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New low-frequency waves and negative mass instability in dusty plasmas

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Abstract. Low-frequency dusty plasma waves with frequencies much smaller than the frequency of charging collisions of plasma particles with dust particles are considered, taking into account elastic and charging collisions of plasma particles with dust and with neutrals. The usual dust sound waves with an upper frequency equal to the dust plasma frequency are found to be present only for wavelengths much smaller than the plasma particle effective mean free path due to the effective collision frequency. The effective collision frequency is found to be inversely proportional to the square root of the product of the charging frequency and the frequency of particle momentum losses, involving processes due to elastic plasma particle–dust collisions, and collisions with neutrals. It is shown that when the wavelength of the wave is much larger than the mean free path for effective collisions the properties of the waves are different from those previously considered. A negative mass instability is found in this domain of frequencies when the effective mean free path of ions is larger than the effective mean free path of electrons. In the absence of neutrals this appears to be possible only if the temperature of ions exceeds the electron temperature. This can occur in laboratory experiments and space plasmas but not in plasma-etching experiments. In the absence of instability a new dust oscillation, a dust charging mode, is found the frequency of which is almost constant over a certain range of wavenumbers. It is inversely proportional to the dust mass and charging frequency of the dust. A new dust electron sound wave is found for frequencies less than the frequency of the dust charging mode. The velocity of the dust electron sound wave is determined by the electron temperature but not the ion temperatures, as for the usual dust sound waves, with the electron temperature exceeding the ion temperature substantially.

1. Introduction

The main feature of dusty plasmas is rapid dust charging followed by continuous electron and ion recombination on dust particles, which in the absence of the photoelectric effect give rise to negatively charged dust grains.

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The dust grain will be considered here to be much smaller in size than the Debye shielding radius, where the limiting orbit theory approach is shown to describe the charging process [1, 2]. The charging time is very small, and is inversely proportional to the charging frequency ν_{ch}, and may be estimated from [1]

$$\nu_{ch} \approx \omega_{pi} \frac{a}{d_i},$$

where $\omega_{pi} = \sqrt{4\pi n_i e^2/m_i}$ is the ion plasma frequency (singly ionized ions assumed), a is the size of the grain (assumed to be spherical), $d_i = v_{Ti}/\omega_{pi}$ is the ion Debyeradius and $v_{Ti} = \sqrt{T_i/m_i}$ is the ion mean thermal velocity. The charging process is on a much shorter timescale, the corresponding frequency is larger than that given above by a factor of $\sqrt{m_i/m_e}$, but the final stage of charging is determined by this expression. The charging is accompanied by recombination of electrons and ions on dust particles and, in the final stage when the dust particle charge reaches its equilibrium value, the recombination rate becomes constant. This constant rate should be compensated by a constant ionization rate (production of electron–ion pairs) under constant plasma density conditions.

A dusty plasma is an open system for which the general statement for equilibrium systems that the dielectric permittivity $\epsilon_{\mathbf{k},\omega}$ for $\omega = 0$ cannot be 0 is not valid. This opens the possibility for the attraction of dust particles having the same sign as proposed in [3]. The physical reason for the attraction is the force of bombardment of electrons and ions which leads to dust charging. When two dust particles are close to each other, one of the dust particles shields the bombardment from the side of the other dust particle. The bombardment force, as has been shown by numerical simulation [2], can then exceed the electrostatic forces. The attraction force is inversely proportional to the square of the interdust distance. As predicted by limiting orbit theory [4, 5] and confirmed by numerical simulation [2], a long-range electrostatic repulsive force inversely proportional to the cube of the interdust distance also appears. In the present paper we restrict ourselves to a range of wavevectors much larger than that of the order of the inverse length for which the long-range dust repulsion and dust attraction is larger than the dust inertia. The new low-frequency waves and the negative mass instability considered in this paper correspond to frequencies larger than those related both to dust attraction and long-range repulsion.

