The Problem of Noncounterfactual Conditionals

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I defend a formulation of the Ramsey Test with a condition for accepting negations of conditionals. It is implicit in the assumptions of the triviality theorems of Gärdenfors, Harper, and Lewis; and it allows for a unified proof of those theorems, from weaker assumptions about belief revision. This leads to a proof of McGee's thesis that iterated conditionals do not obey modus ponens.

1. Introduction. Our ability to embed indicative conditionals in larger sentences is a reason to analyze them as expressing some sort of genuine logical connective. However, embedding conditionals under negations provides some of the best evidence against the most popular analysis, the material conditional. In this article, I will show how negations of conditionals also help formulate the main challenges for stronger theories of the indicative conditional. Although the basic challenge primarily focuses on simple conditionals, it leads to a result about iterated conditionals: they do not obey modus ponens.

2. Troubles with Negating Conditionals. Consider this adaption of Grice's ([1967] 1989) example of an election in which the candidates are Heath, Thorpe, and Wilson. Suppose one election watcher gets information that Thorpe will lose and says, "If Wilson loses, then Heath will win." A second person responds, "That's not the case." The second speaker's response is puzzling on the material conditional analysis, where the conditional is

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equivalent to the disjunction "Either Wilson or Heath wins." For that would commit the second speaker to affirming that Wilson will lose, even though he might well be unsure of that. Grice, crediting this sort of observation to Bromberger, tries to save the material conditional analysis of the indicative by rejecting the claim that disagreement always amounts to affirming the truth-functional negation. Indeed, he suggests that denying a conditional often amounts to affirming the conditional with its consequent negated.

A more plausible strategy, if it can be carried out, is to reject the material conditional analysis in favor of some other theory that gets the right interaction with negation. Any such proposal faces a difficulty of its own. Suppose the second person got the information that Heath will lose and affirms, "If Wilson loses then Heath will lose." Even though it sounds like the two speakers disagree, it seems that neither speaker was wrong if it turns out that Wilson wins. Grice would have it that this means that both speakers spoke truly, supporting the material conditional analysis. Gibbard (1981) concludes that Ramsey ([1929] 1990) was right in claiming that conditionals do not express proper objects of beliefs but, rather, dispositions to revise one's beliefs.

Ramsey's basic idea was that in evaluating whether to accept a conditional, an agent should check whether he accepts the consequent upon revising his beliefs to accept the antecedent. Several philosophers—especially Stalnaker ([1968] 1981, [1970] 1981)—have claimed that one can endorse this positive suggestion by Ramsey, while abandoning his negative conclusion that conditionals are not objects of belief. In the next section I shall present a glimpse of a way of articulating such a theory. Then I shall show how the theory runs into problems, not by considering apparent disagreements between different persons but, rather, by examining a similar problem that arises by considering the potential belief states of a single agent.

3. The Ramsey Test. As a step toward making things more precise, consider a language containing a conditional connective $>$, as well as negation $\neg$, conjunction $\wedge$, and $\supset$ for the material conditional. The letters A, B, C range over the sentences not containing $>$. Greek ϕ, ψ range over all sentences. The constant $\perp$ stands for some truth-functional contradiction.

A theory K is a set of sentences closed under the language's consequence relation: $K = Cn(K)$. The Expansion of K by ϕ, written $K + \phi$, is $Cn(K \cup \{\phi\})$. The exact nature of the consequence relation Cn is to be determined by the structure imposed on the $>$ connective by the Ramsey Test. However, it is assumed that Cn is an extension of classical consequence. For instance, it obeys modus ponens and the deduction theorem for the material conditional, so $\psi \in K + \phi$ if and only if (iff)

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$\phi \supset \psi \in K$. Also, it obeys Monotonicity: if ϕ entails ψ, then $K + \psi \subseteq K + \phi$. If $\{\phi, \neg\phi\} \subseteq K$ then K is inconsistent and contains every sentence. If $\neg\phi \notin K$ then both K and $K + \phi$ are consistent, and ϕ is said to be compatible with K.

