STUDENT ARTICLE

Reaction–diffusion systems for algorithmic composition*

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Reaction–diffusion systems were first proposed by mathematician and computing forerunner Alan Turing in 1952. Originally intended as an explanation of plant phyllotaxis (the structure and arrangement of leaves in plants), reaction–diffusion now forms the basis of an area in biology which is as important as DNA research in the field of biological morphogenesis (Kauffman 1993). Reaction–diffusion systems were successfully utilised within the fields of computer animation and computer graphics to generate visually naturalistic patterning and textures such as animal furs (Turk 1991). More recently, reaction–diffusion systems have been applied to methods of half-tone printing, fingerprint enhancement, and have been proposed for use in sound synthesis (Sherstinsky 1994). The recent publication *The Algorithmic Beauty of Seashells* (Meinhardt 1995) uses various reaction–diffusion equations to explain patterned pigmentation markings on seashells. This article details an example of the application of reaction–diffusion systems to algorithmic composition within the field of computer music. The patterned data produced by reaction–diffusion systems is used to create a naturalistic soundscape in the piece *cicada*.

1. TURING'S MORPHOGENESIS

The continuing development of computing technology increasingly provides for the application of more powerful algorithms which can be used creatively by composers, animators and other artists. This article introduces the use of reaction–diffusion systems within algorithmic composition. Algorithms that are derived from the theoretical modelling of natural dynamic systems, such as reaction–diffusion, chaos theory, neural nets and genetic algorithms, provide composers with new methods with which to utilise structures and forms derived from nature within their work.

Reaction–diffusion systems were first proposed by Alan Turing in 1952 in his only published work on biology, *The Chemical Basis of Morphogenesis* (Turing 1952). This work is cited more often 'than the rest of his works taken together' (Saunders 1992) (despite the fact that his work on the logical requirements of algorithms for the Turing Machine now forms the basis of the digital logic used in computing and digital electronics). His theory of embryology sought to address what was one of the greatest puzzles in modern biology, that of the morphogenetic field: how a growing mass of cells divided into the spatial symmetries which were the basis of organic form and structure. Turing sought an answer to the pattern formation in the morphogenetic field through the oscillations of chemical reactions, which led to his 'diffusion–reaction' model (Turing 1952). The model used partial differential equations to describe the nonlinear reaction and diffusion of chemicals in biological morphogenesis. Similar models based on the Turing work have become known in biology as 'reaction–diffusion systems'.

> Turing's question is simple. If an organism starts from a single cell and grows to some mass of cells, all of which are identical, how do those cells ever become different and set up spatially ordered patterns? (Kauffman 1993)

2. ONE-DIMENSIONAL REACTION–DIFFUSION SYSTEMS

Turing proposed partial differential equations to describe the reaction and diffusion over time of two dynamic chemical agents, a and b, within a theoretical ring of cells. He named the two chemical agents morphogens as they represent the dynamic activity of gene placement during the process of morphogenesis. The equations were proposed by Turing and are here presented in the notational form used by Greg Turk (Turk 1991). These equations are shown below where the change to morphogen concentration is described over time.

$$\partial a/\partial t = F(a, b) + D_a \nabla^2 a, \tag{1}$$

$$\partial b/\partial t = G(a, b) + D_b \nabla^2 b. \tag{2}$$

Here, $\partial a/\partial t$, $\partial b/\partial t$ are the change of the concentration of morphogens a and b over time, F, G are functions of the local concentrations of a and b, D_a, D_b are the rates of diffusion for a and b, and $\nabla^2 a$, $\nabla^2 b$ are the Laplacian differential operators, i.e. the divergence of the gradient of the concentration of a and b.

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In order to observe the reaction–diffusion behaviour described by the partial differential equations under computer simulation, a pair of discrete linear equations is derived and used to plot the state of the system in small time steps, or iterations. Changes in morphogen concentrations at each step are recorded in data arrays where the position in space of each cell is given by the array index i. The discrete linear versions of the above equations required for iterative simulation of reaction–diffusion are given below.

$$\Delta a_i = s(16 - a_i b_i) + D_a(a_{i-1} + a_{i+1} - 2a_i), \quad (3)$$

$$\Delta b_i = s(a_i b_i - b_i - \beta_i) + D_b(b_{i-1} + b_{i+1} - 2b_i). \quad (4)$$

Here, i is the array index, Δa_i is the change in concentration of morphogen a for indexed cell a_i, Δb_i is the change in concentration of morphogen b for indexed cell b_i, s is the rate of reaction, D_a is the rate at which a diffuses, D_b is the rate at which b diffuses, and β_i is the random perturbation indexed to each cell (16 is equal to the initial morphogen concentration of a (4) times that of b (4)).