The aim of the present paper is to investigate the plasma dispersion properties in the low-frequency domain, for frequencies much smaller than the charging frequency, where the charging changes the plasma dispersion substantially, but the dust attraction and long-range electrostatic repulsion are still unimportant. We also investigate the possibility of the existence of a negative mass instability in this range of frequencies. The goal is to find new wave modes and the criteria for instability. We find a new dust charging mode and a new electron dust sound wave mode for frequencies much larger than those for which dust attraction occurs. The term 'negative mass instability' in this range of frequencies is used in the sense of an effective negative mass of the dust particles. In the usual response of dust particles the dispersion relation contains (in the dielectric permittivity, in the case where only the dust inertia is taken into account), the term $(-\omega_{pd}^2/\omega^2)$. Here $\omega_{pd}^2 = 4\pi n_d Z_d^2/m_d$, is the square of the dust plasma frequency, m_d is the mass of dust particles, n_d is their density and Z_d is their charge in units of electron charge. The negative mass of the dust particles means $\omega_{pd}^2 < 0$. We intend to show that in

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the low-frequency limit, for frequencies much less than the frequency corresponding to particle recombination on dust, the dust response has a form $-(\omega_{pd}^2/\omega^2)F(k^2)$, where the factor of $F(k^2)$ can be negative for sufficiently low values of k^2 and in this sense leads to an effective negative mass analogous to the usual gravitational instability.

The paper is organized as follows. We first write a general dispersion relation for a dusty plasma in the low-frequency limit. We then find new dust modes and necessary conditions for the development of the negative mass instability; its threshold and critical wavenumber. The properties of a new dust charging acoustic mode for conditions under which the negative mass instability does not develop are discussed. The role of the attractive force between dust particles is estimated and applications to laboratory experiments, as well as new problems are discussed.

2. Basic equations

To fix the notation let us define all frequencies including the recombination and the charging frequency, which depend on the dust density. Let P represent the charge on a dust particle relative to the electron charge per unit volume

$$P = \frac{n_d Z_d}{n_e}, \tag{1}$$

where n_d and n_e represents the dust and electron density, respectively. For simplicity we will assume that the plasma ions are singly ionized and write the charge neutrality condition in the form

$$n_i = n_e(1 + P). \tag{2}$$

In etching processes P is equal to or larger than 1, while in space plasmas (planetary rings, for example) $P \ll 1$, $P \approx 1$ can also occur in astrophysics (i.e. comet tails). The recombination frequency is of the order of $\bar{\nu} \approx P\nu_{ch}$. The rate of plasma particle reaction on the charging process is highest for $\nu \ll \bar{\nu}$, where $1/\nu$ is the characteristic time of the dynamical processes.

We will consider the dispersion properties of a dusty plasma using the limiting orbit charging process for frequencies much less than both ν_{ch} and $\bar{\nu}$. The effective 'dust hydrodynamics' will be used to describe plasma particle motion for collision frequencies of plasma particles with dust particles much larger than binary plasma collisions. The local plasma particle distribution originates from collisions with dust and the neutral component but not binary collisions. The charge of an individual dust particle, given by the limiting orbit approach, is determined by the equation

$$\exp(-z) = \frac{(1+P)}{\sqrt{\tau}}\sqrt{\frac{m_e}{m_i}}(\tau + z), \tag{3}$$

where $z = Z_d e^2/(aT_e)$, a is their radius and $\tau = T_i/T_e$ is the ratio of the electron to ion temperatures. In etching experiments $\tau \approx 10^{-2}$, while in space plasmas $\tau \approx 1$ or even as large as 10. Figure 1 shows the dependence of z on τ and P for a hydrogen plasma. The charging frequency in limiting orbit theory is given by

$$\nu_{ch} = \frac{\omega_{pi}^2 a}{\sqrt{2\pi}v_{Ti}}(1 + \tau + z), \tag{4}$$

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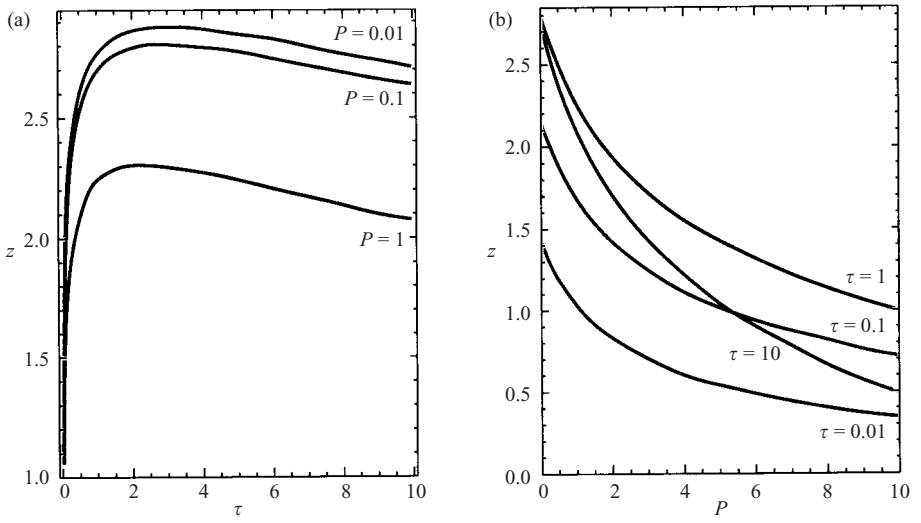


