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Localized coherent nonlinear wave structures in dusty plasma with non-thermal ions

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Abstract. In the present research paper, the characteristics of dust-acoustic solitary waves (DASWs) and double layers (DLs) are studied. Ions are treated as non-thermal and variable dust charge is considered. The Korteweg–de Vries equation is derived using a reductive perturbation method. It is noticed that compressive solitons are obtained up to a certain range of relative density δ $(= n_{i0}/n_{e0})$ beyond which rarefactive solitons are observed. The study is further extended to investigate the possibility of DLs. Only compressive DLs are permissible. Both DASWs and DLs are sensitive to variation of the non-thermal parameter.

1. Introduction

The past decade has witnessed a rapid growth in the field of dusty plasmas. In a dusty plasma, a third charged species with diameter ranging from nanometers to several hundred micrometers is present. The history, occurrence and characterization of dusty plasmas in space and laboratory environments are well described and documented in recent publications [1, 2]. Collective processes, such as low-frequency modes in dusty plasma, have received a great deal of attention over the last 15 years. The presence of charged dust particles introduces many interesting features to collective processes and modifies the propagation of waves, instabilities, etc. in a dusty plasma. Many low-frequency modes have been investigated, in particular dust ion-acoustic waves (DIAWs) [3, 4], dust-acoustic waves (DAWs) [5], dust-Coulomb waves (DCWs) [6], dust-lattice waves [7], etc. These are the normal modes of unmagnetized dusty plasmas and have been described in detail in the literature [1, 2]. In most of the normal modes studied the variation of charge has not been taken into account. Varma et al. [8] were the first to study electrostatic oscillations in the presence of grain-charge perturbations in dusty plasmas.

The linear theory of waves, based on the small-amplitude approximation, breaks down completely when the amplitude becomes large. At this stage, nonlinear effects play an important role, leading to the formation of coherent nonlinear wave structures. The presence of nonlinearity, dispersion and dissipation when adequately balanced manifest themselves in the formation of coherent nonlinear wave structures such as solitons, shock waves, double layers (DLs), vortices, etc. The most widely

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investigated of these structures are dust-acoustic solitons (DASs) and dust-acoustic solitary waves (DASWs), for which the methodology and physics issues are well presented in a recent book by Shukla and Mamun [1]. Some form of the reductive perturbation technique is used to derive a Korteweg–de Vries (KdV) or nonlinear Schrödinger equation (NLSE). In most of the investigations the KdV model has been used to study DASs and DASWs. However, Amin et al. [9] used a reductive perturbation technique to derive a NLSE and studied the modulational instability of DAWs and DIAWs. For the sake of simplicity, dust was considered as cold with a constant charge on it. The introduction of a finite dust temperature has been considered in recent investigations [10, 11]. However, it has been pointed out that the dust charge should be considered as a dynamical variable owing to a variety of mechanisms [12, 13]. This will result in the modification of the collective properties of a dusty plasma. Ignoring the dust temperature, Xie et al. [12] studied DASWs and DLs in a dusty plasma with variable dust charge and ions of two temperature with Boltzmann distributions. However, it has been found that electron and ion distributions play a crucial role in the formation of nonlinear structures. Several investigations have recently been reported in plasmas with non-Maxwellian particle distributions [14–22]. Gill et al. [23] studied the properties of ion-acoustic solitons and DLs in a plasma consisting of warm positive and negative ions with different mass concentrations, charged states and non-thermal electrons.

Among the nonlinear structures, DLs are of considerable importance because of their relevance to cosmic applications [24–28]. In space plasmas they are considered as a source of the Earth's aurora as well as being responsible for auroral electron precipitation. Furthermore, DLs provide a mechanism for the acceleration of a charged particle. Among the other nonlinear structures such as solitons, DLs are the least studied [29–32]. Recently, DLs and DASs in dusty plasmas have attracted a great deal of attention [12, 23, 33–37]. Several recent reviews on these nonlinear wave structures can be found in the literature [38, 39].

Numerical simulations have established beyond doubt that particles may follow distributions other than a Maxwellian distribution. As mentioned above, the electron and the ion distributions play a crucial role in characterizing the physics of the wave structures. They offer a considerable increase in the richness and variety of wave motion that can exist in a plasma and, furthermore, significantly influence the conditions required for the formation of solitons and DLs. Clearly there is a departure from Boltzmann distributions. Several investigations have explored the role played by such non-Maxwellian distributions on solitons in dusty as well as multi-species plasmas. With the observation of solitary wave structures with a density depression, the role of non-thermal electron distributions on the characterization of solitary wave/solitons was reported in [40–42]. Non-thermal distributions are a common feature of the auroral zone [43, 44] and yet the mechanism for the formation of non-thermal particle distributions in space plasmas is still a central problem. The aim of the present investigation is to study the effect of non-thermal ion distributions on the solitary wave behavior as well as DLs in a dusty plasma with dust charge considered as a variable.