We use theories to represent the belief states of an idealized agent, who accepts all the logical consequences of his beliefs. Such an agent has a policy for revising his beliefs on the basis of new information (observations or suppositions; we do not here decide on the exact interpretation). This is captured by a Revision function $*$ which takes a prior belief state K into a posterior belief state $K * A$ on the basis of the input A. (Note that although conditionals are in belief states, we are not here considering them as inputs to revision, although one might do so.)

The Ramsey Test is formulated in terms of the Revision operation. Here is a version that is restricted to simple conditionals (those containing no conditional parts):

RT. If $\neg A \notin K$, then

$$A > C \in K \text{ if } C \in K * A,$$

$$\neg(A > C) \in K \text{ if } \neg C \in K * A.$$

This formulation is restricted to conditionals that are "noncounterfactual," in that their antecedents are compatible with belief. (This is not to say that we don't use indicatives whose antecedents are rejected or that the Ramsey Test shouldn't be extended to cover them.) Additionally, RT is rather minimal in that it gives only sufficient conditions for accepting conditionals and their negations. If the first ("positive") clause were strengthened to a necessary condition, then accepting $A > \neg C$ would suffice for accepting $\neg(A > C)$, which is very plausible. If the "negative" clause were strengthened to a necessary condition, accepting $\neg(A > C)$ would suffice for accepting $A > \neg C$, which is a subject of controversy in the study of counterfactual subjunctive conditionals.[1]

Although there is a case to be made for richer formulations of the Ramsey Test, this version suffices for the purposes of this paper. The Ramsey Test constrains the logical behavior of the conditional by means of properties of the revision function. Thanks to the clause for negations, RT is incompatible with the material conditional as a reasonable interpretation of $A > C$. Because the negation of a material conditional entails

1. Alternately, the negation clause could be strengthened to: $\neg(A > C) \in K$ if $C \notin K * A$. So if $\neg A \notin K$ then either $A > C \in K$ or $\neg(A > C) \in K$, although this is implausible as one can be in suspense about an indicative conditional. Rott (1989) endorses the biconditional of this without the restriction to $\neg A \notin K$ (which requires a qualification that $\perp \notin K$).

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its antecedent, interpreting RT as about the material conditional gives the unacceptable result that one is in suspense about something only if revising by it leads to being in suspense about everything.[2] While it looks like progress is being made, the Ramsey Test turns out to conflict with seemingly plausible principles of belief revision.

4. Triviality Results for Simple Conditionals. We say that a theory K is *Nontrivial* in case there are sentences A, C such that $A \wedge C$, $A \wedge \neg C$, $\neg A$ are each compatible with K. (Equivalently, K is compatible with pairwise incompatible B, C, D; let $A = B \vee C$.) The Ramsey Test is clearly trivial if it only holds in belief states that are more opinionated than this.

We will discuss some principles of Revision that lead to the triviality of the Ramsey Test, but a good deal of the work in proving the theorem can be done without those details. Consider this version of a principle from Stalnaker [1975] 1981, for accepting $>$ conditionals on the basis of material conditionals.

Direct Argument. If $\neg A \notin K$, then

$$A > C \in K \text{ if } A \supset C \in K, \text{ and}$$

$$\neg(A > C) \in K \text{ if } A \supset \neg C \in K.$$

The heart of the triviality proof for RT is contained in the following lemma, which doesn't say anything about the Ramsey Test.

Lemma. K is not Nontrivial if for every B, the Expansion $K + B$ satisfies the Direct Argument.

Proof. Suppose for reductio that K satisfies the Nontriviality condition, so there are sentences A, C such that $A \wedge C$, $A \wedge \neg C$, $\neg A$ are each compatible with K. From the Nontriviality of K, $\neg A \notin K + (A \supset C)$. Applying the Direct Argument to $K + (A \supset C)$, we get $A > C \in K + (A \supset C)$. By the Monotonicity of Cn, $K + (A \supset C) \subseteq K + \neg A$. Therefore, $A > C \in K + \neg A$. A similar argument shows that $\neg(A > C) \in K + (A \supset \neg C) \subseteq K + \neg A$. But this makes $K + \neg A$ inconsistent, contradicting the supposition that $\neg A$ is compatible with K. This proves the Lemma.

