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Plasma effects in electromagnetic field interaction with biological tissue

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Abstract. Theoretical analysis is presented of the nonlinear behavior of charge carriers in biological tissue under the influence of varying low-intensity electromagnetic (EM) field. The interaction occurs because of the nonlinear force arising due to the gradient of the EM field intensity acting on free electrons in the conduction band of proteins in metabolically active biological cell membrane receptors leading to a redistribution of charge carriers. Field dependence of the resulting dielectric constant is investigated by a suitable modification to include an additional electronic contribution term to the three-term Debye model. The exogenous EM field propagating in this nonlinear cellular medium satisfies the nonlinear Schrödinger equation and can be affected significantly. Resulting field effect can be substantially augmented and effective rectification/demodulation can occur. Possible implications of this modification on biological processes in white and grey matter are discussed.

1. Introduction

The effect of the radio frequency power absorbed in biological tissue, such as the head of a mobile telephone user, is currently a topic of great concern. This electromagnetic (EM) field is, in general, so weak that it cannot cause any significant Joule heating effects in the head of the mobile telephone user, but the effect of this weak field exposure on the biological processes may still be non-negligible, as suggested by some experiments on the biological effects of EM exposures (e.g. Preece et al. 1999). The physical basis of such putative interactions is not yet fully understood, despite its vital importance. How does an EM field with precisely known characteristics (e.g. frequency, intensity) interact with cellular matter having reasonably well-defined physical characteristics (conductivity, dielectric constant, charge distribution, volume) is a question which has been asked by many physical scientists, a problem that has been intensively studied, at both the theoretical and experimental levels, for more than a century. It has been observed again and again that not only are the laboratory results consistent with Maxwell equations but it is almost always possible, from theory, to predict the interaction between the given EM field and matter having well-defined characteristics. The same approach when applied to biological systems exposed to Extremely Low Frequency (ELF) fields implies the study of Faraday interactions, membrane capacitive responses and nonlinear effects

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such as surface effects on channels, secondary solitons signals and resonance-like mechanisms such as cyclotron resonance. This research paper investigates on the associated nonlinear effects, such as signal amplification, presumably at the receptor site by treating the cellular medium as a nonlinear dynamical system. Important aspect of this approach is the signalling event across the membrane that may be solitons-like, which are pulses or waves that propagate with no discernable energy loss. Their appearance in biological systems has yet to be established, but they nevertheless represent a prime source of speculation (Boot et al. 1958; Davydov 1971). Some efforts have already been made to understand this biophysical process by using classical and quantum models (Bianco et al. 1997; Moggia et al. 1997 Davydov 1979). The common feature of these models is the study of the dynamics of messenger ions in the presence of an incident EM field (exogenous field) and the endogenous electric field produced by the molecules at the binding/interaction site. These models assume the endogenous field to be a complex function of space but independent of time. In actual situations, for example, due to the biochemical processes, which fuel the living state of the cell (basal metabolism), the process is time-dependent. Therefore, this field should be admitted as EM in nature rather than purely electrostatic. Further, theoretical dosimetry, i.e. calculation of the EM energy absorbed by humans in radiation fields, has become increasingly important in the domain of bioelectromagnetics as the use of EM devices in our society has increased. Since such calculations are difficult and complicated, a number of EM analysis techniques have been used by researchers in the field to obtain dosimetric data and understanding of absorption characteristics. Both analytical and numerical EM techniques have been applied extensively (Durney 1980). Not only this, the basic concepts of dielectric phenomena in biological materials and their interpretation in terms of interactions at the cellular level have been a topic of interest since decades. Attempts have been made to collect and tabulate the dielectric constant and the loss factor of biological materials in the frequency range of 10 KHz to 10 GHz by Stuchly and Stuchly (1980) and in the range of 10 Hz to 20 GHz by Gabriel et al. (1996) using a parametric model based on the Cole–Cole formulation (Gabriel et al. 1996), which provides a physical model for material properties as a function of frequency, others being Debye and Lorentz models, latter being inadequate to describe the material properties at broad frequency ranges (Mrozowski and Stuchly 1997; Eleiwa and Elsherbeni et al. 2001).

Our main concern in the present work is to highlight the importance of the unattached electrons (free electrons), which are present in the conduction band of proteins in the cell membranes and their effect on the exogenous field in the cellular media by a suitable modification to the existing Debye expression of dielectric constant. The importance of these unattached electrons has been discussed by Fröhlich (1980), who suggested that these might even contribute to plasma modes in the proteins of the cell membranes. Such electrons might find themselves, for instance, in the conduction band of proteins through charge transfer mechanisms, for which densities of up to $10^{18}\,\mathrm{cm}^{-3}$ have already been reported (Pethig 1978). Our aim is to study the effect of these electrons on the exogenous field by utilizing effective dielectric constant of the cellular media, which includes a frequency-dependent linear contribution that arises due to these unattached electrons. A similar concept was raised by Davydov (1971) in his theory of molecular excitations. Besides contributing to the linear part of the effective dielectric constant, these electrons also

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contribute to the nonlinearity resulting in a field-dependent (exogenous field intensity dependent) effective dielectric constant, which is also investigated here. The basis of this interaction is the nonlinear force arising due to the gradient of the EM field intensity acting on the free electrons. The nonlinear force so generated may lead to the redistribution of charge carriers and the modified density satisfies the acoustic-type equation (ion-acoustic under quasi-neutrality condition), driving force being the nonlinear force. The exogenous field propagating in this nonlinear medium may undergo several physical processes and in certain situations the exogenous field may be enhanced radically. This will be equivalent to a higher amount of exposure after the first nonlinear interaction at the cell level, thus greatly increasing the possibility of non-negligible biological effects. These effects may take place either through the cyclotron resonance process (Liboff et al. 1991), through classical processes as studied by using Fokker–Planck analysis of the Langevin–Lorentz equation (Moggia et al. 1997) or through quantum processes predicted by Zeeman–Stark modelling (Bianco et al. 1997).

