minimum innovation size required for patents into a Schumpeterian growth model. We show that to satisfy the patentability requirement for minimum innovation size, each research and development (R&D) firm targets only industries in which the incumbent's technology is of sufficient obsolescence. This is because the technological gap between innovator and incumbent is greater in industries using older technologies. Although the increase in minimum innovation size reduces the number of industries targeted for R&D, it also increases the amount of R&D investment directed at those targeted industries. Consequently, introducing a minimum innovation size has a nonmonotonic (or negative) effect on the aggregate flow of innovations. Further, by deriving the endogenous long-run distribution of innovation size, we show that an increase in minimum innovation size reduces the mean innovation size. This implies that even if the patent office only grants patents for superior innovations, it causes innovators to produce generally inferior-quality innovations.

Keywords: Innovations, Intellectual Property Rights, Productivity Distribution

1. INTRODUCTION

In many countries, patent offices have some notion of a *minimum innovation size* that they require for patentability; i.e., the patent office grants a patent only for a sufficiently superior invention compared with prior inventions. For example, in Japan, Europe, and the United States, the patent office grants a patent only if an invention satisfies several statutory requirements, referred to as patentability requirements. One of these patentability requirements is that there is a minimum innovation size required for patents, known as an inventive step (nonobviousness). Although the expression "inventive step" is mainly used in Europe, it is synony-

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mous with "nonobviousness," as used in the United States. Hereafter, we use the term "minimum innovation size" to signify an inventive step (nonobviousness).

Reports by the United States Federal Trade Commission (FTC) (2003) and the National Academy of Sciences (NAS) (2004) recommended substantially strengthening the minimum innovation size requirement.[1] The FTC (2003) and NAS (2004) proposed raising the overall quality of patents by reducing the number of improperly issued patents. According to Lemley and Shapiro (2005), a high proportion of issued patents have little or no commercial success, which generally implies low patent quality. Thereafter, in the case of KSR International Co. v. Teleflex Inc. (2007), the United States Supreme Court dealt with the issue of the requirement for minimum innovation size.[2] Following this court decision, the criteria used to determine whether an invention satisfies a statutory requirement of minimum innovation size were expanded, which made patenting more difficult in the United States.

Inspired by these developments, I analyze how strengthening the minimum innovation size affects innovations. Specifically, I introduce a minimum innovation size requirement for a patent into the Schumpeterian growth model in Aghion and Howitt (1998, Chap. 3). I then determine the effects of strengthening the minimum innovation size on the quantity and quality of innovations (represented by the aggregate flow and mean or median size of innovations, respectively). Hereafter, the size of innovations (or the innovation size) is "the technological gap between innovator and incumbent."

By incorporating a statutory requirement of minimum innovation size into the model of Aghion and Howitt (1998, Chap. 3), we show that each research and development (R&D) firm optimally chooses the industry in which it seeks to obtain a patentable innovation size. Consequently, R&D firms target only those industries employing sufficiently obsolete technologies. This is because the technological gap between innovator and incumbent is wider in industries using older technologies when the innovator introduces the frontier technology into the industry's existing products. However, although an increase in minimum innovation size reduces the proportion of industries targeted by R&D, it also increases the R&D investment that flows into the targeted industries, because each innovator becomes a long-term survivor. There then exists either an inverted U-shaped or a negative relationship between minimum innovation size and the aggregate flow of innovations. Notably, I show that the minimum innovation size requirement plays the role of patent length, so that R&D firms must wait to conduct R&D in industries whose technological levels are currently frontier technology until the technologies become sufficiently obsolete. Therefore, the patent holders do not have to be afraid of creative destruction until their technologies are obsolete. An increase in minimum innovation size then requires a higher degree of obsolescence of the incumbents' technological levels to achieve the patentable innovation size. I show that an increase in minimum innovation size invokes a longer no-risk period for creative destruction, and therefore implies the reinforcement of intellectual property rights (IPR) protection.

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Furthermore, I show that the distribution of innovation size converges to a Pareto distribution in the long run.[3] Surprisingly, using the distribution, an increase in minimum innovation size reduces the mean innovation size, but increases the median innovation size. An increase in minimum innovation size, of course, has a positive effect on both the mean and median innovation size. However, an additional negative effect on the mean and median is as follows. As targeted industries decrease in number, there is a negative effect on the number of applications filed for patents. The amount of knowledge spillover then declines because the patent applicants must disclose detailed information about their inventions. Innovators combine the public knowledge, which grows via disclosed information about past inventions, with existing products, and thus the size of innovations becomes generally less effective. Because of the dominant negative effect of the reduction in knowledge spillover, an increase in minimum innovation size reduces the mean innovation size. In contrast, the median innovation size increases because of the dominant, usually positive effect of minimum innovation size. These results imply that a substantial strengthening of the minimum innovation size requirement following the U.S. Supreme Court's decision in KSR v. Teleflex may result in a reduction in patent quality in the long run. This is counter to the predictions of the FTC (2003) and NAS (2004), which had suggested an increase in patent quality via the strengthening of the minimum innovation size requirement.

The remainder of the paper is organized as follows. In Section 2, I discuss the difference between minimum innovation size and patent breadth and survey the related literature, both empirical and theoretical. In Section 3, I outline the endogenous growth model of Aghion and Howitt (1998, Chap. 3), which incorporates minimum innovation size. In Section 4, I show that there is a unique balanced-growth equilibrium and derive the effect of an increase in minimum innovation size on the aggregate flow of innovation. In Section 5, I derive the productivity distribution and compare this with the empirical productivity distribution. In Section 6, I use the result from Section 5 to derive the distribution of innovation size and compare this with the corresponding empirical distribution. Hence, I determine how an increase in minimum innovation size affects the mean and median size of innovations. Moreover, I also examine the effect on the variance of innovation size. In Section 7, I examine the desirability of minimum innovation size in terms of welfare. In Section 8, I discuss some issues arising with (1) industry choice by R&D firms, (2) the dispersion of the productivity distribution, and (3) the robustness of the results under the generalized utility function. Section 9 concludes.

2. RELATED LITERATURE

First, this section surveys the several forms of patent protection and discusses the difference between a minimum innovation size requirement and patent breadth.[4] A patent system has three main characteristics that determine the degree of patent protection, namely patent length, patent breadth, and the patentability requirements. Patent length determines how long the patent holder has an exclusive

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right. There are two types of patent breadth, lagging breadth and leading breadth. Lagging (leading) breadth characterizes the set of inferior (superior) technologies that infringe the patent.[5] Last, patentability requirements are the statutory requirements that an invention must satisfy to receive a patent. One of these patentability requirements is minimum innovation size; i.e., the patent office only grants a patent for technology that is superior to prior technologies.

In general, lagging breadth works as a protection against imitators, whereas leading breadth and minimum innovation size work as a protection against future innovators. There are similarities between leading breadth and minimum innovation size, in the sense that the very existence of the leading breadth and the minimum innovation size requirement positively affects patent lifetime. Because of the existence of leading breadth, future innovators would infringe the patent and thus must license with followers (the patent holders of past innovations) in order to produce the state-of-the-art product [O'Donoghue (1998), O'Donoghue et al. (1998), O'Donoghue and Zweimüller (2004)]. Hence, the patent holder can earn a share of the profit flow even though there is creative destruction.

In contrast, the existence of a statutory requirement for minimum innovation size extends the lifetime of a patent in a number of different ways. For example, the future innovators must invent a sufficiently superior product to receive a patent, and this increases the expected duration of the patent until creative destruction arises, given the difficulty of success in superior invention [O'Donoghue (1998), O'Donoghue and Zweimüller (2004), Koléda (2008), the present analysis]. The key difference between the minimum innovation size requirement and leading breadth is the license agreement. If a patentable innovation does not infringe the prior patent, the latest patent holder does not need to license with past innovators, as in Scotchmer and Green (1990), O'Donoghue (1998, Sect. 4), O'Donoghue and Zweimüller (2004, Sect. 3.1), Koléda (2008), and the present analysis. However, if patentable innovations infringe the prior patent because of stronger leading breadth, the latest patent holder must license with past innovators to produce the state-of-the-art product, as in O'Donoghue (1998, Sect. 5) and O'Donoghue and Zweimüller (2004, Sect. 3.2).

Second, I survey empirical and theoretical studies of the impact of IPR protection on the flow of innovations that give an inverted U-shaped or negative relationship. Although there is little empirical evidence that strengthening IPR protection increases innovation size, theory predicts that strengthening IPR protection in the form of minimum innovation size always increases innovation size.

Empirical studies support the notion of an inverted U-shaped relationship between IPR protection and the flow of innovations. For example, Qian (2007) uses panel data on pharmaceutical industries in the OECD and an IPR index constructed by Ginarte and Park (1997).[6] Qian (2007) finds that an inverted U-shape best describes the relationship between patent strength and innovation flows. Elsewhere, although Sakakibara and Branstetter (2001) find no evidence that protection-enhancing shifts have a positive impact on innovation flows, Lerner (2009) identifies a negative impact on innovation flows.[7] Hence, there is some empirical support for our theoretical predictions.

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Our finding that an increase in minimum innovation size has both positive and negative effects on mean innovation size relates most closely to the work of Sakakibara and Branstetter (2001). They show that protection-enhancing shifts display no clear evidence of a positive effect on innovation size (patent quality), using patent citations as a measure of quality. Moreover, based on measures of the average claim per U.S. patent, the total number of U.S. patent claims, and the total number of international patent classification (IPC) classes in U.S. patents, Sakakibara and Branstetter (2001) conclude that no empirical evidence exists that protection-enhancing shifts increase innovation size. These results for no clear relationship between patent protection and the size of innovation in Sakakibara and Branstetter (2001) may result from some of the competing forces affecting the size of innovation, as shown in our analysis.

Theoretical studies closely related to the present study include O'Donoghue and Zweimüller (2004) and Koléda (2008), which both find that an increase in minimum innovation size has a nonmonotonic effect on innovation flows. However, the mechanisms driving this result differ. More specifically, O'Donoghue and Zweimüller (2004) and Koléda (2008) both show that an increase in minimum innovation size may reduce innovation flows by lowering the probability of successful R&D in all industries. In our model, the effect works by reducing the number of industries targeted for R&D.

No existing study predicts that an increase in minimum innovation size exerts a negative effect on innovation size. Koléda (2008) assumes that each R&D firm draws its innovation size from an *exogenous* Pareto distribution.[8] Hence, tightening the statutory requirement for minimum innovation size only increases the scale parameter (the minimum innovation size) of the Pareto distribution, thereby increasing the mean and median innovation size.[9] By contrast, our model generates an *endogenous* Pareto distribution, and we show that tightening minimum innovation size increases both the scale and shape parameters of the Pareto distribution, and thereby reduces the mean innovation size. In O'Donoghue and Zweimüller's (2004) model, the distribution of innovation size is degenerate because innovation size equals the minimum innovation size requirement for all industries. Hence, an increase in minimum innovation size increases the mean innovation size. However, in practice, the distribution of innovation size is not degenerate [Scherer and Harhoff (2000), Scherer et al. (2000), Harhoff et al. (2003), Silverberg and Verspagen (2007)], and our model usefully generates a nondegenerate distribution.

My paper also relates to theoretical studies concerning a nonmonotonic relationship between IPR protection and economic growth, such as Furukawa (2007, 2010), Horii and Iwaisako (2007), Chu et al. (2012a, 2012b), Davis and Şener (2012), Gangopadhyay and Mondal (2012), Chu and Pan (2013), Iwaisako and Futagami (2013), and Suzuki (2015). However, none of these studies considers the effect of IPR protection on the productivity distribution. Accordingly, our investigation of the productivity distribution not only is novel but also reveals that strengthening IPR protection in the form of an increase in minimum innovation size reduces the mean innovation size.

