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Particle simulations of the nonlinear electron dynamics in the classical Pierce diode

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Abstract. The classical Pierce model consists of a one-dimensional plasma-filled diode into which a monoenergetic electron beam is injected. Particle-in-cell (PIC) simulations are performed with the BIT1 code, involving a large number of superparticles. The evolution of slightly perturbed uniform equilibria into the related nonlinear attractor states for a broad range of α values, where $\alpha = \omega_{\mathrm{pe}} d/u_0$ is the diode control parameter, is investigated. A period-doubling route to chaos in the classical Pierce diode for α slightly below 5π is analyzed in detail.

1. Introduction

The physical model of interest here is the 'classical Pierce diode' [1], a one-dimensional electrostatic system consisting of two parallel electrodes externally connected by a short circuit: an emitter at $x = 0$ and a collector at $x = d$. A monoenergetic beam of electrons is injected from the emitter with density n_0 and velocity u_0. Electrons hitting the emitter or the collector are completely absorbed, making the system dissipative. The region between the electrodes is filled with a neutralizing background of immobile ions with density n_0. The equilibrium and stability properties of the classical Pierce diode depend only on the dimensionless normalized length, or 'Pierce parameter', $\alpha = \omega_{\mathrm{pe}} d/u_0$, where $\omega_{\mathrm{pe}} = \sqrt{n_0 e^2/(\varepsilon_0 m_{\mathrm{e}})}$ is the electron plasma frequency of the electrons at emission.

This dissipative model system exhibits rich dynamical behavior and has several distinctly different operating regimes. For specific domains of the control parameter α, the linear-perturbation behavior of the uniform equilibrium is dominated by an oscillatory unstable mode which evolves into weakly nonlinear period-N oscillations without electron reflection. The term period-N oscillation describes periodic repetition of the patterns of N different maximum and minimum values. With decreasing α, these oscillations undergo a period-doubling cascade to chaos. Such regimes have already attracted considerable attention and were intensely studied by many authors either theoretically [2, 3] or by experiment [4]. This is due to their important role, e.g., in generating high-power microwaves [5] or producing light-ion beams for inertial-confinement fusion [6].

In this work we study the evolution of slightly perturbed uniform Pierce-diode equilibria into the related nonlinear attractor states for a broad range of α values. These investigations are carried out by means of particle-in-cell (PIC) simulations. As a benchmark of our code, we first reproduced in detail the period-doubling

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route to chaos for α slightly below 3π. In both the linear and nonlinear regimes, we found excellent agreement with the results obtained previously by Godfrey [2] from numerical integration of a full fluid model, and by Hörhager and Kuhn [7] from a three-harmonic fluid analysis. (These reproductive studies are not presented here.) After successful benchmarking of our code, analogous results are presented for α domains in which these phenomena were not considered in detail before.

2. Basic properties of our PIC simulations

For our PIC simulations, the 1d3v electrostatic bounded-plasma code BIT1 [8] was used. Compared with earlier PIC studies of the classical Pierce diode, the number of superparticles has been increased by about one order of magnitude (up to 10^5). The physical properties of the particles in the interelectrode space, the boundary conditions and the properties of the external-circuit parameters corresponded to the model of the classical Pierce diode. For different values of α, the electron-beam emission velocity u_0 and the interelectrode distance d were kept at the constant values $u_0 = 1 \times 10^6$ m s^{-1} and $d = 10^{-1}$ m. The Pierce parameter α was varied by changing the electron-beam emission density n_0. The plasma potential in the middle of the diode, $\Phi_{d/2}(t)$, was measured as the main diagnostic parameter.