Such situations are common in the interaction of high-power laser beams with the plasma electrons (Kruer 1986). The exogenous field propagating in this nonlinear region thus satisfies the NLS equation, where the depth of nonlinear potential depends upon the strength of the exogenous fields. In an analogous physical situation involving nonlinear coupling of a large amplitude EM pump wave with low-frequency collisional modes in the ionospheric plasma, it has been verified experimentally (theoretically proposed as ionospheric stimulated EM technique in late seventies) that the threshold values for stimulated EM emissions are exceeded in ionospheric heating experiments (Stenflo 1990). The theory explaining the propagation of EM waves in plasmas was extended to include magnetic and heating effects treating the lower part of the ionosphere as a magnetized collisional plasma. Laboratory and magnetospheric experiments indicated that the underlying strongly nonlinear processes could have a universal character (Stenflo 1990) and that the usual fluctuation spectrum could be significantly changed by the presence of a strong pump wave (Stenflo 2004). This leads to defining a new approach to understand the interactions between the ionospheric plasma and the EM waves. This links to a secondary motivation of the present work, which is to explore the application of some nonlinear EM analysis developed for modelling of plasma processes (as also in Mahmoud and Sharma 2001; Shukla and Sharma 2001) to biological tissues.

This research paper is organized as follows. Section 2 derives the model equations and the effective dielectric constant of the cellular media starting with the basic Debye equations. Possible effects of the nonlinear processes have been studied in the context of mobile phone and human head interaction in Sec. 3. Section 4 concludes the research paper.

2. Model equations for nonlinear dielectric constant

Consider an EM field of frequency ω_0 denoted by

$$\mathrm{E}_0 = \mathrm{E}'_0(r, z)\exp(i\omega_0 t) \tag{1}$$

propagating in a dielectric medium. This component could be generated by an external source, or it could be the endogenous field, as postulated by Bianco et al.

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(1997) and Moggia et al. (1997). In his work "Polar Molecule", Debye (1929) presented a generalization of how materials with orientational polarizability behave in the region where the dielectric polarization is relaxing, i.e. the situation where the period of the EM wave is comparable to the alignment time of the molecule. When the applied frequency is much greater than the reciprocal of the alignment time, the relative permittivity is called the infinite relative dielectric constant ε_∞, and represents the atomic and electronic polarization. For much lower frequencies it becomes ε_s, the static relative dielectric constant. The frequency-dependent relative permittivity of the medium, when assumed to be of the form

$$\varepsilon(\omega) = \varepsilon_\infty + f(\omega),$$

where $f(\omega)$ is a function of frequency and reduces to $f(0) = \varepsilon_s - \varepsilon_\infty$ when $\omega -> 0$, can be written as

$$\varepsilon^*(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + i\omega\tau} = \varepsilon' - i\varepsilon''. \tag{2}$$

The real and imaginary parts of (2) are given as

$$\varepsilon' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2\tau^2}, \tag{3}$$

and

$$\varepsilon'' = \frac{\omega\tau(\varepsilon_s - \varepsilon_\infty)}{1 + \omega^2\tau^2}. \tag{4}$$

Equations (2)–(4) are known as Debye equations. Let $\omega_r = 1/\tau$ be the cyclic relaxation frequency, then for conductive materials, including the energy loss due to conductivity, gives

$$\varepsilon^*(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (i\omega/\omega_r)} - \frac{i\sigma_i}{\omega\varepsilon_0} = \varepsilon' - i\varepsilon''. \tag{5}$$

The modified imaginary part of the relative permittivity then becomes

$$\varepsilon'' = \frac{\omega\tau(\varepsilon_s - \varepsilon_\infty)}{1 + \omega^2\tau^2} + \frac{\sigma_i}{\omega\varepsilon_0}. \tag{6}$$

Equation (6) thus involves the sum of the dielectric loss and energy loss due to ionic conductivity. The loss due to latter is relatively high at low frequencies and low at high frequencies. The general form of Debye formulation for the relative complex permittivity and which is generally used to investigate EM interaction with biological tissues is given as

$$\varepsilon^*(\omega) = \varepsilon_\infty + \left[\sum_{n=1}^{N} \frac{A_n}{1 + i\omega\tau_n}\right](\varepsilon_s - \varepsilon_\infty) + \frac{\sigma_i}{i\omega\varepsilon_0}, \tag{7}$$

where ε_∞ is the relative permittivity (essentially $\varepsilon^*/\varepsilon_0$) at $\omega\tau_n \gg 1$, ε_s is the relative permittivity at $\omega\tau_n \ll 1$ and σ_i is the static ionic conductivity. The Debye coefficients (ε_s, σ_i, A_n, τ_n) have been tabulated by Mrozowski and Stuchly (1997). However, a much more convenient Debye formulation, in which σ_i is implicitly considered, is

$$\varepsilon^*(\omega) = \varepsilon'_\infty + \left[\sum_{n=1}^{N} \frac{\Delta\varepsilon'_n}{1 + i\omega\tau'_n}\right], \tag{8}$$

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Table 1. Three-term Debye parameters for the frequency range of 500 MHz to 20 GHz for brain white and grey matter.

Tissue	ε'_∞	$\Delta\varepsilon'_1$	$\Delta\varepsilon'_2$	$\Delta\varepsilon'_3$	τ'_1(s)	τ'_2(s)	τ'_3(s)	Maximum % error in real part	Maximum % error in imaginary part
Brain (white matter)	5.338 e +00	3.004 e +01	2.090 e +00	7.050 e +01	7.181 e −012	2.257 e −011	1.156 e −009	0.55	0.64
Brain (grey matter)	5.830 e +00	4.216 e +01	2.754 e +00	1.371 e +02	7.187 e −012	2.246 e −011	1.399 e −009	0.56	0.62