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Finally, this work also relates to the theoretical analysis of the productivity distribution, as in Di Matteo et al. (2005), Benhabib et al. (2014), Lucas and Moll (2014), Perla and Tonetti (2014), König et al. (2016). For the most part, these focus on the effects on the productivity distribution of innovation and/or imitation, social networks, and learning from others. Hence, my analysis of the relationship between IPR protection and the productivity distribution is novel, such that the productivity distribution evolves over time because of sequential innovations and from the optimal industry choices made by R&D firms.

3. THEORETICAL FRAMEWORK

Let us consider the Schumpeterian growth model in Aghion and Howitt (1998, Chap. 3), in which time is continuous, to examine the effect of the minimum innovation size requirement on the quantity and quality of innovations. We focus on the balanced-growth equilibrium, in which all endogenous variables grow at constant rates.

3.1. Households

We assume a representative household with intertemporally additive preferences and a constant rate of time preference $\rho > 0$. Assuming a linear instantaneous utility function, the rate of interest r corresponds to ρ.[10]

3.2. Final Goods

The final good, which we take as the numeraire, is produced under perfect competition, according to the production function

$$Y(t) = L^{1-\alpha} \int_0^1 A(i,\ t)^{1-\alpha} x(i,\ t)^{\alpha} di, \tag{1}$$

where $\alpha \in (0,\ 1)$, $Y(t)$ is gross output at time t, L is the flow of labor input, $x(i,\ t)$ is the output flow of the intermediate product $i \in [0, 1]$, and $A(i,\ t)$ is a productivity parameter for the latest version of the intermediate product i. The final good can be used interchangeably for consumption as an intermediate input or as an R&D input. The inverse demand function for $x(i,\ t)$ derived from (1) is $p(i,\ t) = \partial Y(t)/\partial x(i,\ t) = \alpha L^{1-\alpha} A(i,\ t)^{1-\alpha} x(i,\ t)^{\alpha-1}$.

3.3. Intermediate Goods

One unit of intermediate product is produced using one unit of the final good. The monopolist in industry i chooses quantity $x(i,\ t)$ to maximize profit $\pi(i,\ t) = p(i,\ t)x(i,\ t) - x(i,\ t)$.[11] Equilibrium profit is $\pi(i,\ t) = \tilde{\pi} A(i,\ t)$, where

$$\tilde{\pi} \equiv (1-\alpha)\, \alpha^{\frac{1+\alpha}{1-\alpha}} L.$$

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3.4. R&D and Innovations

Innovations result from R&D activity. Following Aghion and Howitt (1998, Chap. 3), let us consider technological spillover by assuming that each innovation creates a new intermediate product by incorporating a productivity parameter that equals the frontier technology parameter $\bar{A}(t) \equiv \max\{A(i,\ t)|i \in [0,\ 1]\}$. In short, the productivity level $A(i,\ t)$ can always jump to the frontier level $\bar{A}(t)$ if the R&D firm succeeds in R&D in any industry i. The R&D structure in Aghion and Howitt (1998, Chap. 3) also implies that $\bar{A}(t)$ is a public good.

The Poisson arrival rate $\phi(i)$ of innovation (or creative destruction) in industry i is $\phi(i) = \bar{\lambda}(i)n(i)$, where $\bar{\lambda}(i) > 0$ is a productivity parameter for the R&D targeting of industry i, and $n(i) \equiv N(i,t)/\bar{A}(t)$ is the flow of the productivity-adjusted final good devoted to the R&D targeting of industry i, where $N(i,t)$ is the flow of the final good using for the R&D targeting of industry i at date t. Following Jones and Williams (2000) and Chu et al. (2012a), we assume that $\bar{\lambda}(i) = \lambda n(i)^{\gamma-1}$, where $\lambda > 0$ is a productivity parameter for R&D and $\gamma \in (0,\ 1)$ represents the standard negative externality arising from intratemporal duplication within each industry. Hence, at the aggregate level, the rate of innovation in industry i is $\phi(i) = \lambda n(i)^{\gamma}$, which displays decreasing returns to scale. See Kortum (1993) and Thompson (1996) for supporting evidence. Horii and Iwaisako (2007) provide the microfoundations for this type of negative externality.

Following Aghion and Howitt (1998, Chap. 3), we assume that the growth rate of $\bar{A}(t)$ is proportional to the flow of aggregate innovations. Specifically, the rate of technological progress is[12]

$$g \equiv \frac{\dot{\bar{A}}(t)}{\bar{A}(t)} = \sigma I, \tag{2}$$

where $\sigma > 0$ is the spillover coefficient and I denotes the flow of aggregate innovations. In actual economies (such as Japan, the United States, and Europe), patent applicants must disclose information about their inventions, with this information made public 18 months after the application is filed, in sufficient detail to enable a person having ordinary skill in the art to replicate the invention. Hence, as implied by (2), the more frequently innovations are created (and filed for patent), the greater is I, and the higher the rate of growth (g) of public knowledge $\bar{A}(t)$.[13]

3.5. The Minimum Innovation Size

I introduce a minimum innovation size requirement for a patent. Suppose that nonpatented technology can be imitated immediately at no cost. For a patent to be granted, each R&D firm must invent a sufficiently superior technology. Specifically, we assume that the patent office grants a patent on the innovator's vintage technology if and only if the ratio of that vintage to the incumbent's vintage exceeds $\chi \geq 1$. That is, to be granted a patent, at any point in time t, each R&D firm must invest in industry i so that $\bar{A}(t)/A(i,t) \geq \chi$. Note that $\bar{A}(t)/A(i,t)$ represents the innovation size at date t when R&D firms target the industry. We

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can interpret the parameter χ as the strength of the patent's minimum innovation size requirement, as enforced by the patent office.

Under these circumstances, R&D firms must make a decision about industry choice in order to attain the patentable innovation size. At time t, for a given $\bar{A}(t)$, each R&D firm optimally targets all industries for which the inequality $\bar{A}(t)/\bar{A}(v) \geq \chi$ is satisfied, where $\bar{A}(v)$ is the level of the incumbent's vintage. Satisfaction of this inequality ensures that production profits do not fall to zero because of Bertrand competition from imitators. Hence, an incumbent with vintage technology $\bar{A}(v)$ that entered the product market at time v does not have to be afraid of creative destruction during a period of length T. We obtain the value of T by $\bar{A}(v+T)/\bar{A}(v) = \chi$ as follows:[14]

$$T = \frac{1}{g} \ln \chi. \tag{3}$$

We refer to T as the *effective patent length*. Because each new entrant survives with certainty during the period T, the statutory requirement for minimum innovation size plays the role of patent length.[15] From (3), effective patent length T is determined by two variables, χ and g. The variable χ determines the *distance* to the frontier technology, where R&D firms do not target any incumbents in the region $\bar{A}(t)/A(i,t) < \chi$. We then obtain $\partial T/\partial \chi > 0$, as the arrival time T to the distance χ increases with the distance χ for a given speed g. In contrast, the variable g determines the *speed* of obsolescence to run the given distance χ; thus $\partial T/\partial g < 0$. This is because the arrival time T to the distance χ decreases as the speed g increases.

Strengthening the minimum innovation size, which is represented by an increase in χ, implies the reinforcement of patent protection.[16] This is because the minimum lifetime for each patent increases because of the increase in the long-term effective patent length. In the following section, we rigorously establish the result that $dT/d\chi > 0$.

4. EQUILIBRIUM AND MINIMUM INNOVATION SIZE

The value of a patent is the expected present value of the associated future profits. However, this discounting must assume that during the first T periods, there is no risk of creative destruction, but that after period T, the innovator may experience creative destruction. Hence, the productivity-adjusted value of a patent in industry i at time t, represented by the value of a patent divided by the vintage technology $\bar{A}(t)$, is

$$\begin{aligned} V(i) &= \int_t^{T+t} e^{-r(s-t)} \tilde{\pi}\, ds + e^{-rT} \int_{T+t}^{\infty} e^{-(r+\phi(i))[s-(T+t)]} \tilde{\pi}\, ds \\ &= \left(\frac{\tilde{\pi}}{r+\phi(i)}\right)\left[1 + \left(\frac{\phi(i)}{r}\right)\left(1 - e^{-rT}\right)\right]. \end{aligned}$$

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R&D firms choose their R&D expenditure to maximize expected profit. Hence, for all targeted industries, we obtain the following free-entry condition:

$$\lambda^{\frac{1}{\gamma}}\phi^{\frac{\gamma-1}{\gamma}}\left(\frac{\tilde{\pi}}{r+\phi}\right)\left[1+\left(\frac{\phi}{r}\right)\left(1-e^{-rT}\right)\right]=1, \tag{4}$$

where

$$\lambda^{\frac{1}{\gamma}}\phi^{\frac{\gamma-1}{\gamma}}=\lambda n^{\gamma-1}=\bar{\lambda}$$

represents R&D productivity, including negative externalities.

Let Δ denote the proportion of all industries targeted for R&D, i.e., the proportion of industries that use sufficiently obsolete technology, whose vintage is older than T.[17] The inflow of new untargeted industries equals the flow of aggregate innovations I. An untargeted firm remains so for T periods. The outflow of industries also equals I. Thus, at any time, the proportion of untargeted industries $(1-\Delta)$ is equal to the cumulative flow of new firms over the previous T periods; i.e.,

$$1-\Delta=IT. \tag{5}$$

From (2), (3), and (5), we obtain the following equilibrium expression for the proportion of targeted industries:

$$\Delta=1-\frac{1}{\sigma}\ln\chi. \tag{6}$$

To ensure that Δ is positive, we require that $\chi\in[1,e^{\sigma})$. The remainder of the paper mainly assumes that $\chi\in[1,e^{\sigma})$.

LEMMA 1. *An increase in minimum innovation size reduces the proportion of industries targeted for R&D.*

Proof. From (6), $d\Delta/d\chi<0$. ■

Furthermore, the aggregate flow of innovations can be written as

$$I=\Delta\phi, \tag{7}$$

which implies that an increase in the minimum innovation size has a negative effect on the aggregate rate of innovation.

From (2), (3), (6), and (7), we obtain

$$T=\frac{\ln\chi}{\sigma\left(1-\frac{1}{\sigma}\ln\chi\right)\phi}. \tag{8}$$

We can use the free-entry condition (4) and the expression for effective patent length (8) to obtain the equilibrium values of ϕ and T. To simplify analysis of the effect of χ on I, we define $\varphi\equiv\phi^{-1}$ and the iso-I line, which, from (5) and (7)

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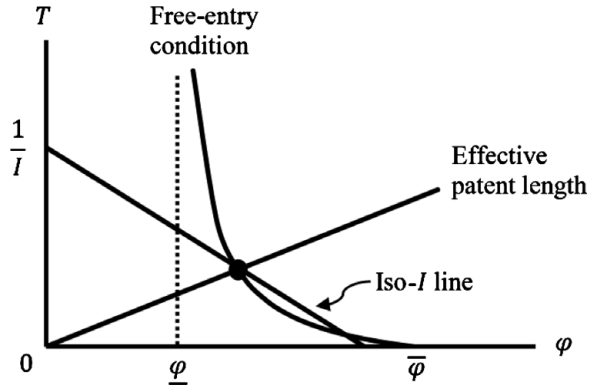


FIGURE 1. The existence of a unique balanced-growth equilibrium.

and the definition of φ, is given by

$$T = \frac{1}{I} - \varphi. \tag{9}$$

Figure 1 shows that there exists a unique balanced-growth equilibrium. The sign of the slope of the free-entry condition (4) is negative (see Appendix A.1). This is because a longer time T implies that each new entrant definitely makes profits over this longer period, and hence, the innovation rate ϕ increases (i.e., φ decreases) as a result of an increase in the incentives for R&D. In contrast, the sign of the slope of the effective patent length (8) is positive. This is because an increase in the innovation rate ϕ (i.e., a decrease in φ) raises the growth rate g of public knowledge $\bar{A}(t)$. Consequently, the effective patent length T decreases because incumbent vintages become obsolete more quickly.