Figure 1. Dependence of z on (a) the dust charge P and (b) the ion to electron temperature ratio.

and is of the order of unity for $\tau \approx 1$ and $z \approx 1$. For most low-temperature plasmas where dust is present, a neutral gas component is also present. To describe its density, n_n, we will use a dimensionless value $N = n_n/n_e$. In most etching experiments $N \approx 10^6$.

We consider only the electrostatic modes in a dusty plasma in the low-frequency limit using perturbed hydrodynamic equations where only the interactions of plasma particles with dust and neutrals are taken into account. The dust will be treated in the collisionless limit, neglecting dust–dust collisions and dust–plasma particle collisions. This is valid provided the dust particles are much heavier than plasma particles. Under these conditions the equations determining the perturbation in the plasma particle momentum and continuity are

$$(-i\omega + \tilde{\nu}_{e,i})\mathbf{u}_{e,i} = -v_{Te,i}^2 i\mathbf{k}\frac{\delta n_{e,i}}{n_{e,i}} \mp \frac{e_{e,i}}{m_{e,i}}\frac{\mathbf{k}}{k}E \tag{5}$$

$$(-i\omega + \bar{\nu}_{e,i})\frac{\delta n_{e,i}}{n_{e,i}} + i\mathbf{k}\cdot\mathbf{u}_{e,i} = -\bar{\nu}_{e,i}\frac{\delta n_d}{n_d} - \delta\bar{\nu}_{e,i}. \tag{6}$$

The subscripts e, i and d denote electron, ion and dust, respectively, $\mathbf{u}$ is the velocity perturbation (the velocity in the equilibrium state is zero, therefore the perturbation is the velocity itself) and $-i\omega$ describes the inertia which in the low-frequency limit will be neglected. The first term on the right-hand side of (5) describes the pressure of electrons and ions, respectively, δn denotes the deviation of the density from its equilibrium value n, and the effective frequencies, $\tilde{\nu}$ in the momentum equation and $\bar{\nu}$ in the continuity equation, are different.

The generation of electrons and ions is taken into account in the continuity equation by the term, which from the balance in the equilibrium state, is given by $\bar{\nu}n_d$. The linear perturbation of the recombination term in the continuity equation gives the terms on the right-hand side of (6). The first of them is due to variations of dust density, while the second is due to variations of dust charges. The frequencies

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$\tilde{\nu}$ and $\bar{\nu}$ can be found from the limiting orbit theory approach. The frequency $\tilde{\nu}$ also contains collisions of plasma particles with neutrals, $\nu_n^{e,i}$. The limiting orbit theory approach also provides expressions for the deviations of the charging collision frequencies about their equilibrium values, $\delta\bar{\nu}_{e,i}$. We find

$$\tilde{\nu}_e = N_n n_e \sigma_n^{(e)} v_{Te} + \nu_{ch} \frac{P(\tau+z)}{z(1+\tau+z)} \left(4 + z + \frac{2z^2}{3} e^z \ln\frac{d}{a}\right), \tag{7}$$

$$\tilde{\nu}_i = N_n n_e \sigma_n^{(i)} v_{Ti}(1+P) + \nu_{ch} \frac{P}{(1+P)z(1+\tau+z)} \left(z + \frac{4}{3}\tau + \frac{2z^2}{3\tau} \ln\frac{d}{a}\right), \tag{8}$$

$$\bar{\nu}_e = \nu_{ch} \frac{P}{z} \frac{(\tau+z)}{(1+\tau+z)}; \quad \bar{\nu}_i = \frac{\bar{\nu}_e}{1+P}, \tag{9}$$

$$\delta\bar{\nu}_e = -z\frac{\delta Z_d}{Z_d}\bar{\nu}_e; \quad \delta\bar{\nu}_i = \frac{z}{\tau+z}\bar{\nu}_i \frac{\delta Z_d}{Z_d}. \tag{10}$$