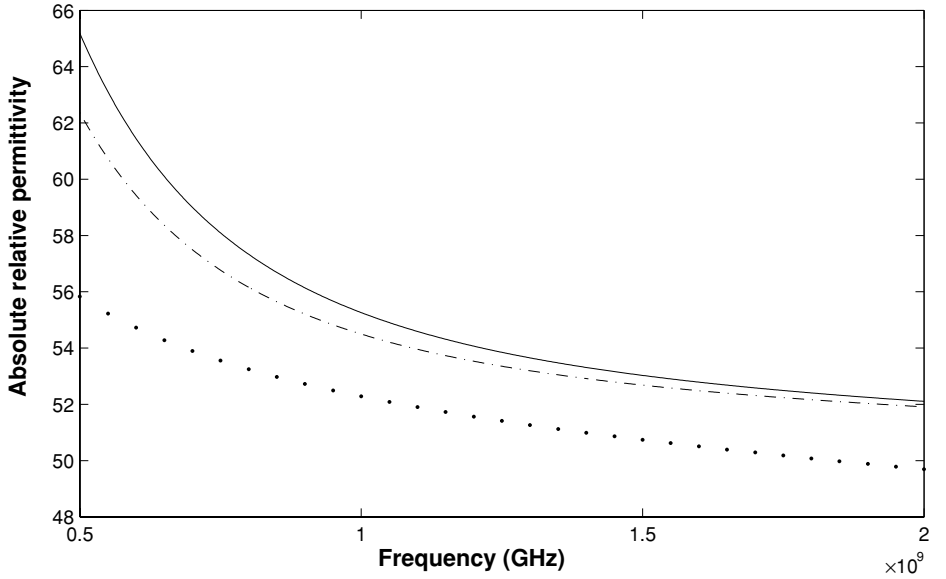


Figure 1. Variation of the absolute value of relative permittivity with frequency in the range of 500 MHz to 2 GHz for grey matter using the three-term Debye model (solid curve) and experimental data obtained using the four Cole–Cole analysis (dotted curve). Dash-dot curve represents three-term Debye model with electronic contribution.

where $\Delta\varepsilon'_n = A'_n(\varepsilon'_s - \varepsilon'_\infty)$, ε'_s and ε'_∞ represent the values of the relative permittivity at very low and very high frequencies, respectively, and N is the order (number of terms) of the Debye relation. The results using this technique are consistent with four-term Cole–Cole formulation. For brain tissue, the three-term Debye parameters for the frequency range of 500 MHz to 20 GHz using the above technique are reproduced in Table 1. Figures 1 and 2 compare the variation of the absolute value of relative permittivity with frequency in the range 500 MHz to 2 GHz for grey and white matter using the three-term Debye model (solid curves) and experimental data obtained using the four Cole–Cole analysis (dotted curves). Figures 3 and 4 compare the variations of conductivity with frequency in the range 500 MHz to

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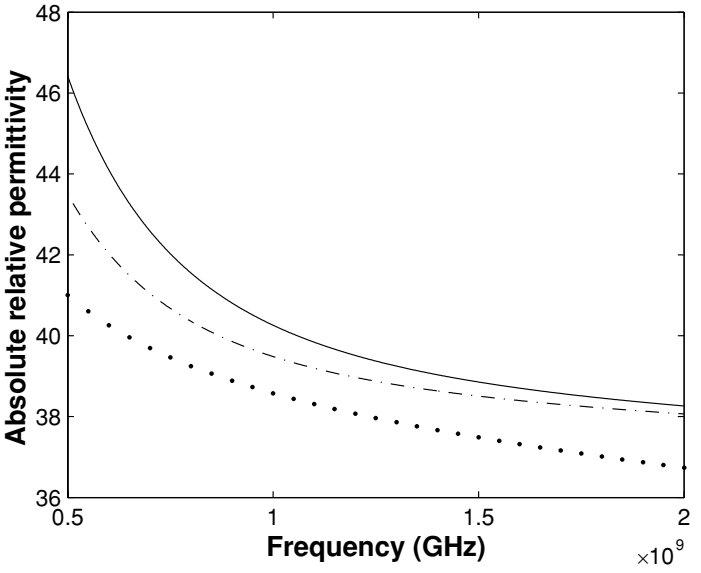


Figure 2. Variation of the absolute value of relative permittivity with frequency in the range of 500 MHz to 2 GHz for white matter using the three-term Debye model (solid curve) and experimental data obtained using the four Cole–Cole analysis (dotted curve). Dash-dot curve represents three-term Debye model with electronic contribution.

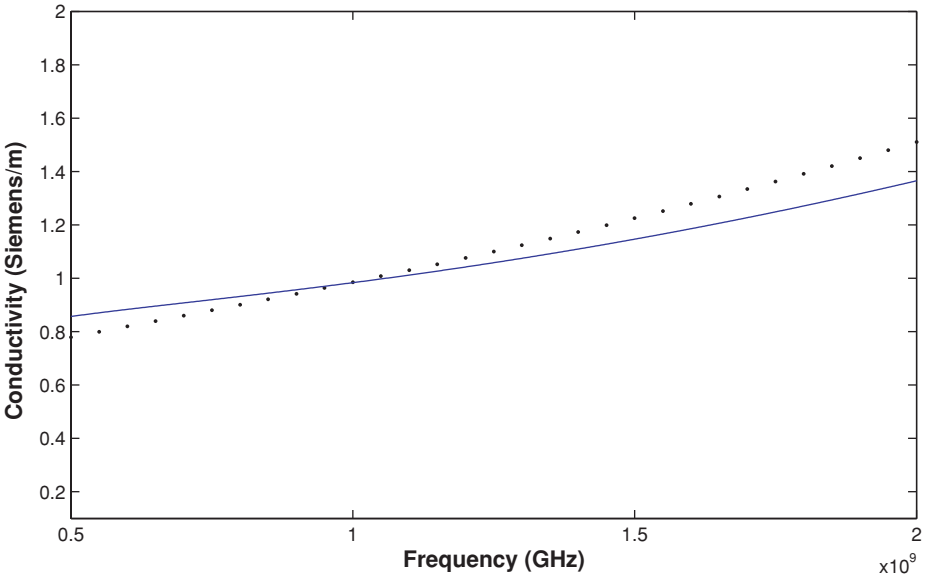


Figure 3. Variation of conductivity with frequency in the range of 500 MHz to 2 GHz for grey matter using the three-term Debye model (solid curve) and experimental data obtained using four Cole–Cole analysis (dotted curve).