We want a way of representing the belief states of an agent who uses a conditional in accord with the Ramsey Test. Let a belief revision model $\langle \mathbf{K}, * \rangle$ consist of a set $\mathbf{K}$ of theories paired with a Revision function $*$,

2. Suppose there are some A, C, and K such that $\neg A \notin K$ but $C \in K * A$. By the material interpretation of $>$, the negation clause of RT gives that $\neg(A \supset \neg C) \in K$, hence $A \in K$.

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which is *closed under RT and Revision*: if $K \in \mathbf{K}$, then K obeys RT, and if $\neg A \notin K$, then $K * A \in \mathbf{K}$. Given a proper specification of the Revision function, the closure condition ensures that if $\mathbf{K}$ contains a theory representing an agent's prior belief state, then it contains all the potential posterior belief states that might come from it.

What is the nature of the Revision function? There is a popular theory of belief revision according to which Revision simply goes by Expansion when the input is consistent with prior belief: $K * A = K + A$, if $\neg A \notin K$. This is defended in Gärdenfors 1988 as the minimal change to be made to K to accept A. If we adopt this rule, then RT is consistent only in trivial cases. But this assumption about Revision is stronger than what is needed for the triviality result. It turns out to be instructive to explore a couple sets of weaker assumptions, in order to facilitate comparison with some influential triviality proofs and to get a better understanding of the tenability conditions for the Ramsey Test.

Since it is assumed that belief revision models are closed under Revisions, if the latter go by Expansion then $\mathbf{K}$ is *closed under Expansions*: if $K \in \mathbf{K}$ and $\neg A \notin K$, then $K + A \in \mathbf{K}$. The rule that Revision goes by Expansion can be split in two:

> **Inclusion.** $K * A \subseteq K + A$
> **Preservation.** $K + A \subseteq K * A$, if $\neg A \notin K$.[3]

Inclusion says one should not leap to any conclusions that do not follow from drawing the consequences of adding A to one's old beliefs. Preservation says one should accept A without losing any old beliefs. We only need a weaker version of Preservation, just applying to beliefs not containing conditionals.

> **Basic Preservation (BP).** If $\neg A \notin K$ and $C \in K + A$, then $C \in K * A$.

The proof we give below requires either Inclusion, or closure under Expansions, but not both.

> **Main Theorem.** For any $\langle \mathbf{K}, * \rangle$ closed under Revisions and RT, where $*$ obeys Basic Preservation, no member of $\mathbf{K}$ is Nontrivial if either: (E) $\mathbf{K}$ is closed under Expansions, or (I) $*$ obeys Inclusion.
>
> *Proof*. We show that the Direct Argument holds for every $H \in \mathbf{K}$, in order to demonstrate that for every $K \in \mathbf{K}$, the Direct Argument

3. Gärdenfors and others define "Preservation" as $K \subseteq K * A$ if $\neg A \notin K$, but whenever they invoke it they also assume that $A \in K * A$, so the terminological difference is harmless. This use of "Inclusion" is standard in the literature following Gärdenfors 1988, such as Bradley 2007. Note that Inclusion holds vacuously when $\neg A \in K$.

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holds for $K + B$. The Lemma then establishes that no $K \in \mathbf{K}$ is Nontrivial.

Suppose $\neg A \notin H$. If $A \supset C \in H$, then $C \in H + A$. So $C \in H * A$ follows from Basic Preservation, and thus $A > C \in H$ by RT. Similarly, if $A \supset \neg C \in H$, then $\neg(A > C) \in H$ by RT. This shows that the Direct Argument holds for H.

If $\neg B \in K$, then $K + B$ is inconsistent and the Direct Argument holds vacuously for it. So suppose $\neg B \notin K$. For case (E), it follows immediately from closure under Expansions that $K + B$ satisfies the Direct Argument.

