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Mathematics in Context: A Case in Early Nineteenth-Century Korea*

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Argument

This paper aims to show how a nineteenth-century Korean scholar's mathematical study reflects the Korean intellectual environment of his time by focusing on the rule of false double position and the method of root extraction. There were two major trends in Korean mathematics of the early nineteenth century: the first was "Tongsan," literally "Eastern Mathematics," which largely depended on Chinese mathematics of the Song and Yuan period adopting counting rod calculation; the second trend was Western mathematics, which was transmitted by the Jesuits and their Chinese collaborators from the late sixteenth century. There was also an intellectual transition in late eighteenth-century Korea when mathematics, which had been of only minor interest for Confucian scholars, became an important part of Confucian pursuits. We can gain an insight into the history of mathematics in Korea by examining and understanding Hong Kil-chu's (1786–1841) mathematical studies and the context of the academic world of his time.

Introduction

Recently discovered texts by a nineteenth-century Korean scholar of mathematics, Hong Kil-chu 洪吉周 (1786–1841), provide an example of how mathematical ingenuity is linked to the social context of Korean mathematics.[1] There were two main trends in Korean mathematics of the early nineteenth century. One was "Tongsan" 東算, literally "Eastern Mathematics," which largely depended on the Chinese mathematics of the Song 宋 and Yuan 元 period adopting counting rod calculation; the other was Western mathematics, which had been transmitted from the late sixteenth century by the Jesuits and their Chinese collaborators. Hong's mathematics reflects the Korean context of mathematical science not only in that the counting rod operation he used was unique to Korea from the fifteenth to the nineteenth century, but also in that his mathematics flourished along with the wide understanding of Western learning from the late eighteenth century in Korea.

* I use the McCune-Reischauer system for romanizing all Korean words, except for the English articles in which authors' names and titles have already been romanized in other ways. For Chinese and Japanese, the Pinyin and the Hephurn romanization systems, respectively, are used.

[1] I have published two articles on Hong's mathematics in Korean (Jun Yong Hoon 2003; 2004a).

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It is often said that counting rods were a device characteristic of Korean mathematics in the pre-modern period (Kim Yong-un and Kim Yong-guk 1977, 276–281).[2] The fate of Zhu Shijie's 朱世傑 *Introduction to Mathematical Studies* (*Suanxue qimeng* 算學啓蒙, 1299), which mainly relies on counting rod calculation, epitomizes the different histories of mathematics in Korea and China. While the Korean King Sejong 世宗 (r. 1419–1499) earnestly studied this book during his reign,[3] Chinese mathematics progressively moved to abacus calculation due to its practical efficiency. The use of counting rods to solve higher order equations gradually fell into oblivion (Li Yan and Du Shiran 1987, 187–189). As abacus calculation became widespread in Chinese society, the *Introduction to Mathematical Studies*, which had enabled advanced mathematical achievements such as the method of the celestial element (*tianyuanshu* 天元術), as well as the use of counting rods disappeared from Chinese society.[4] This did not change until Ruan Yuan 阮元 (1764–1849) printed the *New Edition of the Introduction to Mathematical Studies* (*Xinbian Suanxue qimeng* 新編算學啓蒙), which was a copy of a Korean edition of 1660.[5] In Korea, on the contrary, they continued to use counting rod calculation and "Tongsan" became the term usually used to identify mathematics in the Chosŏn 朝鮮 Kingdom of 1392–1896 as the Korean one.[6]

Western mathematics was first introduced into Korea in the seventeenth century and initially had a minor influence on Korean scholars of mathematics. In the second half of the eighteenth century, however, the influence grew owing to the vast inflow of Western knowledge from the early eighteenth century and the earnest study that Korean scholars devoted to it.[7] Texts such as *Elements of Geometry* (*Jihe yuanben* 幾何原本, 1607) and *Collected Basic Principles of Mathematics* (*Shuli jingyun* 數理精蘊, 1723), which mainly contained Western mathematics, became the mandatory texts for mathematical studies of Korean scholars. Furthermore, Korean scholars experienced a change of intellectual

[2] On the history of calculating instruments, including counting rods and the abacus in China, and the way they operated, see Lam Lay Yong 1996 and Martzloff 1997, 209–216.

[3] On King Sejong and his mathematical activity, see Kim Yong-woon 1973a, 22–23; two more serial introductions are Kim Yong-woon 1973b and 1973c.

[4] The abacus was also widely used in Japan from the early seventeenth century. Yet, counting rods were also used for solving numerical equations adopted in the *Introduction to Mathematical Studies* (Mikami Yoshio 1913, 156–159).

[5] For the domination of commercial mathematics and the decline of counting rod methods in China, see Li Yan and Du Shiran 1987, 175–189. In the 1930s, Fujitsuka first detected stories of re-exportation and re-publication of the *Introduction to Mathematical Studies* (Fujitsuka Chikashi 1975, 400–401).

[6] This term was proposed by Kawahara Hideki to refer to the mathematics from the mid-seventeenth to early eighteenth centuries to describe the creative activities of Korean mathematicians with two properties, the counting rod calculation and the method of the celestial element. Whereas Kawahara divides the period of Chosŏn Kingdom into three sub-periods and uses the term Tongsan to refer to the central period, I would like to expand the periodization of the Tongsan period to refer to the entire period of the Kingdom, because the counting rod calculation method lasted all through the Kingdom (see Kawahara Hideki 1998).

[7] For Western geometry and its influence on Korean scholars in the eighteenth century, see O Yŏng-suk 2004 and Han Yŏng-ho 2001.

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paradigm from the late eighteenth century, and the study of Western knowledge, which included mathematics, sparked enthusiasm for mathematical studies.

Hong Kil-chu was born into an environment that favored the pursuit of mathematics. It was at the culmination of the Tongsan era when Western mathematics became widespread. The new paradigm that prevailed among Korean scholars for mathematical study provided a favorable background for Hong's own pursuit. From this common paradigm, he thought that mathematics was a desirable subject to investigate for his Confucian learning. He was devoted to the study of mathematics and invented a number of insightful algorithms. Using the counting rod calculation and thinking within this framework, he invented some creative algorithms for linear equations and extraction of higher order roots. This paper will study how the Korean context shaped mathematics in the early nineteenth century, by focusing on Hong's work.

1. Mathematics in Korea in the Early Eighteenth Century

Mathematics in Korea was totally dependent upon Chinese mathematical texts until the mid-seventh century when Korean authors began to produce their own writings. Although there is no record in Korean literature about the texts used in the period of three Kingdoms – Koguryŏ 高句麗 (37 BC–668 AD), Paekche 百濟 (18 BC–660 AD), and Silla 新羅 (1 BC–668 AD) – it is believed that Chinese mathematical texts must have been used by a number of mathematicians in bureaucratic organizations of each Kingdom, in relation to taxes, land surveys, accounting, and calendars (Kim Yong-un and Kim Yong-guk 1977, 44).

The first Chinese texts in Korean literature appeared at the end of the seventh century. According to the *Historical Record of the Three Kingdoms* (*Samguk sagi* 三國史記, 1146), United Silla 統一新羅 (668–935) reconstructed governmental institutions modeled after those of the Tang 唐 Dynasty (618–907). The doctor of mathematics (算學博士), who was employed by the government, lectured on the *Canon of Interpolation* (*Zhuijing* 綴經), the *Three Extractions* (*Samkai* 三開), the *Nine Chapters* (*Jiuzhang* 九章), and the *Six Chapters* (*Liuzhang* 九章). Among the four texts, *Canon of Interpolation* and *Nine Chapters* are clearly different names of Chinese classics of mathematics: *Method of Interpolation* (*Zhuishu* 綴術, attributed to Zu Geng 祖暅, ca. 450–520) and *Nine Chapters on the Mathematical Art* (*Jiuzhang suanshu* 九章算術), respectively. The remaining two texts cannot be identified, but *Six Chapters* seems to be a digest of *Nine Chapters*, and *Three Extractions* a beginner's version of *Canon of Interpolation* (Kim Yong-un and Kim Yong-guk 1977, 82–84).

The Koryŏ 高麗 Kingdom (918–1392), which incorporates the Song (960–1279) and the Yuan (1279–1368) dynasties, adopted the mathematical system of the Tang Dynasty and the mathematical texts in United Silla continued to be used in mathematical education. The Royal Academy (*Kukchagam*, 國子監) was established in 992 and several decades later the mathematical section (算學) was installed employing two doctors of

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mathematics. Mathematics was an optional course initially but soon became mandatory for all students in the Royal Academy, even those who did not major in mathematics. According to the *History of Koryŏ* (*Koryŏsa* 高麗史, 1454), among the texts of United Silla, only one text, *Six Chapters*, was replaced by a new book, *Xie Family* (*Xiejia* 謝家), which is believed to be the other name of *Xie Chawei's Mathematical Manual* (*Xie Chawei suanjing* 謝察微算經, ca. 1050) (Li Wenlin, Xu Zelin and Feng Lisheng 1999, 74). So it can be said that the main mathematical text in the Koryŏ Kingdom was still *Nine Chapters on the Mathematical Art*. The demand for mathematicians seems to have been limited during the Koryŏ period as mathematicians could get only lower positions in governmental offices with Rank 9 (*kup'um*, 九品). Mathematics was regarded merely as a practical tool for governing the nation and therefore no meaningful original works were expected (Kim Yong-woon 1973a, 21–22).

However, it is claimed that mathematics in the Koryŏ Kingdom prepared for the next Kingdom, Chosŏn 朝鮮 (1392–1896) (Kim Yong-un and Kim Yong-guk 1977, 131–132). Though there was little distinct progress from the institutional point of view, broad learning and advanced Chinese mathematics in the Song and Yuan period was introduced in the later Koryŏ period. In particular, students were sent to China to learn calendrical astronomy and related mathematics, and they were asked to purchase astronomical and mathematical books from China (ibid., 130). In the process of introducing the *Works and Days Calendar* (*Shoushili* 授時曆, 1280), composed by the astronomer and mathematician Guo Shoujing 郭守敬 (1231–1316) of Yuan Dynasty, Korean mathematicians learned advanced mathematics of the time such as the numerical solution of higher equations, the summation of arithmetic series of higher order, and higher interpolations (Li Wenlin, Xu Zelin, and Feng Lisheng 1999, 74). At this time, newly introduced Chinese mathematical texts seem to have been in circulation. Modern research has shown that *Yang Hui's Methods of Computation* (*Yang Hui suanfa* 楊輝算法, ca. 1270), *Introduction to Mathematical Studies*, and *Explanations of Arithmetic* (*Xiangming suanfa* 詳明算法, 1373), three key texts in the Chosŏn Kingdom, were introduced in the late Koryŏ period; the mathematical education system was renewed with these books soon afterwards (1430 AD) (Kim Yong-un and Kim Yong-guk 1977, 130).

Until early in the seventeenth century, which corresponds to the Ming 明 Dynasty (1368–1644), Korean mathematicians did not produce any original work but only studied Chinese mathematical texts of the Song and Yuan period. Yet, it was an important period for Korean mathematicians of later periods that Korean scholars cultivated a Korean mathematics that was different from the Chinese. King Sejong, who was interested in developing mathematical and astronomical learning, reformed the recruiting system of bureaucrats in ten sections (十學). The mathematical section required mastering new textbooks such as *Yang Hui's Methods of Computation*, *Introduction to Mathematical Studies*, *Explanations of Arithmetic*, *Mathematical Manual of the Five Government Departments* (*Wucao suanjing* 五曹算經), and *Field Arithmetic* (*Disuan* 地算). Zhen Luan's 甄鸞 *Mathematical Manual of the Five Government Departments* (written

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around the sixth century AD), which belonged to *Ten Books of Mathematical Classics* (*Shibu suanjing* 十部算經) of the Tang Dynasty, was then approved as a textbook to prepare for the state examination. It dealt with practical problems useful for official work, such as calculating cultivated areas, transporting supplies, exchanging and trading, collecting taxes, and counting income and expenditures (Li Yan and Du Shiran 1987, 95). This book can be called a digest of *Nine Chapters on the Mathematical Art* (Kim Yong-un and Kim Yong-guk 1977, 173). *Field Arithmetic* is not identified, but seems to have been a text on land surveying.