The organization of the paper is as follows. Section 2 gives the basic equations governing dust fluids, electron and ion distributions, as well as dust charge dynamics. In Sec. 3, we use a reductive perturbation method to derive a KdV equation and in Sec. 4 we derive the modified KdV (mKdV) equation. Section 5 deals with the description of DLs and Sec. 6 concludes the paper.

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2. Basic equations

We consider a dusty plasma with extremely massive, highly negatively charged dust grains, electrons and non-thermal ions. Charge conservation implies the following equilibrium equation:

$$n_{i0} = Z_{d0} n_{d0} + n_{e0}, \tag{1}$$

where n_{i0}, n_{e0} and n_{d0} are the unperturbed ions, electrons and dust number densities, respectively, and Z_{d0} is the unperturbed number of charges on the dust grain.

The following equations are the normalized continuity, momentum and Poisson's equations that govern the dynamics of the dusty plasma:

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = Z_d \frac{\partial \phi}{\partial x}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = Z_d n_d + n_e - n_i, \tag{4}$$

where n_d, u_d, ϕ and Z_d are the dust number density, dust particle velocity, electrostatic potential and the variable charge number of dust grains, respectively.

For the sake of convenience, we use here the procedure of Xie et al. [12]. The electrons and ions obey the Boltzmann distribution and non-thermal distribution, respectively, as follows:

$$n_e = \frac{n_{e0}}{Z_{d0} n_{d0}} \exp(\beta_1 \phi), \tag{5}$$

$$n_i = \frac{n_{i0}}{Z_{d0} n_{d0}} (1 + \beta\phi + \beta\phi^2) \exp(-\phi). \tag{6}$$

Parameters β_1 and β are defined, respectively, as follows:

$$\beta_1 = \frac{T_i}{T_e}, \quad \beta = \frac{4\alpha}{(1 + 3\alpha)}. \tag{7}$$

In (2), (3) and (4), the dust charge density n_d, dust particle velocity u_d, variable charge number of dust grain Z_d, distance x and time t are normalized by n_{d0}, the acoustic speed $c_d = \sqrt{(T_i Z_{d0}/m_d)}$, Z_{d0}, the Debye length $\lambda_D = \sqrt{(T_i/4\pi Z_{d0} n_{d0} e^2)}$ and the inverse of the plasma frequency $\omega_{pd}^{-1} = \sqrt{(m_d/4\pi n_{d0} Z_{d0}^2 e^2)}$, respectively. The electrostatic potential ϕ is normalized by T_i/e and α is the non-thermal parameter. We have followed the model of Cairns et al. [40,41], where ions are assumed to follow a non-thermal particle distribution. The parameter α in the distribution function $f_i(v_i)$ defines the shape of the distribution and expresses the deviation from the Maxwellian state. This form of the distribution is convenient for the description of various observed particle distributions. For example, when $\alpha = 0$ we obtain a Maxwellian distribution and when $\alpha \to 1$ the distribution resembles two counter streaming beams with a cold core distribution [42].

The dust charge variable Q_d is governed by the charge current balance equation [7]

$$\left(\frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x} \right) Q_d = I_e + I_i. \tag{8}$$

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It must be noted that the charging currents originate from the electrons and ions reaching the grain surface. The dust charging time is of the order of 10^{-9} s, while the dust motion time is of the order of 10^{-3} s. Thus, the dust charge can quickly reach the local equilibrium at which the currents from the electrons and ions to the dust grains are balanced. The current balance equation implies

$$I_{\mathrm{e}} + I_{\mathrm{i}} \approx 0. \tag{9}$$

We further suppose that the streaming velocities of electrons and ions are much smaller than the thermal velocities. The electron and ion currents are

$$\begin{aligned} I_{\mathrm{e}} &= -e\pi r^2 \left(\frac{8T_{\mathrm{e}}}{\pi m_{\mathrm{e}}}\right)^{1/2} n_{\mathrm{e}} \exp\left(\frac{e\Phi}{T_{\mathrm{e}}}\right), \\ I_{\mathrm{i}} &= e\pi r^2 \left(\frac{8T_{\mathrm{i}}}{\pi m_{\mathrm{i}}}\right)^{1/2} n_{\mathrm{i}} \left(1 - \frac{e\Phi}{T_{\mathrm{i}}}\right), \end{aligned} \tag{10}$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ. From the current balance equation and following Xie et al. [12], we have