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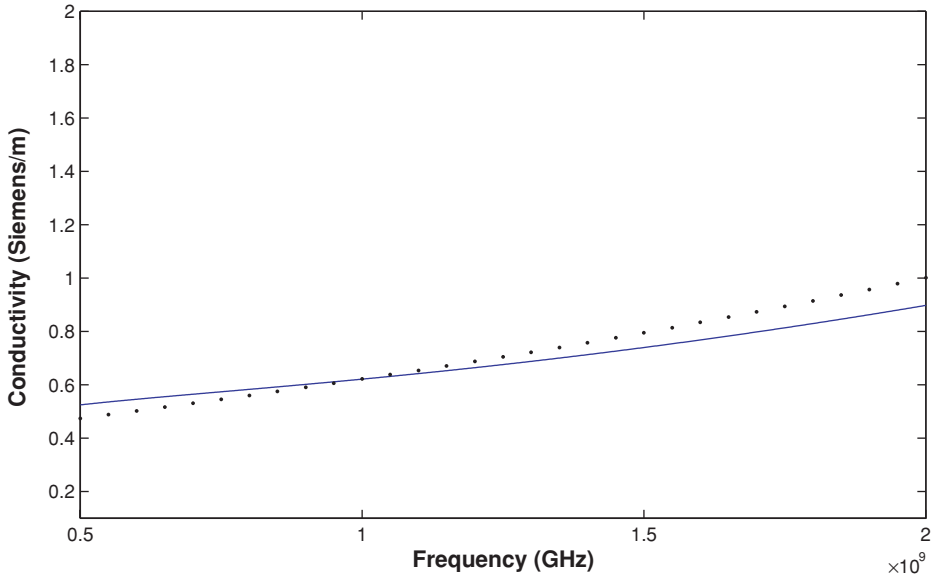


Figure 4. Variation of conductivity with frequency in the range of 500 MHz to 2 GHz for white matter using the three-term Debye model (solid curve) and experimental data obtained using the four Cole–Cole analysis (dotted curve).

2 GHz again for grey and white matter using the three-term Debye model (solid curves) and experimental data obtained using the four Cole–Cole analysis (dotted curves).

The Cole–Cole model has the form

$$\varepsilon^*(\omega) = \varepsilon_\infty + \sum_{n=1}^{4} \frac{\Delta\varepsilon_n}{1 + (i\omega\tau_n')^{1-\alpha_n}} + \frac{\sigma}{i\omega\varepsilon_0},$$

where ε_∞ is the high-frequency permittivity, τ_n' is the relaxation time, $\Delta\varepsilon_n$ is the pole amplitude and α_n is the pole broadening.

In the absence of nonlinear forces and polarization forces, the dielectric constant of the cellular medium can be written (Davydov 1971) as

$$\varepsilon_c = \varepsilon_r - \frac{\omega_{po}^2}{\omega_0^2}, \tag{9}$$

where ε_r is the normal linear complex effective dielectric constant and ω_{p0} is the electron plasma frequency given by $\omega_{p0}^2 = \frac{N_0 e^2}{m_e \varepsilon_0}$. Here e and m_e are the charge and effective mass of the electron, respectively, N_0 is the number of electrons per unit volume and $\varepsilon_0^{-1} = 36\pi \times 10^9$. When the EM field is non-uniform, a nonlinear force acts on the electrons. This force arises due to the interaction of the drift velocity of the electrons (caused by the electric vector of the EM field) with the magnetic vector of the EM field, plus a force due to the gradient of the drift velocity (Boot et al. 1958). After using the expression for drift velocity and vector identities, the

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magnitude of this nonlinear force in the presence of this EM field is given by

$$\vec{F}_e = -\frac{e^2}{4m_e}\left(\frac{1}{\omega_0^2}\vec{\nabla}\left(\vec{E}_0 \cdot \vec{E}_0^*\right)\right). \tag{10}$$

The nonlinear force on ions is less by a factor of m_e/m_i in comparison with that on electrons (where m_i is the ion mass); therefore, it is also not taken into account in further analysis. In the presence of this nonlinear force, following equations can be defined for diffusion of charge carriers, both electrons and ions:

$$m_e\frac{\partial \vec{V}_e}{\partial t} = -e\vec{E}_s - \frac{\gamma_e K_B T_e}{N_e}\vec{\nabla}N_e + \vec{F}_e, \tag{11}$$

$$m_i\frac{\partial \vec{V}_i}{\partial t} = e\vec{E}_s - \frac{\gamma_i K_B T_i}{N_i}\vec{\nabla}N_i. \tag{12}$$

The indices e and i refer to electrons and ions respectively, T_e and T_i are the corresponding temperatures, K_B is Boltzman's constant, E_s is the space charge field and other symbols have their usual meanings. Owing to space charge effects, electrons and ions move almost with the same velocity, i.e. $V_e \cong V_i \cong V$ in an almost electrically neutral medium and $N_e \cong N_i \cong N$, where V is the diffusion velocity of electrons and ions and N is the modified electron and ion density in the presence of the EM fields. This implies that space charge is not so strong that it can stop the diffusion process of charged particles, but is sufficiently strong to keep electrons and ions together (by Coulomb attraction between electrons and ions, they tend to move collectively). This condition can be achieved when the Debye length is much less than the beam width r_0 (this is generally achieved in a macroscopic description of charged but quasi-neutral matter, as used here). Adding (11) and (12) and neglecting electron mass in comparison to ion mass, the following equation is obtained:

$$\frac{\partial \vec{V}}{\partial t} = -\frac{c_s^2\,\vec{\nabla}N}{N} - \frac{e^2}{4m_e m_i}\left(\frac{1}{\omega_0^2}\vec{\nabla}(\vec{E_0}\cdot\vec{E_0^*})\right), \tag{13}$$

where $c_s^2 = (\gamma_e k_B T_e + \gamma_i k_B T_i)/m_i$ is the modified sound speed (ion sound speed). For further analysis, (13) may be supplemented by the equation of continuity:

$$\frac{\partial N}{\partial t} + \vec{\nabla}\cdot\left(N\vec{V}\right) = 0. \tag{14}$$

Equations (13) and (14) when combined give

$$\frac{\partial^2}{\partial t^2}\left(\ln\frac{N}{N_o}\right) = c_s^2\nabla^2\left(\ln\frac{N}{N_o}\right) + \frac{e^2}{4m_e m_i}\left(\frac{1}{\omega_0^2}\nabla(\vec{E_0}\cdot\vec{E_0^*})\right), \tag{15}$$

where N_o is the electron density in the absence of the EM field. One of the terms from $\nabla(NV)$ contributes negligibly in (15) because of the pure transverse nature of the exogenous field in the cellular media. Equation (15) is the driven ion acoustic equation in the presence of the field considered here. In order to obtain the modified density response in the presence of this EM field, (15) needs to be solved together with the wave equation for the field by making use of the full set of Maxwell's equations. Obtaining an exact analytical solution is very difficult, but a numerical solution can be obtained by using, for example, a pseudo-spectral method for space-wise

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integration in a periodic box and a finite difference scheme with predictor–corrector for the evolution in time.