Some time later, however, the number of mathematical texts for state examination shrank to three books, eliminating *Mathematical Manual of the Five Government Departments* and *Field Arithmetic*. After *Grand Code of Managing the Nation* (*Kyŏngguk taejŏn* 經國大典, 1485), the national code of the Chosŏn Kingdom until reform in the eighteenth century called for only three texts: *Yang Hui's Methods of Computation*, *Introduction to Mathematical Studies* and *Explanations of Arithmetic*, these three books dominated Korean mathematics until Western mathematics came into use. These texts were reprinted and remained popular. One hundred copies of *Yang Hui's Methods of Computation* were printed in 1433 and *Introduction to Mathematical Studies* was reprinted at least twice, in 1433 and 1660 (Li Wenlin, Xu Zelin, and Feng Lisheng 1999, 81). *Explanations of Arithmetic* is also believed to have been reprinted in Korea (Kim Yong-un and Kim Yong-guk 1977, 174).

The Chosŏn Kingdom established a well organized civil service examination for high-class literati. To become a high-ranking official, one had to take the erudite examination (*mun'gwa* 文科), or the military examination (*mugwa* 武科), while middle-class people (*chungin* 中人) who worked in such fields as interpreter of foreign languages, medicine, law, mathematics, and astronomy needed to pass miscellaneous examinations (*chapkwa* 雜科). The literati class who were eligible for the erudite and military examination were called "*yangban*" 兩班 (two orders) implying the two orders – erudite and military (文班, 武班). Technical official posts were not highly respected and were commissioned mostly to middle class people. High-ranking government officials and the families who produced these officials were called *sadaebu* 士大夫 (the gentry) and were highly respected in society. So the literati class were devoted to mastering Neo-Confucian philosophy, which was regarded as state learning and was required for passing the state examination. In this vein, until early in the seventeenth century, Korean Neo-Confucianism, the so called *Sŏngnihak* 性理學 (Study of Human Nature and Principle), flourished among the literati class, while mathematics was employed by mathematical officials of middle class and was applied to practical use in governing the nation. It is said that the practical use of mathematics stole mathematical curiosity from Korean mathematicians and left them with only the mannerism of elementary arithmetic (Kim Yong-un and Kim Yong-kuk 1977, 224).

In Korea, Neo-Confucianism was defined as "a doctrine that seeks to establish an ethical basis for an enlightened, Confucian political order through substantiation of the premise that the nature of man is fundamentally good, but it is also a metaphysical

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system of thought that endeavors to find the roots of this premise in the natural order of the cosmos" (Lee Ki-baik [1984] 1993, 217). Neo-Confucians divide all existence into two inseparable components, *i* 理 and *ki* 氣 (*li* and *qi* in Chinese). The one, *i*, is a patterning or formative element that accounts for what things are and how they behave, or normatively should behave, while the other, *ki*, is the concretizing and energizing element. The two are interdependent and inseparable, since *i* could not exist concretely without *ki*, and *ki* would be but formless and directionless energy without *i*. Based on this dualism, two distinct schools of Neo-Confucian thought developed in Korea, one giving primary emphasis to *i*, the other arguing the primacy of the role of *ki*. Korean Neo-Confucianism held Zhu Xi's 朱熹 (1130–1200) philosophical scheme in high esteem, while technical scientific knowledge such as astronomy and mathematics was only of secondary interest. To Confucians in Korea, ethical, social, and metaphysical problems were always more important (Kim Yung Sik 1992, 97).

By the time *Introduction to Mathematical Studies* was reprinted in 1660, most mathematical texts had disappeared and competent mathematicians were rare. According to Kim Si-jin 金始振 (1618–1667), the mayor of Chŏnju 全州, only *Explanations of Arithmetic* and its elementary methods survived. *Nine Chapters on the Mathematical Art* had long been missing and there was no one to consult about it (Kim Si-jin [1660] 1993, 1126).

There is no adequate study of what activated the restoration and the new study of mathematics in late seventeenth-century Korea, but the social demand for mathematics was probably the trigger. With the devastation caused by invasions from Japan (1592–1593, 1597–1598) and Manchu (1627, 1636–37), the Koreans were confronted with a need to reconstruct and reform their society. Land ruined by these invasions needed to be surveyed, so survey techniques and related mathematical knowledge became necessary. These conditions could have stimulated the study of mathematics. Kim Si-jin, who endeavored to restore and reprint *Introduction to Mathematical Studies*, had been appointed as head of the national surveying team (Sillokch'ŏng [1677] 1978, vol. 36, 343, 6 August 1662).

Around 1660, in his effort to restore mathematical texts, Kim Si-jin was able to acquire a handwritten version of *Yang Hui's Methods of Computation* and a printed version (1433) of *Introduction to Mathematical Studies*. He restored the missing pages of the latter with help from Im Chun 任濬 (1647–?), a county magistrate who was acquainted with "the method of the celestial element" (*tianyuanshu* 天元術). He also added a new chapter concerning trigonometric surveying borrowing from the *Yang Hui's Methods of Computation* (Kim Si-jin [1660] 1993, 1126).[8]

The restoration and study of three major Chinese texts, *Explanations of Arithmetic*, *Yang Hui's Methods of Computation*, and *Introduction to Mathematical Studies*, enabled mathematicians from both the middle class and the *sadaebu* class to create original works

[8] The missing pages explained the procedure of solution by the method of the celestial element.

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from the mid-seventeenth century.[9] The fact that these three main texts contained advanced mathematics that had been established in the Song and Yuan period and was based on the use of counting rod calculation made Korean mathematics different from Chinese. The titles – *Commentary to the New Edition of Introduction to Mathematical Studies* (*Sinp'yŏn sanhak kyemongju* 新編算學啓蒙註) and *Interpretation of the Explanations of Arithmetic* (*Sangmyŏng sugyŏl* 詳明數訣) – suggest that these books contained the studies and explanations of *Introduction to Mathematical Studies* and *Explanations of Arithmetic*, which are no longer extant. The direction of the mathematical study of the day can be seen in a text written by a middle-class mathematician Kyŏng Sŏn-jing 慶善徵 (1616–?), *Calculation Method in the Writings of Contemplation* (*Muksajip sanbŏp* 默思集算法), which assembled the contents of all three major texts in explanation while taking the structure and classification of mathematical branches in *Introduction to Mathematical Studies* (Kawahara Hideki 1998, 49–51).[10]

The *sadaebu* mathematicians also produced creative writings. In his *Summary of the Nine Branches of Numbers* (*Kusuryak* 九數略), Choe Sŏk-chŏng 崔錫鼎 (1646–1715) reconstructed the calculation method with counting rods and practical mathematics according to his philosophy of the *Book of Change* (*Yijing* 易經) (Kawahara Hideki 1998, 53). Cho T'ae-gu 趙泰耉 (1660–1723) constructed his *Narrow View of Mathematical Text* (*Chusŏ kwan'gyŏn* 籌書管見, 1718) in nine chapters following the *Nine Chapters on the Mathematical Art*, even though it was missing at that time. He reclassified mathematical operations and designed problems suitable for beginners.

Concerning "the method of the celestial element" with counting rods calculation, books of Korean mathematicians of the day show the highest point of application.[11] Pak Yul's 朴繘 (1621–?) *Origin of Mathematics* (*Chuhak ponwŏn* 籌學本原), of which only some fragments remain in another book, shows a systemized explanation of the "method of extracting roots by iterated multiplication" (*zengcheng kaifangfa* 增乘開方法) (Kawahara Hideki 1998, 58–60). The work of another middle class mathematician Hong Chŏng-ha's 洪正夏 (1684–?) *Writings of Nine and One* (*Kuiljip* 九一集, ca. 1714) is praised as "one big jump over the method of the celestial element" (ibid., 65). In this book, he not only dealt with a huge number of problems, about five hundred, but he also skillfully applied the method of the celestial element in his solution. Hence, Korean mathematics from the late seventeenth to the early eighteenth centuries could display

[9] For books and mathematical studies in this period, see Kim Yong-un and Kim Yong-kuk 1977, 231–256; Kawahara Hideki 1998, 47–66.

[10] For a detailed analysis of *Calculation Method in the Writings of Contemplation*, see Li Jianzong 2003.

[11] "The method of celestial element" is a general method of obtaining equations from given conditions. "Celestial element" means the unknown number in the problem. In this technique, "establish the celestial element as such and such" means "let x be such and such," and a polynomial or polynomial equation is usually indicated by using the character 元 (yuan, element) at the first degree term. When the equation that governs the unknown is obtained, the next step is to solve the equation and find the unknown number by use of the "method of extracting roots by iterated multiplication" (*zengcheng kaifangfa* 增乘開方法) (Kawahara Hideki 1998, 40–44).

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two distinctive characteristics: "the method of the celestial element" and "counting rod calculation."

Several Chinese texts of the Ming 明 and Qing 清 periods have been introduced into Korea. Among them, the *Systematic Treatise on Arithmetic* (*Suanfa tongzong* 算法統宗, 1592) by Cheng Dawei 程大位 (1533–1606) was the most notable one. This book was a practical mathematics text. Using abacus calculation as the main means of computation, chapters were arranged according to the layout of *Nine Chapters on the Mathematical Art* to cover all branches of traditional mathematics.[12] Books written by Korean scholars after the mid-seventeenth century frequently employed problems and solutions from this book.[13] *Complete Books on Mathematical Method* (*Suanfa quanshu* 算法全書, 1739) by Jiang Shoucheng 蔣守誠, which is no longer extant but is believed to have dealt with abacus calculation, was also used in Korea (Li Yan [1931] 1998, 411–412). Hong Tae-yong 洪大容 (1731–1783) listed this book as one of the references in his *Requirement for the Understanding of Mathematics* (*Chuhae suyong* 籌解需用, 1773).[14]

By the early eighteenth century, some Koreans led the way to learning Western mathematics. Among them was Choe Sŏk-chŏng, who cited the *Instruction for Calculation in Common Script* (*Tongwen suanzhi* 同文算指, 1614).[15] Mathematical studies in Korea at the time, however, were not dependent on the influence of Western mathematics, which had already prevailed in China. It was in the next generation that a well-established mathematical tradition laid the ground for Korean mathematics to develop in a Korean way with the inflow of Western mathematics.

2. The Introduction of Western Mathematics

In the late seventeenth century, Korean scholars tried to learn Western calendrical knowledge and other related subjects. The *Treatise on Calendrical Science According the New Western Methods* (*Xiyang xinfa lishu* 西洋新法曆書, 1645), which was later re-edited with the title *New Methods in Mathematics* (*Xinfa suanshu* 新法算書, 1666), and *First Collection of Writings on Heavenly Learning* (*Tianxue chuhan* 天學初函, 1626) were hailed as the key texts during the period between 1654, the year Korea adopted the Qing Calendar

[12] Abacus calculation, which the *Systematic Treatise on Arithmetic* depended on, was hardly accepted in Korea until the early twentieth century. For a general introduction to this book, see Li Yan and Du Shiran 1987, 185–189; Li Zhaohua 1993.

[13] Choe Sŏk-chŏng quoted some paragraphs from this book, and Pae Sang-yŏl 裵相說 (1759–1789) also, in his *Transcriptional fragments of Mathematics* (*Sŏgye swaerok* 書計瑣錄, 1786), frequently paraphrased problems and solutions in this book (see Kim Yong-un and Kim Yong-guk 1977, 237; Lin Kenhui 2003, 45–47).

[14] For the analysis of Hong Tae-yong's mathematics, see Hong Yiting 2003.

[15] Though Choe Sŏk-chŏng adopted some parts of Western mathematics in his book, the *Summary of the Nine Branches of Numbers*, he still insisted on the priority of traditional mathematics and, especially, various numerical methods of calculation used in nine branches of traditional mathematics, which he classified into four basic types entrusting the genesis of four entities (*sixiang* 四象) in the *Book of Change* (see Kawahara Hideki 1996).

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(*Shixianli* 時憲曆) based on Western methods, and the early eighteenth century.[16] There are hints about some efforts by the Korean government and its scholars to obtain and study these books and related subjects. Yet, how far they understood them and to what extent these books circulated has not been studied adequately except for the suggestion that Korean scholars' understanding of Western mathematics remained limited until the early eighteenth century.