3. RANDOM PERTURBATION

In equations (3) and (4), the 'random perturbation value' β is central to the whole process of reaction–diffusion. It is this value that simulates the random disturbances within a biological system which trigger the oscillations in chemical reactions. It is the standing waves resulting from such oscillations that produce the patterning of morphogen concentrations. Turing used the analogy of the electronic oscillator to explain chemical oscillation:

> The situation is very similar to that which arises in connexion [sic] with electrical oscillators. It is usually easy to understand how an oscillator keeps going when once it has started, but on a first acquaintance it is not obvious how the oscillation begins. The explanation is that there are random disturbances always present in the circuit. Any disturbance whose frequency is the natural frequency of the oscillator will tend to set it going. The ultimate fate of the system will be a state of oscillation at its appropriate frequency, and with an amplitude (and a wave form) which are also determined by the circuit. The phase of the oscillation alone is determined by the disturbance. (Turing 1952)

The 'disturbance' in the system Turing uses to trigger chemical oscillation is represented in the equations by the quantity β_i. The amount of random disturbance within a reaction–diffusion system must not be so great as to cause the system to become 'unbalanced'. Too great a change in the system will cause a cascading effect of ever-increasing values that will deny the simulation of chemical oscillation. In equations (3) and (4) all morphogen concentrations are given the value 4, so that at the beginning the system is in equilibrium with all morphogen values being equal. Each cell within the system is assigned an individual random perturbation value β_i that randomly varies around the value 12, so that β_i will have the value $12 \pm c$, where c is a real number representing the amount of random perturbation in the system. The value of c lies within certain limits: c must be greater than zero for any change to occur within the system. The upper bounds to the value c, where $\beta = 12 \pm c$, depends upon the value of the other equation variables, so that c is bounded by a function of s, D_a, and D_b. For instance, the graph in figure 1 plots the upper bounds of the value of c against the rate of reaction s, where diffusion rates D_a and D_b are constant. The graph shows values which cause the system to become unbalanced.

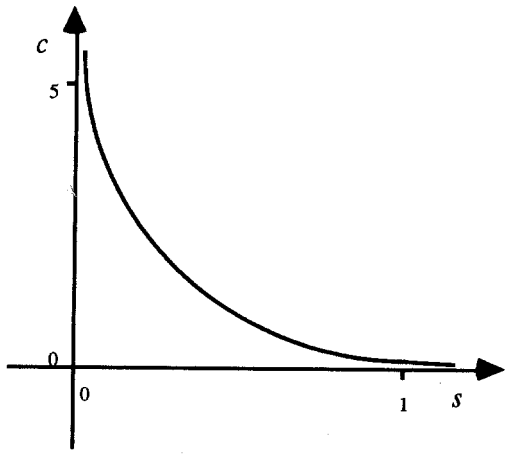


Figure 1. The value of c, where $\beta = 12 \pm c$, plotted against the rate of reaction s.

Whether the system becomes unbalanced or not depends on a number of different relationships between the equation variables, such as that between s and β described above. A similar relationship which must be maintained in the system in order to simulate stable oscillation is between the two diffusion rates D_a and D_b. If one morphogen is diffusing at a significantly faster rate than the other, eventually one morphogen will dominate the whole system. More generally, balance and stability can be maintained in the system by considering the two parts of the equations: the reaction part, $F(a, b)$ and $G(a, b)$, which contains the amount of random perturbation and the rate of reaction, and the diffusion part, $D_a\nabla^2 a$ and $D_b\nabla^2 a$, which contains the diffusion rates and the rule defining the Laplacian (see below). The changes to morphogen concentrations in the system as a whole must generally offset each other for stability to be maintained. Within the value limits that permit oscillation, different 'tunings' of the variables in the system produce different kinds of patterning – some produced by the formation of standing waves, and others that continue to shift.