$$\alpha_1 \delta (1 + \beta\phi + \beta\phi^2) \exp(-\phi)(1 - \psi) - \exp(\beta_1(\phi + \psi)) = 0, \tag{11}$$

where $\psi = e\Phi/T_{\mathrm{i}}$, $\alpha_1 = \sqrt{\beta_1/\mu_{\mathrm{i}}}$, $\delta = n_{\mathrm{i}0}/n_{\mathrm{e}0}$ and the mass ratio of ions to electrons is by $\mu_{\mathrm{i}} = m_{\mathrm{i}}/m_{\mathrm{e}} \simeq 1843$. By using (11) we can calculate the dust charge $Q_{\mathrm{d}} = C\Phi$, where C is the capacitance of the dust grain ($C = r$) and

$$Z_{\mathrm{d}} = \frac{\psi}{\psi_0} \tag{12}$$

where $\psi = \psi_0$ at $\phi = 0$ is the dust surface floating potential with respect to the unperturbed plasma potential at an infinite distance from the dust grain surface. In (11), by substituting $\phi = 0$ to determine the value of ψ_0, we have

$$\alpha_1 \delta (1 - \psi_0) - \exp(\beta_1 \psi_0) = 0. \tag{13}$$

Now, by expanding ψ near ψ_0, $Z_{\mathrm{d}1}$ from (12) can be obtained as follows:

$$Z_{\mathrm{d}1} = \gamma_1 \phi_1, \quad \gamma_1 = \frac{\gamma_{\mathrm{b}}}{\gamma_{\mathrm{a}} \psi_0}, \tag{14}$$

where

$$\gamma_{\mathrm{a}} = [\alpha_1 \delta + \beta_1 \exp(\beta_1 \psi_0)]$$

and

$$\gamma_{\mathrm{b}} = -[\beta_1 \exp(\beta_1 \psi_0) - \alpha_1 \delta (1 - \psi_0)(\beta - 1)].$$

Similarly, $Z_{\mathrm{d}2}$ can be obtained as follows:

$$Z_{\mathrm{d}2} = \gamma_1 \phi_2 + \gamma_2 {\phi_1}^2, \quad \gamma_2 = \frac{\gamma_{\mathrm{c}}}{\gamma_{\mathrm{a}} \psi_0}, \tag{15}$$

with

$$\gamma_{\mathrm{c}} = \gamma_{\mathrm{c}1} + \gamma_{\mathrm{c}2} + \gamma_{\mathrm{c}3},$$

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where

$$\gamma_{c1} = \tfrac{1}{2}[\alpha_1\delta(1-\psi_0) - {\beta_1}^2\exp(\beta_1\psi_0)],$$

$$\gamma_{c2} = \psi_0\gamma_1[\alpha_1\delta(1-\beta) - {\beta_1}^2\exp(\beta_1\psi_0)],$$

$$\gamma_{c3} = -\tfrac{1}{2}(\psi_0\gamma_1\beta_1)^2\exp(\beta_1\psi_0).$$

The value of ψ_0 is always negative for the dust grains that are negatively charged, so the condition

$$1 \geqslant \alpha_1\delta \tag{16}$$

must be satisfied.

3. Small-amplitude solitary waves

A self-consistent dust charge variation with plasma potential would have considerable influence on the small amplitude as well as large-amplitude solitary waves. Here we first study the small-amplitude solitary waves with non-thermal ions. As mentioned earlier, a reductive perturbation method will be used here. In order to investigate the properties of small-amplitude DASWs, we construct a weakly nonlinear theory of DAWs that leads to a scaling of the independent variable through the stretching coordinates as follows:

$$\xi = \epsilon^{1/2}(x - v_0 t) \quad \text{and} \quad \tau = \epsilon^{3/2}t,$$

where ϵ is a small parameter measuring the weakness of dispersion and v_0 is the solitary wave velocity. To strike an appropriate balance between nonlinearity and dispersion terms the dependent variables $n_{\rm d}$, $u_{\rm d}$, ϕ and $Z_{\rm d}$ are expanded from equilibrium values in the power of ϵ as

$$\begin{aligned} n_{\rm d} &= 1 + \epsilon n_{\rm d1} + \epsilon^2 n_{\rm d2} + \cdots, \\ u_{\rm d} &= \epsilon u_{\rm d1} + \epsilon^2 u_{\rm d2} + \cdots, \\ \phi &= \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots, \\ Z_{\rm d} &= 1 + \epsilon Z_{\rm d1} + \epsilon^2 Z_{\rm d2} + \cdots. \end{aligned} \tag{17}$$