It was from the mid-eighteenth century that the change began, from a limited to an extensive and systematic study and understanding of Western knowledge. The political détente between China and Korea fostered interests in Chinese culture and led to the adoption of Western learning among Korean scholars, resulting in a change of intellectual atmosphere in Korean intellectual society. The change began with the stabilization of the Kangxi 康熙 reign (1662–1722) and the establishment of a firm relationship between the two countries in the early eighteenth century.[17] The Kangxi emperor decreed that the border between China and Korea be opened in 1711, and an agreement on the boundary between the two countries was set up the following year.[18] The reconciliatory mood between the two countries was fostered by Korean envoys who were interested in Chinese culture and Western learning. Korean scholars could freely visit Catholic churches or collect books from bookstores in Beijing and they learned about Western science including mathematics and astronomy.[19] They imported many recently published books. Under these circumstances, from the 1720s Korea experienced a vast inflow of Western knowledge on mathematics and astronomy.[20]

As China regularly prohibited subordinate countries from learning calendrical knowledge, *Collected Basic Principles of Mathematics* (*Shuli jingyun* 數理精蘊, 1723) and the *Thorough Investigation of Calendrical Astronomy* (*Lixiang kaocheng* 曆象考成, 1723) were imported into Korea in 1729, six years after their publication.[21] This unusual success can be attributed to the Korean government's struggle to acquire Western calendrical knowledge and related books and also to individual scholars' competition to collect the newest books and to reach Chinese scholarly intellectual standards.[22] The Korean government strived to keep up with the changes of theories and numerical values in the new calendar that was adopted in 1726 by the Chinese government. The Korean government sent mathematical experts to learn calendrical knowledge and urged envoys to travel to China to procure the newest books of related knowledge

[16] About Korean learning of Western calendar and related knowledge, see my dissertation, Jun Yong Hoon 2004b, 12–48.
[17] On the relationship between Qing and Korea around the Kangxi reign, see Choe So-ja 2001.
[18] The detailed process of settlement of boundaries and its meaning are discussed in Kang Sŏk-hwa 1995.
[19] Statistics of Korean visits to Catholic churches and envoys' activities in Beijing are in Wŏn Chae-yŏn 1998.
[20] A general description of the introduction of books into Korea is in Song Il-gi and Yun Chu-yong 1998.
[21] Sŭngmunwŏn [1881] 1978, 1654–1655. For detailed history of introduction of texts on Western sciences and Korean scholars' understanding, see Jun Yong Hoon 2004b, 49–110.
[22] On the philosophical thought and cultural perspectives of Koreans at that time regarding Chinese culture, see Pak Kwang-yong 1995 and Kim Mun-sik 1994.

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(Sillokch'ŏng [1781] 1978, vol. 43, 32, 14 September 1741). Korean envoys also eagerly collected books for their private possession and some of these books were traded for high prices in the domestic intellectual community.[23]

Korean scholars' eagerness to study astronomy and mathematics was a sub-stream of general enthusiasm for learning more about the advanced Qing culture. Pukhak 北學, literally "Northern learning," was a new intellectual trend in the late eighteenth century that favored study of Qing institutions, technology, administration, and evidential research.[24] From the mid-eighteenth century, this intellectual atmosphere began to be shaped by the scholars who had visited Beijing and witnessed advanced Qing culture.[25] Impressed by the development of Qing and exchanges with Qing scholars, many Korean scholars changed their scholarly perspective and began to take an interest in more practical study that would help to develop their country, instead of adhering to Neo-Confucian mental pursuit. At the center of these scholars was Pak Chi-won 朴趾源 (1737–1805). The record of his journey to Beijing in 1780, where he described the contrast between advanced contemporary culture of Qing China and poor conditions in Korea, garnered many followers, including activists such as Pak Che-ga 朴齊家 (1750–?) and Yi Tŏk-mu 李德懋 (1741–1793) to his school. There was widespread interest in late eighteenth-century Korea for keeping abreast with the Chinese cultural standard.

Coinciding with this general interest in Qing culture, studying mathematics and astronomy in the mid-eighteenth century became a popular pursuit, triggering the transformation of the intellectual paradigm. Newly introduced books by Chinese scholars were enthusiastically received and Korean scholars absorbed the Chinese intellectual trend of the time. Mei Wending 梅文鼎 (1633–1721) and Qian-Jia 乾嘉 scholars played a crucial role in reshaping the academic landscape of Korean scholars.

In China during the seventeenth and eighteenth centuries, there emerged criticism of Neo-Confucianism, and more specifically criticism of cosmology and the development of textual studies (Jami 1991a, 100–101; Elman 1984, 26–36). Scholars opposed a "return to the origins" of Confucianism and self-cultivation leading to sagehood through the understanding of cosmological patterns. Evidential scholarship, which had originally been a methodology devised to restore the original version of ancient texts, tended to become a new mode of discourse. In this vein, the importance of "concrete studies," which included technical aspects of learning connected with statecraft, was advanced. The meaning of ancient mathematical and astronomical texts was essential for the return to antiquity; moreover the scientific content of all ancient texts was considered relevant to understanding their meaning. Contemporary mathematics and astronomy, including elements of Western origin, were useful tools for restoring meaning, and as such they were one of the bases of the methodology

[23] An example of book trading can be seen in Hwang Yun-sŏk [1735–1791] 1994–2003, vol. 3, 144b–145a.
[24] For general meaning of Pukhak and the formation of Pukhak school in Korea, see Yu Pong-hak 1995.
[25] On the exchange between Korean and Qing scholars in Beijing in Qian-Jia period, see Elman 1984, 155–156. For the close relationship and exchange between Korean scholar Kim Chŏng-hŭi 金正喜 (1786–1856) and Chinese scholars, see Fujitsuka Chikashi 1975.

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of documentary studies. One of the leading figures of concrete studies Mei Wending undertook the task of compiling a vast volume of mathematical and astronomical studies, which were posthumously edited under the title *Complete Books on Calendrical Mathematics* (*Lisuan quanshu* 曆算全書, 1723).[26] He emphasized mathematical study as a method leading to the immutable truth and argued that the study of mathematics was justified by its social usefulness illustrated by the precedent of Confucian scholars of the antiquity. This attitude was widely shared among scholars during this period (Li Di and Guo Shirong 1988, 190). Pan Lei's 潘耒 (1646–1708) statement in his preface to Mei Wending's *Theory of Rectangular Array* (*Fangchenglun* 方程論, 1690), which dealt with the linear algebra, demonstrates the practical usefulness of mathematical study.

> Although mathematics is the last of the Six Arts (*liui* 六藝), it has wide applications. Without mathematics, it is impossible to understand the measurement of Heaven and the survey of the Earth; it is impossible to regulate taxes and to manage finances; it is impossible to raise armies and dispose troops; it is impossible to administer civil engineering. (Li Di and Guo Shirong 1988, 190; English translation quoted in Jami 1991a, 101)

It is well-known that Mei Wending's approach to mathematics had a strong influence on the following generations of scholars, the so called "Qian-Jia" scholars who participated in the editorial work of imperial publications and adopted this perspective in their classical and mathematical studies.[27] The Qian-Jia school refers to a research community whose members made efforts to rejuvenate Chinese classical studies by means of textual criticism during the two successive reigns of the Emperors Qianlong 乾隆 (1736–1795) and Jiaqing 嘉慶 (1796–1820) (Liu Dun 1994, 106). As one of the "Six Arts" that ancient Confucians were required to study for their moral cultivation and general knowledge, mathematics acquired a certain importance. Masters of the classics became interested in traditional mathematics, and among these, Dai Zhen 戴震 (1724–1777), the most conspicuous figure, rediscovered and compiled the important works of ancient mathematics, many of which had almost vanished and were virtually unknown. Ruan Yuan 阮元 (1764–1849) also contributed to preserving and sorting out the ancient mathematical books. He composed the *Biographies of Astronomers and Mathematicians* (*Chourenzhuan* 疇人傳, 1799), a history of Chinese mathematics which includes more than three hundred professional astronomers and mathematicians from the earliest legendary times to the 1790s. Jiao Xun 焦循 (1763–1820), Wang Lai 汪萊 (1768–1813), and Li Rui 李銳 (1769–1817) were the leading scholars who devoted themselves to the study of traditional Chinese mathematics.[28]

[26] For the general research on Mei Wending and his studies, see Jami 1994; Li Di and Guo Shirong 1988.
[27] On the perspective of Mei Wending and his successors in mathematics, see Horng Wann-sheng 2001.
[28] For mathematical studies of Qian-Jia scholars, especially these three figures, see Horng Wann-sheng et al. 1993.

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As the Qian-Jia scholars adapted mathematics and astronomy to their evidential studies (Elman 1984, 180–184), Korean scholars began to agree that the concrete studies, especially the study of numbers (*shu* 數), was part of Confucian scholarship. Mathematics and astronomy, which had been considered so trivial that they could not attract serious interest from Confucian scholars, now became academic fields worthy of study for carrying out intellectual activities.

> Because the measurement of celestial movement is not beyond the Old Experts' (Xi 羲 and He 和) management and the Yellow Emperor's (Huangdi 黃帝) principle of a right angled triangle is entirely being used in the reckoning method (of celestial bodies), this is the remaining work of our Confucians. (Hong Yang-ho [1843] 2000, 267)[29]

Hwang Yun-sŏk 黃胤錫 (1729–1791), who devoted himself to the study of mathematics and astronomy in Korea, was also assured that the study of numbers should never be considered a shame for a Confucian scholar.

> Since there is the number (數), the number of Harmonics and Calendar will not be beyond this . . . . . The reason and principle of celestial motions are related to a part of Confucian studies. Because the prominent Confucian scholars also reported (to the throne) and discussed this, to study this work together with others would not be hindered. (Hwang Yun-sŏk [1735–1791] 1994–2003, vol. 2, 222)

In this vein, Hong Kil-chu agreed that "The method of Geometry is worth studying by recent Confucian scholars" (Hong Kil-chu 1826, Section 14, 1).

In Korea, however, the introduction of Qian-Jia scholars' mathematical research was quite delayed. It seems that Hong Kil-chu was not able to acquire them. It was the next generation who could obtain Qian-Jia scholars' research. Chinese texts restored and studied by Qian-Jia scholars such as *Nine Chapters on the Mathematical Art*, *Precious Mirror of the Four Elements* (*Siyuan yujian* 四元玉鑑, 1303), *New Steps in Computation* (*Yigu yanduan* 益古演段, 1259), *Mathematical Treatise in Nine Sections* (*Shushu jiuzhang* 數書九章, 1247), and *See Mirror of Circle Measurements* (*Ceyuan haijing* 測圓海鏡, 1248) were acquired and studied by Korean scholars like Nam Pyŏng-gil 南秉吉 (1820–1869) and Yi Sang-hyŏk 李尙爀 (1810–?) in the mid-nineteenth century.[30] Mei Juecheng's 梅穀成 (1681–1763) *Remained Treasure in Red Water* (*Chishui yizhen* 赤水遺珍) was also used in Korea.

Among the various fields of mathematical studies in the nineteenth century, "the method for borrowing roots and powers" (*jiegenfang* 借根方) was a specific topic in

[29] Hong Yang-ho, an enlightened Korean scholar and Ji Yun 紀昀 (1724–1805), a leading Chinese scholar in the Qian-Jia years, exchanged letters from 1795 to 1801. This paragraph was quoted from Hong's letter in 1798. For more about Korean scholars' attitude for exact sciences, see Jun Yong Hoon 2004b, 87–90.

[30] About Nam and Yi's mathematical studies, see Kim Yong-un and Kim Yong-guk 1977, 263–274; Hrong Wann-sheng 2002; Wu Binghong 2003, 11.