The first term in (7) and (8) describes the collisions with neutrals, the second term in (7) and (8) contains both inelastic (recombination) collisions of electrons and ions with dust particles (the terms in parentheses not containing $\ln d/a$) and elastic collisions of electrons and ions with dust particles (the terms in parentheses containing $\ln d/a$).

To find the variation of the charges on dust particles we use the limiting orbit approach for the dust charging equations in the limit $\omega \ll \nu_{ch}$:

$$\frac{\delta Z_d}{Z_d} = \frac{(\tau+z)}{z(1+\tau+z)} \left(\frac{\delta n_e}{n_e} - \frac{\delta n_i}{n_i}\right). \tag{11}$$

The system of equations (5) and (6) are

$$\left(1+k_e^2\right)\frac{\delta n_e}{n_e} = \mathscr{E}_e - \frac{\delta n_d}{n_d} + \frac{(\tau+z)}{1+\tau+z}\left(\frac{\delta n_e}{n_e} - \frac{\delta n_i}{n_i}\right) \tag{12}$$

$$\left(1+k_i^2\right)\frac{\delta n_i}{n_i} = -\mathscr{E}_i - \frac{\delta n_d}{n_d} - \frac{1}{1+\tau+z}\left(\frac{\delta n_e}{n_e} - \frac{\delta n_i}{n_i}\right), \tag{13}$$

where

$$k_e^2 = \frac{k^2 v_{Te}^2}{\bar{\nu}_e\tilde{\nu}_e}; \quad k_i^2 = \frac{k^2 v_{Ti}^2}{\bar{\nu}_i\tilde{\nu}_i}, \tag{14}$$

and

$$\mathscr{E}_e = \frac{iekE}{m_e\bar{\nu}_e\tilde{\nu}_e}; \quad \mathscr{E}_i = \frac{iekE}{m_i\bar{\nu}_i\tilde{\nu}_i}. \tag{15}$$

We also introduce the dimensionless electron and ion plasma frequencies:

$$\Omega_e^2 = \frac{\omega_{pe}^2}{\bar{\nu}_e\tilde{\nu}_e}; \quad \Omega_i^2 = \frac{\omega_{pi}^2}{\bar{\nu}_i\tilde{\nu}_i}. \tag{16}$$

Since the dust will be considered collisionless we can use a simple expression for changes in dust density. For $\omega \gg kv_{Td}$, where v_{Td} is the dust thermal velocity we have:

$$\frac{\delta n_d}{n_d} = -\frac{ieZ_d}{\omega^2 m_d} kE \longrightarrow \left(\varepsilon_{k\omega}^d - 1\right) ik\delta E. \tag{17}$$

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The dispersion relation has the form

$$\epsilon_{\omega,k} = A(k^2) - \frac{\omega_{pd}^2}{\omega^2} B(k^2) = 0, \tag{18}$$

where

$$A(k^2) = 1 + \Omega_e^2 \frac{\left[k_i^2 + \frac{\tau+z}{1+\tau+z} + \frac{1+P}{1+\tau+z} + \frac{P(\tau+z)}{z(1+\tau+z)}\left(k_i^2+1\right)\right]}{\left[\frac{k_e^2(\tau+z)}{1+\tau+z} + \frac{k_i^2}{1+\tau+z} + k_i^2 k_e^2\left(k_i^2+1\right)\right]}$$

$$+ \Omega_i^2 \frac{\left[k_e^2 + \frac{1}{1+\tau+z} + \frac{\tau+z}{(1+\tau+z)(1+P)} + \frac{P(\tau+z)}{(1+P)(\tau+1)_z}\left(k_e^2\right)\right]}{\left[\frac{k_e^2(\tau+z)}{1+\tau+z} + \frac{k_i^2}{1+\tau+z} + k_i^2 k_e^2\right]}, \tag{19}$$

and

$$B(k^2) = 1 + \frac{k_e^2(1+P) - k_i^2}{P\left[\frac{k_e^2(\tau+z)}{1+\tau+z} + \frac{k_i^2}{1+\tau+z} + k_i^2 k_e^2\right]}. \tag{20}$$