Using stretching coordinates and inserting (5), (6) and (17) into (2)–(4), and equating the same powers of ϵ, we obtain a set of equations as follows:

$$n_{\rm d1} = -\frac{\phi_1}{v_0^2}, \quad u_{\rm d1} = -\frac{\phi_1}{v_0} \quad \text{and} \quad Z_{\rm d1} = \gamma_1\phi_1, \tag{18}$$

with

$$\frac{1}{v_0^2} = \gamma_1 + \frac{\beta_1 + \delta(1-\beta)}{\delta - 1}.$$

To next order of ϵ, we obtain

$$v_0\frac{\partial n_{\rm d2}}{\partial\xi} - \frac{\partial n_{\rm d1}}{\partial\tau} - \frac{\partial u_{\rm d2}}{\partial\xi} - \frac{\partial(n_{\rm d1}u_{\rm d1})}{\partial\xi} = 0, \tag{19}$$

$$v_0\frac{\partial u_{\rm d2}}{\partial\xi} - \frac{\partial u_{\rm d1}}{\partial\tau} - u_{\rm d1}\frac{\partial u_{\rm d1}}{\partial\xi} = -\frac{\partial\phi_2}{\partial\xi} - Z_{\rm d1}\frac{\partial\phi_1}{\partial\xi}, \tag{20}$$

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$$\frac{\partial^2\phi_1}{\partial\xi^2} = n_{\mathrm{d}2} + Z_{\mathrm{d}1}n_{\mathrm{d}1} + Z_{\mathrm{d}2} + \phi_2\left(\frac{n_{\mathrm{e}0}}{n_{\mathrm{d}0}Z_{\mathrm{d}0}}\beta_1 + \frac{n_{\mathrm{i}0}}{n_{\mathrm{d}0}Z_{\mathrm{d}0}}(1-\beta)\right)$$

$$+ \frac{\phi_1^2}{2}\left(\frac{n_{\mathrm{e}0}}{n_{\mathrm{d}0}Z_{\mathrm{d}0}}\beta_1^2 - \frac{n_{\mathrm{i}0}}{n_{\mathrm{d}0}Z_{\mathrm{d}0}}\right). \tag{21}$$

After some algebraic manipulation and eliminating all the second-order terms, we obtain the following KdV equation:

$$\frac{\partial\phi_1}{\partial\tau} + A\phi_1\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \tag{22}$$

where

$$A = B\left(\frac{3\gamma_1}{v_0^2} - \frac{3}{v_0^4} - \frac{\beta_1^2-\delta}{\delta-1} - 2\gamma_2\right) \quad \text{and} \quad B = \frac{v_0^3}{2}.$$

The coefficients A and B are the coefficients of nonlinear and dispersive terms, respectively. On introducing the new variable $\chi = \xi - u_0\tau$, u_0 is a constant velocity, and the soliton solution of (22) in the stationary frame is given as

$$\phi_1 = \phi_{\mathrm{m}}\ \mathrm{sech}^2\left[\frac{(\xi-u_0\tau)}{w}\right], \tag{23}$$

where $\phi_m = 3u_0/A$ and $w = 2\sqrt{B/u_0}$ are the soliton amplitude and width, respectively.

The form of A suggests that it is a function of a number of parameters such as $\gamma_1, v_0, \beta_1, \gamma_2$, etc. These parameters further depend on the temperature of electrons and ions, non-thermality, the relative density of ions to electrons and other entries characterizing dusty plasma in a complicated way. Thus, nonlinearity explicitly depends on a number of variables. However, dispersion is a simple function of v_0. Thus, we can strike a delicate balance between nonlinearity and dispersion for a certain range of parameters to obtain a soliton solution given by (23). We have performed such a characterization by performing a numerical computation of ϕ_m as a function of relative density $\delta\ (= n_{\mathrm{i}0}/n_{\mathrm{e}0})$ and the results are shown in Fig. 1. Other parameters have been taken as $u_0 = 1$ and $\beta_1 = 0.001$. Compressive solitons are observed up to a certain range of δ beyond which rarefactive solitons are obtained. The peak amplitude shifts with non-thermality and reverses its sign with increase in δ. Furthermore, there is a singularity for all values of β. All these features are succinctly observed in Fig. 1.