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Korea. In this method, the multiples of the unknown number and its square are taken to form an equation and then the equation is solved to find the unknown number. This was the Western method contained in *Collected Basic Principles of Mathematics*, but this was very similar to the method of the celestial element in Song and Yuan period. Yi Sang-hyŏk in Korea analyzed the method for borrowing roots and powers comparing it with the method of the celestial element in his three books: *Elementary Research on the Method for Borrowing Roots and Powers* (*Ch'agŭnbang monggu* 借根方蒙求, 1854), *Narrow View of Mathematical Method* (*Susul kwan'gyŏn* 數術管見, 1855), and *Assistance to Mathematics* (*Iksan* 翼算, 1868).[31]

3. Mathematical Studies before the Nineteenth Century

Hong Sŏk-chu 洪奭周 (1774–1842) presented Hong Kil-chu, his younger brother, with a reading list for Confucian students in 1810 (Hong Sŏk-chu [ca. 1810] 1984, 45a). This list shows the subjects a young scholar was required to study in order to become a Confucian scholar in those years. Amongst a wide range of subjects, Hong Sŏk-chu listed the following books in the categories of mathematics (*suga* 數家) and astronomy (*ch'ŏmun'ga* 天文家).[32]

Instruction for Calculation in Common Script (*Tongwen suanzhi* 同文算指), 10 volumes, written by Li Zhizao 李之藻 (1565–1630).
Elements of Geometry (*Jihe yuanben* 幾何原本), 6 volumes, translated by Xu Guangqi 徐光啓 (1562–1633).
New Methods in Mathematics (*Xinfa suanshu* 新法算書), 100 volumes, edited by Xu Guangqi.
Complete Books on Calendrical Mathematics (*Lisuan quanshu* 曆算全書), 60 volumes, written by Mei Wending.
Collected Basic Principles of Mathematics (*Shuli jingyun* 數理精蘊), 53 volumes.
Thorough Investigation of Calendrical Astronomy (*Lixiang kaocheng* 曆象考成), 42 volumes.
Second Volume of Thorough Investigation of Calendrical Astronomy (*Lixiang kaocheng* houbian 曆象考成後編), 10 volumes.
Thorough Investigation of Astronomical Instruments (Yixiang kaocheng 儀象考成), 32 volumes.

This list includes most compilations that were considered landmarks of mathematics and astronomy adopting Western knowledge from the seventeenth to early nineteenth centuries. In mathematics, *Elements of Geometry*, the early seventeenth-century translation of Euclid's first six chapters of *Elements*, and *Instruction for Calculation in Common Script*, a combined work of translation of Christopher Clavius' (1538–1612)

[31] For the study of *Elementary Research on the Method for Borrowing Roots and Powers*, see Wu Binghong 2003.
[32] The volume numbers and names of authors were added by Hong himself. Matteo Ricci's cooperation with Li Zhizao and Xu Guangqi was missing from the list. For the study of Hong's reading list, see Yi Sang-yong 1994, 45–66.

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Essentials of Practical Arithmetic and excerpts of Chinese sources, were still at the top of the list.[33] The extended period of time that these books prevailed in Korea, from their introduction in the mid-seventeenth century on, shows how important Korean scholars considered these mathematical textbooks to be. Hong Sŏk-chu identified *Elements of Geometry* as Euclid's *Elements* translated by Xu Guangqi.[34] It seems that Korean scholars, as far as geometry is concerned, were influenced more by the work of Euclid than by the work of Pardies included in *Collected Basic Principles of Mathematics*.

Of all these books, *Collected Basic Principles of Mathematics* and *Thorough Investigation of Calendrical Astronomy* were the most influential volumes by which Korean scholars could learn Western sciences at that time. *Thorough Investigation of Calendrical Astronomy* was a new compendium of Western calendrical astronomy that evolved out of *Treatise on Calendrical Science According the New Western Methods* and *New Methods in Mathematics*, which had required a revision of numerical variables to correct accumulated errors after the Qing Calendar (*Shixianli*) was adopted in 1645. Moreover, due to its systematic description of planetary motions with relevant figures, *Thorough Investigation of Calendrical Astronomy* was considered to be a standard textbook on calendrical astronomy (Hashimoto Keizō 1970b, 73). It was from this book that Sŏ Ho-su 徐浩修 (1736–1799) excerpted many paragraphs into the official historiography of Korean astronomy, *Study of Starry Pattern and Calendar* (*Sangwigo* 象緯考, 1770) (Pak Yong-dae et al. [1770, 1809, 1908] 1980, vol. 1, 78–102).

Collected Basic Principles of Mathematics was also regarded as an essential reference for Korean scholars from the mid-eighteenth century. This mathematical encyclopedia covers almost all mathematical knowledge, Chinese and Western, available in China at the time of its publication (1723), including a presentation of mathematical notations, various mathematical techniques and numerical tables.[35] This book consisted of five sections: the first section (𠂇部), the line section (線部), the surface section (面部), the solid section (體部), and the last section (末部). Regarding the topics, which I will discuss later in this paper, the line section contains the rule of double false position, and the surface section includes the extraction of square roots.

Collected Basic Principles of Mathematics enjoyed lasting success and played a key role in the development of Chinese mathematics in the middle and late Qing period. A modern study explains why: "the use of relevant figures under the analysis of the subject" and "the classification of mathematical topics into the line, surface and solid sections" (Han Qi 1993, 8). In Korea, owing not only to its clear and systematic description of mathematical subject but also to the authorized publication by

[33] About *Instruction for Calculation in Common Script*, see Engelfriet 1998, 321–326.

[34] There is another version of *Elements of Geometry* in *Collected Basic Principles of Mathematics*, which was translated from a work of the French Jesuit, Ignace Gaston Pardies (1636–1673), *Elémens de géométrie*. The work by Pardies is of a nature very different from Euclid's (Engelfriet 1998, 437–438).

[35] For a general introduction to *Collected Basic Principles of Mathematics*, see Qian Baocong [1964, 1981] 1998; Jami 1991b; Han Qi 1993; Martzloff 1997, 163–166.

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imperial patronage, this book was also hailed as a reliable encyclopedia of mathematical knowledge. From the 1760s, this book, together with *Thorough Investigation of Calendrical Astronomy*, was widely circulated among scholars in Seoul area and produced several Korean scholars of mathematics who were respected as "masters" of the book (Jun Yong Hoon 2004b, 83–87).

Thorough Investigation of Astronomical Instruments, which discussed the astronomical instruments as well as the data of field stars, shows clearly that scholars of that time extended their attention to astronomy in order to understand observational data and the use of instruments. And finally *Complete Books on Calendrical Mathematics*, which had been frequently read by Korean scholars in the late eighteenth century, still occupies the main position in the list. As mentioned before, this means that the Korean scholars' opinion of what sources were required for the study of exact sciences in the early nineteenth century could reflect Mei Wending's perspective.[36]

Korean scholars, having studied these books, began to make notable contributions to mathematics and astronomy from the late eighteenth century on. In his *Free Diary* (*Ijaenan'go* 頤齋亂藁), Hwang Yun-sŏk 黃胤錫 (1729–1791) mentioned a dozen Confucian scholars who were engaged in mathematics and astronomy.[37] Including Hwang himself, Hong Tae-yong and Hong Yang-hae 洪景海 wrote books on mathematics. Yi Sŭng-hun 李承薰 (1756–1801), the first Catholic convert in Korea, and Yi Pyŏk 李檗 (1754–1786), a believer in and promoter of Catholicism, were famous for their mastery of geometry. In fact, the Jesuits in Beijing had some doubt about Yi Sŭng-hun's Catholic faith because he expressed his interest in geometry while visiting Beijing to be christened (Kim Ok-hŭi 1998, 84). Yi Ka-hwan 李家煥 (1742–1801), who once occupied the position of Second High State Councilor (*Chwaŭijŏng* 左議政), regretted that the government neglected to train geometricians and disclosed his pride in his broad grasp of geometry: "After my death, Korea would be barren with no more geometricians" (Hwang Sa-yŏng [1801] 1975, 57). It was about the same time that Sŏ Ho-su, who was connected with Hong Kil-chu, wrote vast volumes of mathematical books in which he enlarged and supplemented some materials for *Collected Basic Principles of Mathematics*, *Thorough Investigation of Calendrical Astronomy*, and *Second Volume of Thorough Investigation of Calendrical Astronomy*. *Brief Comment on the Proportion* (*Pirye yaksŏl* 比例約說), *Supplemental Explanation of the Collected Basic Principles of Mathematics* (*Suri chŏngon pohae* 數理精蘊補解), *Supplemental Explanation of the Thorough Investigation of Calendrical Astronomy* (*Yŏksang kosŏng pohae* 曆象考成補解), and *Supplemental Explanation of the Second Volume of Thorough Investigation of Calendrical Astronomy* (*Yŏksang kosŏng hup'yŏn pohae* 曆象考成後編補解) were his own writings. In

[36] Sŏ Ho-su said that he consulted Mei Wending's *Complete Books on Calendrical Mathematics* for more than ten years whenever he confronted problems in studying astronomy (Sŏ Ho-su [1736–1799], 33–35).

[37] Hwang Yun-sŏk left occasional remarks about qualified mathematicians and astronomers around the 1770s. I have summarized here the scattered records in his book. For detailed records, see Jun Yong Hoon 2004b, 82–86.

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addition to these books, he wrote or edited more than ten books on astronomy with his assistants in the National Astronomical Bureau (*Sŏun'gwan* 書雲觀).

Scholars of the next generation such as Kim Yŏng 金泳 (1749–1817), Hong Kil-chu, and Sŏ Yu-bon 徐有本 (1762–1822), also considered mathematics and astronomy to be a serious pursuit. Kim Yŏng, in particular, exemplifies this new generation's involvement in mathematics. It is remarkable that he even planned to revise some part of *Collected Basic Principles of Mathematics*, though it was not actually carried out (Sŏ Yu-bon [ca. 1817] 1992, 643). With his mastery of geometry, Kim Yŏng planned to adapt the line section of *Collected Basic Principles of Mathematics*, which makes up the arithmetic subjects such as proportions and the rule of double false position, into a geometrical text using geometrical expressions. Just as Xu Guangqi did in the preface of *Elements of Geometry*, Kim Yŏng differentiated the numerical method (*shufa* 數法) and the method of measurement (*liangfa* 量法) (Engelfriet 1998, 138–142). While the number (*shu* 數), which is discrete quantity, is manipulated in algebraic or arithmetical manner, the magnitude (*du* 度), which is continuous quantity, is controlled in a geometrical manner. So his intention to adapt the line section of *Collected Basic Principles of Mathematics* means the reconstruction of algebraic algorithm in a geometrical manner. Moreover, the fact that he intended to write a book on projective geometry shows that the level of mathematical inquiry in Korea had reached that far by the early nineteenth century.

Hong Kil-chu was also a member of this generation. In addition to many books available to him, Hong was fortunate to come across mentors and co-workers that stimulated him to cultivate his mathematical study. The drive towards exact sciences, which had been experienced by his prior generation and contemporary scholars, created a favorable environment where Hong's mathematics could thrive.

4. Work and Personal Background of Hong Kil-chu

As a member of one of the noblest families in the eighteenth century, Hong Kil-chu had many higher officials among his relatives even though he himself could not pass the state examination for a governmental post.[38] His grandfather and elder brother had been Chief State Councilor (*Yŏngŭijŏng* 領議政) and Second High State Councilor respectively, and his father also had occupied a high office. His family was so high ranking that King Chŏngjo 正祖 (r. 1777–1800) arranged for his daughter to marry Hong's younger brother. Hong was provided with the best education that his family could afford. From his childhood he was taught by both his mother and elder brother who were equally respected as the foremost scholars of the day. His mother was known for her erudition and literary talent and his elder brother was also known as one of the

[38] Even though he did not pass the state examination, he was summoned for official posts several times owing to his forefathers' contribution to the government. But he resigned from them in a short while. About Hong's life, see the biography by his son, Hong U-gŏn 洪祐健 (1811–?) (Hong U-gŏn ca. 1906, book. 7).

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leading scholars in Confucianism and literature.[39] His household collection of books supplied resources for his self-education after he gave up the state examination at the age of 26 (1811) devoting himself entirely to scholarship.[40]

In addition to a joint work with his brothers, *Collection of Eternal Glory of Triple Joy* (*Yŏnggasamijip* 永嘉三怡集, 1824) and another well-known work, *My Thought Somebody Will Follow* (*Suksunyŏm* 孰遂念, 1826), Hong also authored a collection of three volumes, *Composition A at the Top of the Hill* (*Hyŏnsu kapko* 峴首甲藁, 1816), *Binding B at Jade Green Stone* (*P'yorong ŭlch'ŏm* 縹瑢乙幟, 1836), and *Chest C of Tranquil Dew* (*Hanghae pyŏngham* 沆瀣丙函, edited posthumously by his son in 1842).[41] These volumes contain an immense variety of literary writings, articles on Confucian philosophy, biographies of his acquaintances and various other kinds of writings, and reveal a lifetime spent in scholarship. Hong sought and achieved a writing style unique enough to be called "transcendental" by modern scholars who have studied his works.[42]

His mathematical studies deserve attention because they were among his main pursuits from his childhood to the end of his life and they demonstrate his scholarly ingenuity.[43] Hong wrote two kinds of mathematical books: *New Methods of Geometry* (*Kiha sinsŏl* 幾何新說) and *Explanations and Examples for Arc and Angle* (*Hogak yŏllye* 弧角演例). *Unsorted Fragments of Supplement for Geometry* (*Kiha chapswaebo* 幾何雜碎補) is a small appendix he added to *New Methods of Geometry*. His mathematical works mainly depend on two kinds of precedent knowledge: traditional Chinese mathematics and newly introduced Western mathematics. There are traditional subjects and problems of Chinese mathematics scattered in *New Methods of Geometry*. In his two main mathematical books there are Western subjects and terms in geometry and spherical trigonometry, borrowed from such Chinese compendia of Western mathematics as *Elements of Geometry*, *Collected Basic Principles of Mathematics*, and *Thorough Investigation of Calendrical Astronomy*.