Before going into numerical schemes for modeling, one can obtain some physical insight into the nonlinear processes, under certain approximations. For example, in the steady state (or in the adiabatic case), from (15), an analytical expression for N can be obtained as

$$N \cong N_0 \exp\left[-\alpha_0 \vec{E}_0 \cdot \vec{E}_0^*\right], \tag{16}$$

where

$$\alpha_0 = \frac{e^2}{4m_e m_i c_s^2 \omega_0^2}.$$

The value of the exponent in (16) is a measure of the square of the ratio of the electron drift speed and the electron thermal speed. By using (9) and (15), the field-dependent effective dielectric constant of the cellular medium, denoted by $\varepsilon_{\rm cf}$, can be expressed as $\varepsilon_{cf} = \varepsilon_c + \phi_0(\vec{E}_0 \vec{E}_0^*)$, where ϕ_0 is the nonlinear part of the dielectric constant and is given by

$$\phi_0 = \frac{\omega_{p0}^2}{\omega_0^2}\left[1 - \exp\left(-\alpha_0 \vec{E}_0 \cdot \vec{E}_0^*\right)\right]. \tag{17}$$

For the same value of the ratio of electron drift speed to electron thermal speed, the nonlinear part of the effective dielectric constant, as given by (17), varies inversely to the exogenous field frequency. Therefore, also at relatively lower frequencies, these nonlinear effects should be important.

The wave equation governing the electric field in the nonlinear dielectric medium (assuming cylindrical symmetry) can be written as

$$\frac{\partial^2 E_0'}{\partial z^2} + \frac{\partial^2 E_0'}{\partial r^2} + \frac{1}{r}\frac{\partial E_0'}{\partial r} + \frac{\omega_0^2}{c^2}\varepsilon_{\rm cf} E_0' = 0. \tag{18}$$

The following analysis is also valid for cartesian geometry. In writing (18), the term $\nabla(\nabla.E)$ has been neglected (WKB approximation), which is justified as long as $\left(\omega_{p0}^2/\omega_0^2\right)(1/\varepsilon_{\rm cf})\ \ln\ \varepsilon_{\rm cf} \ll 1$. On expressing the solution of (18) as $E_0' = A_0^0(\mathrm{r}, \mathrm{z})$ $\exp(-ik_0 z)$, where $k_0 = \frac{\omega_0}{c}\sqrt{\varepsilon_c}$, the following equation is obtained

$$-2ik_0\frac{\partial A_0^0}{\partial z} + \frac{\partial^2 A_0^0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0^0}{\partial r} + \frac{k_0^2}{\varepsilon_0}\left[\phi_0(A_0^0 A_0^{0*})\right] \cdot A_0^0 = 0. \tag{19}$$

The term $\frac{\partial^2 A_0^0}{\partial z^2}$ is omitted when compared to the term $-2ik_0\frac{\partial A_0^0}{\partial z}$. This implies that the characteristic distance of field variation along the z direction is much smaller than the wavelength. This equation also applies for the case of quadratic nonlinearity and (17) can be made to match this form by expanding the exponential and neglecting higher order nonlinear terms in electric field intensity. In quadratic nonlinearity case, it can be exactly solved by the inverse scattering transform method of soliton theory (Zakharov et al. 1972). However, for arbitrary nonlinearity, or if more than one wave is involved (for example the coupling of exogenous and endogenous fields), this soliton method does not work. In this case, (19) can be solved as follows. We express (Akhmanov et al. 1968):

$$A_0^0 = A_0(r, z)\exp[-i(k_0 S_0(r, z))] \tag{20}$$

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Here, S_0 is the additional eikonal and $A_0(r,z)$, $S_0(r,z)$ are real functions of r and z. Substituting A_0^0 from (20) in (19) and separating the real and imaginary parts, we obtain

$$2\frac{\partial S_0}{\partial z} + \left(\frac{\partial S_0}{\partial r}\right)^2 = \frac{\phi_0}{\varepsilon_c} + \frac{1}{k_0^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right), \tag{21}$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S_0}{\partial r}\frac{\partial A_0^2}{\partial r} + A_0^2\left(\frac{\partial^2 S_0}{\partial r^2} + \frac{1}{r}\frac{\partial S_0}{\partial r}\right) = 0. \tag{22}$$

The solutions of (21) and (22) can be written as follows (Akhmanov et al. 1968):

$$S_0 = \frac{r^2}{2}\frac{1}{f_0}\frac{df_0}{dz} + \Phi_0(z), \tag{23}$$

$$A_0^2 = \frac{E_0^2}{f_0^2} F(r/r_0 f_0). \tag{24}$$

Here $F(r/r_0 f_0)$ is the general function which characterizes the intensity of the exogenous field at the entrance level at $z = 0$ as a function of the radial coordinate (at $z = 0, f_0 = 1$ but for z greater than zero, the value of f_0 is different from 1). Substituting for S_0 and A_0^2 from (23) and (24) in (22), it reduces to

$$\frac{\partial A_0^2}{\partial z} + r\left(\frac{1}{f_0}\frac{df_0}{dz}\right)\frac{\partial A_0^2}{\partial r} + 2\left(\frac{1}{f_0}\frac{df_0}{dz}\right)A_0^2 = 0. \tag{25}$$

Using (23)–(25), one can numerically calculate the intensity profiles of the exogenous field in the cells by using the appropriate boundary conditions and initial field profile. In order to have an appreciation of the field profile in the brain, for convenient analysis a Gaussian profile is chosen although the exogenous filed in the brain tissue may not be strictly Gaussian.