4. Derivation of the modified KdV equation

The singularity occurs when the coefficient of the nonlinear term of (22) is zero, as shown in Fig. 1. For this case the KdV equation (22) is no longer valid. We derive a modified (mKdV) equation that governs the dynamics of small-amplitude DLs when A becomes zero. In such cases we must include higher-order nonlinearity. For this purpose we also include higher-order nonlinear effects, which are important in the study of DLs. This demands the introduction of new stretching coordinates

$$\xi = \epsilon(x - v_0t) \quad \text{and} \quad \tau = \epsilon^3 t,$$

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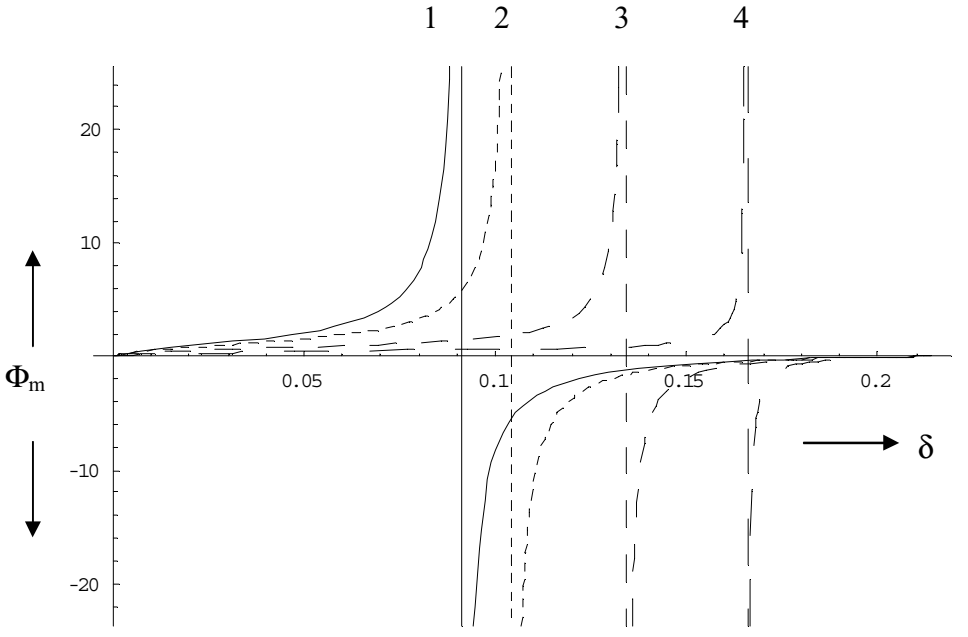


Figure 1. Variation of peak amplitude ϕ_m in the KdV equation with δ for different values of β and with $u_0 = 1$ and $\beta_1 = 0.001$. Curve 1 corresponds to $\beta = 0.0$, curve 2 to $\beta = 0.01$, curve 3 to $\beta = 0.3$ and curve 4 to $\beta = 0.5$.

where ϵ is a small expansion parameter. Using the above stretching coordinates and substituting (17) into (2)–(4), and comparing the coefficients of ϵ^3, we have

$$n_{\mathrm{d}2} = \frac{u_{\mathrm{d}2}}{v_0} + \frac{u_{\mathrm{d}1}^2}{v_0^2}, \tag{24}$$

$$v_0 \frac{\partial u_{\mathrm{d}2}}{\partial \xi} - u_{\mathrm{d}1} \frac{\partial u_{\mathrm{d}1}}{\partial \xi} = z_{\mathrm{d}1} v_0 \frac{\partial u_{\mathrm{d}1}}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi}, \tag{25}$$

and

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = z_{\mathrm{d}1} n_{\mathrm{d}2} + z_{\mathrm{d}2} n_{\mathrm{d}1} + z_{\mathrm{d}3} + n_{\mathrm{d}3} + \phi_3 \left(\frac{n_{\mathrm{e}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} \beta_1 - \frac{n_{\mathrm{i}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} (\beta - 1) \right)$$
$$+ \phi_1^3 \left(\frac{n_{\mathrm{e}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} \frac{\beta_1^3}{6} + \frac{n_{\mathrm{i}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} \frac{1}{2} \left(\beta + \frac{1}{3} \right) \right) + \phi_1 \phi_2 \left(\frac{n_{\mathrm{e}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} \beta_1^2 - \frac{n_{\mathrm{i}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} \right). \tag{26}$$