Hong's central works on mathematics are included in *New Methods of Geometry*, a work which displays his abilities. This book consists of three parts – "Manipulation of two supposed values" (*Ssangch'u ŏksan* 雙推臆算), "Elementary research on root extraction" (*Kaebang monggu* 開方蒙求), and "Occasional copies of unsorted fragments"

[39] About his mother and elder brother, see Kim Mi-ran 1994; Kim Ch'ŏl-bŏm 1992; Kim Mun-sik 1996.
[40] On his household collection of books, see Yi Sang-yong 1994, 8–12.
[41] Including all these writings mentioned above, the complete collection of Hong's writings was published in the late nineteenth century by his grandson. Chŏng In-bo wrote a review of it in 1931, which includes some valuable quotations from it. However, this complete collection disappeared sometime around 1940. In the 1990s some scholars who were studying classical literature happened to find the original manuscripts of these three volumes and used them privately until the Yonsei University Library decided to reveal them to the public in 2003. For Chŏng's first review of the complete collection, see Chŏng In-bo [1931] 1983, 41–42. About the rediscovery of Hong's manuscripts, see Chŏng Min 1991.
[42] Studies on Hong Kil-chu's literature can be found in bibliographies attached to articles in the special edition of *Yŏlsang kojŏn yŏn'gu*, vol. 17 (2003).
[43] On Hong's mathematics, see my two articles (Jun Yong Hoon 2003 and 2004a).

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(*Chapswae such'o* 雜鎖隨鈔).[44] Hong solves a variety of difficult mathematical problems in these three parts by the use of advanced algorithms. In the first part, he deals with problems such as proportions, excess and deficit (double false position), and linear equations. In the second part, the simple algorithms he designed are employed to solve the extraction of square, cubic, fourth and fifth roots, and problems of quadratic equations involving sum, difference, and product of two sides of a rectangle. The third part mainly expounds his ideas on an array of topics that are largely related to Euclidean geometry. Here he proposes new methods for solving problems such as triangulation, geometrical rules in a circum-circle, the division of a line into mean and extreme ratio (*lifen zhongmoxian* 理分中末線 in *Elements of Geometry*), various kinds of inscribed polygons, and making a half-area-square or circle of a given.

The other mathematical book by Hong, *Explanations and Examples for Arc and Angle*, is also notable in Korean history of mathematics as it is entirely devoted to the spherical trigonometry imported from the West via China.[45] Hong extracted the theorems of spherical trigonometry from *Thorough Investigation of Calendrical Astronomy* which mainly addresses astronomical calculations. In comparison with Mei Wending's study on spherical trigonometry, which was available to Korean scholars, Hong's research in *Explanations and Examples for Arc and Angle* is not so advanced.[46] He aimed at making the complicated procedures not fully explained in *Thorough Investigation of Calendrical Astronomy* understandable for beginners. On the ninety different cases composed by the combination of three knowns and one unknown, he explained the detailed procedure to the solution of the unknown.[47] This book was the first book to deal with spherical trigonometry in the history of Korean mathematics. Although there are several books written by Korean scholars that include spherical trigonometry such as *Outline of the Qing Calendar* (*Sihŏn kiyo* 時憲紀要, 1860) and *Gathered Remnant of Mathematics* (*Sanhak sŭbyu* 算學拾遺, 1860s), not only are those books written fifty years later than Hong's *Explanations and Examples for Arc and Angle* but they only feature spherical trigonometry as a minor part. Hong seems to have been the first scholar in Korea to open a new field and to offer a new objective, spherical trigonometry in the pursuit of mathematical

[44] The original manuscript of *New Methods of Geometry* might have been written before he was 30 years old (1816), because the title is listed in the table of contents of the *Composition A at the Top of the Hill* (1816), even though this newly opened manuscript does not include the text. Fortunately we have another manuscript of *New Methods of Geometry* within his different collection of writings, *My Thought Somebody Will Follow*, which was edited in 1826.

[45] According to Hong himself, he finished the manuscript of *Explanations and Examples for Arc and Angle* in 1814 and revised it in 1836. This book is now in *Chest C of Tranquil Dew*.

[46] Mei Wending's three main works on spherical trigonometry, *Elements of Spherical Triangles* (*Husanjiao juyao* 弧三角舉要), *Geodesy* (*Huanzhong shuchi* 環中黍尺) and *Surveying Solids* (*Qiandu celiang* 塹堵測量), were circulated among Korean scholars from late 1770s (see Hwang Yun-sŏk [1735–1791] 1994–2003, vol. 4, 384; Jun Yong Hoon 2004b, 91–92). About Mei Wending's study on spherical trigonometry, see Liu Dun 1993a, 565, 605–607, 655–656; Li Di and Guo Shirong 1988, 172–180.

[47] I analyzed his spherical trigonometry and tracked down the history of the introduction of trigonometry from the West via China and into Korea (see Jun Yong Hoon 2004a, 293–298).

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study. I believe that it was Hong who first brought this subject to the attention of Korean mathematics and that his work allowed scholars to accept this field as an important area for pure mathematical studies independent of its astronomical applications.

Hong was not isolated. Teachers and associates who lived nearby, including relatives and close friends, frequently exchanged ideas and stimulated his study of mathematics. His environment was filled with the ferment of the new intellectual paradigm that was flourishing in Korea in the late eighteenth century. This trend toward the exact sciences like astronomy and mathematics fostered Hong's aptitude for mathematics.

As a matter of fact, his mother was a gifted mathematician who could invent new algorithms while reading the *Introduction to Mathematical Studies* and she was the first person to promote Hong's talent for mathematics. Those who read his works commonly agreed that his mother's instruction led him to mathematical pursuits as well as literary ones (Hong Han-ju [ca. 1906] 1984, 165). Hong grappled with mathematical problems from childhood and filled papers with exercises. He wrote:

> I have studied mathematics since I was seven or eight years old. By the age of 10, I mastered *ying nü* 盈朒 (excess and deficit, double false positional problems), *fang cheng* 方程 (system of linear equations) and *gou gu* 句股 (right-angled triangle). . . . After 20, I absorbed myself again in mathematics.[48]

Before his twenties, he studied and practiced such topics as *ying nü*, *fang cheng*, and *gou gu*, the common titles of sections that comprised traditional mathematical texts. He continued to enlarge and revise his mathematical writings even into his final years. He described his labors in confident terms:

> In this book, I completely swept away the old method and invented new method. Though I do not boast that I am above the ancients, readers will be able to witness what I strove for.[49]

Hong's family lineage on his mother's side was the Taegu Sŏ family 大邱 徐氏 (Sŏ Family from Taegu district), one of the most noble and outstanding families in the eighteenth and early nineteenth centuries. His maternal grandfather, Sŏ Hyŏng-su 徐逈修 (1725–1779) was related to Sŏ Ho-su, who is regarded as one of the best mathematicians and astronomers in the Korean history of science (Yu Kyŏng-no 1999, 242–255).[50] They were fourth-degree cousins. The Taegu Sŏ family was noted

[48] I quote this from Chŏng In-bo [1931] 1983, 42. Chŏng quoted this from Hong's preface of the final edition of the *New Methods of Geometry*, which was edited in the complete collection in the late nineteenth century. However, since the entire volume of the complete collection disappeared sometime later, we can not identify the quotations in the extant manuscript of the *New Methods of Geometry* which has no preface.

[49] I quote this from Chŏng In-bo [1931] 1983, 42.

[50] Yu singled out Sŏ Ho-su and his father Sŏ Myŏng-ŭng as one of the three most outstanding couples in Korean history of exact science. Yi Sun-chi 李純之 (1406–1465) and Kim Tam 金淡 (1416–1464), two associates for

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for its study of mathematics and astronomy. Sŏ Myŏng-ŭng 徐命膺 (1716–1787), an authoritative scholar on the study of the *Book of Change*, assigned his family members to study mathematics.[51] Owing to his mathematical ability, Sŏ Ho-su, the first son of Su Myŏng-ŭng, was in charge of the National Astronomical Bureau and churned out books on mathematics and astronomy. Sŏ Yu-bon 徐有本 (1762–1822), the first grandson of Sŏ Myŏng-ŭng, also followed his forefathers' footsteps, devoting himself to mathematics. He produced a book on geometry, *Elementary Research on Geometry* (*Kiha monggu* 幾何蒙求), in 1808 (Sŏ Yu-bon [1808] 1992, 495).[52]

The Hong and the Sŏ family members were friends. Sŏ Yu-gu 徐有榘 (1764–1845), the second son of Sŏ Ho-su, and Hong Sŏk-chu, the elder brother of Hong Kil-chu, had a close relationship through their common experiences of official service in academic ministries such as the Office of Special Advisers (*Hongmun'gwan* 弘文館) and the Office of Royal Decrees (*Yemun'gwan* 藝文館). They discussed their bibliographical studies. It is said that Hong Sŏk-chu adopted Sŏ Yu-gu's perspective (Yi Sang-yong 1997, 110–112). Hong himself also frequently joined the Sŏ brothers in a poetry club where they collected poems, and sometimes traveled together (Cho Ch'ang-nok 2001, 293–296, 307–308). Although no definitive proof of their connection with mathematics has been disclosed, it is likely that the young scholars of the two families frequently exchanged ideas on various topics, including mathematics.

Kim Yŏng, who had a close relationship with both families, may possibly have been the connection between Hong's mathematical studies and the Sŏ family.[53] Kim played an essential role in the mathematical studies of both families and was a self-taught astronomer and mathematician. When the Sŏ family learned by hearsay about Kim's talent, Sŏ Ho-su tested Kim's mathematical capability and was amazed at Kim's superb ability. When King Chŏngjo planned to move his father's tomb in 1789, he faced the critical problem of choosing a favorable day. Because the estimated auspicious time was in the vicinity of midnight and the dating depended on the local time, the determination of the exact local time turned out to be a crucial issue. The King sought an expert to entrust this task. Until Kim was appointed to the Astronomical Bureau Sŏ Ho-su had been engaged in governmental work in astronomy and mathematics. Sŏ recommended Kim Yŏng to carry out the astronomical observations for reckoning the exact local time.

settlement of *Ch'ilchŏngsan* 七政算 (The Calculation of the Seven Celestial Bodies) Calendar in King Sejong era; and Nam Pyŏng-ch'ŏl 南秉哲 (1817–1863) and Nam Pyŏng-gil, prosperous brother writers on astronomy and mathematics in the late nineteenth century are the other two couples.

[51] This family confidently regarded mathematics as their family learning (Sŏ Yu-gu [1822] 1983, 206–208).

[52] Only the preface is available now.

[53] Kim's life can be reconstructed in three kinds of biographies by his contemporaries. Besides two biographies by Hong Kil-chu and Sŏ Yu-bon, respectively, Yu Chae-gŏn 劉在建 (1793–1800) also collected Kim's life story in his *Collection of Observations in Countryside* (*Ihyang kyŏnmullok* 里鄕見聞錄). Chŏng Min presented a brief introduction of Kim's life in modern words (Chŏng Min 1992).

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Even though Kim was already known as a gifted man among the scholars engaged in astronomy and mathematics, his employment was recognized as extraordinary because he had never passed the central state examination for public officials of mathematics or astronomy. It was Hong's grandfather who persuaded the opponents and obtained Kim Yŏng's appointment to the Bureau. After this incident, Kim became quite dependent on Hong's family and also forged a close relationship with the Sŏ family.