Note that (4) is defined for $\underline{\varphi} < \varphi \leq \bar{\varphi}$, where

$$\underline{\varphi} \equiv \left(\lambda^{\frac{1}{\gamma}} \tilde{\pi}/r\right)^{\frac{\gamma}{\gamma-1}}$$

and $\bar{\varphi}$ is the function that satisfies

$$\lambda^{\frac{1}{\gamma}} \tilde{\pi} \varphi^{\frac{1}{\gamma}} = 1 + r\varphi$$

when $T = 0$ (see Appendix A.1). The aggregate innovation rate I is determined by the intercept of (9), as shown in Figure 1.

PROPOSITION 1. *There exists an inverted U-shaped relationship between the minimum innovation size and the aggregate innovation rate if and only if* $1/(1 + r\bar{\varphi}) + (1 - \gamma)/\gamma < 1$. *Otherwise, strengthening the minimum innovation size necessarily lowers the aggregate innovation rate.*

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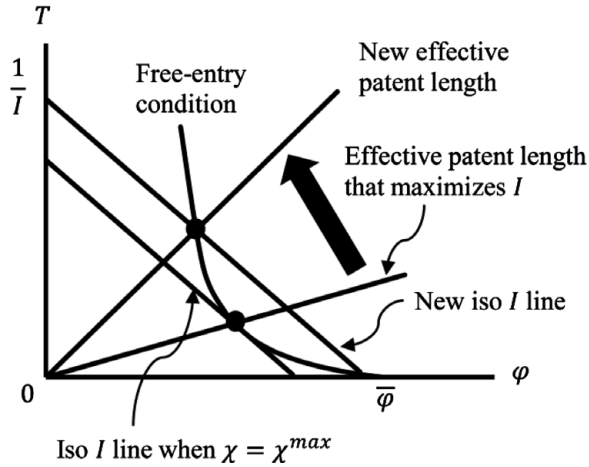


FIGURE 2. The negative impact of strengthening the minimum innovation size requirement on the rate of aggregate innovation when $1/(1+r\bar{\varphi})+(1-\gamma)/\gamma<1$.

Proof. The absolute value of the slope of (9) exceeds that of (4) when $\varphi=\bar{\varphi}$ if and only if $1/(1+r\bar{\varphi})+(1-\gamma)/\gamma<1$ (see Appendix A.2). In this case, there exists a unique interior solution $\chi^{\max}$, which is the value of χ that maximizes I. At this $\chi^{\max}$, (9) is tangential to (4), as shown in Figure 2. Under the constraint that $\chi\in[\chi^{\max},\ e^{\sigma})$, an increase in χ shifts the iso-I line upward, which requires a decrease in I (see Figure 2). In contrast, when $\chi\in[1,\ \chi^{\max})$, an increase in χ shifts the iso-I line downward, which implies an increase in I. For the case of $1/(1+r\bar{\varphi})+(1-\gamma)/\gamma\geq 1$, the effects are described in Appendix A.2. ■

Strengthening the minimum innovation size has two competing effects on the aggregate innovation rate $I=\Delta\phi$: the effects of an increase in χ on Δ and ϕ. As shown in Lemma 1, an increase in χ lowers the proportion of targeted industries Δ; this drives the negative effect on I. However, an increase in χ also induces a greater effective patent length T (see Figure 2). This increases the rate ϕ of innovation in the targeted industries, which drives the positive effect on I. The overall relationship between χ and I critically depends on the degree of negative externality, γ, in each R&D sector.

The necessary condition for $1/(1+r\bar{\varphi})+(1-\gamma)/\gamma<1$ is $\gamma>1/2$.[18] When the negative externality in each R&D sector is sufficiently small, the increase in ϕ induced by an increase in χ tends to be sufficiently large to outweigh the negative effect of Δ on I. Hence, in the case of a large γ, there is a positive relationship between the minimum innovation size χ and the aggregate innovation rate I for $\chi\in[1,\ \chi^{\max})$. However, because of decreasing returns to scale in the innovation rate $\phi=\lambda n^{\gamma}$, an increase in the minimum lifetime T exerts a positive but diminishing effect on ϕ. Hence, when χ is so large that $\chi\in[\chi^{\max},\ e^{\sigma})$, an increase in χ induces the strong negative effect of Δ on I.

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The sufficient condition for $1/(1+r\bar{\varphi})+(1-\gamma)/\gamma \geq 1$ is $\gamma \leq 1/2$. If the negative externality in each R&D sector is sufficiently strong, the increase in ϕ resulting from an increase in χ is relatively small. In this case, the negative effect of Δ (the proportion of targeted industries) on the aggregate innovation rate I outweighs the positive effect of ϕ.

5. THE PRODUCTIVITY DISTRIBUTION AND THE MINIMUM INNOVATION SIZE

In this section, I derive the productivity distribution, as distinct from the size distribution of innovations, and compare it with the empirical distribution. The productivity distribution is required for deriving the size distribution and for providing the economic intuition underlying the finding that strengthening the statutory requirement of minimum innovation size reduces the mean size of innovations.

5.1. The Empirical Productivity Distributions

Empirical evidence indicates that for several countries over several time periods, the distributions of firm-level labor productivity, capital productivity, and total factor productivity (TFP) are unimodal (i.e., they exhibit an inverted U shape). These studies include Di Matteo et al. (2005) for firm-level labor and capital productivity levels in Italy and France, and König et al. (2016) for firm-level TFP in France. According to Di Matteo et al. (2005), high levels of (labor and capital) productivity follow power-law distributions. Furthermore, König et al. (2016) argue that a power-law function not only fits the distribution of high-TFP firms but also approximates that of low-TFP firms.

5.2. The Theoretical Productivity Distribution

In this subsection, I derive the productivity distribution, which evolves over time because of the development of sequential innovations and the optimal industry choices made by each R&D firm. I then show that under certain conditions, the minimum innovation size requirement leads to a unimodal productivity distribution in the long run.

Let $F(\cdot,t)$ denote the cumulative distribution of the (absolute) productivity parameter $A(i,t)$ at time t. Let any $A>0$ be the frontier technological level in some previous period $t_0 \geq 0$, and define $\Phi(t) \equiv F(A,t)$. Then $\Phi(t_0)=1$ because no industry can have a productivity parameter that exceeds the frontier technological parameter at time t_0. Consider the inflow and outflow of the industries with respect to $\Phi(t)$ for each point of time $t \geq t_0$. The inflow is obviously zero, but there is an outflow if the R&D firms succeed in R&D. Hence, the cumulative distribution $\Phi(t)$ obeys the differential equation

$$\dot{\Phi}(t) = -\phi\Omega(t), \tag{10}$$

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where $\Omega(t)$ is the proportion of industries that satisfy the following two conditions: (a) R&D is conducted and (b) the productivity parameter $A(i,t)$ is less than A. For some small $t \geq t_0$, the productivity level A is around the level of the frontier technology $\bar{A}(t)$. Specifically, the productivity parameter A is in the R&D-untargeted region $A \in (\bar{A}(t)/\chi,\ \bar{A}(t)]$ for $t \in [t_0,\ t^*)$, where $t^* \equiv t_0 + T$ is the time to reach the R&D-targeted point $A = \bar{A}(t^*)/\chi$ for industries using technology A. Then, for $t \in [t_0,\ t^*)$, the proportion of industries that satisfy condition (b) exceeds the proportion of industries targeted by R&D, i.e., $\Phi(t) > \Delta$ for $t \in [t_0,\ t^*)$. Thus, the value of $\Omega(t)$ is determined only by the proportion of industries satisfying condition (a); i.e., $\Omega(t) = \Delta$ for $t \in [t_0,\ t^*)$. By contrast, for some large $t \geq t_0$, the productivity level A is far from frontier technology $\bar{A}(t)$ given the growth of $\bar{A}(t)$. Thus, the productivity parameter A is in the R&D-targeted region $A \in (0,\ \bar{A}(t)/\chi]$ for $t \in [t^*,\ \infty)$; therefore, $\Phi(t) \leq \Delta$ for $t \in [t^*,\ \infty)$. Hence, for $t \in [t^*,\ \infty)$, all industries that satisfy condition (b) are targeted for R&D; i.e., $\Omega(t) = \Phi(t)$. In sum, we can rewrite (10) as

$$\dot{\Phi}(t) = \begin{cases} -\phi\Phi(t) & \text{for } t \in [t^*,\ \infty) \\ -I & \text{for } t \in [t_0,\ t^*). \end{cases} \tag{11}$$

Note that $t^* \equiv t_0 + T$ also represents the time at which the same proportion of industries satisfy conditions (a) and (b); i.e., $\Phi(t^*) = \Delta$. During the period $t \in [t_0, t^*)$, governed by the differential equation $\dot{\Phi}(t) = -I$, $\Phi(t)$ monotonically converges to $\Phi(t^*) = \Delta$, given the initial condition $\Phi(t_0) = 1$. However, after time t^*, given the initial condition $\Phi(t^*) = \Delta$, $\Phi(t)$ is governed by the differential equation $\dot{\Phi}(t) = -\phi\Phi(t)$ and monotonically converges to zero.

According to (11), by using the initial condition $\Phi(t_0) = 1$, for $t \in [t_0,\ t^*)$, the solution to the differential equation for $\Phi(t)$ is

$$\Phi(t) = 1 - I(t - t_0) \quad \text{for } t \in [t_0,\ t^*). \tag{12}$$

Given the other initial condition $\Phi(t^*) = \Delta$, for $t \in [t^*,\ \infty)$, the solution to the differential equation for $\Phi(t)$ is

$$\Phi(t) = \Delta e^{-\phi(t-t^*)} \quad \text{for } t \in [t^*,\ \infty). \tag{13}$$

In summary, from (12) and (13), we obtain

$$\Phi(t) = \begin{cases} \Delta e^{-\phi(t-t^*)} & \text{for } t \in [t^*,\ \infty) \\ 1 - I(t - t_0) & \text{for } t \in [t_0,\ t^*). \end{cases} \tag{14}$$

Solving (2) yields

$$\bar{A}(t) = Ae^{g(t-t_0)}. \tag{15}$$

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From (15) and the definition $t^* \equiv t_0 + T$, (14) can be rewritten as

$$\Phi(t) = \begin{cases} \Delta(\chi a)^{\phi/g} & \text{for } t \in [t^*, \infty) \\ 1 + \frac{I}{g} \ln a & \text{for } t \in [t_0, t^*), \end{cases} \tag{16}$$

where $a \equiv A/\bar{A}(t) \in (0, 1]$.

Next, we derive the support of the distribution (16). Given (6) and (16), the value of the relative productivity parameter a that satisfies $\Phi(t^*) = \Delta$ is $1/\chi$. Noting that $a = 1$ at time t_0 converges monotonically to zero because of growth in $\bar{A}(t)$, the support of distribution (16) is

$$\Pr(A(i,t) \leq A) \equiv \Phi(t) = \begin{cases} \Delta(\chi a)^{\phi/g} & \text{for } a \in (0, 1/\chi] \\ 1 + \frac{I}{g} \ln a & \text{for } a \in (1/\chi, 1] \end{cases}$$

for all $A \geq \bar{A}(0)$, or alternatively,

$$F_{a(i,t)}(a) \equiv \Pr(a(i,t) \leq a) \equiv \Phi(t) = \begin{cases} \Delta(\chi a)^{\phi/g} & \text{for } a \in (0, 1/\chi] \\ 1 + \frac{I}{g} \ln a & \text{for } a \in (1/\chi, 1] \end{cases} \tag{17}$$

for all $a \geq \bar{A}(0)/\bar{A}(t)$, where $a(i,t) \equiv A(i,t)/\bar{A}(t) \in (0, 1]$. As t approaches infinity, $\bar{A}(0)/\bar{A}(t)$ converges to zero. Thus, the distribution of the relative productivity parameter $a(i,t)$ monotonically converges to the time-invariant cumulative distribution function (17). Differentiating (17) with respect to a yields the following probability density function for $a(i,t)$:

$$f_{a(i,t)}(a) \equiv \frac{dF_{a(i,t)}(a)}{da} = \begin{cases} \frac{I}{g} \chi^{\phi/g} a^{(\phi-g)/g} & \text{for } a \in (0, 1/\chi] \\ \frac{I}{ga} & \text{for } a \in (1/\chi, 1]. \end{cases} \tag{18}$$

By substituting $g = \sigma I$ and $I = \Delta\phi$ into (18), we obtain the following lemma.