Furthermore, comparing the coefficients of ϵ^4, we obtain

$$-v_0 \frac{\partial n_{\mathrm{d}3}}{\partial \xi} + \frac{\partial n_{\mathrm{d}1}}{\partial \tau} + \frac{\partial u_{\mathrm{d}3}}{\partial \xi} + \frac{2}{v_0} \frac{\partial (u_{\mathrm{d}1} u_{\mathrm{d}2})}{\partial \xi} + 3 \frac{u_{\mathrm{d}1}^2}{v_0^2} \frac{\partial u_{\mathrm{d}1}}{\partial \xi} = 0, \tag{27}$$

$$\frac{\partial u_{\mathrm{d}1}}{\partial \tau} - v_0 \frac{\partial u_{\mathrm{d}3}}{\partial \xi} + \frac{\partial (u_{\mathrm{d}1} u_{\mathrm{d}2})}{\partial \xi} = \frac{\partial \phi_3}{\partial \xi} + z_{\mathrm{d}1} \frac{\partial \phi_2}{\partial \xi} + z_{\mathrm{d}2} \frac{\partial \phi_1}{\partial \xi}. \tag{28}$$

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After eliminating all the second- and third-order terms, we obtain the following mKdV equation:

$$\frac{\partial \phi_1}{\partial \tau} + C\phi_1^2 \frac{\partial \phi_1}{\partial \xi} + A\frac{\partial(\phi_1 \phi_2)}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{29}$$

where

$$A = \frac{3\gamma_1 v_0}{2} - \frac{3}{2v_0} - \gamma_2 v_0^3 - B\left(\frac{n_{e0}}{n_{d0} z_{d0}}\beta_1^2 - \frac{n_{i0}}{z_{d0} n_{d0}}\right), \tag{30}$$

$$B = \frac{v_0^3}{2}, \tag{31}$$

$$C = \frac{3}{2}\left[\frac{5}{2v_0^3} - \frac{3\gamma_1}{v_0} + \frac{\gamma_1^2 v_0}{2} + \frac{4}{3}\gamma_2 v_0 - \gamma_3 v_0^3 - \frac{v_0^3}{3}\left[\frac{n_{e0}}{n_{d0} z_{d0}}\frac{\beta_1^3}{2} + \frac{3n_{i0}}{2n_{d0} z_{d0}}\left(\beta + \frac{1}{3}\right)\right]\right]. \tag{32}$$

At $A = 0$, the stationary solution of (29) can be written as

$$\phi_1 = \phi_{1m}\ \text{sech}\left[\frac{(\xi - u_0 \tau)}{L_1}\right], \tag{33}$$

where $\phi_{1m} = \sqrt{6u_0/C}$ and $L_1 = \sqrt{B/u_0}$ are the peak amplitude and width of the soliton, respectively. Solitons exist when $C > 0$.

5. Double layer solution

When $A \to 1$ but $A \neq 0$ and using $A\phi_2 \to D\phi_1/2$ [12] in (29), we obtain the following equation:

$$\frac{\partial \phi_1}{\partial \tau} + C\phi_1^2 \frac{\partial \phi_1}{\partial \xi} + D\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial \phi_1^3}{\partial \xi^3} = 0. \tag{34}$$

We present a DL solution associated with the mKdV equation. For this, we introduce a variable $\zeta = \xi - M\tau$ in a stationary frame, where M is a Mach number. After integration (34) leads to the following equation:

$$\frac{1}{2}\left(\frac{\partial \phi}{\partial \zeta}\right)^2 + V(\phi) = 0, \tag{35}$$

where $V(\phi)$ is the Sagdeev potential given by

$$V(\phi) = -\frac{M}{2}P\phi^2 + \frac{Q}{3}\phi^3 + \frac{R}{4}\phi^4. \tag{36}$$

Here,

$$P = \frac{1}{B}, \quad Q = \frac{D}{2B}, \quad R = \frac{C}{3B}. \tag{37}$$

For simplicity, we have introduced ϕ for ϕ_1. While deriving (35), we have used the following boundary conditions:

$$\phi \to 0, \quad \frac{\partial \phi}{\partial \zeta} \to 0, \quad \frac{\partial^2 \phi}{\partial \zeta^2} \to 0 \quad \text{as } |\zeta| \to \infty. \tag{38}$$

Furthermore, (35) and (36) can be considered as the equations of motion of a particle of unit mass under the action of the potential $V(\phi)$. They may also be regarded as

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the equations of anharmonic oscillation, provided ζ and ϕ are interpreted as time and space coordinates, respectively. For the DL solution, the Sagdeev potential should be negative between $\phi = 0$ and $\phi = \phi_{2m}$, where ϕ_{2m} is some extreme value of potential. Additional boundary conditions that $V(\phi)$ must satisfy for the DL solution are as follows:

$$V(\phi) = 0 \quad \text{at } \phi = 0 \quad \text{and} \quad \phi = \phi_{2m}, \tag{39}$$

$$V'(\phi) = 0 \quad \text{at } \phi = 0 \quad \text{and} \quad \phi = \phi_{2m}, \tag{40}$$

$$V''(\phi) = 0 \quad \text{at } \phi = 0 \quad \text{and} \quad \phi = \phi_{2m}. \tag{41}$$

