Feedback

On Artzt parabolas of a triangle [1]: Andrew Jobbings writes: The 'curve-stitching' construction (see Figures 1 and 3 in [1]) may be extended by continuing each sequence of points in both directions outside the original triangle ABC, as shown in Figure 1. This defines the whole parabola, not just that part within triangle ABC.

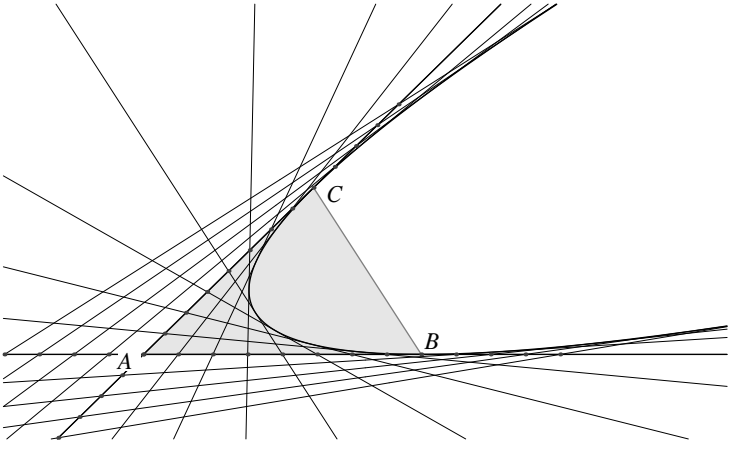


FIGURE 1

As proved in [2], in homogeneous barycentric coordinates the A-parabola has equation $x^2 - 4yz = 0$. Thus the parabola passes through the points $M_a = (2 : 1 : 1)$, $P_b = (4 : 1 : 4)$ and $P_c = (4 : 1 : 4)$, in addition to $B = (0 : 1 : 0)$ and $C = (0 : 0 : 1)$. Similarly for the B- and C-parabolas.

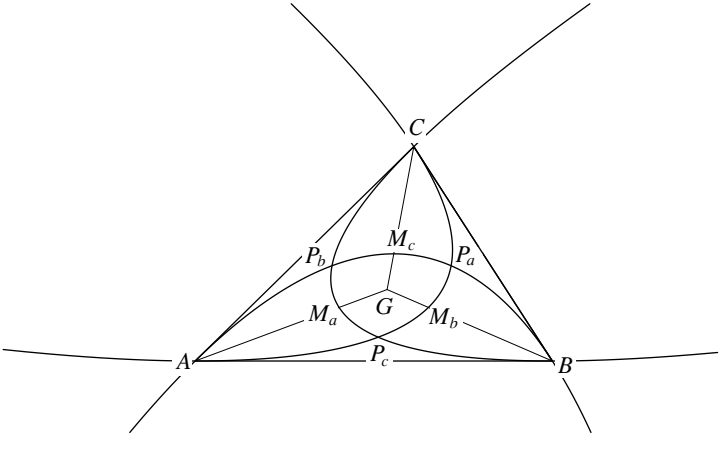


FIGURE 2

It follows that, for example, M_a is the point where one parabola meets the median through A, and that the other two parabolas intersect at P_a, which

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is also on the median. The points and parabolas are shown in Figure 2, where G is the centroid of triangle ABC.

The two triangles shown in Figure 9 in [1] (which seems to be missing one of the parabolas) are the triangles $P_aP_bP_c$ and $M_aM_bM_c$.

Define the homothecies θ_P and θ_M to have centre G and scale factors -3 and 4 respectively. Then triangle $P_aP_bP_c$ is homothetic to triangle ABC via θ_P, and triangle $M_aM_bM_c$ is homothetic to triangle ABC via θ_M.

The results about areas are an immediate consequence.

Also, since the circles C_1, C_2 and C_2 (see Figure 13 in [1]) are circumcircles of these triangles, the results concerning these circles also follow at once.

The Artzt ellipse $\mathscr{E}_{\text{Artzt}}$ (see Figure 15 in [1]) has centre G and passes through P_a, P_b and P_c, as shown in Figure 3. Also shown is the Steiner ellipse $\mathscr{E}_{\text{Steiner}}$, which is the unique ellipse with centre G circumscribing the triangle ABC.

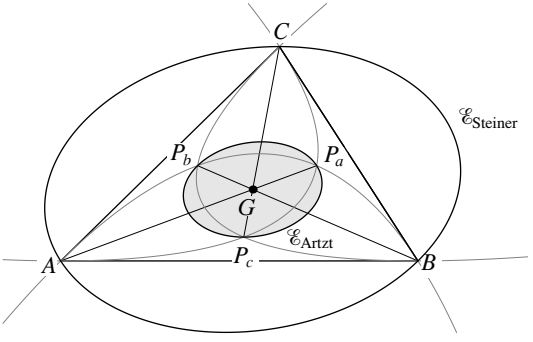


FIGURE 3

The homothecy θ_P maps $\mathscr{E}_{\text{Artzt}}$ to $\mathscr{E}_{\text{Steiner}}$, so the area of $\mathscr{E}_{\text{Artzt}}$ is $\frac{1}{9}$ of the area of $\mathscr{E}_{\text{Steiner}}$.