3. Negative mass instability

Let us denote the ratio of the square of the effective mean free path of ions to the square of the mean free path of electrons divided by $1+P$ by

$$F = \frac{\lambda_i^2}{(1+P)\lambda_e^2}. \tag{21}$$

The effective mean free path is determined by $\lambda_{e,i} = \sqrt{\lambda_{ch}^{e,i}\lambda_m^{e,i}}$, where $\lambda_{ch}^{e,i} = v_{Te,i}/\bar{\nu}_{e,i}$ is the mean free path for the charging process for electrons and ions respectively, and $\lambda_m^{e,i} = v_{Te,i}/\tilde{\nu}_{e,i}$ is the mean free path for momentum losses of electrons and ions, respectively. Using previous definitions F is also given by

$$F(\tau, P, N) = \frac{k_i^2}{k_e^2(1+P)}. \tag{22}$$

The condition for a negative mass instability to develop is to have the value of B negative. We may write B in the following form:

$$-B = \frac{k_{cr}^2 - k_i^2}{k_0^2 + k_i^2}, \tag{23}$$

where k_{cr} is the critical value of the wavenumber. The instability can only develop for $k_i < k_{cr}$ with

$$k_{cr}^2 = F\frac{(1+P)(1+\tau+z) + P_\tau}{zP(1+\tau+z)} - \frac{1+P}{P} - \frac{(\tau+z)}{(1+\tau+z)}\frac{(z+1)}{z} \tag{24}$$

$$k_0^2 = \frac{F(1+P) + \tau + z}{1+\tau+z}. \tag{25}$$

A necessary condition for the development of the instability is that the coefficient of F in the numerator be positive,

$$P < P_{cr} = 1 + \tau + z(P_{cr}, \tau). \tag{26}$$

The solution of this equation is different for different ion to electron temperature

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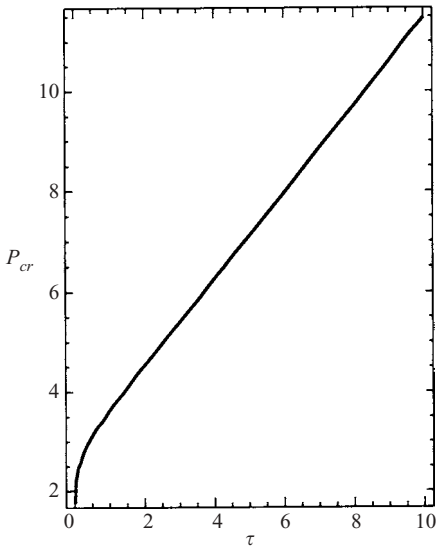


Figure 2. Solution of (26) showing P_{cr} as a formation of the ion to electron temperature ratio.

ratio. The results for a hydrogen plasma are given in Fig. 2. For $P < P_{cr}$ the instability will develop if F exceeds some critical value $F > F_{cr}$

$$F_{cr} = \frac{1+\tau+z}{1+\tau+z-P}\left[1+\frac{Pk_i^2}{1+P}+\frac{P(\tau+z)}{(1+P)(1+\tau+z)}\right]. \tag{27}$$

We find that a negative mass instability can develop when the effective mean free path for ions is larger than the mean free path for electrons. The physical interpretation of this result is that electrons under these conditions will not be able to reach the dust as quickly as the ions and the dust will be discharged. This is the source driving the instability. More explicitly, the instability will develop if

$$\frac{\lambda_i^2}{\lambda_e^2} > (1+P)F_{cr}. \tag{28}$$

For laboratory plasmas $P \geqslant 1$ and $\tau \ll 1$. From (3), (26) and (27) it is readily seen that

$$F_{cr} \simeq \frac{P_{cr}}{(P_{cr}-P)}\left[1+\frac{P(P_{cr}-1)}{(1+P)P_{cr}}+\frac{Pk_i^2}{1+P}\right] \geqslant 1. \tag{29}$$