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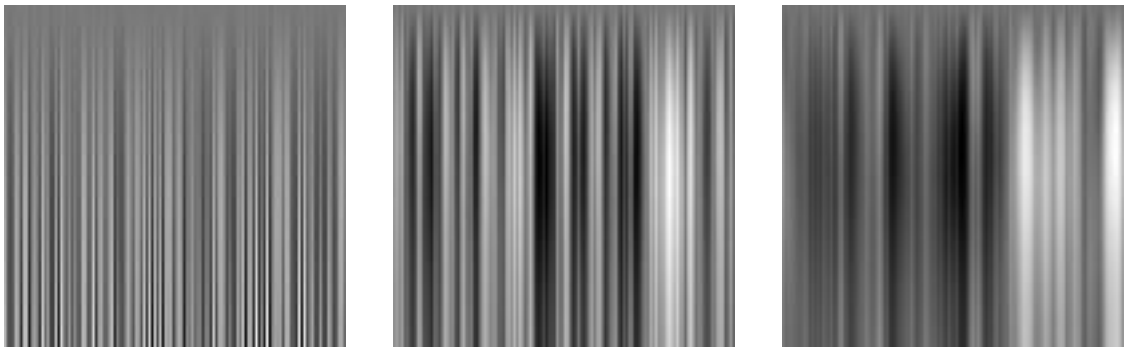


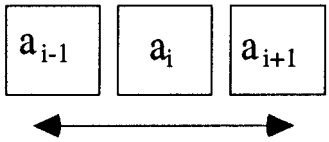
Figure 2. Three examples of one-dimensional reaction–diffusion patterns produced while increasing the rate of D_b with D_a, s, and c remaining constant. The patterns (left to right) were generated with $D_b = 0.001$, $D_b = 0.025$, and $D_b = 0.55$. The patterns represent a ring of 128 cells in the horizontal direction where the concentration of morphogen b is represented in greyscale. The vertical direction shows the changes in concentration over time with time zero at the top.

An interesting aspect of the chemical oscillation simulated in the reaction–diffusion system is that a system can be purposefully made unstable to cause a high degree of activity within the system. A compensating adjustment can then be made to the system in order to reverse the instability and keep the morphogen concentrations within the bounds of simulated reaction and diffusion.

4. THE LAPLACIAN

The second part of the partial differential equations (equations (1) and (2)) for one-dimensional (1D) reaction–diffusion contain Laplacian operators ∇^2, or differential operators. The Laplacian gives a 'measure of the concentration of morphogens in neighbouring cells' (Turk 1991), a value for the divergence or flux of the gradient of morphogen concentration at each particular cell. In the above equations the discrete Laplacian for morphogen a is given by:

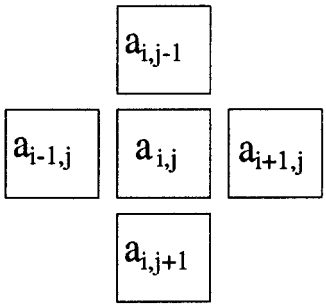
$$\nabla^2 a_i = (a_{i-1} + a_{i+1} - 2a_i). \tag{5}$$



It can be seen above that this Laplacian operator gives a regional average value of concentration. In the case of a single cell where neighbouring cells are found to have a high concentration of a, then $\nabla^2 a$ will be positive and morphogen a diffuses from the neighbouring cells toward the cell. If neighbours have lower concentrations, then $\nabla^2 a$ will be negative and a will diffuse away from the cell.

The effect of the Laplacian operators in the equations is to create regions of high and low morphogen concentrations so that, after many iterations, patterns of a and b can be observed across the (theoretical) cellular membrane. Other 'Laplacian-like' functions could be defined to change how this value is calculated and which neighbouring cells are considered, in the same way as one might redefine the 'rule' of a cellular automaton, and thereby produce different patterning outcomes. The partial differential equations for a 2D reaction–diffusion system are the same as those for 1D reaction–diffusion (equations (1) and (2)), except that the Laplacian is defined to account for neighbouring cells in two directions instead of one:

$$\nabla^2 a_i = a_{i+1,j} + a_{i-1,j} + a_{i,j+1} + a_{i,j-1} - 4a_{i,j} \tag{6}$$



5. TWO-DIMENSIONAL REACTION–DIFFUSION EQUATIONS

The discrete linear equations for a 2D reaction–diffusion system are given below.

$$\Delta a_{i,j} = s(16 - a_{i,j} b_{i,j}) + D_a(a_{i+1,j} + a_{i-1,j} + a_{i,j+1} + a_{i,j-1} - 4a_{i,j}), \tag{7}$$

$$\Delta b_{i,j} = s(a_{i,j} b_{i,j} - b_{i,j} - \beta_{i,j}) + D_b(b_{i+1,j} + b_{i-1,j} + b_{i,j+1} + b_{i,j-1} - 4b_{i,j}). \tag{8}$$

