# Feedback

**On 'On the frequency of Friday the thirteenth': Henry Ricardo writes**: In the May 1933 issue of the *American Mathematical Monthly*, B. H. Brown posed the problem (E 36) of showing that the thirteenth of the month is more likely to be Friday than any other day of the week. In his eleven-line featured solution (December, 1933), Raphael Robinson showed that in a 400 year period, the thirteenth of the month occurs 4800 times, distributed as follows:

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| 687 | 685 | 685 | 687 | 684 | 688 | 684 |

The problem later became one of the "Four Hundred 'Best' Problems (1918-1950)" that constituted *The Otto Dunkel Memorial Problem Book*. This was Part II of the August-September 1957 issue of the *Monthly*.

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**On 105.16: Nick Lord writes**: Des MacHale's neat elementary proofs that, if $p$ is prime, then $2^n + p$ is composite for infinitely many values of $n$ (Theorem 2) with a similar result for $2^n - p$ (Theorem 1), set me thinking about the underlying general results.

Suppose that $a$ and $b$ are coprime positive integers with $a > 1, b > 0$.

*Generalisation of Theorem* 2: $a^n + b$ is composite for infinitely many values of $n$.

*Proof*: Let $p$ be any prime divisor of $a + b$. Then $a$ and $b$ are coprime so, by Fermat's little theorem, $a^{p-1} \equiv 1 \pmod{p}$. Let $n = 1 + r(p-1)$ for $r = 1, 2, \ldots$. Then $a^n = a\left(a^{p-1}\right)^r \equiv a \pmod{p}$, so that

$$a^n + b = \left(a^n - a\right) + (a + b) \equiv 0 \pmod{p}$$

is composite.

*Generalisation of Theorem* 1: $a^n - b$ is composite for infinitely many values of $n$.

*Proof*: The same proof works with prime $p$ dividing $a - b$, unless $a - b = \pm 1$. In the latter case $a^3 - b \geqslant 5$ and we choose any prime divisor, $p$, of $a^3 - b$. Then $a$ and $b$ are coprime so, by Fermat's little theorem, $a^{p-1} \equiv 1 \pmod{p}$. Let $n = 3 + r(p-1)$ for $r = 1, 2, \ldots$. Then $a^n \equiv a^3 \pmod{p}$, so that $a^n - b = \left(a^n - a^3\right) + \left(a^3 - b\right) \equiv 0 \pmod{p}$ is composite.

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**On 105.17 and 105.18: Chris Starr writes**: The Notes in [1] and [2] discuss ideal multigrades of order 5, that is, sets of 6 integers such that the sums of the zeroth to the fifth powers are equal for each set.

The parametric formula asserted by Tyke in [1, p. 286] can be derived from Theorem 4 in [2, p. 297] by using the following transformation:

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$$a \to a - 3b$$

$$b \to a + b$$

$$c \to a - 5b.$$

*References*

1. Adam McBride, A tale of two sixes, *Math. Gaz.* **105** (July 2021) pp, 285-290.
2. Chris Starr, Notes on Listener Crossword 4595 by Elap, *Math. Gaz.* **105** (July 2021) pp. 291-298.

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**On 105.19: Nick Lord writes**: I greatly enjoyed this short note linking Fermat's last theorem with a mensuration problem about cones which provides a lovely glimpse of the frontiers of mathematics for young students. Here, I just add two further thoughts.

First, the same situation considered in the note, but filling hypercones (or simplexes) in $\mathbb{R}^n$ ($n \geqslant 3$) rather than cones, leads to the replacement of equation (1) of the note with $(h - h_1)^n + h_2^n = h^n$ which also has no solutions in rationals, by the general case of Fermat's last theorem.

Second, the idea of the note leads to interesting problems in number theory for other shapes. For a closed, hollow hemisphere of radius $r$ filled with liquid from its base to a depth $x$ and then inverted to fill a cap of height $y > x$, we have the following analogue of equation (1) of the note:

$$C(y) + C(r - x) = \tfrac{2}{3}\pi r^3, \qquad (*)$$

where $C(t) = \frac{1}{3}\pi\left(3rt^2 - t^3\right)$ is the volume of a spherical cap of height $t$. Substituting this formula into $(*)$ and simplifying leads to

$$3r^2x - 3ry^2 - x^3 + y^3 = 0.$$

This equation *does* have integer solutions, for example $x = 9$, $y = 39$, $r = 155$. Substituting $X = r - x$ produces the symmetrical equivalent equation $2r^3 - 3r\left(X^2 + y^2\right) + X^3 + y^3 = 0$, and it would be good to know more about its integer solutions.

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**On 105.23: Henry Ricardo writes**: Toward the end of his interesting article, Clement Falbo writes that [proving the Goldbach conjecture] "would make some other prime number theorems easier to prove." In my brief note in the June-July 2005 issue of *The American Mathematical Monthly* (p. 492), I proved that Goldbach's Conjecture implies Bertrand's Postulate. This result was submitted independently by Yoshihiro Tanaka at about the same time. Later, in a Letter to the Editor of *Mathematical Spectrum* (January 2014: Vol. 46, No. 2, p. 91), Guido Lasters also stated and proved the result.

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