LEMMA 2. *In the long run, the distribution of the relative productivity $a(i,t)$ converges to the following time-invariant density function:*

$$f_{a(i,t)}(a) = \begin{cases} \frac{1}{\sigma} \chi^{1/\sigma\Delta} a^{1/\sigma\Delta - 1} & \text{for } a \in (0, 1/\chi] \\ \frac{1}{\sigma a} & \text{for } a \in (1/\chi, 1]. \end{cases} \tag{19}$$

In the absence of a minimum innovation size, the long-run productivity distribution is identical to those in Aghion and Howitt (1998, Chap. 3) and Howitt (1999, 2000); i.e.,

$$f_{a(i,t)}(a) = \tfrac{1}{\sigma} a^{1/\sigma - 1} \quad \text{for } a \in (0, 1], \quad \text{when } \chi = 1. \tag{20}$$

In this case, the density function (20), which is a power-law distribution, does not exhibit an inverted U-shape. The slope of (20) depends only on σ: when

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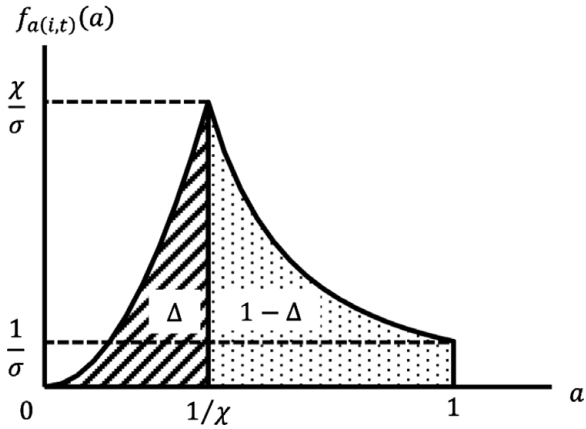


FIGURE 3. The density function $f_{a(i,t)}(a)$ for relative productivity $a(i,t)$ when $1/\sigma\Delta > 2$. The area shaded with diagonal lines corresponds to the proportion of industries targeted for R&D, Δ. The dotted area corresponds to the proportion of untargeted industries, $1-\Delta$.

$\sigma < 1$, the density function slopes upward; when $\sigma > 1$, the density function slopes downward; when $\sigma = 1$, the distribution is uniform.

However, (19) implies that introducing the minimum innovation size requirement into Aghion and Howitt's (1998, Chap. 3) model yields for the long-run productivity distribution a unimodal (inverted-U) shape under the condition $1/\sigma\Delta > 1$.[19] The left-hand side of the productivity distribution (19) has a power-law tail with an exponent of $1/\sigma\Delta - 1$, whereas the right-hand tail is described by the inverse (reciprocal) distribution. Hence, a unimodal distribution emerges if and only if $1/\sigma\Delta > 1$. The exponent of the right-hand tail of (19) is –1 (for the inverse distribution), and this may be inconsistent with the empirical distribution. For example, according to Di Matteo et al. (2005), the density function exponents for labor and capital productivity in Italy and France range from -2.1 to -4.6.

Figure 3 illustrates the density function $f_{a(i,t)}(a)$ of the relative productivity parameter $a(i,t)$ when $1/\sigma\Delta > 2$. For $1/\sigma\Delta > 2$, we require either $2\sigma < 1$ or $\chi > e^{(2\sigma-1)/2}$ when $2\sigma \geq 1$. Any industry i for which $a(i,t) \leq 1/\chi$ is targeted for R&D. Hence, the left-hand tail of the density function, i.e., $f_{a(i,t)}(a)$ for $a \in (0,\ 1/\chi]$, represents the distribution of $a(i,t)$ for the targeted industries. Hence, we have $\int_0^{1/\chi} f_{a(i,t)}(a)da = \Delta$, as shown in Figure 3. The right-hand tail of the density function, i.e., $f_{a(i,t)}(a)$ for $a \in (1/\chi,\ 1]$, represents the distribution of $a(i,t)$ for the untargeted industries.

5.3. The Economic Intuition behind the Long-Run Productivity Distribution

In this subsection, I explain the intuition behind the sign of the slope of the density function (19). First, I focus on the region of the density function that relates to the

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industries targeted for R&D, i.e., where $a \in (0,\ 1/\chi]$. Equations (18) and (19) imply that $1/\sigma\Delta - 1 \gtreqless 0 \Leftrightarrow \phi - g \gtreqless 0$. Thus, the slope of the density function for the targeted industries, located in the region $a \in (0,\ 1/\chi]$, depends on the sign of $\phi - g$. Given ϕ, an increase in the growth rate (g) of public knowledge $\bar{A}(t)$ helps to lower the relative productivity $a(i,t)$ of the targeted industries when R&D firms fail to develop innovation. Thus, in the $a \in (0,\ 1/\chi]$ region, greater knowledge growth raises the proportion of industries that have relatively low values of $a(i,t)$. This tends to make the slope of (19) negative for $a \in (0,\ 1/\chi]$. However, given g, an increase in the rate of innovation ϕ tends to induce the targeted industries to move from region $a \in (0,\ 1/\chi]$ to region $a \in (1/\chi,\ 1]$. That is, although industries move into the targeted-for-R&D region, they leave more quickly because of the increased rate of innovation ϕ. This tends to make the slope of (19) positive for $a \in (0,\ 1/\chi]$. This is because fewer industries with lower $a(i,t)$ values accumulate in the region $a \in (0,\ 1/\chi]$. Equation (18) implies that the overall effect on the slope of the density function depends on the sign of $\phi - g$. When $\phi - g > 0 \Leftrightarrow 1/\sigma\Delta - 1 > 0$, the slope of the density function in the region where the targeted industries are located ($a \in (0,\ 1/\chi]$) is positive, as in Figure 3. For $1/\sigma\Delta > 1$, we require either $\sigma < 1$ or $\chi > e^{\sigma-1}$ when $\sigma \geq 1$. This requires g to fall relative to ϕ when there are fewer spillovers ($\sigma < 1$) or when the targeted proportion (Δ) is small, such that $\chi > e^{\sigma-1}$ because of a strong requirement for a minimum innovation size, despite the greater spillover ($\sigma \geq 1$).

For $a \in (1/\chi,\ 1]$, the sign of the slope of (19) is unambiguously negative. This is because the industries located in region $a \in (1/\chi,\ 1]$ are untargeted for R&D, and public knowledge $\bar{A}(t)$ grows over time. Given the growth of $\bar{A}(t)$, relative productivity $a(i,t)$ falls *exponentially* at the rate g. Hence, relative productivity differentials tend to narrow across untargeted industries over time; i.e., for $a \in (1/\chi,\ 1]$, $a(i,t)$ is a Cauchy sequence. Therefore, in the region $a \in (1/\chi,\ 1]$, a smaller proportion of industries are accumulated in the relatively high-productivity region, as in Figure 3.

6. THE DISTRIBUTION OF INNOVATION SIZE AND THE MINIMUM INNOVATION SIZE

6.1. The Empirical Size Distribution of Innovations

Using data on patent citations and monetary values, Silverberg and Verspagen (2007) found that the distribution of innovation size resembles a lognormal distribution overall, but that it has a fat right tail better represented by a Pareto distribution. Scherer and Harhoff (2000), Scherer et al. (2000), and Harhoff et al. (2003) obtained similar results using data on patent values, royalties, and changes in asset values in startup companies; i.e., the distribution of innovation size resembles a lognormal distribution overall compared with a Pareto distribution.

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6.2. The Theoretical Size Distribution of Innovations

First, I show that in the models with and without minimum innovation size, innovation size (quality improvement) follows a Pareto distribution, which is to some extent consistent with the empirical distribution. Second, I show that an increase in minimum innovation size reduces the mean size of innovations.

To derive the distribution of innovation size, let us focus on the distribution of inverse relative productivity $\hat{a}(i,t) \equiv a(i,t)^{-1} = \bar{A}(t)/A(i,t)$. The value of $\hat{a}(i,t)$ represents the *feasible* size of innovations if industry i were targeted for R&D at date t. By transforming $a(i,t)$ into $\hat{a}(i,t) \equiv a(i,t)^{-1}$ in (19), we obtain the following density function for $\hat{a}(i,t)$:

$$f_{\hat{a}(i,t)}(\hat{a}) = \begin{cases} \left(\frac{1}{\sigma}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)} & \text{for } \hat{a} \in [\chi,\ \infty) \\ \frac{1}{\sigma\hat{a}} & \text{for } \hat{a} \in [1,\ \chi)\,. \end{cases}$$

Given that R&D is conducted only in the industries characterized by $\hat{a}(i,t) \geq \chi$, the distribution of the *equilibrium* size of innovations must be the conditional density function for $\hat{a}(i,t)$ given that $\hat{a}(i,t) \geq \chi$ (i.e., $\hat{a} \in [\chi, \infty)$), which is derived as follows:

$$\begin{aligned} f_{\hat{a}(i,t)}(\hat{a}\,|\hat{a}\,(i,t) \geq \chi) &= \frac{\left(\frac{1}{\sigma}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)}}{\Pr(\hat{a}\,(i,t) \geq \chi)} \\ &= \left(\frac{1}{\sigma\Delta}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)}. \end{aligned}$$

This yields the following lemma.

LEMMA 3. *In the long run, innovation size converges to a time-invariant conditional density function of the inverse of relative productivity* $\hat{a}(i,t) \equiv a(i,t)^{-1}$*:*

$$f_{\hat{a}(i,t)}(\hat{a}\,|\hat{a}\,(i,t) \geq \chi) = \left(\tfrac{1}{\sigma\Delta}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)} \quad \text{for } \hat{a} \in [\chi,\ \infty)\,. \tag{21}$$

Equation (21) represents a Pareto distribution with a scale parameter of χ and a shape parameter of $1/\sigma\Delta$. The statement in Lemma 3 that innovation size follows the Pareto distribution is, to some extent, supported empirically. Silverberg and Verspagen (2007) demonstrate that the Pareto distribution is relevant only for the right-hand tail.

The model in Aghion and Howitt (1998, Chap. 3) without a minimum innovation size is consistent with a Pareto distribution for innovation size. Substituting $\chi = 1$ into (21) yields the Pareto distribution with a scale parameter of unity and a shape parameter of $1/\sigma$. In Aghion and Howitt's (1998, Chap. 3) model with a minimum innovation size, the scale and shape parameters are $\chi \in [1,\ e^{\sigma})$ and $1/\sigma\Delta$, respectively.

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PROPOSITION 2.

(i) Under the assumption $1/\sigma\Delta > 1$, *an increase in the minimum innovation size reduces the mean innovation size.*

(ii) An increase in the minimum innovation size raises median innovation size.

Proof.

(i) Given $1/\sigma\Delta > 1$, the mean of (21) has a finite value:

$$\hat{a}^* \equiv \int_{\chi}^{\infty} \hat{a}\, f_{\hat{a}(i,t)}\left(\hat{a}\,|\hat{a}\,(i,t) \geq \chi\,\right) d\hat{a} = \frac{\chi}{1-\sigma\Delta}. \tag{22}$$

Differentiating (22) with respect to χ yields

$$\frac{d\hat{a}^*}{d\chi} = \frac{-\sigma\Delta}{(1-\sigma\Delta)^2} < 0.$$

(ii) The median of (21) is

$$\hat{a}^{**} \equiv 2^{\sigma\Delta}\chi, \tag{23}$$

where $\hat{a}^{**}$ is a unique value that satisfies $\int_{\chi}^{\hat{a}^{**}} f_{\hat{a}(i,t)}(\hat{a}|\hat{a}(i,t) \geq \chi)d\hat{a} = 1/2$.