Here, i, j are the 2D array indices, $\Delta a_{i,j}$ is the change in concentration of morphogen a for indexed cell $a_{i,j}$, $\Delta b_{i,j}$ is the change in concentration of morphogen b for indexed cell $b_{i,j}$, s is the rate of reaction, D_a is the rate at which a diffuses, D_b is the rate at which b

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diffuses, and $\beta_{i,j}$ is the random perturbation indexed to each cell, i.e. $\beta_{i,j} = 12 \pm c$.

6. ALGORITHMIC COMPOSITION AND ARTIFICIAL INTELLIGENCE

> By an intelligent system, we mean a system that works cooperatively with the user, providing useful levels of automated reasoning to support laborious and tedious tasks (such as working out an appropriate stream of synthesis parameters for each desired single sound) and to aid the user in exploring possible alternatives when designing a certain sound. (Miranda 1995)

Machine intelligence is often discussed by composers who work within algorithmic composition and computing. Eduardo Miranda has applied the term 'artificial intelligence' to describe computer systems used in composition, defining the role of an intelligent system as that of 'automated reasoning' (Miranda 1995). In the article *Metamusical Composing with Computers* (Worrall 1995), David Worrall suggests compositional methods where the computer may be 'a collaborator in the creative process itself'; he describes a method of composition he terms 'procedural composition', in which the composer designs a procedure that the computer can implement in order to generate the whole or part of a composition. The patterned data produced by reaction–diffusion systems has been successfully applied to spatial and temporal parameters in the algorithmic composition of music, including MIDI composition and various methods of sound synthesis (Martin 1996). Interestingly, the origins of reaction–diffusion systems are linked to some early practical ideas about artificial intelligence. Turing's interest in the biological problem of the morphogenetic field was driven by his belief in the inevitable development of machine intelligence.

Turing was known for his public comments about an 'electronic brain' and of 'thinking machines' (which offended some of his scientific colleagues) (Hodges 1985). Turing believed that the function of the organic brain could be understood in the same way as inorganic phenomena, i.e. in terms of physical and chemical processes. The task to which he applied one of the first working computers (Ferranti Mark I (1951)) involved the calculations necessary to develop his 'mathematical theory of embryology' (Turing 1951).

> Our new machine is to start arriving on Monday. I am hoping as one of the first jobs to do something about 'chemical embryology'. In particular I think one can account for the appearance of Fibonacci numbers in connection with fir-cones. (Turing 1951)

For Turing, the key to gaining insight as to how the brain worked was to explain how the intrinsic natural structures of the brain were formed. In correspondence with the neuro-physiologist J. Z. Young, Turing wrote:

> The brain structure has to be one which can be achieved by the genetical embryological mechanism, and I hope that this theory that I am now working on may make clearer what restrictions this really implies. What you tell me about growth of neurons under stimulation is very interesting in this connection. It suggests means by which the neurons might be made to grow so as to form a particular circuit, rather than to reach a particular place. (Turing 1951)

7. NATURE AND ALGORITHMIC COMPOSITION

Systems and software that have been developed for algorithmic composition are increasingly making use of processes and systems which imitate natural structures and forms, or processes that produce a resemblance of the detail of natural configurations such as fractals, which are actually purely mathematical in origin. In the article, *Nature*, *Music*, *and Algorithmic Composition* (Leach and Fitch 1995), the authors place new emphasis on the relationship between algorithmic composition and the modelling of natural systems by drawing a parallel between the self-modifying 'laws of nature' and those of musical structure.

> This article is a first step into an investigation of the relationship between music and natural growth. We suspect that the future of algorithmic composition lies in interactive systems, based on groups of fundamental natural processes that can be iterated to form an account of how a structure changes over time. (Leach and Fitch 1995)

Their article suggests that music can be produced automatically by the use of natural dynamic systems based on 'the axiom that music mimics the way nature behaves' (Leach and Fitch 1995). As reaction–diffusion systems have been developed within biology to explain how spatial symmetries and segmentations occur during organic growth, they may be well suited to the task of organising the spatial and structural changes over time which occur in music. How the organic patterning produced by reaction–diffusion is applied to form the spatial and temporal dimensions of music raises questions as to what compositional methods best translate the innate patterning. The composers Iannis Xenakis and John Cage have developed their practice of algorithmic composition in

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different and contrasting ways. Their use of algorithmic composition was not to automatically compose but rather a method of inquiry into the spatial and temporal substance of music.