It is likely that Kim Yŏng taught the Hong brothers mathematics and astronomy and so Kim probably became a close friend with Hong Kil-chu. The latter wrote a short biography of Kim including a eulogy (Hong Kil-chu 1836, book 2). In it, he recalled that he had discussed mathematical problems with Kim since his own childhood. Kim's influence on Hong Kil-chu's mathematical studies is well-confirmed by a line in *New Methods of Geometry*. On the problem of finding the length of one side of three squares inscribed in a given circle, Hong adopted Kim's method (Hong Kil-chu 1826, 41). Kim also frequently joined Sŏ Yu-bon and discussed mathematical problems for long hours. Kim once persuaded Sŏ Yu-bon to concentrate on mathematics, and to study *Elements of Geometry* (Sŏ Yu-bon [ca. 1817] 1992, 642–644). In his biography of Kim Yŏng written in 1817, Sŏ Yu-bon recalled that their close friendship had lasted more than a decade until Kim's death. Considering the relationships among the three of them, Kim Yŏng, Hong Kil-chu, and Sŏ Yu-bon, it is natural to claim that Hong's mathematical studies were closely related with the Sŏ family's mathematical activity as well as that of Kim Yŏng.

5. Specialty and Ingenuity

New Methods of Geometry, containing his newly designed algorithms, demonstrates Hong Kil-chu's mathematical ingenuity. In addition to the algorithms in the first and second parts, the main topics of this section, Hong introduced in the third part more than twenty different methods for various problems which could not be grouped into one category. He examined precedents before he proposed his own inventions. For some problems he adopted "other methods" (*ilsŏl* 一說) including Western methods (*sŏbŏp* 西法). For an inscribed polygon problem, as mentioned above, where three squares are inscribed in a circle, he adopted Kim Yŏng's method together with his own. This suggests that he openly discussed his new inventions with others.

Among the many new methods proposed in *New Methods of Geometry*, the most notable ones are in the first and second part: manipulation of two supposed values (the rule of double false position) and method of extracting higher order roots. Hong tried to find simple and easy algorithms suitable for counting rod operations, and took pride in the fact that his new methods were easy for everyone to understand and operate with counting rods.

In the first part of *New Methods of Geometry*, "Manipulation of two supposed values," he proposed an insightful method different from that of *Collected Basic Principles of*

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Mathematics and even from that of *Introduction to Mathematical Studies*. These two books were conventional texts upon which Hong and his contemporaries based their general knowledge of mathematics. As expected, he borrows all 13 problems dealt with in this part from *Collected Basic Principles of Mathematics*. While the problems are spread into the usual categories of problems like proportions, double false positional problems, and system of linear equations, etc., his new "method of manipulation of two supposed values" covers all of these categories.

As suggested by its title, "Manipulation of two supposed values," the method consists in taking two supposed numerical values and working on them.

(Problem) Here is a mass of thread which weighs 369 *kŭn* 斤. When allotted to A, B, C and D with the decrement ratio of 8/10, how much does each get?

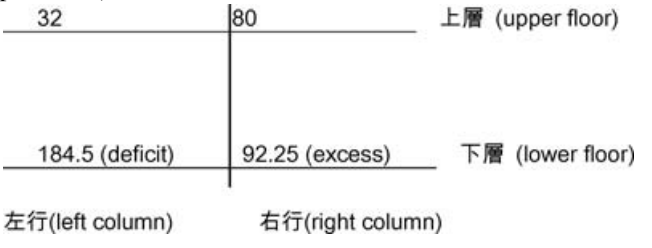
For this problem, Hong first adopts 80 and 32 as D's share. Other shares are then derived in accordance with the defined ratio. The procedure Hong proposes is as follows:

(Procedure to solution)
If D gets 80 *kŭn* (C earns 100 *kŭn*, B 125 *kŭn*, and A 156.25 *kŭn*), the total weight of thread should be 461.25 *kŭn*, 92.25 *kŭn* more compared to the original weight.
If again D earns 32 *kŭn* (C earns 40 *kŭn*, B 50 *kŭn*, and A 62.5 *kŭn*), total weight of thread should be 184.5 *kŭn*, 184.5 *kŭn* less compared to the original weight.[54]

(Modern recapitulation)
D = 80, C = 100, B = 125, A = 156.25, sum total=461.25, excess = 92.25
D = 32, C = 40, B = 50, A = 62.5, sum total=184.5, deficit = 184.5

This time, line up (yŏl, 列) the 80 *kŭn* and the excess 92.5 *kŭn* on the right. Line up the 32 *kŭn* and the deficit 184.5 *kŭn* on the left.

(Modern recapitulation)

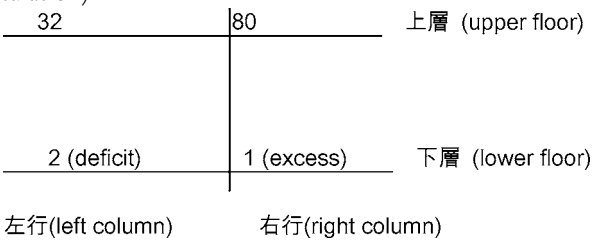


If [one] abbreviates the lower floors of both columns (兩行下層) each other (Use the bigger as the denominator, the smaller as the numerator, and abbreviate according to the rules), 92.25 *kŭn* is reduced to 1 portion (分), 184.5 *kŭn* to 2 portions.

[54] Descriptions with small letters in parenthesis are original footnotes by Hong Kil-chu.

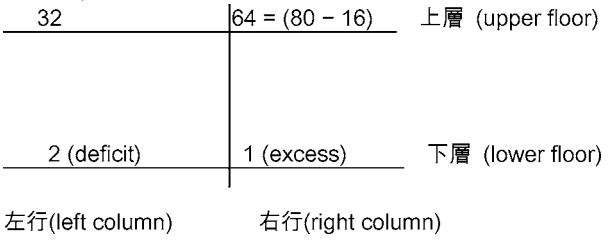
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(Modern recapitulation)



If [one] adds them, [he] earns 3 portions (If one [of the differences between the total weights and the original weight] is excess and the other is deficit, add them. If both are excess and both are deficit, then subtract each other).
If [one] obtains the difference of upper floor values (At the upper floor, regardless of the combination of the values, one excess and one deficit, both excess, and both deficit, subtract each other), 48 *kŭn*, the remainder, becomes the value of 3 portions, 32 *kŭn* of 2 portions, and 16 *kŭn* of 1 portion.
If [one] subtracts 16 *kŭn*, the value of 1 portion, from 80 *kŭn*, the upper right, the remainder will be 64 *kŭn*, which is D's share.

(Modern recapitulation)



C's share 80 *kŭn*, B's share 100 *kŭn*, A's share 125 *kŭn* can be deductively obtained.

Hong's method above is an application of what we call "the rule of double false position." Though he was proud that he expanded the applicability of the method and made it simpler to operate, this basically originated from the method used in *Collected Basic Principles of Mathematics*, which adopted the Western procedure of operation. Hong said:

> This method is roughly the same as x*u tao ying nü* 虛套盈朒 (excess and deficit with void multiplication) [in the *Collected Basic Principles of Mathematics*].[55] However, it does not use any cross multiplication (*wei cheng* 維乘) but only reduction. It is applicable to the problems dealt in the line section [of the *Collected Basic Principles of Mathematics*], from *gui chu* 歸除(the fractional numbers) to *fang cheng*(system of linear equations) instead of the methods appropriate for each section. It is very easy for calculation and really extensive for

[55] It seems to be a miswriting of *shuang tao ying nü* 雙套盈朒 (excess and deficit with double cross multiplications), the chapter 6 of the line section.

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> application. Definitely this is the mathematician's short cut and a necessary way. (Hong Kil-chu 1826, 2)

Collected Basic Principles of Mathematics actually used the rule of double false position to solve the problems in the chapter of "*ying nü*" (excess and deficit).[56] What Hong did with the original method was to transform this to avoid a cross multiplication and apply it to the problems of the other chapters. All thirteen problems dealt with in the part of "Manipulation of two supposed values" were borrowed from various chapters of *Collected Basic Principles of Mathematics*, "from *gui chu* (the fractional numbers) to *fang cheng* (system of linear equations)" in his words above, as well as from the "*ying nü*" chapter.

Considering the rule of double false position in *Nine Chapters on the Mathematical Art*, which adopted almost the same algorithm as Hong's and had a wide applicability but used the cross multiplication in the procedure, the history of this method in China and Korea needs to be mentioned to understand why Hong regarded his algorithm and its wide application as his own achievement. Of the valuable achievements in Chinese mathematics, the rule of double false position in *Nine Chapters on the Mathematical Art* and later Chinese mathematicians' studies on this topic gathered the most attention from modern researchers (Qian Baocong [1927, 1983] 1998; Lam Lay Yong 1974; Li Jimin 1982; Li Jimin 1990; Liu Dun 1993b, 179–182). On the twenty problems in the "*ying bu zhu*" 盈不足 (excess and deficit) chapter of *Nine Chapters on the Mathematical Art*, Liu Hui 劉徽 (3rd century AD) presented an analysis of the method called "the *ying bu zhu shu*" 盈不足術 (the method of excess and deficit), which is now believed to be the same as "the rule of double false position" of Western mathematics. The cross multiplication of the guesses with excess and deficit are used in the procedure. Yang Hui 揚輝 (13th century AD) also explained and expanded this method as given in *Nine Chapters on the Mathematical Art* and suggested alternative and simpler solutions for some problems (Lam Lay Yong 1974, 50).

It is often said that the rule of double false position has "all-round capability" in its applications, because guesses can be given at the first step to make false positions even when the problems in appearance do not include any false position (Li Jimin 1990, 203). That is why the "*ying nü*" chapter of *Nine Chapters on the Mathematical Art* could cover problems that belong to various mathematical branches such as fractions, proportions, and linear equations. In the period after Liu Hui, however, from the third century to the end of Ming dynasty, the application became narrower though the method did not disappear (Qian Baocong [1927, 1983] 1998, 48).

Nine Chapters on the Mathematical Art and Liu Hui's and Yang Hui's studies on the rule of double false position might not have been accessible to Korean scholars for a long time. As mentioned before, *Nine Chapters on the Mathematical Art* had been missing in Korea until it was reintroduced in the mid-nineteenth century, long after

[56] This chapter contains two subchapters dealing with problems solved by the rule of double false position using single cross multiplication (*danfa* 單法) and double cross multiplications (*shuangtao* 雙套) in the procedure.

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its restoration by Chinese scholars in the late eighteenth century.[57] There has been no evidence that Yang Hui's *A Detailed Analysis of Mathematical Method in the Nine Chapters* (*Xiangji jiuzhang suanfa* 詳解九章算法) was introduced into Korea. In Hong's time, there were certainly books accessible to Korean scholars such as *Introduction to Mathematical Studies* and *Systematic Treatise on Arithmetic*, which contained the rule of double false position. However, these books only provided limited examples to be solved by the elementary algorithm of cross multiplication without advanced mathematical analysis. The problems of a different nature, which earlier were solved by adopting the rule of false position in *Nine Chapters on the Mathematical Art*, employed different methods appropriate to their nature: the proportional expression on proportional problems and the elimination on linear equations. It was the same in Korean mathematicians' books. Hong Chŏng-ha's *Writings of Nine and One* also contained one chapter dealing with the rule of double false position, but it dealt with only thirteen problems which provided double false positions in appearance (Hong Chŏng-ha [1684–?] 1984, 136–145).

In the West, it was the Arabs who made the ordinary rule of false position known to European scholars (Smith [1925] 1958, vol. 2, 437–439). This method spread and was widely used in the European world in the twelfth century. When the Chinese were introduced to Western mathematics in the seventeenth century, they witnessed the rule of double false position in *Instruction for Calculation in Common Script* and *Collected Basic Principles of Mathematics*, from which Hong learnt the basic idea of his algorithm (Qian Baocong [1927, 1983] 1998, 43–47). Like Chinese texts, *Collected Basic Principles of Mathematics* used the rule of double false position only when the problem itself has false positions. Other problems like proportions and linear equations employed different methods appropriate to the problems. So it can be said that Hong's transformation and wide application of the method to the problems of other mathematical branches was a rediscovery of the "all-round capability" of the rule of double false position, which had once been proposed by Chinese mathematics in *Nine Chapters on the Mathematical Art*. With the limited information of the history of this method, Hong was confident that his application of the method was quite new and simpler than the one he had known earlier.