3. Implications of the linear and nonlinear mechanisms

Several implications are possible on account of the above-mentioned changes in the dielectric constant profile and consequently on the exogeneous EM field. The dielectric constant is made up of two parts, linear and nonlinear (17). In the linear part using (9) and using the relation $\omega_{p0}^2 = \frac{N_0 e^2}{m_e \varepsilon_0}$ in $k_0 = \frac{\omega_0}{c}\sqrt{\varepsilon_c}$, a variation of k_0 can be obtained with N_0. Figure 5 illustrates the variation of wave number k_0 with electron density N_0 for white (solid) and grey (dash-dot) matter at various frequencies in the range of 500 MHz to 2 GHz. It is seen from the above two relations that the wavelength depends on the dielectric constant, which, in turn, depends on the number density. If the number density is reduced the wavelength changes accordingly. This, in turn, affects the properties of the exogenous field in the cell. It is found that k_0 linearly decreases with N_0. N_0 has been calculated for a range within which ω_p^2/ω_0^2 vary between 0.4 and 4.9 ($\omega_0 = 2\pi \times 10^9$ Hz). The dash-dot curves in Figs 1 and 2 illustrate the variation of the relative dielectric constant with frequency from 500 MHz to 2 GHz with electronic contribution to the real part of the dielectric constant as determined from the four-term Debye model. The graphs indicate that the results for both white and grey matter with this

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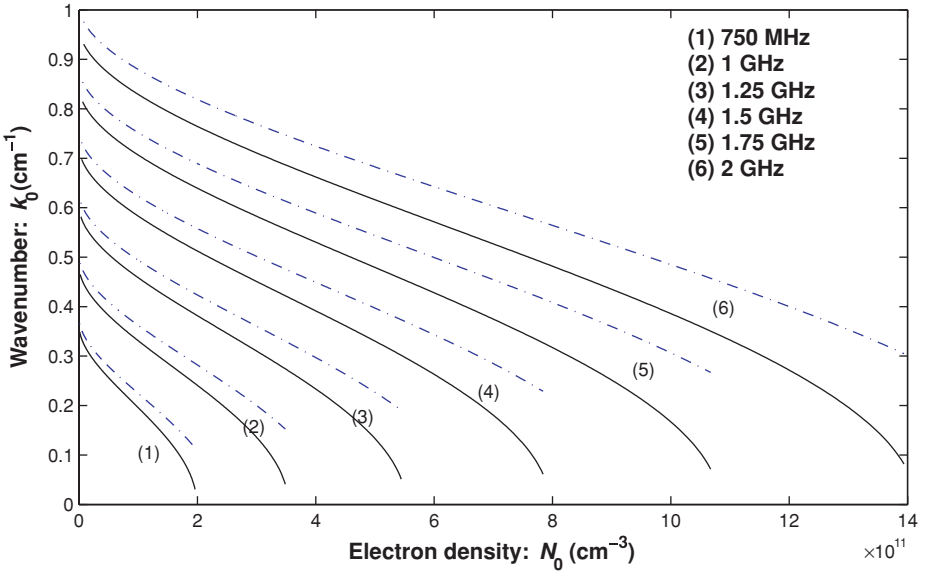


Figure 5. Variation of wave number k_0 with electron density N_0 for white (solid) and grey (dash-dot) matter.

correction are closer to the experimental variation obtained from four Cole–Cole analysis. For both the graphs, the peak specific absorption rate (SAR) value is assumed to be 1.2 W/kg. Figures 9 and 10 illustrate the variation of skin depth for white and grey matter in the frequency range of 500 MHz to 2 GHz. From (17) the dependence of the nonlinear part of the dielectric constant can be studied. It is seen that the magnitude of the nonlinear part, ϕ, depends on the number density, frequency of the exposure field and the power flux. The value of ϕ has been calculated for the following set of parameters: $\omega_0 = 2\pi \times 10^9$ Hz, $Te = T_i = 300$ K and $\omega_{p0}^2/\omega_0^2 = 4.8$, 3.2 and 1.61. Though the magnitude of ϕ is small and ϕ/ε_c is 0.027 for $\varepsilon_r = 5$, powerflux $= 10\,\mathrm{mW\,cm^{-2}}$ and $\omega_{p0}^2/\omega_0^2 = 4.8$, but it may have important consequences. Figure 6 shows the variation of ϕ with power flux. In Fig. 6 variation of ϕ at possible higher power flux has also been shown. It is seen that it may lead to saturation as for higher values of power, electron density N will be depleted in the cellular media. Point D in the graph, where log(powerflux) $= 6.9126$, corresponds to the peak SAR value of 1.2 W/kg for the solid curve. As mentioned above, though the value of ϕ is very small, it may have several important consequences. Without going into the mathematical details, some of them can be understood as follows.

One of the consequences of these nonlinear phenomena is that the modulation index of the exogenous field is changed while propagating in biological tissue. This is on account of the fact that different parts of the modulated signal envelope after propagating in this nonlinear medium contribute differently for nonlinear properties of $\varepsilon_{\mathrm{cf}}$. Therefore, the resultant envelope will be distorted due to these processes and the depth of the modulation will change. It is possible that this idea could also be exploited for diagnostic purposes.

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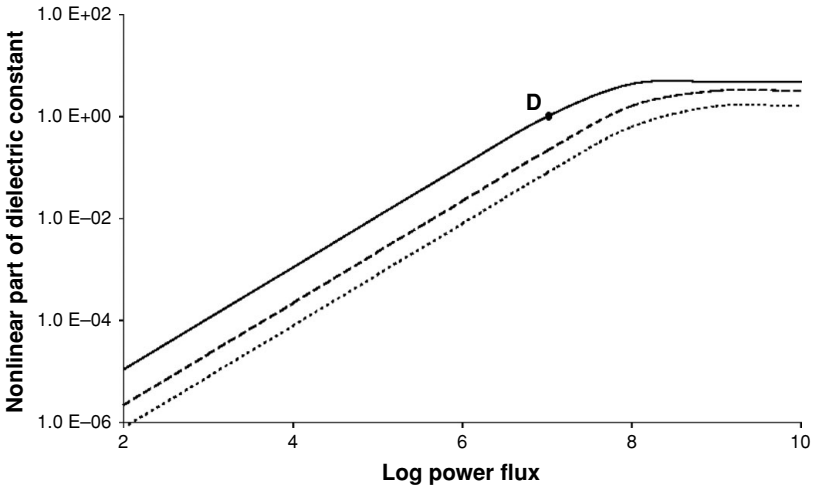


Figure 6. Variation of nonlinear part of dielectric constant, ϕ with power flux for different values of ω_{p0}^2/ω_0^2. Solid curve, dotted curve and semi-dotted curves are for $\omega_{p0}^2/\omega_0^2 = 4.8, 3.2, 1.61$, respectively, when $\omega_0 = 2\pi \times 10^9$ Hz, $\varepsilon_r = 5$.