3. Selected results and summary

The PIC simulations performed for $\alpha_0 \leqslant \alpha \leqslant 5\pi$ retrieve the oscillatory decaying solution, as expected from linear stability analysis [9]. (Here $\alpha_0 \approx 4.9335\pi$ is the point of marginal stability for linear perturbations of the uniform equilibrium.) When α is decreased below α_0, the linear dominant eigenmode becomes oscillatory unstable and saturates as a finite-amplitude steady-state period-1 oscillation. The first subharmonic bifurcation appears at $\alpha/\pi \approx 4.9258$. For α lightly below this value, the linear instability first saturates as an unstable period-1 solution and only then transforms to the stable period-2 oscillation. The period-2 oscillation is replaced with a period-4 oscillation at $\alpha/\pi \approx 4.9231$, followed by a period-8 oscillation at $\alpha/\pi \approx 4.9225$ and a period-16 oscillation at $\alpha/\pi \approx 4.92235$. Rapidly decreasing α separation between adjacent bifurcation points did not allow the identification of higher periodicities.

Regular nonlinear oscillations give way to a chaotic attractor at $\alpha/\pi \approx 4.9223$. Time histories of $\Phi_{d/2}(t)$ with periodicity from 1 up to 8 are shown in Fig. 1(a). Using the method of delays [10], the phase-plane portraits of the attractors were constructed from the time histories of $\Phi_{d/2}(t)$. The phase-plane portraits of attractors of periodicity 1, 2, 4 and 8 are illustrated in Fig. 1(b). In Figs. 1(c) the finite Fourier transform (FFT) power spectra of time histories of $\Phi_{d/2}(t)$ with periodicity from 1 up to 8 are shown. The sum of intensities associated with the new frequency components arising for the period-N oscillation was compared with the total intensity of the period-$(N-1)$ oscillation. As the periodicity of the modulation increases, this ratio approaches the universal constant factor 0.1525 (corresponding to a logarithmic difference of -8.17 dB) [11]. For a representative value of α within the chaotic band, an average maximum Lyapunov exponent $\langle\lambda\rangle \approx 2.5 \times 10^4$ s^{-1} was calculated. This result confirms the chaotic character of the nonlinear oscillations in question.

At $\alpha/\pi \approx 4.92065$, the chaotic attractor suffers an *interior crisis* and gives way to stable period-3 oscillations. With decreasing α, the stable fixed points undergo

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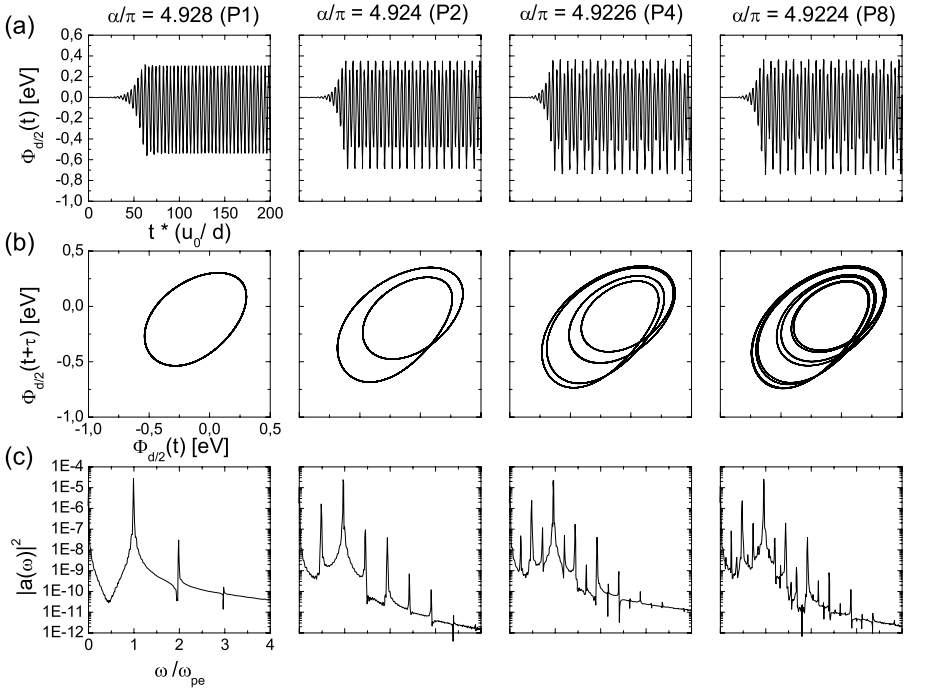