Now the ratio of the area of $\mathscr{E}_{\text{Steiner}}$ to the area of triangle ABC is $4\pi\sqrt{3} : 9$.

Therefore the ratio of the area of $\mathscr{E}_{\text{Artzt}}$ to the area of triangle ABC is $4\pi\sqrt{3} : 81$.

References

1. John Sharp, Artzt parabolas of a triangle, *Math. Gaz.* **99** (November 2015) pp. 444-463.
2. Nikolaos Dergiades, Conics tangent at the vertices to two sides of a triangle, Forum Geom. 10 (2010) pp. 41-53. Available at: http://forumgeom.fau.edu/FG2020volume10/FG201007.pdf

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On Constructing the 15th pentagon that tiles the plane [1]: Andrew Jobbings writes: The pentagonal tile may also be considered as two 'half squares' and two 'half equilateral triangles', of suitable sizes, placed together as shown in Figure 1.

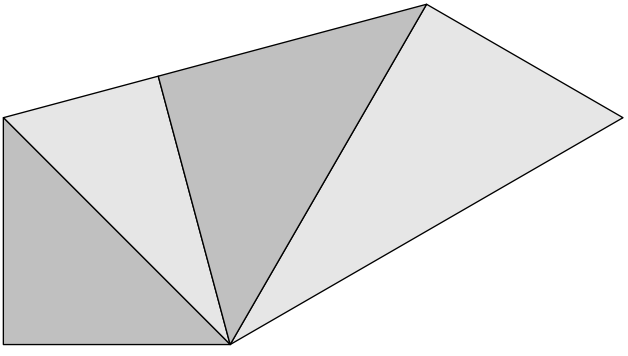


FIGURE 1

Figure 2 shows that this gives the correct lengths and angles.

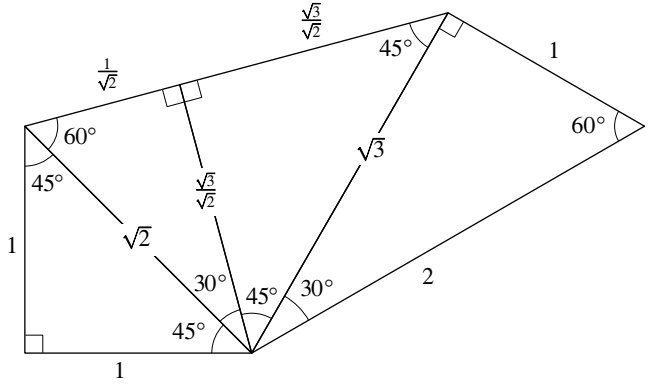


FIGURE 2

Note that to calculate the side lengths nothing more than Pythagoras' Theorem is needed.

This decomposition gives a simple method of constructing the tile (see Figure 3).

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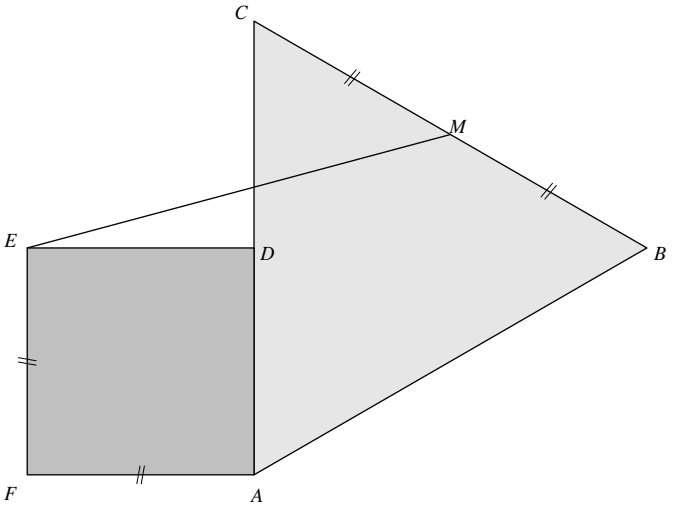


FIGURE 3

Construct an equilateral triangle ABC with sides of length 2, and a square $ADEF$ with sides of length 1 and with D on CA, as shown.

Now construct the midpoint M of BC. Then the tile is $ABMEF$ because $AE = \sqrt{2}, AM = \sqrt{3}$ and $\angle EAM = 75°$.

Reference

1. Nick Lord, Constructing the 15th pentagon that tiles the plane, *Math. Gaz.* **100** (March 2016) pp. 154-158.

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