For case (I), suppose that $\neg A \notin K + B$. It follows from Inclusion that $\neg A \notin K * B$. Suppose the hypothesis of the first case of the Direct Argument holds for $K + B$: $A \supset C \in K + B$. Since $\neg B \notin K$, it follows by Basic Preservation that $A \supset C \in K * B$. Because $K * B \in \mathbf{K}$, it satisfies the Direct Argument, so $A > C \in K * B$. Therefore, by Inclusion applying to conditionals, $A > C \in K + B$. Likewise, supposing $A \supset \neg C \in K + B$ gives us that $\neg(A > C) \in K * B \subseteq K + B$. Hence $K + B$ obeys the Direct Argument. This completes the proof of the Theorem.

5. Inclusion, Iteration, and Indexicality.

5.1. Inclusion. Is the Ramsey Test itself to blame for the triviality theorem, or does the fault lie with the auxiliary assumptions? It is difficult to argue with Basic Preservation, as a condition on sentences not containing conditionals. Hence the defender of the Ramsey Test will incline to identifying the culprits as the assumptions of the two cases of the Main Theorem: closure under Expansions, and Revision obeying Inclusion. Each version of the proof shows that Revision does not go by Expansion, but separately they teach different lessons.

While our closure conditions gave us all the belief states we needed to represent an agent's revisions of belief, we haven't ensured that those are the only theories in **K**. So it is not clear that all of the elements of **K** are reasonable candidates for belief states, unless obeying the Ramsey Test somehow suffices for being a reasonable belief state. For instance, nothing we have said yet rules out that our models contain some K containing a sentence interpretable as "I think, but I don't exist."

Arguably, this gap between the formalization and its intended purpose is exploited in the version using closure under Expansions. The closure condition imposes the Ramsey Test on all Expansions of the elements of **K**, even though we are not assuming that those Expansions are reachable by Revision from the theories in **K** which represent reasonable belief states.

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As Rott (1989) has observed, this should not be accepted by a defender of the Ramsey Test who is willing to drop the rule that Revision goes by Expansion.

The Ramsey Test should be understood as a constraint on which sets of sentences count as rational states of belief, rather than a logical principle holding of every set of sentences. But whatever constraint the Ramsey Test imposes on acceptable belief states, it should be closed under Revisions. A lesson of the other case of the Main Theorem is that if we wish to keep RT, then we must give up the application of Inclusion to conditionals.[4] The Ramsey Test makes the acceptance of conditionals a form of ampliative reasoning: conditionals are accepted in revised belief states without being entailed by the prior belief plus the input to belief revision.

5.2. Iteration. The proof of the Main Theorem only made use of simple conditionals (containing no conditionals parts). But it leads immediately to lessons about the logic of iterated conditionals once we extend the Ramsey Test to conditionals with conditional consequents. Suppose we modify the positive clause of RT to read: if $\psi \in K * A$ then $A > \psi \in K$. Modus ponens for $>$ tells us that if $A > \psi \in K$, then $\psi \in K + A$. These together entail that if $\psi \in K * A$, then $\psi \in K + A$, which is Inclusion. So we cannot have both of the Ramsey Test and modus ponens for $A > \psi$, where ψ contains a conditional. McGee (1989) has already given counterexamples to modus ponens for iterated conditionals, and here we give another one that fits the structure of the Main Theorem more closely.

Return to the election example in which Wilson, Thorpe, and Heath are the candidates in an election. If one is unsure which of them will win, then the following two iterated conditionals are jointly acceptable, both intuitively and by the iterated Ramsey Test:

(i) If Thorpe loses, then Heath wins if Wilson loses.
(ii) If Heath loses, then it's not the case that Heath wins if Wilson loses.

But if these conditionals obey modus ponens, then they are jointly incompatible with the scenario compatible with belief where Wilson wins. For in that scenario the antecedents of each are satisfied, so together they entail both

4. Other principles of revision that hold for sentences without conditionals may also fail for beliefs containing conditionals, notably Preservation, which is targeted by Bradley 2007. Several of Bradley's arguments invoke a principle he calls "Conditional Contradiction"; it would not be acceptable to those who think $\bot > C$ is valid, since it then gives the result that every K is inconsistent. Bradley endorses a version of RT without any qualification that $\neg A \notin K$, but the negation clause needs to be restricted to the case of $\bot \notin K * A$.