Differentiating (23) with respect to χ yields

$$\frac{d\hat{a}^{**}}{d\chi} = 2^{\sigma\Delta}(1-\ln 2) > 0.$$

■

Surprisingly, Proposition 2 states that even though the minimum innovation size required for patenting requires superior innovations, strengthening it reduces the mean size of innovations.[20] However, it increases the median size of innovations. The reason is as follows. Two competing forces affect the mean and median innovation size: scale (χ) and shape ($1/\sigma\Delta$). Strengthening the minimum innovation size requirement directly increases the minimum innovation size: the mean and median innovation size then increase with the scale (χ). However, strengthening the minimum innovation size requirement also increases the shape parameter $1/\sigma\Delta$ by reducing Δ. This exerts negative effects on the mean and median because an increase in the shape parameter $1/\sigma\Delta$ shifts the weight in the Pareto distribution from the tail area to the area around the scale parameter χ (see Figure 4).[21] This implies that there is a reduction in the proportion of industries that use extremely obsolete technologies around $a(i,t) = 0$. This results from a decline in the growth of public knowledge relative to the innovation rate ϕ [see Figure 5 and (18) and (19)]. That is, because the fall in g lowers the rate at which vintage technologies for industries in the targeted region ($a \in (0,\ 1/\chi]$) become obsolete, each targeted industry can escape to the untargeted region ($a \in (1/\chi,\ 1]$) before its technologies become obsolete with extremely low relative productivity $a(i,t) = 0$. Thus, strengthening the minimum innovation size requirement reduces the accumulation of industries around $a(i,t) = 0$ (see Figure 5). This corresponds to the Pareto distribution in Figure 4, which has a lower weight in the tail.

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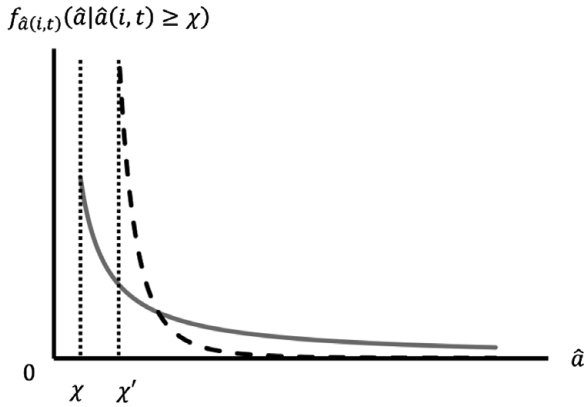


FIGURE 4. The effect of an increase in minimum innovation size on the distribution of innovation size, $f_{\hat{a}(i,t)}(\hat{a}|\hat{a}(i,t) \geq \chi)$, when $1/\sigma\Delta > 1$. The solid curve represents the distribution with a minimum innovation size χ. The dashed curve represents the distribution with a minimum innovation size $\chi' > \chi$.

Because each R&D firm combines public knowledge $\bar{A}(t)$ with existing products, innovations are less effective on both the mean and median because of the reduced proportion of industries around $a(i,t) = 0$. Consequently, Proposition 2 states that the overall effect of an increase in χ on the mean innovation size is negative because of the dominant negative effect of the increase in the Pareto distribution's shape parameter. In contrast, the median increases with χ because of the dominant positive effect of the increase in the scale parameter.

The parameter restriction $1/\sigma\Delta > 1$ for the result of part (i) for Proposition 2 should be realistic. As in Howitt (2000), we choose the following values. From Mankiw et al. (1992), the long-run growth rate of output per person in the United States over the period 1960–1985 is $g = 0.02$. From Caballero and Jaffe (1993), the average rate of creative destruction is $I = \Delta\phi = 0.036$, because they estimate that the typical U.S. firm that does not innovate loses value at a rate of 3.6 percent per annum. These specific values of g and I imply that the spillover coefficient is $\sigma = 0.556 < 1$. Therefore, the assumption $1/\sigma\Delta > 1$ holds given that $\sigma < 1$ and $\Delta \in (0, 1]$.

6.3. Variance of the Size Distribution of Innovations

To understand precisely the effect of minimum innovation size on the distribution of innovation size, we now consider the variance of innovation size. Under the assumption that $1/\sigma\Delta > 2$, from the Pareto distribution (21), we obtain the finite

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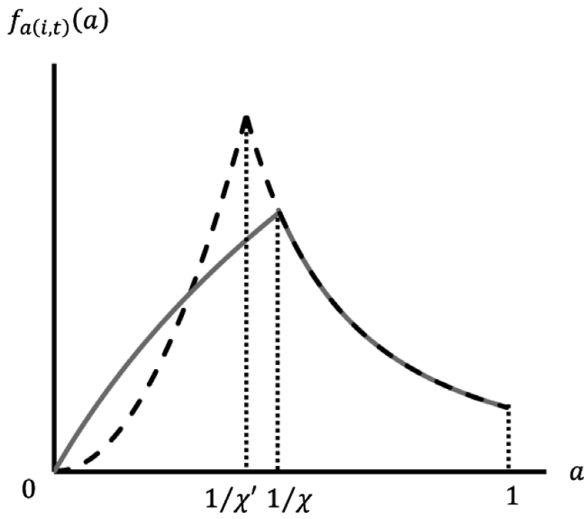


FIGURE 5. The effect of an increase in minimum innovation size on the relative productivity distribution, $f_{a(i,t)}(a)$. The solid curve represents the distribution when $1/\sigma\Delta \in (1, 2)$ and when the minimum innovation size is χ. The dashed curve represents the distribution when $1/\sigma\Delta > 2$ and when the minimum innovation size is $\chi' > \chi$.

variance value for innovation size $\hat{a}(i, t)$:[22]

$$\hat{a}_{\text{Var}} \equiv \int_{\chi}^{\infty} \hat{a}^2 f_{\hat{a}(i,t)}\left(\hat{a} \,|\hat{a}\,(i,t) \geq \chi\,\right) d\hat{a} - \left(\hat{a}^*\right)^2$$

$$= \frac{\chi^2(\sigma\Delta)^2}{(1-2\sigma\Delta)\,(1-\sigma\Delta)^2}. \tag{24}$$

PROPOSITION 3. *Under the assumption* $1/\sigma\Delta > 2$, *an increase in minimum innovation size reduces the variance of innovation size.*

Proof. The assumption $1/\sigma\Delta > 2$ ensures a finite value of variance of innovation size. Differentiating (24) with respect to χ yields

$$\frac{d\hat{a}_{\text{Var}}}{d\chi} = \left[\frac{1}{(1-2\sigma\Delta)\,(1-\sigma\Delta)^2}\right]^2$$
$$\times \left[-2\sigma\Delta\chi\,(1-2\sigma\Delta)\,(1-\sigma\Delta)^3 - 2\chi(\sigma\Delta)^2\right.$$
$$\left.\times\left\{(1-\sigma\Delta)^2 + (1-2\sigma\Delta)\,(1-\sigma\Delta)\,\right\}\right].$$

From the assumption $1/\sigma\Delta > 2$, we yield $d\hat{a}_{\text{Var}}/d\chi < 0$. ■

Similarly to the mean and median, there are two competing effects on the variance of innovation size: scale (χ) and shape ($1/\sigma\Delta$). An increase in minimum innovation size χ has a positive effect on the variance $\hat{a}_{\text{Var}}$ via the scale parameter

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χ. Hence, as in Koléda (2008), if the innovation size follows an exogenous Pareto distribution, a minimum innovation size increases the variance $\hat{a}_{\text{Var}}$. But in our model, innovation size follows an endogenous Pareto distribution, so that an increase in the minimum innovation size has an additional negative effect on the variance $\hat{a}_{\text{Var}}$ of the Pareto distribution because of the increase in the shape parameter $1/\sigma\Delta$. This is because an increase in shape $(1/\sigma\Delta)$ shifts the weight in the Pareto distribution from the tail to the scale parameter; i.e., many R&D firms attain an innovation size around the minimum innovation size, and it is then less likely that we will observe very large innovation size (see Figure 4). Therefore, an increase in shape $(1/\sigma\Delta)$ ensures that the distribution of innovation size approaches the degenerate distribution, which reduces the variance $\hat{a}_{\text{Var}}$. For example, at the limit of $\chi \to e^{\sigma}$, $\Delta \to 0$, and the shape parameter $1/\sigma\Delta \to \infty$. In this case, the Pareto distribution (21) converges to the degenerate distribution; i.e., any innovation size observed is at the minimum innovation size χ. Then, according to (24), the variance $\hat{a}_{\text{Var}}$ converges to zero. In our model, an increase in the minimum innovation size χ reduces the variance $\hat{a}_{\text{Var}}$ of innovation size because of the dominant negative effect of the shape parameter $1/\sigma\Delta$.

7. WELFARE ANALYSIS

This section considers the desirability of a statutory requirement for minimum innovation size from a steady-state welfare perspective. That is, we focus on the comparative statics of the steady-state value of welfare.[23] Suppose that the economy is at the balanced-growth equilibrium, and the convergence of the productivity distribution has already taken place at date t. The utility of the representative household at the balanced-growth equilibrium is

$$u(t) \equiv \int_t^{\infty} e^{-r(s-t)} C(s)\, ds = \frac{C(t)}{r-g}, \tag{25}$$

where $C(t)$ represents the aggregate consumption level in an economy at date t. The condition $r - g > 0$ must hold from the transversality condition. From (25), the welfare level is determined by the level of consumption $C(t)$ and the economic growth rate g. The level of $C(t)$ is determined by the final goods market-clearing condition:

$$C(t) = Y(t) - \int_0^1 x(i,t)\, di - \Delta n \bar{A}(t) \; = yA(t) - \Delta n \bar{A}(t), \tag{26}$$

where

$$y \equiv \alpha^{\frac{2\alpha}{1-\alpha}} \left(1-\alpha^2\right) L$$

is a composite parameter and $A(t) \equiv \int_0^1 A(i,t)di$ represents the aggregate (average) productivity across industries. Note that the term $yA(t)$ represents gross

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domestic product (GDP) at date t. Substituting (26) into (25), we obtain

$$u(t) = \bar{A}(t)\left(\frac{c}{r-g}\right), \tag{27}$$

where $c \equiv ya^* - \Delta n$ is the productivity-adjusted steady-state level of aggregate consumption and $a^* \equiv \int_0^1 a f_{a(i,t)}(a)da = (\frac{1}{\sigma})[1 - (\frac{1}{1+\sigma\Delta})(\frac{1}{\chi})]$ represents the steady-state value of the aggregate (average) relative productivity $a(i,t)$. Given that the level of public knowledge $\bar{A}(t)$ is historically determined, we examine the effect of χ on $c/(r-g)$.

To discern the difference between growth- and welfare-maximizing policies, we consider the marginal effect of an increase in minimum innovation size at the growth-maximizing level on the steady-state welfare. As shown in Proposition 1, the economic growth rate g and aggregate innovation rate I are maximized by either $\chi = \chi^{\max}$ or $\chi = 1$. As the empirical result in Qian (2007) implies an interior innovation-maximizing policy, we focus on the case of $\chi^{\max}$. As shown in Proposition 1, the necessary condition for the existence of interior growth-maximizing policy $\chi^{\max}$ is that the negative externality in R&D sector is sufficiently small so that $\gamma > 1/2$. The parameter restriction $\gamma > 1/2$ is also realistic, given that Thompson (1996) reports that the average value of γ across industries is 0.84.