By using these boundary conditions in (36), we obtain

$$M = -\frac{R}{2P}\phi_{2m}^2, \tag{42}$$

and

$$\phi_{2m} = -\frac{2}{3}\frac{Q}{R}. \tag{43}$$

Using (42) and (43) in (36), we obtain

$$V(\phi) = \frac{R}{4}\phi^2(\phi_{2m} - \phi)^2. \tag{44}$$

The DL solution of (36) with (44) is given by

$$\phi = \frac{\phi_{2m}}{2}\left[1 - \tanh\sqrt{-\frac{R}{8}}\phi_{2m}(\xi - M\tau)\right]. \tag{45}$$

Equation (45) implies the existence of a DL whenever R is negative. Furthermore, this characterizes the nature of the DLs. The system will support compressive and rarefactive DLs depending on the sign of Q. If Q is positive, a compressive DL exists, whereas for negative Q a rarefactive DL exists. The thickness of the DL is given by

$$d = \frac{4}{|\phi_{2m}|}\left(-\frac{2}{R}\right)^{1/2}. \tag{46}$$

Earlier investigations have shown that both compressive and rarefactive DLs are admissible in a dusty plasma [45]. We have chosen the parameters as $u_0 = 1$ and $\beta_1 = 0.001$. Figure 2 displays $V(\phi)$ versus ϕ for the above chosen set of parameters, and apparently compressive DLs are obtained. The increase in the non-thermal parameter decreases the amplitude as well as the depth of the potential well. Further increases in β leads to the disappearance of the DLs. Figures 3 and 4 display the peak amplitudes of solitons and DLs, respectively, as a function of δ $(= n_{i0}/n_{e0})$ for three values of β. An interesting result is that in both cases an increase of δ and β leads to an increase of ϕ_{1m} and ϕ_{2m}, respectively. However, the width of the DLs first increases with δ and subsequently falls with any further increase in δ. The same behavior is observed for different β but the width decreases with an increase in non-thermality, as shown in Fig. 5.

6. Conclusions

In this paper, we have studied the role of the non-thermal parameter β both on DASs and DLs in a dusty plasma with a variable dust charge and consisting of

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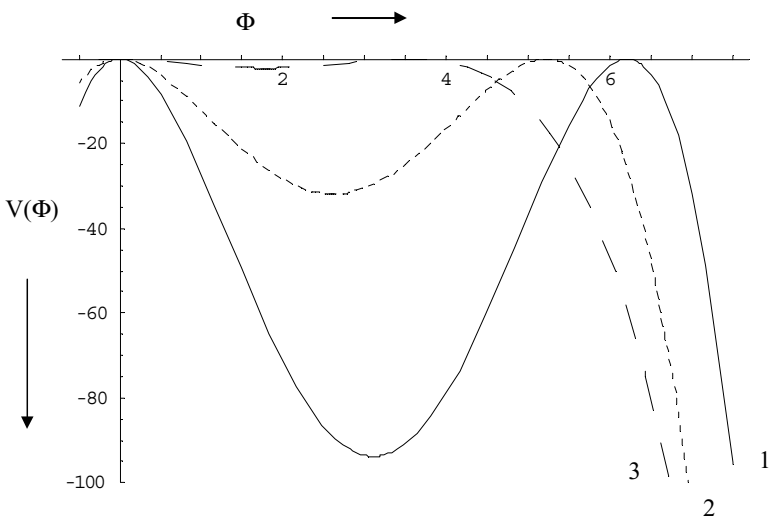


Figure 2. Variation of the quasi-potential $V(\phi)$ in the DLs with ϕ for different values of β and with $u_0 = 1$ and $\beta_1 = 0.001$. Curve 1 corresponds to $\beta = 0.0$, curve 2 to $\beta = 0.1$ and curve 3 to $\beta = 0.3$.