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(iii) If Wilson loses then Heath wins

and its negation. Like McGee's examples, this has no bearing on whether simple conditionals like (iii) obey modus ponens.

5.3. Indexicality. We formulated the Ramsey Test and revision rules in a sentential framework, without considering the truth conditions of conditionals. But when we ask what propositions conditionals express, some aspects of our treatment of the Ramsey Test may look questionable. The problem comes out most vividly in the interpersonal case of two agents whose observations are jointly consistent (with $\neg A$), yet one endorses $A > C$ while the other accepts $\neg(A > C)$. Do we really want to say that the agents disagree about a *conditional fact* which does not supervene on the mundane facts?

Some have thought it more reasonable to follow Stalnaker ([1975] 1981) in saying although those two conditionals are incompatible with each other on a fixed interpretation of the conditional, that interpretation is indexical to a belief state. So despite appearances, the two agents do not really disagree with each other, since they are in different belief states. The indexical theory of conditionals also helps explain why, in the intrapersonal triviality proof, conditionals seem not to obey standard rules of belief revision. Properly formulated, the rules of belief revision apply to propositions. Since a conditional sentence expresses different propositions relative to different belief states, its acceptance conditions across revisions in belief will differ from a sentence that is not indexical.

If the indexical strategy is adopted for endorsing the Ramsey Test, one still ends up with the conclusion that iterated conditionals do not obey modus ponens. The indexical approach requires the theorist to answer the following question. In evaluating $A > (B > C)$, which should one check for upon revising by A: the acceptance of the proposition that $B > C$ expresses relative to the original belief state, or the proposition that $B > C$ expresses relative to the state revised by A? Harper (1981) suggests that the latter answer is the correct one, and the result is that the conditional expresses a different semantic value at the two places it occurs in $A > (B > C)$.

It can be seen that if we follow Harper in this, then we will get the intuitively correct result, that both (i) and (ii) should be acceptable to somebody unsure of whether Wilson, Heath, or Thorpe will win. (Whereas we will *not* get this result if the other account gives a *necessary* condition for accepting $A > (B > C)$.) So $A > (B > C)$ and A will not generally entail the proposition that $B > C$ expresses as a stand-alone sentence.[5] Hence,

5. Let X be the proposition which (iii) expresses relative to unrevised belief K. Suppose

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Stalnaker ([1975] 1981) is incorrect to analyze indicative conditionals by means of making his semantic analysis relative simply to a belief state. The Ramsey Test shows that the context for evaluating an iterated conditional shifts in the middle of the sentence.

6. Related Triviality Theorems. Two of the most influential triviality proofs about the Ramsey Test are those of Lewis ([1976] 1981) and Gärdenfors (1986, 1988). These proofs proceed from different enough assumptions that one might wonder how much they really have in common. It turns out that we can use the Main Theorem of this paper to generalize both of them, as well as a theorem by Harper (1976) which has not received as much attention.

Gärdenfors doesn't work with a clause for accepting negations of conditionals but, rather, with belief-contravening revisions. Gärdenfors's proof relies on the claim that **K** is closed under Expansion along with the following:

G1. $A > C \in K$ iff $C \in K * A$.
G2. If $\bot \in K * A$, then $\bot \in K$ or $\bot \in Cn(\{A\})$.

An interesting feature of Gärdenfors's proof, which he attributes to Alchourrón and Makinson, is that G1 entails a property of Revisions that suffices for a triviality proof without conditionals. The triviality of G1 follows from that as a corollary. It turns out that the only requirements of Revision are G2 and Basic Preservation.[6]

However, by focusing on the Ramsey Test rather than Revision in the

there is a world w compatible with K in which Wilson wins and X is false. Let H be the belief state that comes from revising K by the proposition that Thorpe loses. By the rule for minimal belief revision, w is compatible with H. So (i) will not be accepted at K if it is a necessary condition that X be accepted at H. By the Ramsey Test, (iii) is accepted at H, so the proposition Y which it expresses is distinct from X. If the Ramsey Test for (i) checks whether Y is accepted relative to H, then (i) will be accepted relative to K; let Z be the proposition (i) expresses in that case. Since Z and its antecedent are both true at w but X is false there, (i) does not obey modus ponens with respect to the proposition its consequent expresses on its own. If the supposition is false, so all the worlds compatible with belief where Wilson wins are ones where X is true, then it is (ii) that does the job relative to the revision by Heath losing.