We show the effect of a marginal increase in χ from $\chi^{\max}$ on the steady-state welfare. A slight increase in χ at $\chi^{\max}$ has no growth effect; i.e., $dg/d\chi|_{\chi=\chi^{\max}} = 0$. Thus, we need only examine the level effect. The level effect is decomposed into the effects of aggregate (relative) productivity a^* and those of (productivity-adjusted) aggregate R&D expenditure Δn:

$$\left.\frac{dc}{d\chi}\right|_{\chi=\chi^{\max}} = y\left.\frac{da^*}{d\chi}\right|_{\chi=\chi^{\max}} - \left.\frac{d\Delta n}{d\chi}\right|_{\chi=\chi^{\max}}. \tag{28}$$

First, we show that the aggregate productivity effect on the steady-state welfare is positive:

$$\left.\frac{da^*}{d\chi}\right|_{\chi=\chi^{\max}} = \frac{\Delta(\chi^{\max})}{(1+\sigma\Delta(\chi^{\max}))^2}\left(\frac{1}{\chi^{\max}}\right)^2 > 0, \tag{29}$$

where $\Delta(\chi^{\max}) = 1 - (1/\sigma)\ln\chi^{\max}$. This is because an increase in minimum innovation size causes a reduction in the proportion of industries that use extremely obsolete technology around $a(i,t) = 0$, given the negative effect of Δ on the growth $g = \sigma\Delta\phi$ of public knowledge, as shown in Figure 5. Hence, the productivity-adjusted GDP (ya^*) increases with χ, which has a positive effect on the steady-state welfare.

Second, we show that the effect of (productivity-adjusted) R&D expenditure on the steady-state welfare is negative. From the R&D production function $\phi = \lambda n^{\gamma}$ and the equation for aggregate innovation $I = \Delta\phi$, the productivity-adjusted R&D

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expenditure n for a targeted industry can be written as

$$n = (\phi/\lambda)^{\frac{1}{\gamma}} = (I/\lambda\Delta)^{\frac{1}{\gamma}}.$$

Hence, the aggregate R&D expenditure is

$$\Delta n = \Delta^{-\frac{1-\gamma}{\gamma}}(I/\lambda)^{\frac{1}{\gamma}}.$$

Noting that $dI/d\chi|_{\chi=\chi^{\max}} = 0$, we differentiate Δn with respect to χ at $\chi = \chi^{\max}$:

$$\left.\frac{d\Delta n}{d\chi}\right|_{\chi=\chi^{\max}} = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{\sigma}\right)\left(\frac{1}{\chi^{\max}}\right)\left(\frac{\phi(\chi^{\max})}{\lambda}\right)^{\frac{1}{\gamma}} > 0, \qquad (30)$$

where $\phi(\chi^{\max})$ is the innovation rate ϕ for the targeted industries when $\chi = \chi^{\max}$. Thus, the marginal increase in χ from $\chi = \chi^{\max}$ reduces the allocation of final goods to consumption because of the increase in aggregate R&D expenditure Δn, which has a negative effect on the steady-state welfare. This results from the decreasing returns to scale of R&D technology, $\phi = \lambda n^{\gamma}$.

In sum, there are two competing forces influencing the level effect on the steady-state welfare: the aggregate (relative) productivity a^* and the (productivity-adjusted) aggregate R&D expenditure Δn. Noting that $d\Delta n/d\chi|_{\chi=\chi^{\max}} \to 0$ as $\gamma \to 1$, we obtain $dc(t)/d\chi|_{\chi=\chi^{\max}} > 0$ and thus $du(t)/d\chi|_{\chi=\chi^{\max}} > 0$ under a very small degree of negative externalities such that $\gamma \to 1$. In contrast, we can show that the negative effect of Δn on steady-state welfare is dominant under a certain condition. Specifically, if $y/(1+\sigma)^2 \leq [(1-\gamma)/\gamma][1/\sigma][(\bar{\varphi})^{-1}/\lambda]^{1/\gamma}$ holds, we obtain the result $dc(t)/d\chi|_{\chi=\chi^{\max}} < 0$, and thus $du(t)/d\chi|_{\chi=\chi^{\max}} < 0$ (see Appendix A.3).

Compared with Proposition 2, a notable feature of the welfare analysis is that an increase in minimum innovation size χ increases the aggregate (relative) productivity a^*, which has a positive effect on welfare [see (29)]. In this sense, a smaller average innovation size is desirable in terms of welfare, as this implies that many vintage technologies are close to the frontier technology, and thus the level of GDP is higher. Specifically, if $\gamma \to 1$, an increase in the minimum innovation size χ from $\chi^{\max}$ raises the steady-state level of welfare $du(t)/d\chi|_{\chi=\chi^{\max}} > 0$, given the aggregate productivity effect $da^*/d\chi|_{\chi=\chi^{\max}} > 0$. However, if $y/(1+\sigma)^2 \leq [(1-\gamma)/\gamma][1/\sigma][(\bar{\varphi})^{-1}/\lambda]^{1/\gamma}$, the positive effect of a^* on welfare is relatively small. Therefore, an increase in the minimum innovation size χ from $\chi^{\max}$ reduces the steady-state level of welfare from the strong negative effect of aggregate R&D expenditure Δn. Note that the composite parameters y and $\bar{\varphi}$ do not include the parameter σ. Hence, to satisfy the parameter condition $y/(1+\sigma)^2 \leq [(1-\gamma)/\gamma][1/\sigma][(\bar{\varphi})^{-1}/\lambda]^{1/\gamma}$, e.g., we require a sufficiently small spillover coefficient $\sigma \to 0$; this leads to very large $d\Delta n/d\chi|_{\chi=\chi^{\max}} > 0$. But a sufficiently large spillover coefficient σ also

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satisfies $y/(1+\sigma)^2 \leq [(1-\gamma)/\gamma][1/\sigma][(\bar{\varphi})^{-1}/\lambda]^{1/\gamma}$ because $\sigma/(1+\sigma)^2 \to 0$ as $\sigma \to \infty$.

8. DISCUSSION

In this section, we discuss a number of issues arising from (1) optimal industry choice by R&D firms, (2) the dispersion of the productivity distribution, and (3) nonlinear utility.

8.1. Optimal Industry Choices

I have shown that the existence of a minimum innovation size required for patenting affects the optimal industry choices of new entrants. When new entrants attempt to combine frontier technology with existing products, they must target industries that use inferior technologies to satisfy the statutory requirement for a minimum innovation size. As a result, new entrants optimally target those industries far from the technological frontier. However, this is nothing more than one possibility. In this subsection, I discuss several other possibilities for optimal industry choices by new entrants.

First, when we introduce the state-dependent applicability of frontier technology into our model, new entrants choose to target those industries whose technologies are in the middle range. In Aghion and Howitt (1998, Chap. 3), new entrants always attain the frontier technology $\bar{A}(t)$ for all industries if they succeed in R&D and thus the innovation size is greater in those industries whose technologies are far from the frontier. However, this model setup may be somewhat unrealistic. Instead, we assume the state-dependent applicability of the frontier as follows. R&D firms can combine the frontier technology $\bar{A}(t)$ with the products in industries such that $a(i,t) \in [1/\psi,\ 1]$, where $\psi > 1$, whereas they cannot apply the frontier technology for those industries such that $a(i,t) \in (0,\ 1/\psi)$; i.e., the innovation size is zero for $a(i,t) \in (0,\ 1/\psi)$. Hence, the feasible innovation size is $\hat{a}(i,t) \in [1,\ \psi]$. In short, the frontier technology is only applicable to industries whose technologies are close to the frontier. In this case, R&D firms optimally target industries such that $a(i,t) \in [1/\psi,\ 1/\chi]$, where $\psi > \chi$ must hold to ensure the existence of patentable innovation. Therefore, the equilibrium innovation size is $\hat{a}(i,t) \in [\chi,\ \psi]$. In sum, new entrants enter industries whose technologies are in the middle range, $a(i,t) \in [1/\psi,\ 1/\chi]$. However, in this situation, the model fails to provide an interior balanced-growth equilibrium. This is because once industries enter the region $a(i,t) \in (0, 1/\psi)$, they will never be targeted by R&D firms. In the transition, there exists an inflow of industries into the region $a(i,t) \in (0, 1/\psi)$ because of obsolescence; however, there is no outflow of industries from $a(i,t) \in (0, 1/\psi)$ as a result of no R&D. Hence, in the balanced-growth equilibrium, all industries are in a no-R&D trap, $a(i,t) \in (0, 1/\psi)$. That is, the economy always falls into the no-growth trap.[24]

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We could avoid this technical problem via the following setup. In industries where $a(i,t) \in (0, 1/\psi)$, new entrants improve the productivity, which is drawn randomly from the distribution of the region $a(i,t) \in [1/\psi,\ 1/\chi]$. This provides a flow of industries from $a(i,t) \in (0, 1/\psi)$ to $a(i,t) \in [1/\psi,\ 1/\chi]$ and obtains an interior balanced-growth equilibrium. However, this is merely an ad hoc assumption, because the innovation size targeting the industries $a(i,t) \in (0, 1/\psi)$ depends on the distribution in the region $a(i,t) \in [1/\psi,\ 1/\chi]$, which includes the statutory requirement for a minimum innovation size χ in the support of the distribution. Thus the policy parameter χ directly affects the *feasible* innovation size in $a(i,t) \in (0, 1/\psi)$. An increase in χ directly reduces the expected innovation size in industry $a(i,t) \in (0, 1/\psi)$, given that industry productivity jumps to some $a(i,t) \in [1/\psi,\ 1/\chi]$. This exerts an additional negative force of χ on the mean (median) innovation size. As future research, we then should consider the analysis of the state-dependent applicability of frontier technology in a more realistic setup.

Second, in actual economies, a large share of new firms may enter industries that are close to the frontier. We can demonstrate this possibility using the model in Li (2001). Li (2001) develops the quality-ladder model of Grossman and Helpman (1991, Chap. 4) under constant-elasticity-of-substitution (CES) preferences. Given a CES structure, the demand for each industry's product positively depends on the quality of the industry's product [see (2) in Li (2001)]. Thus, the production profit positively depends on the quality of the industry's product [see (5) in Li (2001)]. This yields different values for patents across industries [see (8) in Li (2001)]. Hence, under a linear R&D production function with constant R&D productivity, as assumed in Grossman and Helpman (1991, Chap. 4), the R&D firms optimally target industries whose quality is frontier across industries.[25] This result lies totally opposite our analysis.

When the quality-ladder type R&D process [Grossman and Helpman (1991, Chap. 4)] and endogenous innovation size of the quality ladder [Aghion and Howitt (1992, Sect. 6)] are introduced into our model, R&D takes place in all industries. In order to satisfy minimum innovation size, R&D firms must choose the quality-ladder R&D process, per Grossman and Helpman (1991, Chap. 4), for industries close to the frontier. The innovation size of the quality ladder would equal the minimum innovation size under some strict minimum innovation size requirement, as in O'Donoghue and Zweimüller (2004). However, R&D firms would choose the R&D process in Aghion and Howitt (1998, Chap. 3) for industries far from the frontier. A frontier technology then evolves as a result of quality-ladder type R&D. These R&D choices are very similar to König et al.'s (2016) model. An increase in the minimum innovation size also raises the innovation size of the quality-ladder region, as in O'Donoghue and Zweimüller (2004). However, it has a negative effect on the probability of success in patentable innovation via quality-ladder R&D given the difficulty of superior innovation. The evolution of frontier technology will then decelerate. This accounts for the smaller innovation size for the region discussed in Aghion and Howitt (1998, Chap. 3), given the lower

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obsolescence speed, as shown in our analysis. The overall effect of the increase in minimum innovation size on the mean and median values is then unclear, and a detailed analysis is beyond the scope of the current paper. We thus highlight future research concerning the analysis of the minimum innovation size requirement in a model combining Grossman and Helpman (1991, Chap. 4) and Aghion and Howitt (1998, Chap. 3).

8.2. Coefficient of Variation for the Productivity Distribution

In this subsection, I analyze the coefficient of variation, which is the ratio of the standard deviation to the mean, of the productivity distribution. In Proposition 2, I showed that an increase in minimum innovation size reduces the mean innovation size. This results from the increase in the mean value of relative productivity, i.e., $da^*/d\chi > 0$. As I have already shown the effect of minimum innovation size on mean productivity in Section 7, I focus here on the dispersion of productivity using the coefficient of variation.