Figure 1. (a) The time histories of $\Phi_{d/2}(t)$ from periodicity P1 up to P8, (b) The respective phase-plain portraits, and (c) the FFT power spectra (from left to right).

a period-doubling cascade to form, at $\alpha/\pi \approx 4.9201$, three chaotic bands. At $\alpha/\pi \approx 4.91995$ these merge into a single narrow chaotic region. When α is further decreased, the bifurcation scheme repeats in the opposite direction: the chaotic attractor is replaced with stable period-6 oscillations followed by period-3 oscillations. At $\alpha/\pi \approx 4.91924$, the period-3 attractor is slightly shifted with α varying by less than $2.5 \times 10^{-5}\pi$. When the control parameter is further decreased, the periodic window is succeed by a sequence of period-doubling bifurcations. Subsequently, a set of three chaotic bands is formed which at $\alpha/\pi \approx 4.91895$ merge into a single band. The basic features of the nonlinear oscillatory regime for α below 5π are summarized in the bifurcation diagram of Fig. 2.

At $\alpha \approx 4.9178$, the chaotic attractor collides with the unstable fixed point. In Fig. 3, the point is encircled by a small loop which is visible to the lower left of the static equilibrium. Slightly below the critical value of α, the attractor suffers a *boundary crisis*: the phase-plane trajectory oscilates chaotically around the old attractor region. The oscillations persist only for a limited number of periods, before they are interrupted by virtual cathode formation. During this stage, a significant part of the electrons is either reflected by or trapped inside the potential hills. Consequently, the average value of the external current is reduced by about 30%.

4. Conclusions

A period-doubling route to chaos in the classical Pierce diode was studied by PIC simulations involving a large number of superparticles. We found out that the

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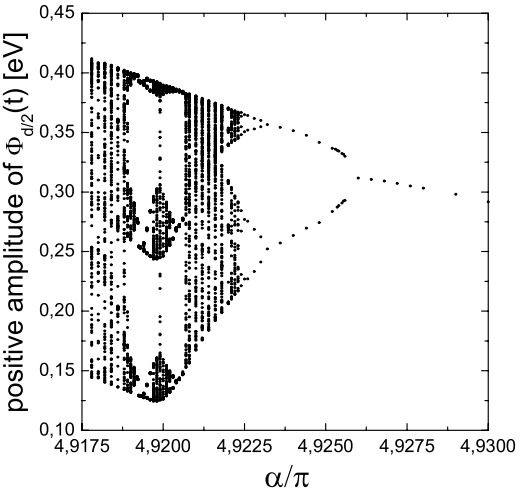


Figure 2. Bifurcation diagram of the nonlinear oscillatory regime for α slightly below 5π.

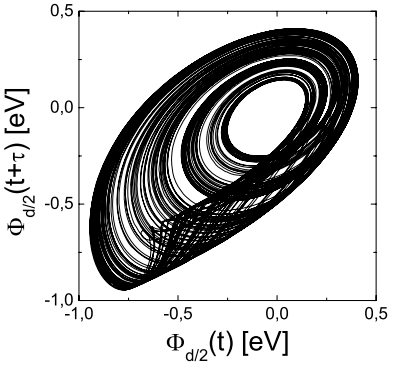


Figure 3. Boundary crisis ($\alpha = 4.9178\pi$). The chaotic attractor collides with the unstable fixed point (encircled by a small loop located to the lower left of the static equilibrium).

nonlinear (non-reflective) steady-state oscillations are localized in an extremely narrow interval of the diode control parameter α. With a further decrease of α below a critical value, the oscillations are interrupted by virtual cathode formation. In this case, the single-cold-fluid model of the classical Pierce diode turns out to be incorrect, because it does not consider electron reflection. Thus, the analysis requires a fully kinetic description, such as ours.

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