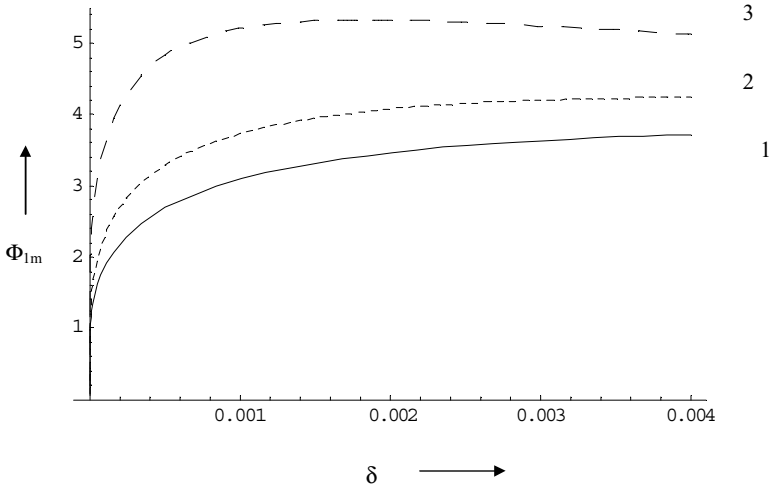


Figure 3. Variation of peak amplitude ϕ_{1m} in the mKdV equation with δ for different values of β and with $u_0 = 1$ and $\beta_1 = 0.001$. Curve 1 corresponds to $\beta = 0.01$, curve 2 to $\beta = 0.03$ and curve 3 to $\beta = 0.05$.

non-thermal ions. We have derived KdV and mKdV equations using a reductive perturbation method. The results obtained have several ramifications as follows.

(i) It is concluded that when the variable charge is introduced, then there is the possibility of both types of solitary waves for non-thermal distributions ($\beta \neq 0$). In the case of non-thermal distributions, the range of δ for a positive peak amplitude depends upon the non-thermal parameter β. It is noticed that there is the possibility of both types of solitary waves for variable charge as

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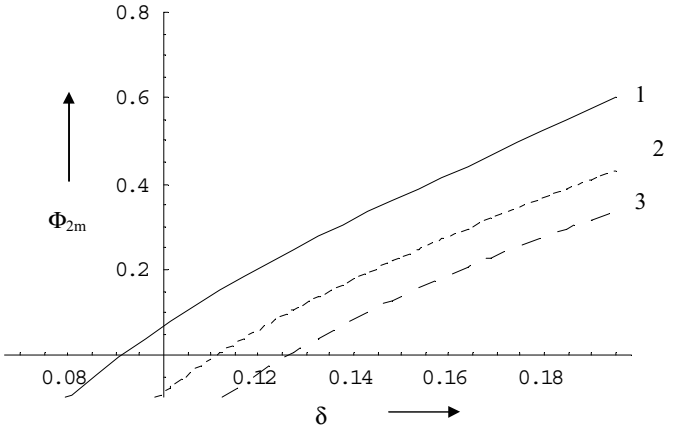


Figure 4. Variation of peak amplitude ϕ_{2m} in the DLs with δ for different values of β and with $u_0 = 1$ and $\beta_1 = 0.0001$. Curve 1 corresponds to $\beta = 0.0$, curve 2 to $\beta = 0.15$ and curve 3 to $\beta = 0.25$.

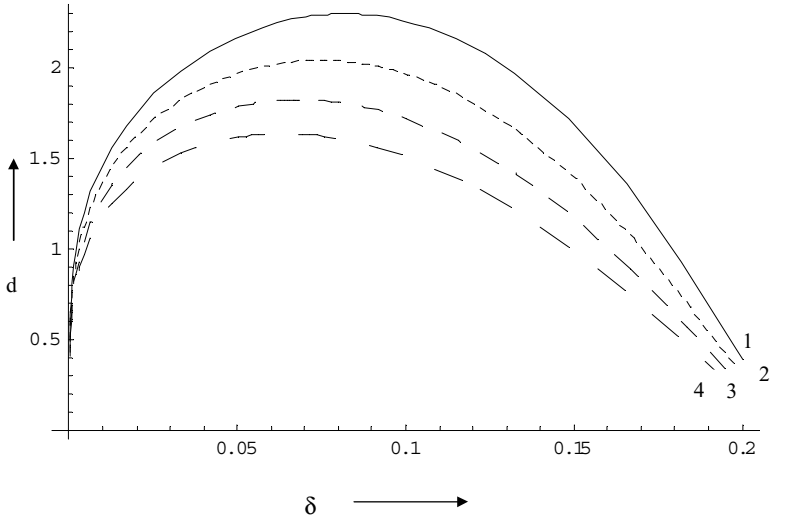


Figure 5. Variation of the thickness d of the DLs of small-amplitude solitons with δ for different values of β and with $u_0 = 1$ and $\beta_1 = 0.001$. Curve 1 corresponds to $\beta = 0.0$, curve 2 to $\beta = 0.05$, curve 3 to $\beta = 0.1$ and curve 4 to $\beta = 0.15$.

compared with constant dust charge for which only one type of solitary wave is obtained [14].

(ii) Only compressive DLs are observed. The non-thermal parameter β plays a crucial role in the existence of DLs.

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