Secondly, on account of the nonlinearity in the medium, the exogeneous field may also break up into filamentary structures as a high-power laser beam propagates in the nonlinear medium (Akhmanov et al. 1968). For typical parameters, the threshold for this can be calculated following Akhmanov et al. (1968). Growth rates and thresholds for the stimulated scattering and modulational instabilities for a homogeneous magnetized plasma taking into account the effect of nonlinear current density have also been investigated by Yu et al. (1974). For typical exogenous field and cell parameters ($\omega_0 = 2\pi \times 10^9$ Hz, for a transverse scale size, $r_0 = 1.2\lambda_0$, $T_e = T_i = 300$ K and $\omega_{p0}^2/\omega_0^2 = 4.8$), the critical field ($E_{0cr}$) is of the order of 9.0167 V/m and the corresponding power flux density is of the order of 10 mW cm^{-2}. This is of the order of the typical power flux density incident on the head of a mobile phone user, using current equipment. When the incident field exceeds this value, division into filamentary structures in the cell material can occur. From Table 1 for normalized electric field of 19.10, variation of f with normalized z illustrates two filaments with sizes of 0.0134 m and 0.012 m (for white matter) and 0.0092 m and 0.011 m (for grey matter) respectively (r_0 is of the order of 0.01 approximately). Figure 7 illustrates the variation of the axial exogenous field amplitude with distance of propagation in the human head from 0 to 9.0 cm for quadratic nonlinearity (first-order approximation to ponderomotive nonlinearity) for white and grey matter. Figure 8 illustrates the transverse field profiles in white matter and grey matter obtained numerically solving (25).

The possible resulting biological effects in the head of a mobile telephone user may now be briefly discussed. Specifically, the effect on binding of a ligand ion to a receptor protein in the presence of incident exogenous field, with and without the filamentary structures, will be considered. In the absence of filamentary structures it has been shown in Bianco et al. (1997) that at a typical EM exposure flux of around 10 mW cm^{-2}, the relative change (time average) in the binding probability

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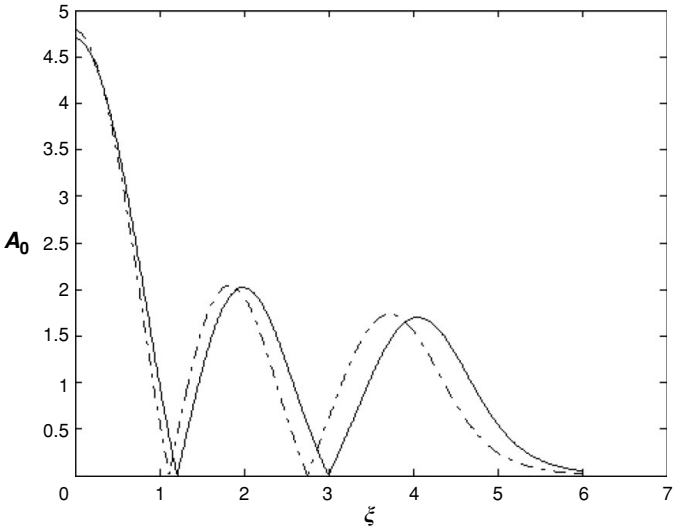


Figure 7. Variation of the axial field amplitude for the exogenous field with transverse variable assuming cylindrical symmetry in the human head for white matter (dash-dot) and grey matter (solid). Numerical solution for (25) for ponderomotive saturating nonlinearity.

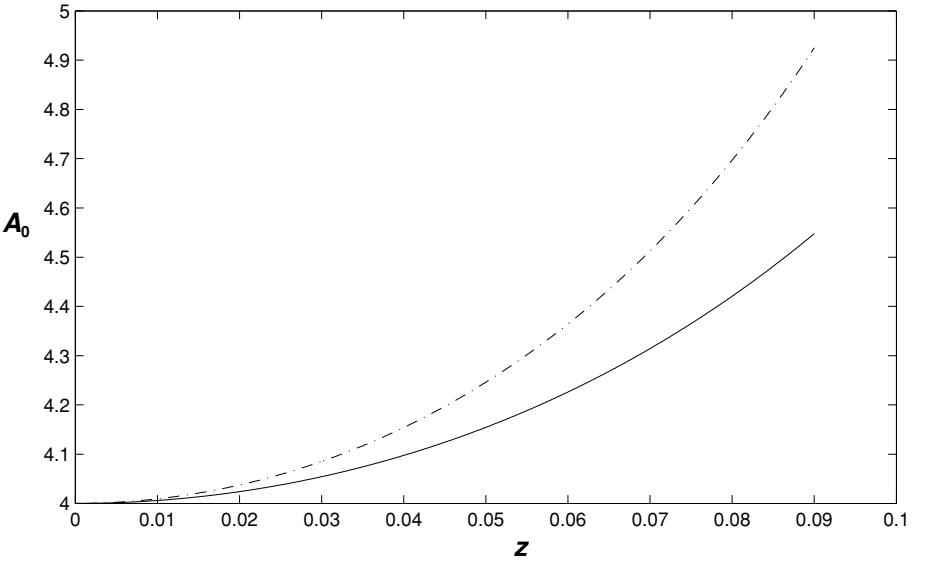


Figure 8. Variation of the axial field amplitude (in V m^{-1}) for the exogenous field with distance of propagation (in meters) in the human head (maximum size 14 cm) from 0 to 9 cm for quadratic nonlinearity for 1-GHz frequency. Starting field $E = 4\,\mathrm{V\,m^{-1}}$. Threshold field $= 1.3\,\mathrm{V\,m^{-1}}$.

is around 9%. In the presence of the filamentary structures and with a similar incident flux density, this value will be around 12% in the filaments (assuming a value of $f_0 = 0.65$). Therefore, an enhancement factor of about 33% in this binding probability is predicted in the filaments, as compared to the case without