For space plasmas, $P \ll 1$ and $\tau \geqslant 1$, and using the same argument as above, results in

$$F_{cr} \simeq \frac{P_{cr}}{P_{cr}-P} \geqslant 1. \tag{30}$$

Expressing (28) in terms of collision frequencies, the negative mass instability will develop in laboratory or space plasmas if

$$\frac{\tilde{\nu}_e}{\tilde{\nu}_i} \geqslant \left(\frac{T_e}{T_i}\right)\left(\frac{m_i}{m_e}\right). \tag{31}$$

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For a laboratory hydrogen plasma $T_e/T_i \geqslant 1$ and $m_i/m_e \simeq 1840$ so that $\tilde{\nu}_e/\tilde{\nu}_i \geqslant 1.84 \times 10^3$. Keeping only the neutral component in the collision frequency and employing elementary kinetic theory results for the neutral collisions [6], we obtain

$$\frac{\tilde{\nu}_e}{\tilde{\nu}_i} = \frac{1.7 \times 10^{11}}{4 \times 10^9} \sqrt{\frac{T_e}{T_i}},$$

which is less than 1840. The charging frequency ratio contribution is much smaller, of order 1, as is readily verified. Hence no negative mass instability develops in laboratory plasmas under etching conditions. Only in the much lower-frequency domain, when the dust–dust attraction forces are taken into account, can the other type of instability related to these forces develop [5]. In space plasmas with no neutral contribution to the collision frequency and the charging collision frequency ratio contribution is also of the order of unity. For instability to develop, one must have $T_i/T_e \simeq m_i/m_e$ ($T_i/T_e \simeq 1840$ for a hydrogen plasma). However, these conclusions depend on the limiting orbit theory results for the collision frequencies.

4. The new dust charging mode and the new electron dust sound wave mode

The condition where the mean free path of electrons is much larger than the mean free path of ions is often met in laboratory plasmas and etching experiments. Under those conditions the negative mass instability will not develop. For wavelengths much less then the effective mean free paths of either electrons or ions, the dispersion properties of a dusty plasma will change drastically and will be quite different from those expected if account of charging and other effective collisions of plasma particles with dust particles is not considered. First, we check that for wavelengths much less then the effective mean free paths we introduced above, the usual results known for dust sound waves are recovered.

In the limit $k_i^2 \gg 1$ and $k_e^2 \gg 1$ the coefficient B approaches unity and A approaches a value which corresponds to the Debye screening. Then the dust waves will be the usual dust sound waves with

$$\omega^2 = \omega_{pd}^2 \frac{1}{1 + \Omega_e^2/k_e^2 + \Omega_i^2/k_i^2}. \tag{32}$$

There always exists a domain of wavenumbers where the usual dust sound waves exist since the k values for which the relation $k = k_i$ is less than the inverse Debye length by a factor a/d_i where d_i is the ion Debye radius, can always be satisfied. Nevertheless, the latter ratio is not very small in existing experiments and it is of interest to consider the case where the $k_e^2 \gg 1$ inequality is not satisfied.

Since in many cases the condition $k_i \approx 1$ corresponds to values of k much larger than $k_e \approx 1$, we can consider the case $k_i \ll 1$ but $k_e \gg 1$. Then we find the approximate expression for A and B:

$$B \approx 1 + \frac{(1+P)(1+\tau+z)}{P(\tau+z)} \tag{33}$$

$$A \approx 1 + \Omega_i^2 \frac{(1+\tau+z)}{(\tau+z)} + \frac{\Omega_e^2}{k_e^2}\left(1 + \frac{1+P}{\tau+z}\right), \tag{34}$$

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which leads to a new dust mode

$$\omega^2 = \omega_{pd}^2 \frac{1 + \frac{(1+P)(1+\tau+z)}{P(\tau+z)}}{1 + \Omega_i^2 \frac{(\tau+z+1)}{(\tau+z)} + \frac{\Omega_e^2}{k_e^2}\left(1 + \frac{1+P}{\tau+z}\right)}. \tag{35}$$

For $k_e \gg \Omega_e/\Omega_i$ this leads to a new dust oscillation mode related to the charging process. For $\Omega_i \gg 1$

$$\omega \approx \frac{\omega_{pd}}{\Omega_i} \sqrt{\frac{\tau + z + \frac{(1+P)}{P}(1+\tau+z)}{1+\tau+z}}. \tag{36}$$