6. Here is a proof of a slightly stronger result than Gärdenfors's. (For one thing, it does not use his assumption that $A \in K * A$ even when $\neg A \in K$. The case where $\neg A \notin K$ is captured by our BP. To prevent confusion, it should be noted that Gärdenfors 1986 uses "Inclusion" to denote what we here call "Preservation.") The closure of **K** under G1 entails Monotonicity of Revisions (MR): for $K, H \in \mathbf{K}$, if $K \subseteq H$ and $C \in K * A$, then $C \in H * A$. And MR, BP, and G2 entail that no member of **K** is Nontrivial. If K, A, C satisfy Nontriviality, then BP yields $C \in (K + A \supset C) * A$. From MR, $C \in (K + \neg A) * A$. Similarly, $\neg C \in (K + A \supset \neg C) * A$, so $\neg C \in (K + \neg A) * A$. But by Nontriviality and G2, $\bot \notin (K + \neg A) * A$.

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abstract, we may use Gärdenfors's G1 and G2 to give a variation of both cases of the Main Theorem of this paper. Indeed, we may dispense with G2 and belief-contravening revisions and use case (*E*) of the Main Theorem to derive Gärdenfors's result about G1. To do so, assume that **K** is closed under Expansions $K + \phi$ by conditionals. From G1 holding for all $K \in \mathbf{K}$, it follows that

H1. $((A > C) \wedge (A > \neg C)) \supset (A > \bot) \in K$.

From G2 also holding in all members of **K**, it follows that

H2. If $\neg A \notin K$, then $\neg(A > \bot) \in K$.

(Suppose for reductio that H2 is false: $\neg A \notin K$ but $\neg(A > \bot) \notin K$. Hence A and $K + (A > \bot)$ are each consistent. By closure under Expansions, G1 applies to $K + A > \bot$, so $\bot \in (K + A > \bot) * A$, which contradicts G2.) Harper 1976 proves a version of the Main Theorem basically using H1, H2, and G1.[7] But if G1, H1, and H2 hold in all $K \in \mathbf{K}$, so does RT, just given the closure of **K** under Revisions. Hence the Main Theorem entails these results and distinguishes the two cases: closure under Expansions and Revision obeying Inclusion.

Ramsey's original test was a probabilistic one. A probability P is a function from sentences into real numbers obeying the laws that $P(\phi) \geq 0$, $P(\phi \vee \neg\phi) = 1$, and $P(\phi \vee \psi) = P(\phi) + P(\psi)$ if ϕ, ψ are incompatible. A conditional probability is equal to a ratio of probabilities $P(\psi/\phi) = P(\phi \wedge \psi)/P(\phi)$, if $P(\phi) > 0$.

Let the operation $P_\phi(\psi)$ be the posterior probability assigned to ψ upon revising prior P by ϕ. In its probabilistic form, the Ramsey Test tells us that $P(A > C) = P_A(C)$, if $P(A) > 0$. The standard rule for probabilistic revision says that it obeys

Conditionalization. $P_\phi(\psi) = P(\psi/\phi)$, if $P(\phi) > 0$.

Combined with the rule that P_A goes by conditionalization, the Ramsey Test gives us an equation often named after Adams's 1975 development of Ramsey's idea.

Adams's Thesis (AT). $P(A > C) = P(C/A)$ if $P(A) > 0$.