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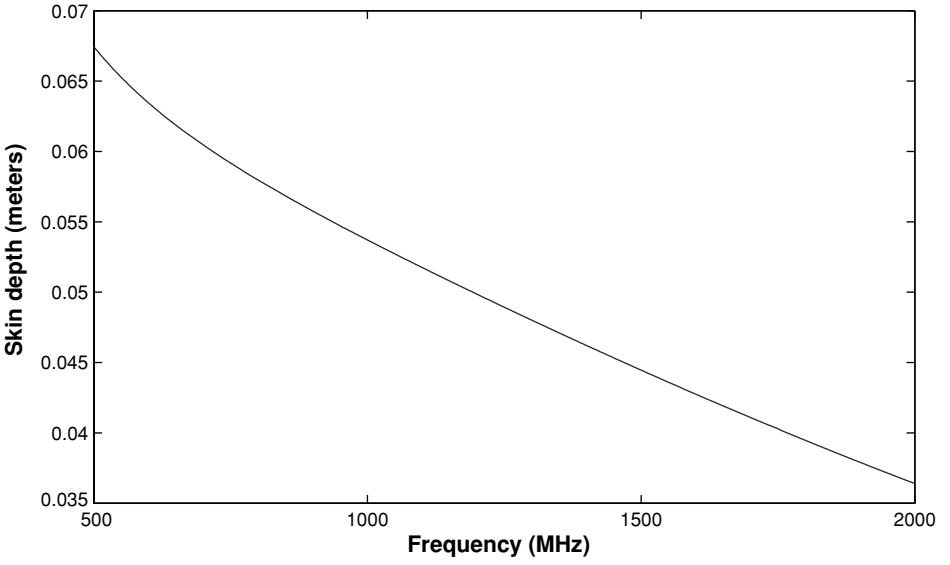


Figure 9. Variation of skin-depth in meters with frequency in the range of 500 MHz to 2 GHz for white matter using the three-term Debye model.

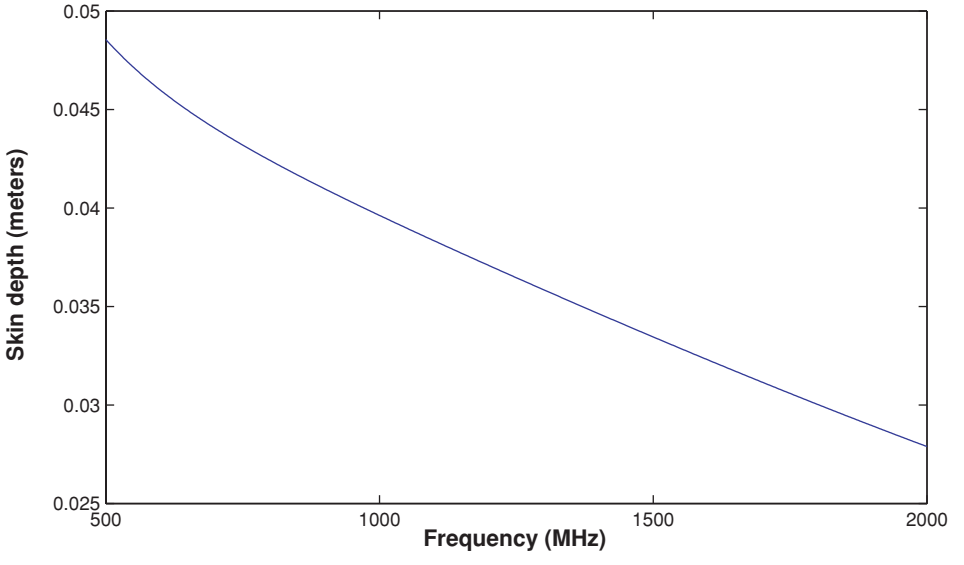


Figure 10. Variation of skin-depth in meters with frequency (500 MHz to 2 GHz) for grey matter using the three-term Debye model.

filamentation. In any case, the nonlinear mechanism described here is capable of causing demodulation of an exogenous field, even in the absence of the putative endogenous field.

In addition, the physical process associated with the diffusion of ions in the presence of nonlinear forces, and governed by the diffusion equation ((16)), may

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have significant consequences. Because this diffusion process is transient in nature and depends upon the field intensities, the binding process is expected to be complex; this forming the future work.

Finally, mention may be made of the relevance of the present investigation to a recent experiment to study the effects of low-intensity radio frequency EM fields on electrical activity in rat hippocampal slices. EM exposures of fields at 700 MHz have produced significant changes in evoked and spontaneous electrical activity in the hippocampal slices in the absence of any detectable increase in temperature (Tattersall et al. 2001). Tattersall's more recent experiments (Tattersall et al. 2009) have already demonstrated that the effects produced can be explained by localized heating by short pulses caused by the interaction of the EM fields with the biological tissue thus affecting neurotransmission in the brain tissue.

They have proposed a possible source of these changes through mechanisms that convert the incident radio frequency field into a much lower frequency signal, which may be capable of influencing the neuronal excitability. Without going into much detail, the conversion of exposure field into a much lower frequency signal can also be explained by using (10). On account of the beating of two sidebands of an amplitude-modulated signal (exposure field), (10) gives a nonlinear force at beat wave frequency $\omega_1 - \omega_2$ (here $\omega_1 \approx \omega_0, \omega_2 \approx \omega_0 \pm \Omega, \Omega$ being the modulation frequency). This generates a low-frequency signal as required in the experiment of Tattersall et al. (2001) Moreover, in the Tattersall experiment, the incident wavefront is essentially planar at the brain slice exposure area, so $r_0 > \lambda_0$. Assuming the parameters $\omega_0 = 2\pi \times 10^8$ rad s^{-1}, $r_0 = 1.2\lambda_0$, $T_e = T_i = 300$ K and $\omega_{p0}^2/\omega_0^2 = 4.8$, the critical field for filamentation is found to be $E_{0\mathrm{cr}} = 0.5 \times 10^{-4}$ V cm^{-1} for a power flux of 0.00176 mW cm^{-2}. This is in the range of the power flux used by Tattersall et al. (2001).

4. Conclusions

The linear and nonlinear contribution to the effective dielectric constant of cellular medium may affect the wavelength of the exposure field depending upon the value of ω_{p0}^2/ω_0^2 and hence the electron density (N_0). The nonlinear part of the dielectric constant may lead to the distortion of the signal (demodulation effects), the filamentation process and the conversation to a low-frequency signal. These effects may have biological consequences. Some of these effects like the measurement of ε (linear and nonlinear) for different biological tissues should not be difficult to measure. Therefore, biological experimentation should not be difficult to test some of the formulae derived here.

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