It is significant that Hong developed his idea through the use of counting rods which had long been obsolete in Chinese mathematics. One must consider where Hong carries out his algorithm. As recapitulated in modern form above, he simulates the whole procedure of his method on a calculation board with counting rods in his hands. He says "This time, line up (*yŏl*, 列) the 80 *kŭn* and the excess 92.5 *kŭn* on the right. Line up the 32 *kŭn* and the deficit 184.5 *kŭn* on the left." He obtains the ratio of "lower floor" values and the difference of "upper floor" values. Then he finally finds the solution by subtracting appropriate portions from upper floor values. This operation can be laid out easily on the counting board which is square partitioned.

[57] On the restoration of *Jiuzhang suanshu* in China, see Li Yan and Du Shiran 1987, 227–230. Nam Pyŏng-gil in Korea left *An Analysis of the Nine Chapters on Mathematical Art* (*Kujangsulhae* 九章術解) in 1864 (Kim Yong-un and Kim Yong-guk 1977, 266–267).

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Using counting rods, four values can be laid out in 2 × 2 matrix on the counting board and the remaining procedure also can be understood in terms of counting rod operation.

It is widely agreed that counting rod calculation was "the main stream calculation method" in the history of Korean mathematics from the fifteenth to the nineteenth centuries (Kim Yong-un and Kim Yong-guk 1977, 281). The fact that the three Chinese texts, *Yang Hui's Methods of Computation*, *Introduction to Mathematical Studies* and *Explanations of Arithmetic*, which Korean mathematics totally depended upon during that period, were based on counting rod calculation method shaped Korean mathematics.[58] *Introduction to Mathematical Studies*, in particular, played a key role in providing the Koreans expertise in mathematics and counting rod calculation.[59] That the *Introduction to Mathematical Studies* was a fundamental text of traditional mathematics in Hong's days is revealed in an anecdote concerning Hong's mother.

> My mother loved mathematics. Reading *Introduction to Mathematical Studies* and coming to the sections on equal division, abbreviation, positive and negative, and right-angled triangles, she said, "Why did they follow such difficult and complicated ways?" And she solved them by her new algorithms. Later compared with *Collected Basic Principles of Mathematics*, her new algorithms turned out to be totally matching and correct.[60]

The quotation above shows that *Introduction to Mathematical Studies* occupied a central position as the most important manual for traditional mathematics even though *Collected Basic Principles of Mathematics*, which incorporated Western mathematics, had been introduced into Korea and was used in Hong's days. Therefore a mathematician in the late eighteenth century, mastering *Introduction to Mathematical Studies*, must have been skillful with counting rod calculation. Hong was no exception. As can be seen from the quotation, he usually used counting rod calculation unless he needed to draw figures.

> When the problem did not fit into counting rod calculation, I drew figures with indications of *kap* 甲 and *ŭl* 乙, which afterward overflowed baskets. (Chŏng In-bo [1931] 1983, 42)

Hong's new algorithms for extracting roots are also evidence of the relationship between Korean specialties in counting rod operation and mathematical ingenuity. In the second part of *New Methods of Geometry*, "Elementary research on root extraction,"

[58] Though the Korean version of the *Explanations of Arithmetic* was a reprint from an early Ming version in 1373, this book does not refer to abacus calculation.

[59] *Yang Hui's Methods of Computation* and *Introduction to Mathematical Studies* were recognized as the most influential Chinese texts in the Chosŏn Kingdom (see Kim Yong-un and Kim Yong-guk 1977, 178–192). Kawahara picked up *Yang Hui's Methods of Computation* prior to *Introduction to Mathematical Studies*, because the former retained the original ideas of extracting roots, which later developed into the general method for solving higher order equations.

[60] The biography of Hong's mother was written by his elder brother (Hong Sŏk-chu 1824, 8b–9a).

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which deals with the extraction of square, cubic, 4th and 5th roots and the solving of quadratic equations, he proposes new methods for extracting roots. In these methods, using only easy addition, subtraction, multiplication, and division allows one to reach the solution. Below is the procedure of square root extraction:

(Problem)
When the square is 441 *ch'ŏk* 尺,[61] obtain the length of one side.
(Procedure to Solution) (Modern recapitulation)

Place(置) 441 *ch'ŏk*. 441
Dividing itself into halves (自半) gains 220 *ch'ŏk* 5 *ch'on* 寸.[62] 441/2 = 220.5
Subtract 1 *ch'ŏk* from the value above. (Remainder 219 *ch'ŏk* 5 *ch'on*)[63] 220.5 − 1 = 219.5
Subtract again 2 *ch'ŏk*. (Remainder 217 *ch'ŏk* 5 *ch'on*) 219.5 − 2 = 217.5
Subtract again 3 *ch'ŏk*. (Remainder 214 *ch'ŏk* 5 *ch'on*) 217.5 − 3 = 214.5
Subtract again 4 *ch'ŏk*. (Remainder 210 *ch'ŏk* 5 *ch'on*) 214.5 − 4 = 210.5
Subtract again 5 *ch'ŏk*. (Remainder 205 *ch'ŏk* 5 *ch'on*) 210.5 − 5 = 205.5
Subtract again 6 *ch'ŏk*. (Remainder 199 *ch'ŏk* 5 *ch'on*) 205.5 − 6 = 199.5
Subtract again 7 *ch'ŏk*. (Remainder 192 *ch'ŏk* 5 *ch'on*) 199.5 − 7 = 192.5
Subtract again 8 *ch'ŏk*. (Remainder 184 *ch'ŏk* 5 *ch'on*) 192.5 − 8 = 184.5
Subtract again 9 *ch'ŏk*. (Remainder 175 *ch'ŏk* 5 *ch'on*) 184.5 − 9 = 175.5
Subtract again 10 *ch'ŏk*. (Remainder 165 *ch'ŏk* 5 *ch'on*) 175.5 − 10 = 165.5
Subtract again 11 *ch'ŏk*. (Remainder 154 *ch'ŏk* 5 *ch'on*) 165.5 − 11 = 154.5
Subtract again 12 *ch'ŏk*. Remainder (142 *ch'ŏk* 5 *ch'on*) 154.5 − 12 = 142.5
Subtract again 13 *ch'ŏk*. (Remainder 129 *ch'ŏk* 5 *ch'on*) 142.5 − 13 = 129.5
Subtract again 14 *ch'ŏk*. (Remainder 115 *ch'ŏk* 5 *ch'on*) 129.5 − 14 = 115.5
Subtract again 15 *ch'ŏk*. (Remainder 100 *ch'ŏk* 5 *ch'on*) 115.5 − 15 = 100.5
Subtract again 16 *ch'ŏk*. (Remainder 84 *ch'ŏk* 5 *ch'on*) 100.5 − 16 = 84.5
Subtract again 17 *ch'ŏk*. (Remainder 67 *ch'ŏk* 5 *ch'on*) 84.5 − 17 = 67.5
Subtract again 18 *ch'ŏk*. (Remainder 49 *ch'ŏk* 5 *ch'on*) 67.5 − 18 = 49.5
Subtract again 19 *ch'ŏk*. (Remainder 30 *ch'ŏk* 5 *ch'on*) 49.5 − 19 = 30.5
Subtract again 20 *ch'ŏk*. (Remainder 10 *ch'ŏk* 5 *ch'on*) 30.5 − 20 = 10.5
Next time, [you] are supposed to subtract 21 *ch'ŏk*, but the remainder is insufficient to the value of designated subtraction (當減之數).
If [you] double(倍之) the remainder 10 *ch'ŏk* 5 *ch'on*, you will gain 21 *ch'ŏk*, which is exactly sufficient (恰足) to the value of designated subtraction, therefore, each side of the square is 21 *ch'ŏk*.

The first distinction of Hong's method above is that the operation applied at each step is only a simple subtraction. With any numbers, this easy calculation yields a final answer, though the number of steps increases exponentially as the original square increases linearly. Of course Hong explained that the complex manipulations in case

[61] The length units are also used to denote areas.
[62] 1 *ch'ŏk* equals to 10 *ch'on*.
[63] Descriptions with small letters in parenthesis are original footnotes by Hong Kil-chu.

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roots can not be integers. Though such cases require extra handling of adding and dropping digits, the basic algorithm of numerical manipulation remains constant.[64]

Furthermore, the same easy operation using only addition, subtraction, multiplication, and division is consistently applied in extracting higher order roots. Only the first division of original number and the numbers to subtract in each step are different according to the order of root. In extracting cube root, for example, he divides the original value by 6, drawing on the serial sum to subtract at each step.

(Problem)
When the cube is 343 *ch'ŏk* 尺,[65] obtain the value of one side.

(Procedure to solution)
Place (置) 343 *ch'ŏk*.
Dividing by 6 gains 57 *ch'ŏk* and 1/6 *ch'ŏk*.
Subtract 1 *ch'ŏk* from the value above. (Remainder 56 *ch'ŏk* and 1/6 *ch'ŏk*)
Subtract 3 *ch'ŏk*, the sum of 1 *ch'ŏk* and 2 *ch'ŏk*. (Remainder 53 *ch'ŏk* and 1/6 *ch'ŏk*)
Subtract 6 *ch'ŏk*, the sum of 1 *ch'ŏk*, 2 *ch'ŏk* and 3 *ch'ŏk*. (Remainder 47 *ch'ŏk* and 1/6 *ch'ŏk*)
Subtract 10 *ch'ŏk*, the sum of 1 *ch'ŏk*, 2 *ch'ŏk*, 3 *ch'ŏk* and 4 *ch'ŏk* (Remainder 37 *ch'ŏk* and 1/6 *ch'ŏk*)
Subtract 15 *ch'ŏk*, the sum of 1 *ch'ŏk*, 2 *ch'ŏk*, 3 *ch'ŏk*, 4 *ch'ŏk* and 5 *ch'ŏk*. (Remainder 22 *ch'ŏk* and 1/6 *ch'ŏk*)
Subtract 21 *ch'ŏk*, the sum of 1 *ch'ŏk*, 2 *ch'ŏk*, 3 *ch'ŏk*, 4 *ch'ŏk*, 5 *ch'ŏk* and 6 *ch'ŏk*, [then] the remainder is 1 *ch'ŏk* and 1/6 *ch'on*.
The remainder can not cope with the value of designated subtraction(當減之數).
If [you] sextuple(六倍之) the remainder, you will gain 7 *ch'ŏk*, which is exactly suitable(恰當) for the last number of designated subtraction(當減之末數)(Since the designated subtraction is the sum of 1, 2, 3, 4, 5, 6 and 7, 7 *ch'ŏk* is the last number of designated subtraction), therefore, it is each side of the cube.

(Modern recapitulation)
343
$343/6 = 57 + 1/6$
$(57 + 1/6) - 1 = 56 + 1/6$
$(56 + 1/6) - (1 + 2) = 53 + 1/6$
$(53 + 1/6) - (1 + 2 + 3) = 47 + 1/6$
$(47 + 1/6) - (1 + 2 + 3 + 4) = 37 + 1/6$
$(37 + 1/6) - (1 + 2 + 3 + 4 + 5) = 22 + 1/6$
$(22 + 1/6) - (1 + 2 + 3 + 4 + 5 + 6) = 1 + 1/6$

[64] In that case, add two digits to the original value before extracting the root, and at the end drop one digit of the final value, then you will get the root with decimal fraction. If you want a more precise value, add four or six digits before extracting and drop 2 or 3 digits at the end.
[65] The length units are also used to denote volumes.