> If we received (and in fact, we do receive) signals from intrastellar, galactic space, well, it would be necessary to know how to distinguish these from noise . . . to see if they are structured, if they are coherent, and if this coherency is meaningful or not. By meaningful, I mean to say if it comes from natural sources (which is to say, from nature itself) or if it comes from other beings who would resemble man. It would be necessary to go back, well before all structures, before all forms of thought which we have received from civilization and schooling, and to get to pre-rational, pre-logical, pre-structural, pre-syntactical situations. (Xenakis 1985)

The patterned data produced by reaction–diffusion systems could be considered 'meaningful' in Xenakis' terms as it is a simulation of organic growth, a form 'from nature itself'. Reaction–diffusion essentially describes an organic process and natural behaviour rather than a formalised structure. John Cage proposed that music can find form in 'nature's manner of operation', in terms of natural processes.

> But in recent years, the last twenty-five, we have moved from an interest in structure to an interest in process, away from the division of a whole into parts and close to something without beginning, middle, and end, something like weather. (Cage 1993)

In his early work Cage was concerned with rejecting the dominance of pitch and harmonic organisation, and substituted instead an organisation based on rhythm. Later in his career, Cage sought to create alternative musical structures, which he described as 'making my responsibility that of asking questions instead of making choices'. His use of indeterminacy through chance operations and other processes to create alternative musical structures allowed for a music that could reflect the forms and variation found in nature.

> Musical habits include scale, modes, theories of counterpoint and harmony, and the study of the timbres, singly and in combination of a limited number of sound producing mechanisms. In mathematical terms these all concern discrete steps. They resemble walking – in the case of pitches, on steppingstones [sic] twelve in number. This cautious stepping is not characteristic of the possibilities of magnetic tape, which is revealing to us that musical action or existence can occur at any point or along any line or curve or what have you in total sound-space; that we are, in fact, technically equipped to transform our contemporary awareness of nature's manner of operation into art. (Cage 1968)

8. CICADA

> . . . in the history of Western music, considerable interest has been shown by composers in structural principles superficially more nearly extramusical. One of the most obvious of these, and one which is familiar and generally accepted by most listeners, is the "imitation of nature," a musical tradition with a respectable history, if not always given equal recognition to the so-called higher forms of music. (Hiller 1959)

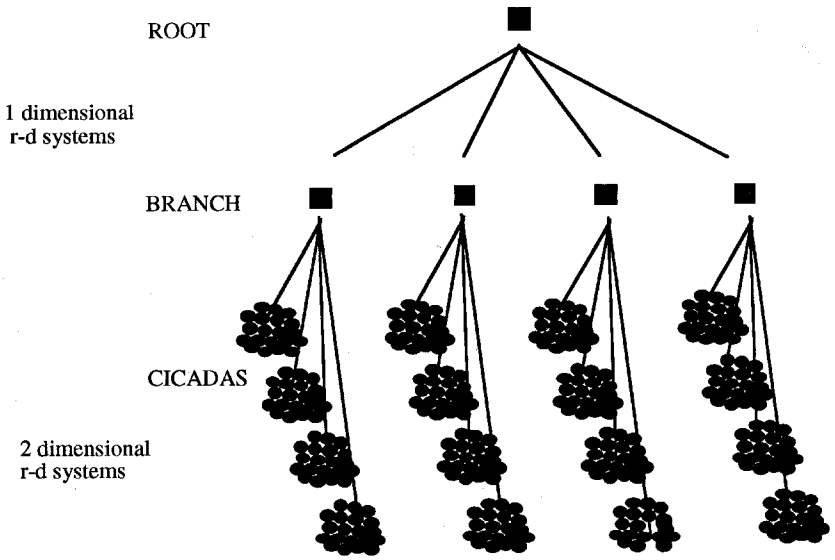


Figure 3. Layered branching structure of r–d systems.