Since $\Omega_i \gg 1$ this frequency is much less than the dust plasma frequency and since Ω_i is inversely proportional to $\bar{\nu}_i\tilde{\nu}_i$, the frequency of the new dust mode will be proportional to $\bar{\nu}_i\tilde{\nu}_i$. From the latter it can be seen that its presence is determined by the charging process. For $k_e \ll \Omega_e/\Omega_i$ we find a new dust sound mode the velocity of which will be determined by the electron temperature (but not the ion temperature as is the case for the usual dust acoustic mode of $T_i \ll T_e$). We shall call this new acoustic mode an electron dust sound wave. The dispersion relation for it is

$$\omega^2 \approx k^2 v_{eds}^2, \tag{37}$$

where

$$v_{eds}^2 = \frac{T_e P Z_d}{m_d} \frac{z + \tau + \frac{(1+P)(1+\tau+z)}{P}}{\tau + z + 1 + P}. \tag{38}$$

Thus, we find that the charging process can change the modes of a dusty plasma substantially for wavelengths less than the effective mean free path of the plasma particles due to the charging process.

5. Limitations of results obtained

We first discuss the limitations arising from the process of electrostatic dust long-range repulsion in a plasma. The long-range electrostatic repulsion potential is not screened by plasma particles and creates long-range interactions corresponding to low values of wavenumbers [5]. Thus, the results presented here are limited not only by upper values of wavenumbers but also by lower values. For $\tau \ll 1$ the new charging dust modes appear for $k \ll a/d_i\sqrt{\tau}$ when the collisions with neutrals are negligible, while the conditions for the dust long-range repulsion to be negligible contains on the right-hand side an additional factor of the order of P^2/τ, which implies that for $P \approx 1$ the new dust mode exists but for $P \ll 1$ the domain of existence is limited by the relation $P > \sqrt{\tau}$. If $P \ll \sqrt{\tau}$ the ratio of the minimum wavevector of this mode to the maximum wavevector is $P/\sqrt{\tau}$. Comparing the larger value of the electron dust sound wave with the effects of dust unscreened repulsion we obtain a condition where $k \gg a/d_e^2$, the latter can be neglected while the electron dust sound wave exists for $k \ll Pa/d_e d_i \tau$ (for $\tau \ll 1$). This shows that there always exists a range of wavevectors for the electron dust sound mode if $\tau \ll 1$ and $P \ll \tau^2$. These conditions are always fulfilled in etching experiments. All of these constraints are softened when neutrals determine the mean free path since they do not influence the dust long-range interaction appreciably. For $\tau \gg 1$ the electron sound wave and the negative mass instability for long wavelengths exists for $P > \tau$.

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6. Conclusions

The results obtained in the present paper can have applications in both space research and laboratory dust–plasma experiments, including plasma etching. The new modes can be excited using a resonance with an external radio-frequency field by matching the frequencies. On the other hand, the strong dust-sound waves will obviously turn to shock waves and consideration of them is an important issue in supernova shocks and cometary physics. Thus, the result of the present paper can be considered as a starting point for research in a dust-shock wave with applications to many astrophysical problems. Our main conclusions are the following:

(1) the new mode can be used as a laboratory dusty plasma diagnostic, and other applications in plasma etching;

(2) the negative mass instability develops for sufficiently large τ; and

(3) the absence of a negative mass instability does not imply the absence of other dust instabilities including the electrostatic dust attraction instability which in most cases can be complementary to the negative mass instability (see [5]).

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References

[1] Havnes, O. and Tsytovich, V. N. 1993 *Comment. Plasma Phys. Control. Fusion* **15**, 267.

[2] Khodataev, Ya, Tsytovich, V. N. and Bingham, R. 1996 *Comment. Plasma Phys. Control. Fusion* **17**, 249.

[3] Tsytovich, V. N. 1994 *Comment. Plasma Phys. Control. Fusion* **15**, 349.

[4] Bernstein, I. B. and Rabinovich, I. N. 1959 *Phys. Fluids* **2**, 112.

[5] Tsytovich, V. N., Bingham, R., Resendes, D. and Khodataev, Ya K. 1996 *Comment. Plasma Phys. Control. Fusion* **17**, 287.

[6] Ginzburg, V. L. 1970 *The Propagation of Electromagnetic Waves in Plasmas*, 2nd edn. Oxford: Pergamon.

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