From the relative productivity distribution (19), the standard deviation of relative productivity $a(i,t)$ is

$$a_{\mathrm{SD}} \equiv \sqrt{a^{**} - (a^*)^2}, \tag{31}$$

where a^{**} and a^* are given by

$$a^{**} \equiv \int_0^1 a^2 f_{a(i,t)}(a)\, da = \left(\frac{1}{2\sigma}\right)\left[1 - \left(\frac{1}{1+2\sigma\Delta}\right)\left(\frac{1}{\chi}\right)^2\right] \tag{32}$$

and

$$a^* \equiv \int_0^1 a f_{a(i,t)}(a)\, da = \left(\frac{1}{\sigma}\right)\left[1 - \left(\frac{1}{1+\sigma\Delta}\right)\left(\frac{1}{\chi}\right)\right]. \tag{33}$$

Therefore, from (31), (32), and (33), the coefficient of variation is

$$a_{\mathrm{CV}} \equiv \frac{a_{\mathrm{SD}}}{a^*} = \sqrt{z-1}, \tag{34}$$

where $z \equiv a^{**}/(a^*)^2 \geq 1$. Note that (34) also represents the coefficient of variation for the absolute productivity $A(i,t)$. This is because the variance of $A(i,t)$ is $[a^{**} - (a^*)^2][\bar{A}(t)]^2$ and the mean of $A(i,t)$ is $a^*\bar{A}(t)$.

PROPOSITION 4. *An increase in minimum innovation size reduces the coefficient of variation of both relative productivity $a(i,t)$ and absolute productivity $A(i,t)$.*

Proof. See Appendix A.4.

The intuition behind Proposition 4 is as follows. According to Figure 5, an increase in minimum innovation size reduces that proportion of industries whose

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technological levels are around $a(i,t) = 0$, given the deceleration of g. This causes a reduction in the dispersion of productivity.

8.3. Nonlinear Utility

If I introduce constant relative risk aversion (CRRA) utility into the model, the interest rate r is endogenous; i.e., $r = \theta g + \rho = \theta\sigma\Delta\phi + \rho$, where $\theta > 0$ is the inverse of the intertemporal elasticity of substitution. This extension affects only the patent value, and thus the free-entry condition (4). An increase in χ has a negative effect on r via the reduction of Δ. This effect has the power to raise the patent value, and thus has a positive effect on ϕ. However, an increase in ϕ in itself has a negative effect on patent value via the increase in r, as in the effect of creative destruction. This somewhat discourages R&D activity when χ increases. Even though we assume the household has CRRA preferences, we believe that an increase in χ raises ϕ, and then may yield a result similar to Proposition 1 under somewhat modified parameter conditions. Then again, the results for Propositions 2, 3, and 4 are independent of the value of ϕ and thus identical results hold under CRRA utility.

9. CONCLUSION

Using the model in Aghion and Howitt (1998, Chap. 3), I investigated the effects of a statutory requirement for minimum innovation size on the quantity of innovations (the aggregate flow of innovations) and the quality of innovations (the mean and median size of innovations). First, I showed that the relationship between IPR protection (minimum innovation size) and the aggregate flow of innovations either has an inverted U shape or is negative. There is empirical support for this proposition. An increase in minimum innovation size (by reinforcing IPR protection) then has two competing effects: a negative effect on the proportion of industries targeted for R&D and a positive effect for R&D investment provided to the targeted industries. Consequently, an increase in minimum innovation size affects the aggregate flow of innovations nonmonotonically or negatively.

Second, I showed that the size distribution of innovations converges to a Pareto distribution in the long run, and that an increase in minimum innovation size reduces the mean size of innovations, but raises the median. An increase in minimum innovation size raises the Pareto distribution's scale parameter (representing the minimum innovation size), which has a positive effect on the mean and median innovation size. However, an increase in the minimum innovation size also increases the Pareto distribution's shape parameter. This has a negative effect on the mean and median innovation size. I showed that strengthening the minimum innovation size requirement reduces (increases) the mean (median) value innovation size. Specifically, my theoretical analysis predicts that substantial strengthening of the minimum innovation size requirement, following the United States Supreme Court's decision in KSR International Co. v. Teleflex Inc., will reduce the number

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of questionable patents (those that cannot satisfy the strict minimum innovation size requirement), as recommended by the FTC (2003) and NAS (2004). However, in the long run, strengthening the minimum innovation size requirement may lower the quality of patentable innovations by lowering the frequency with which inventions are filed for patent, which reduces knowledge spillover.

NOTES

1. The FTC (2003) and the NAS (2004) report the motivated amendment of U.S. patent law. The amendments passed in 2011 led to the establishment of the America Invents Act (AIA). Importantly, the AIA introduced a shift from the first-to-invent principle to the first-to-file principle. Cozzi (2001) and Cozzi and Spinesi (2006) developed an endogenous growth model under the first-to-invent principle.

2. *KSR International Co. v. Teleflex, Inc. et al.*, 127 S. Ct. 1929 (2007). Opinion available at http://www.supremecourt.gov/opinions/06pdf/04-1350.pdf.

3. The model in Aghion and Howitt (1998, Chap. 3) without minimum innovation size is consistent with a Pareto distribution. However, in our analysis, the Pareto distribution incorporates a policy parameter (representing the minimum innovation size required for a patent).

4. The discussion in this section draws mainly on O'Donoghue (1998). See O'Donoghue (1998, Sect. 2) for additional details about patent breadth and the minimum innovation size requirement.

5. The noncumulative innovation model in Gilbert and Shapiro (1990) assumes that wider patent breadth causes a higher flow rate of profits over the patent length. In contrast, the noncumulative innovation model in Gallini (1992) assumes that wider patent breadth results in imitators facing higher fixed production costs. These concepts of patent breadth correspond best to lagging breadth because there are no cumulative innovations.

6. To construct an IPR index, which indicates the strength of IPR protection in each country, Ginarte and Park (1997) used patentability (inventive step, novelty, and industrial applicability) as an indicator.

7. Specifically, Lerner (2009) focuses on observed protection-enhancing policy shifts, such as whether the country offered comprehensive patent protection, the patent length, the cost of awards, and provisions for patent revocation. Sakakibara and Branstetter (2001) focus on the impact of Japan's protection-enhancing shifts, with respect to patent breadth in the manufacturing industry and patent length in the pharmaceutical industry. As discussed in this section, the minimum innovation size and leading breadth display similar properties such that these patent instruments both protect against future innovators and increase the effective patent lifetime. In this spirit, we could potentially apply our analysis of minimum innovation size to the results in Sakakibara and Branstetter (2001) focusing on patent breadth.

8. In using the model in Li (2001, 2003) to conduct welfare analysis, Minniti et al. (2013) assume that the size of innovations follows an exogenous Pareto distribution.

9. If X is a random variable that follows a Pareto distribution, then the probability that X is less than some number x (i.e., the cumulative distribution function) is given by $F_X(x) \equiv \Pr(X \leq x) = 1 - (\frac{x_{\min}}{x})^{\kappa}$ for $x \geq x_{\min}$. The parameters $x_{\min}$ and κ are referred to as the scale parameter and the shape parameter, respectively. If $\kappa > 1$, the mean of X is finite: $\mathrm{E}(X) = (\frac{\kappa x_{\min}}{\kappa - 1})$.

10. Kishi (2015) derives the benchmark transitional dynamics for the basic Aghion and Howitt (1998, Chap. 3) model under a CRRA (constant relative risk aversion) utility function.

11. We assume that each innovator enters into Bertrand competition with the previous incumbent in the sector. Moreover, following Howitt and Aghion (1998), we assume that an incumbent that leaves the industry cannot reenter. Thus, the latest innovator can charge the monopoly price without worrying about competition from the previous incumbent. In contrast, if we allow the latest innovator to compete with the previous incumbent, the equilibrium prices depend on the innovation size, and thus the innovator faces optimal industry choice under a linear R&D production function [Howitt and Aghion (1998)].

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12. We establish that g is the steady-state rate of technological progress in note 19.

13. Strictly speaking, in Aghion and Howitt's (1998, Chap. 3) model, the definition of public knowledge is not $\bar{A}(t) \equiv \max\{A(i,\ t)|i \in [0, 1]\}$, but is given by (2) with the initial condition being $\bar{A}(0) \equiv \max\{A(i,\ 0)|i \in [0, 1]\}$. At each point in time, at least one R&D firm succeeds in combining public knowledge with an existing product. Hence, public knowledge $\bar{A}(t)$ corresponds to frontier technology $\max\{A(i,\ t)|i \in [0, 1]\}$ for all t.

14. To obtain (3), we rewrite $\bar{A}(v + T)/\bar{A}(v) = \chi$ as $e^{gT} = \chi$ and then take natural logarithms of both sides.

15. From (3), the government can indirectly control the effective patent length by adjusting the strength of the minimum innovation size requirement. O'Donoghue (1998) and O'Donoghue and Zweimüller (2004) find that the minimum innovation size has similar effects on the lifetime of the patent. Palokangas (2011) suggests that governments can indirectly control effective patent length, which he defines as the expected time for patent imitation by either facilitating or obstructing imitation. In R&D models of growth incorporating patent length, such as Judd (1985), Iwaisako and Futagami (2003), and Futagami and Iwaisako (2007), the assumption is that governments directly control patent length (the time until expiry).

16. In our model, the degree of IPR protection is determined by the exogenous parameter χ. In contrast, Eicher and García-Peñalosa (2008) develop an R&D based-growth model with *endogenous* strengthening of IPR.

17. For example, if T were 10 years, then as of 2015, any industry with technology dating from 2005 or earlier would be using sufficiently obsolete technology to be included in Δ.

18. Because $\bar{\varphi}$ is an increasing function of r, if r is sufficiently large, the condition $1/(1 + r\bar{\varphi}) + (1 - \gamma)/\gamma < 1$ is satisfied by $\gamma > 1/2$.

19. Because (19) is time-invariant, the mean value $a^* \equiv \int_0^1 a f_{a(i,t)}(a)da = (\frac{1}{\sigma})[1 - (\frac{1}{1+\sigma\Delta})(\frac{1}{\chi})]$ is constant in the steady state. This implies that the growth rate of average productivity $A(t) \equiv \int_0^1 A(i, t)di$, which represents the rate of technological progress, corresponds to that of public knowledge $\bar{A}(t)$; i.e., g is the rate of technological progress in the steady state.

20. Assuming that $1/\sigma\Delta > 1$ ensures that the sign of the slope of the productivity distribution for $a \in (0,\ 1/\chi]$ is positive; see (19). Alternatively, if $1/\sigma\Delta \leq 1$ holds, industries exist using extremely obsolete technology $a(i, t) = 0$. When industries with $a(i, t) = 0$ are targeted, the innovation size becomes of infinite value. This causes the mean value innovation size to be infinite when $1/\sigma\Delta \leq 1$.

21. Figure 4 illustrates the size distribution when $1/\sigma\Delta > 1$. Under this inequality, $f_{\hat{a}(i,t)}(\chi'|\hat{a}(i, t) \geq \chi') > f_{\hat{a}(i,t)}(\chi|\hat{a}(i, t) \geq \chi)$ for $\chi' > \chi$, as in Figure 4.

22. To satisfy the assumption that $1/\sigma\Delta > 2$ with $\sigma = 0.556$, we require that $\Delta < 0.9$.

23. The transitional dynamics of our model is quite complex because the model includes a de facto finite patent length, as in Judd (1985) and Iwaisako and Futagami (2003). Hence, we focus on the welfare analysis in the steady state.