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> Next time, $(1+2+3+4+5+6+7)$ is supposed to be subtracted, but the remainder of former stem is not sufficient for the designated subtraction.
> See how the sextuple of the remainder is. $(1+1/6)\times 6=7$
> The 7, the sextuple of the remainder, is exactly suitable for the last number of designated subtraction.
> Then 7 is the cube root of 343. $7=\sqrt[3]{343}$

Glancing over the above algorithms, it can be seen that Hong's method significantly differs from traditional and modern methods of extracting roots. The basic method of root extraction in traditional mathematics can be traced back to *Nine Chapters on the Mathematical Art*, which contains algorithms for extracting square and cubic root with counting rods, and in which the fundamental principle involved is just the same as that in the method commonly used today (Li Yan and Du Shiran 1987, 50).[66] A developed form of this method, proposed by Jia Xian 賈憲 in the middle of eleventh century, is thought to have opened the ground for advanced Chinese mathematics in the Song and Yuan period represented by the algebraic "method of celestial element" and the "method of four elements" (*siyuanshu* 四元術) (ibid., 121). Although modern historians of mathematics often differentiate this new method, calling it "extracting roots by iterated multiplication," from the old one of *Nine Chapters on the Mathematical Art*, these two methods originated from the same fundamental concept.[67] They may need to be distinguished mathematically, but the procedures of the two methods can be equally represented as a basic formula. For example, the formula for taking the square root of N is $\sqrt{N}=a+b+c$, where a, b, c are the numbers of different order. To obtain each number, operations are needed for eliminating the original value in accordance with the rule of changing numbers step by step. For the extraction of cubic root and higher order roots, the fundamental concept is also the same. Supposing the root of original value would be the combination of numbers of different order, removing original value step by step to obtain the number of each order is the fundamental concept of this method (Martzloff 1997, 223).[68] Even though "one can clearly see that the difference between Jia Xian's method and the old method lies in replacing the squaring, cubing, and other procedures by adding after each multiplication," the difference is only in the process of operation but not in the fundamental concept (Li Yan and Du Shiran 1987, 120).

Hong must have learnt the new method of extracting roots by iterated multiplication as well as the elementary method of *Nine Chapters on the Mathematical Art* through the fragments and further studies of this book adopted in other books. Jia Xian's

[66] A geometrical analysis of the extraction of square root in *Nine Chapters on the Mathematical Art* was proposed by Lam Lay Yong 1970.
[67] On the development of root extraction by Jia Xian and the method of "extracting roots by iterated multiplication," see Lam Lay Yong 1969; Qian Baocong [1959, 1966] 1998, 478–506; Mei Rongzhao 1989.
[68] Martzloff showed that the extraction of square and cubic root are based on the developments of $(a+b+c\ldots)^2$ and $(a+b+c\ldots)^3$ in a geometrical way.

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method was recorded by Yang Hui and later evolved into a universal method for solution of polynomial equations of arbitrary degree (ibid., 127). *Yang Hui's Methods of Computation* and *Introduction to Mathematical Studies* containing this development of the Song and Yuan period were so influential in Korea that Pak Yul, a seventeenth century Korean mathematician, systemized an explanation of the method of "extracting roots by iterated multiplication" in his book, *Origin of Mathematics*.

The method in *Collected Basic Principles of Mathematics*, another possible method Hong might have learnt, was also based on the same fundamental concept and procedure as those traditional ones.[69] At any rate, Hong developed his new method independently from other methods he might have studied.

What is the fundamental concept of his new algorithm? According to a modern mathematician, his method is based on understanding the general sum of serial numbers.[70] In the procedure of square root extraction above, the numbers subtracted at each step are the natural numbers and the remainder at the last step ($r = 10.5$ *ch'ŏk*) is the difference between the half of original square ($a/2 = 220$ *ch'ŏk* 5 *ch'on*) and the serial sum of the natural numbers up to 20.

$r = a/2 - (1 + 2 + 3 + \cdots + 19 + 20)$, where the original value and the remainder of the last step are a and r, respectively.

With the general sum of natural numbers, $S = 1 + 2 + \cdots + n$, a general formula $S = n\ (n + 1)\ /2$ can be recalled in this procedure.

The general sum of serial numbers is also used in the procedure of cubic root extraction. As seen above, the numbers subtracted at each step are the serial sums of natural numbers, and the remainder at the last step ($r = 1$ *ch'ŏk* and 1/6 *ch'ŏk*) is the difference between the one sixth of the original cube ($a/6 = 57$ *ch'ŏk* and 1/6 *ch'ŏk*) and the serial sum of sums of the natural numbers up to 6.

$r = a/6 - [(1 + (1 + 2) + \cdots + (1 + 2 + \cdots + 6)]$, where the original value and the remainder of the last step are a and r, respectively.

From the general sum of sums of natural number, $S = [(1 + (1 + 2) + \cdots + (1 + 2 + \cdots + (n\text{-}1) + n)]$, a general formula $S = n(n + 1)(n + 2)/6$ can be recalled in the procedure of cubic root extraction. Furthermore, the idea of employing serial sums

[69] Wang Ling and Joseph Needham have shown in great detail the correspondence of the method of extracting the square and cube roots found in *Nine Chapters on the Mathematical Art* with Horner's method of solving higher numerical equation, which, I think, can be traced back to the elementary methods adopted in *Collected Basic Principles of Mathematics* (see Wang Ling and Needham 1955).

[70] On the mathematical meaning of Hong's method of extracting roots, I consulted Professor Kim Hong-Jong at the Department of Mathematical Science in Seoul National University. The following descriptions of the serial sums are rendered by him.

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is consistent with his methods for extracting 4th and 5th order of roots.[71] This shows that Hong's invention of root extraction method was based on his understanding of serial sums.

Concerning his idea of serial sum used in root extraction, Hong seems to have been indebted to the research on stacking or piling by Chinese scholars in the Song and Yuan period, some of which were also adopted in *Collected Basic Principles of Mathematics*.[72] In his collected work, the *Yang Hui's Methods of Computation*, which was surely accessible in Korea, Yang Hui dealt with two types of serial sums, triangular stacking (*sanjiaoduo* 三角垛) and cake stacking (*guoziduo* 菓子垛) (Kim Yong-un and Kim Yong-guk 1996, 248–251).[73] These are exactly the same that Hong adopted in extracting cubic and 4th roots, respectively. Zhu Shijie's *Precious Mirror of the Four Elements*, which contained further research on stacking, and *Collected Basic Principles of Mathematics* also deal with these cases. Though it is still uncertain that the *Precious Mirror of the Four Elements* was accessible to Hong Kil-chu, considering that the *Yang Hui's Methods of Computation* was so common that it was a required text for state examination in mathematics and *Collected Basic Principles of Mathematics* was well studied in Korea from the mid-eighteenth century, Hong could be indebted for his new method of root extraction to the study of serial sums in China.

In addition to his mathematical understanding of serial sums, there is another notable factor that prompted Hong's invention of new algorithms. Facility in counting rod operation was a critical need as far as he was concerned. When it comes to the procedure of extracting square root, it may seem troublesome as it requires many steps. However, once this calculation is brought onto the counting board with counting rods, the repetitive operations are quickly accomplished. Each operation can be easily conducted in the wake of the former operation using the result of the former step. In the procedure of calculation above, it can be easily seen that the next step is being operated with the remainder of the former step.

In an episode involving Korean and Chinese mathematicians, we can see how skillfully and quickly Korean mathematicians could perform counting rod calculations. He Guozhu 何國柱, a Chinese mathematician and an envoy to Korea in 1713, was deeply impressed by the skillful operation of counting rods by a Korean mathematician (Hong Chŏng-ha [1684–?] 1984, 493–494). Amazed at his adroit operation for solving the advanced mathematical problems, He Guozhu asked how the Korean mathematicians could solve algebraic equation problems with such ease. "Algebraic equation problems belong to an intermediate standard" was the answer. For Korean mathematicians who were well versed in counting rod calculation and the "method of celestial element" of *Introduction to Mathematical Studies*, solving algebraic equation problems with counting

[71] For the 4th root, he used the serial sums of $1^2+2^2+\cdots+n^2=n(n+1)(2n+1)/6$ and $1^3+2^3+\cdots n^3=[n(n+1)/2]^2$, and for the 5th root, $1^4+2^4+\cdots+n^4=n(n+1)(2n+1)(3n^2+3n-1)/30$.

[72] For the study of series in China, see Li Yan [1954] 1998. About the study of stacking after the Western influence to China, see Tian Miao 2003.

[73] Lam Lay Yong published a detailed analysis of *Yang Hui's Methods of Computation* (Lam Lay Yong 1977).

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rods was a simple operation. In contrast, counting rod operation was unknown to Chinese mathematicians, prompting a request for some counting rods to be brought back to China where this skill had disappeared.

It is obvious that Hong's new method of root extraction was aimed at Korean mathematical learners who were familiar with counting rod calculation.

> It is often said that the method of extracting roots using the side (*lian* 廉, the coefficient of x^2) and the corner (*yu* 隅, the coefficient of x^3) is quite difficult. As to the 3rd and 4th order roots, it is more complex and confusing. Now I invented a simple method that even dull or doltish children can apply. (Hong Kil-chu 1826, 18)[74]

What kind of calculation method could even dull or doltish children apply in his days? It must have been the counting rod calculation.

It seems that Hong did not have in mind the well established mathematicians as the audience of his method. If there were mathematicians skilled in the old methods, they would not willingly follow Hong's new method of root extraction because they already had well established ones. Hong's target may have been the beginners for mathematics who just wanted to learn an easy way to carry with counting rods. The format of his *New Methods of Geometry*, however, does not seem to have been for beginners because only his new ideas were collected in this book without any elementary teaching.

Instead, it is likely that his natural preference for uniqueness led him to strive for new algorithms. As a critic of Hong's literary work in 1866 stated, his "unusual ideas and ingenious composition" was related to the "mastery of mathematics," enjoying unusual methods in mathematics seems to have fostered his exercise of literary uniqueness (Song Paek-ok [1866] 1992, 367). I think that his private satisfaction in mathematical invention, as in literary composition, sustained his mathematical activity throughout his lifetime without concerning any specific audience in mind. It can also be a reason why Hong's mathematical work could not have been widely shared by the next generation. *New Methods of Geometry* was not published during his lifetime and his mathematical work was not well known outside the circle of his friends and family. The eminent mathematicians of the next generation, such as Nam Pyŏng-gil and Yi Sang-hyŏk, did not leave any hint of Hong's work in their books.

Conclusion

The mathematical and academic world of Hong Kil-chu offers an insight into the history of mathematics in Korea. Firstly, traditional Chinese mathematics embedded in the Korean context provided fertile ground for Hong. It is notable that mathematics in Korea during the Tongsan period mainly depended upon traditional Chinese mathematics from the Song and Yuan period, rather than the more recent Ming

[74] For a simulation of root extraction with side and corner, see Martzloff 1997, 224–249.

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period. Hong's new algorithms are one example of some of the precious achievements that Korean scholars contributed. His ingenuity in inventing unusual algorithms for the rule of double false position and the method of root extraction was surely of a nature linked to his specialty in counting rod calculation. He was well versed in counting rod operation and so invented suitable methods using these tools.

Secondly, Hong could use another important mathematical resource, Western mathematics, which Korea had imported from China in the seventeenth and eighteenth century. Western geometry including spherical trigonometry, and lots of algebraic subjects were earnestly studied by Hong and his contemporaries. Beside texts for traditional Chinese mathematics, Hong and his contemporaries studied Chinese compendia of Western mathematics and related knowledge. Western mathematics also supplied Korean scholars with new subjects, such as geometry, which was adopted in Hong's book and functioned as source material that was melded with traditional material and finally forged new ideas in the Korean context.

Finally, Hong's mathematics reflected the intellectual atmosphere of the late eighteenth and early nineteenth centuries in Korea. His predecessors' enthusiasm for learning about the advanced Qing culture in the Qian-Jia period fostered new trends in mathematics and astronomy which were further developed by his contemporaries. Hong started to learn mathematics and engaged himself in mathematical work, sharing the new paradigm that called for an earnest pursuit of the exact sciences as an authorized Confucian investigation. His pursuit of mathematics was also a product of the Korean context of his time.

Whereas highly recognized mathematicians like Nam Pyŏng-gil and Yi Sang-hyŏk in the 1860s have been studied, there has been little attempt to connect these scholars to their seventeenth-century predecessors. This is due to a lack of understanding of the context of the eighteenth and early nineteenth centuries.[75] An understanding of Hong's activities and of the context of his days enables us to outline the long history of Korean mathematics from the fifteenth to the nineteenth century. The persistence of the method of counting rod calculation in traditional mathematics, digestion of newly introduced Western mathematics, and the transition of intellectual paradigms for pursuing mathematics are the main themes that the scholars in the late nineteenth century inherited.

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[75] For a bibliographical introduction to studies on the history of science in Korea, see Kim Yung Sik 1998. I set up a brief bibliography on the history of astronomy in Korea for the web site of the Commission for History of Ancient and Medieval Astronomy, International Union of History and Philosophy of Science (see Jun Yong Hoon 2002).

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