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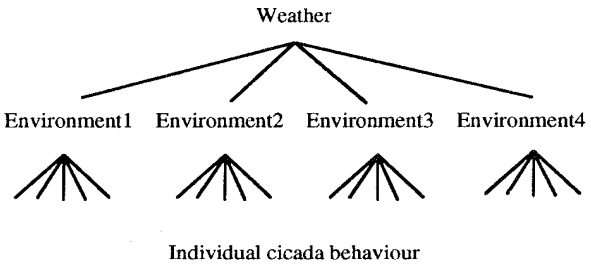


Figure 4. 'Reason' branch structure used in the piece *cicada*.

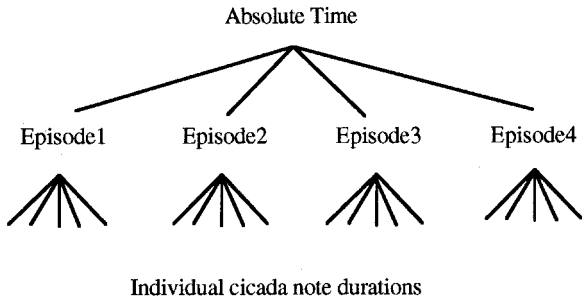


Figure 6. Spatial branch structure used in the piece *cicada*.

The piece *cicada* is an example of algorithmic composition which uses multiple 1D and 2D reaction–diffusion (r–d) systems to control the spatial and temporal parameters of a large sonic composition. The piece has been realised using *Csound* (Vercoe 1986) with an r–d program writing a 'score' for a designated cicada *Csound* instrument. The r–d systems are configured into three branching structures where each structure controls parameters in one of three groupings: time, space, and reason. The reason group of parameters is used to determine the occurrence of sonic events, while the other two groupings provide the spatial and temporal quantities forming each event. The composition is conceptually based on the behaviour and sound of different species of cicada (Moulds 1990).

The voice of each cicada is produced by a simple FM synthesis instrument given individual reaction–diffusion values; the frequencies of the voices are clustered around the species' frequency so that 'beating' difference tones are produced. Each cicada either sings, chirps, clicks, or is silent depending on its environment and the weather conditions which are modelled by reaction–diffusion patterns. This creates episodes of varying dynamic levels and some periods of silence during the piece.

All r–d systems within each tree structure share common equation variables: D_a, D_b, s, and c (where $\beta = \pm c$), so that the equation variables for the root r–d system of each tree are used for all other r–d systems within that tree structure. Each r–d system, however, varies in the random distribution of the random perturbation factor β, so that all r–d patterns will be unique, though the patterns within each tree structure will be similar. Figure 3 shows the branching structure of r–d systems used for each of the three control structures.

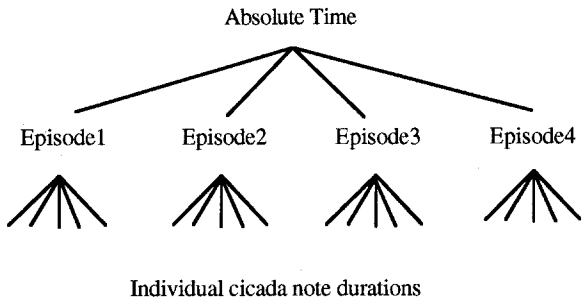


Figure 5. Temporal branch structure used in the piece *cicada*.

Certain features of the cicadas' environment and behaviour are modelled in the composition, such as the warm weather required for them to sing. Other aspects considered within the composition include the following: species of cicada share the same frequency of mating call, and many species will generally group together in one area of trees. Such cicada behaviour is determined in the piece by consulting the r–d values contained within the 'reason' tree: here, the root r–d system gives a value for the prevailing weather conditions, the second level branches provide a value as to the favourability of the environment, and the third level provides a factor for individual cicada behaviour. Values from each of the three levels are consulted to determine whether each cicada sings, chirps, or clicks, or whether it is inactive.

The time branching structure has an r–d system as its root which provides values for the overall duration of the piece. The next level of branches provides durational values for the series of episodes in which the aural behaviour of the cicadas is portrayed. The third level of the time tree structure is used to determine note and rhythmic durations of individual cicadas when they are active.

The spatial tree structure of r–d systems controls elements such as the fundamental frequency and amplitude of each active species, the stereo pan position of where a species of cicadas is clustered, and the individual frequency and amplitude variation of partials for each cicada voice.

In addition to the controls outlined above, values from different tree structures are compared in order to map the sonic events within the piece. For example, the weather value is compared with a species value to determine whether a species will be active during a certain episode. Once active, individual cicadas draw control parameters from the corresponding r–d systems from each of the three

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structures. These values are then written to a *Csound* score.

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