24. This no-growth trap problem arises in the following generalized setup for state-dependent applicability. Succeeding in R&D in an industry where $a(i, t)$, R&D firms attain innovation size $q(a(i, t)) \geq 1$, where the function $q(\cdot)$ satisfies $q(1) = 1$, $q(a(i, t))A(i, t) \leq \bar{A}(t)$, concavity, and has an interior maximum value, i.e., $q(\cdot)$ describes the inverted U-shaped curve. The condition $q(1) = 1$ implies the innovation size is zero for industries at the frontier. The condition $q(a(i, t))A(i, t) \leq \bar{A}(t)$ implies that the new productivity level $q(a(i, t))A(i, t)$ following the innovation must be less than frontier $\bar{A}(t)$. In this situation, for some small χ, R&D takes place for industries whose technologies are in the middle range, given innovation size does not satisfy the minimum innovation size χ for some small $a(i, t)$ and for some large $a(i, t)$. The model then faces a no-growth trap in the balanced-growth equilibrium, because R&D never occurs in industries that entered the region $q(a(i, t)) < \chi$ because of a small $a(i, t)$.

25. To avoid this optimal industry choice, Li (2001) assumes that R&D productivity negatively depends on the industry quality level [see (7) in Li (2001)]. This ensures that R&D takes place in all industries.

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APPENDIX

A.1. THE SHAPE OF THE FREE-ENTRY CONDITION

We confirm that the free-entry condition (4) is a convex function with a negative slope. Solving (4) for T and imposing the definition of φ yields

$$T = -\frac{1}{r}\ln\left[\frac{(1+r\varphi)\left(\lambda^{\frac{1}{\gamma}}\tilde{\pi} - r\varphi^{\frac{\gamma-1}{\gamma}}\right)}{\lambda^{\frac{1}{\gamma}}\tilde{\pi}}\right]. \tag{A.1}$$

(A.1) can be defined over $\underline{\varphi} < \varphi \leq \bar{\varphi}$, where

$$\underline{\varphi} \equiv \left(\lambda^{\frac{1}{\gamma}}\tilde{\pi}/r\right)^{\frac{\gamma}{\gamma-1}}$$

and $\bar{\varphi}$ is the function that satisfies

$$\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} = 1 + r\varphi$$

when $T = 0$. Differentiating (A.1) with respect to φ yields

$$\frac{\partial T}{\partial \varphi} = -\frac{1}{1+r\varphi} - \frac{\left(\frac{1-\gamma}{\gamma}\right)}{\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi} < 0. \tag{A.2}$$

This is because

$$\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi > 0$$

for $\varphi > \underline{\varphi}$, which ensures that the term in the square brackets of (A.1) is positive. Differentiating (A.2) with respect to φ yields

$$\frac{\partial^2 T}{\partial \varphi^2} = \frac{r}{(1+r\varphi)^2} + \left(\frac{1-\gamma}{\gamma}\right)\frac{\frac{1}{\gamma}\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1-\gamma}{\gamma}} - r}{\left(\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi\right)^2} > 0.$$

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This is because we can obtain

$$\frac{1}{\gamma}\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1-\gamma}{\gamma}} - r > 0$$

by using

$$\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi > 0$$

and $\gamma \in (0, 1)$. Hence, the free-entry condition (4) is as in Figure 1.

A.2. NOTE ON PROPOSITION 1

First, from (A.2), we find that the absolute value of the slope of the free-entry condition (4) is smaller than that of the isoinnovation line (9) when $\varphi = \bar{\varphi}$; i.e.,

$$\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=\bar{\varphi}} > -1 \Leftrightarrow 1/(1 + r\bar{\varphi}) + (1 - \gamma)/\gamma < 1, \tag{A.3}$$

and otherwise,

$$\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=\bar{\varphi}} \leq -1 \Leftrightarrow 1/(1 + r\bar{\varphi}) + (1 - \gamma)/\gamma \geq 1. \tag{A.4}$$

The necessary condition for satisfying (A.3) is $\gamma > 1/2$. Because $\bar{\varphi}$ is an increasing function of r, if r is sufficiently large, condition (A.3) is satisfied by $\gamma > 1/2$. The sufficient condition for (A.4) is $\gamma \leq 1/2$.

Second, we graphically confirm that strengthening the minimum innovation size requirement χ necessarily retards aggregate innovation rate I if and only if $1/(1 + r\bar{\varphi}) + (1 - \gamma)/\gamma \geq 1$. Because the absolute value of the slope of the free-entry condition (4) exceeds that of the iso-I line (9) when $\varphi = \bar{\varphi}$ under (A.4), we can illustrate the balanced-growth equilibrium as in Figure A.1. Given (A.4), $\chi = 1$ maximizes I because it minimizes the intercept of the iso-I line. An increase in χ shifts the iso-I line upward and thus decreases I, as shown in Figure A.1.

A.3. NOTE ON THE WELFARE ANALYSIS

First, we confirm that $\Delta(\chi^{\max})/[(1 + \sigma\Delta(\chi^{\max}))^2\chi^{\max}] < 1/(1 + \sigma)^2$ holds. We use this inequality to derive the result $du(t)/d\chi|_{\chi=\chi^{\max}} < 0$ if the parameter condition $y/(1 + \sigma)^2 \leq [(1 - \gamma)/\gamma][1/\sigma][(\bar{\varphi})^{-1}/\lambda]^{1/\gamma}$ holds. Define the function so that $h(\chi) \equiv \Delta/[(1 + \sigma\Delta)^2\chi]$. Differentiating $h(\chi)$ with respect to χ yields

$$\frac{dh(\chi)}{d\chi} = -\left(\frac{1}{1 + \sigma\Delta}\right)^3\left(\frac{1}{\chi}\right)^2\left(\frac{1}{\sigma}\right)\left[1 + (\sigma\Delta)^2\right] < 0$$

for all $\chi \in [1, e^{\sigma})$. Hence, we obtain $h(\chi^{\max}) < h(1)$, i.e., $\Delta(\chi^{\max})/[(1 + \sigma\Delta(\chi^{\max}))^2\chi^{\max}] < 1/(1 + \sigma)^2$.

Second, we show that $du(t)/d\chi|_{\chi=\chi^{\max}} < 0$ if the condition $y/(1 + \sigma)^2 \leq [(1 - \gamma)/\gamma][1/\sigma][(\bar{\varphi})^{-1}/\lambda]^{1/\gamma}$ holds. Substituting $T = 0$ (i.e., $\chi = 1$) into (4), we obtain the minimum value of the innovation rate $\bar{\varphi}^{-1} < \phi(\chi^{\max})$, where $\bar{\varphi}$ is a function that satisfies

$$\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} = 1 + r\varphi.$$

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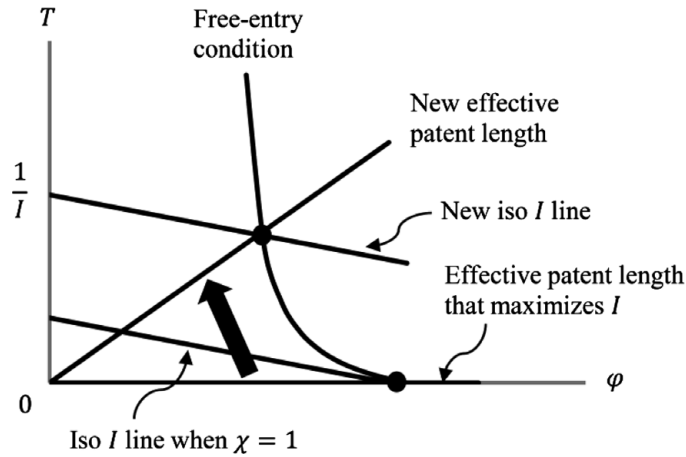


FIGURE A.1. An increase in minimum innovation size χ has a negative impact on aggregate innovation I when $1/(1+r\bar{\varphi})+(1-\gamma)/\gamma \geq 1$.

Hence, from (28), (29), (30), and $\bar{\varphi}^{-1}$, we obtain the following relationships:

$$\begin{aligned}\left.\frac{dc}{d\chi}\right|_{\chi=\chi^{\max}} &= \left(\frac{1}{\chi^{\max}}\right)\left[\frac{y\Delta(\chi^{\max})}{(1+\sigma\Delta(\chi^{\max}))^2}\left(\frac{1}{\chi^{\max}}\right)-\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{\sigma}\right)\left(\frac{\phi(\chi^{\max})}{\lambda}\right)^{\frac{1}{\gamma}}\right] \\ &< \left(\frac{1}{\chi^{\max}}\right)\left[\frac{y\Delta(\chi^{\max})}{(1+\sigma\Delta(\chi^{\max}))^2}\left(\frac{1}{\chi^{\max}}\right)-\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{\sigma}\right)\left(\frac{\bar{\varphi}^{-1}}{\lambda}\right)^{\frac{1}{\gamma}}\right] \\ &< \left(\frac{1}{\chi^{\max}}\right)\left[\frac{y}{(1+\sigma)^2}-\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{\sigma}\right)\left(\frac{\bar{\varphi}^{-1}}{\lambda}\right)^{\frac{1}{\gamma}}\right].\end{aligned} \tag{A.5}$$

The first inequality of (A.5) uses $\bar{\varphi}^{-1} < \phi(\chi^{\max})$. The second inequality uses $\Delta(\chi^{\max})/[(1+\sigma\Delta(\chi^{\max}))^2\chi^{\max}] < 1/(1+\sigma)^2$. From (A.5), if $y/(1+\sigma)^2 \leq [(1-\gamma)/\gamma][1/\sigma][(\bar{\varphi})^{-1}/\lambda]^{1/\gamma}$ holds, we obtain the required result $dc/d\chi|_{\chi=\chi^{\max}} < 0$, and thus $du(t)/d\chi|_{\chi=\chi^{\max}} < 0$.

A.4. PROOF OF PROPOSITION 4

From (34), we show that the sign of $dz/d\chi$ is negative. From $z \equiv a^{**}/(a^*)^2$, the derivative $dz/d\chi$ is given by

$$\frac{dz}{d\chi} = \left(\frac{1}{a^*}\right)^3\left[\frac{da^{**}}{d\chi}a^* - 2a^{**}\frac{da^*}{d\chi}\right]. \tag{A.6}$$

From (32) and (33), we obtain

$$\frac{da^{**}}{d\chi} = \frac{2\Delta}{(1+2\sigma\Delta)^2}\left(\frac{1}{\chi}\right)^3 \tag{A.7}$$

and

$$\frac{da^*}{d\chi} = \frac{\Delta}{(1+\sigma\Delta)^2}\left(\frac{1}{\chi}\right)^2. \tag{A.8}$$

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Noting that $a^* > 0$, according to (A.6), we must show that $[(da^{**}/d\chi)a^* - 2a^{**}(da^*/d\chi)] < 0$. Using tedious algebra, (32), (33), (A.7), and (A.8) yield

$$\left[\frac{da^{**}}{d\chi}a^* - 2a^{**}\frac{da^*}{d\chi}\right] \gtreqless 0 \Leftrightarrow z_1(\chi) \gtreqless z_2(\chi), \tag{A.9}$$

where the functions $z_1(\chi)$ and $z_2(\chi)$ are given by

$$z_1(\chi) \equiv 2(1+\sigma\Delta)^2\chi \tag{A.10}$$

and

$$z_2(\chi) \equiv (1+2\sigma\Delta)^2\chi^2 + 1. \tag{A.11}$$

From (A.10) and (A.11), we show $z_2(\chi)$ is larger than $z_1(\chi)$ when $\chi = 1$:

$$z_1(1) = 2\left(1+2\sigma+\sigma^2\right) < z_2(1) = 2\left(1+2\sigma+2\sigma^2\right). \tag{A.12}$$

Further, we show the slope of $z_2(\chi)$ always exceeds that of $z_1(\chi)$; i.e.,

$$\frac{dz_1(\chi)}{d\chi} = 2\left[(\sigma\Delta)^2 - 1\right] < \frac{dz_2(\chi)}{d\chi} = 2\chi\left[(2\sigma\Delta)^2 - 1\right]. \tag{A.13}$$

From (A.12) and (A.13), $z_1(\chi) < z_2(\chi)$ holds for any $\chi \in [1, e^\sigma)$. Therefore, we obtain the required result, $dz/d\chi < 0$.

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