From the law of probability that $P(\psi/\phi) + P(\neg\psi/\phi) = 1$, it follows from

7. Harper's proof (118–119) is semantic rather than syntactic, and he concludes that the conditional is indexical to belief states. H1 is a consequence of his Lewisian truth conditions for the conditional. H2 corresponds to the claim that if A is compatible with belief, then for each world compatible with belief there is an accessible A world. Note that a standard definition makes $\neg(A > \bot)$ equivalent to $\Diamond A$.

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AT that $P(A > \neg C) = 1$ iff $P(A > C) = 0$. Hence Ramsey's remark that denying a conditional is equivalent to denying its consequent.

Lewis ([1976] 1981) proves that except in trivial cases, AT is not closed under conditionalization. Although the *proof* that Lewis gives involves the full use of AT (in its last step), it turns out that Lewis's *theorem* only requires the extreme cases: if $P(A) > 0$ then $P(A > C) = 1$ if $P(C/A) = 1$, and $P(A > C) = 0$ if $P(C/A) = 0$.[8] From just that weak version of AT and the laws of probability, Lewis's result can be obtained from this paper's Main Theorem, in the following way.

Let **P** be a set of probability assignments satisfying the hypothesis of Lewis's theorem (closure under both AT and conditionalization by sentences not containing conditionals). The main task is to define a belief revision model $\langle \mathbf{K}, * \rangle$ in terms of **P** in such a way as to satisfy the hypothesis of the Main Theorem of this paper. This is easy to do, in a way that captures the qualitative features we need from Lewis's quantitative model.

First, define **K** as the smallest set where for every $P \in \mathbf{P}$, there is a $K \in \mathbf{K}$ such that $K = \{\phi : P(\phi) = 1\}$. We show that defining **K** in this way entails that it is closed under Expansions: if $K \in \mathbf{K}$ and $\neg A \notin K$, then $K + A \in \mathbf{K}$. For each $P \in \mathbf{P}$, consider the corresponding K. (This ensures that we cover each member of **K**, since it is the smallest set meeting the definition.) If $\neg A \notin K$, then $P(\neg A) \neq 1$, and so $P(A) > 0$. Since **P** is closed under conditionalization, it contains P_A, where $P_A(\psi) = 1$ iff $P(\psi/A) = 1$. Moreover, **K** is defined so as to contain $H = \{\psi : P_A(\psi) = 1\}$. By probability, $P(\psi/A) = 1$ iff $P(A \supset \psi) = 1$. So $H = \{\psi : A \supset \psi \in K\}$, which is $K + A$ by the definition of Expansions.

Next, we can define $K * A = K + A$ when $\neg A \notin K$. From this we have that **K** is closed under Revisions, and clearly $*$ satisfies Basic Preservation. Since **P** is closed under AT, it follows that **K** is closed under RT. Finally, suppose for reductio there is some $P \in \mathbf{P}$ that is nontrivial in Lewis's sense: there are A, C such that $P(A \wedge C)$, $P(A \wedge \neg C)$, $P(\neg A)$ are all positive. That supposition entails that there is some $K \in \mathbf{K}$ that is compatible with $A \wedge C$, $A \wedge \neg C$, and $\neg A$. But the Main Theorem tells us that this is impossible, which proves Lewis's result.[9]

8. Bradley (2007) highlights an even weaker condition: if $P(A) > 0$, then $P(A > C) = 1$ if $P(C) = 1$ and $P(A > C) = 0$ if $P(C) = 0$. Bradley shows that the closure of that condition under conditionalization is enough to derive the penultimate step of Lewis's proof, that $P(A > C) = P(C)$. Bradley's proof follows Lewis's technique and does not generalize to the qualitative case.

9. Vann McGee has emphasized to me that in order to prove Lewis's theorem (with full generality) in this way, it is important that "compatibility with K" is a formal notion that does not require that all the members of K can be true. Consider a coin to be tossed infinitely many times. Suppose that our language is rich enough to contain,

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Adams himself takes the triviality results to show that Ramsey was right: a conditional expresses a conditional probability rather than any truth conditional connective. But as in the qualitative case, we may maintain the probabilistic Ramsey Test by suitable restrictions. We may hold on to our standard theory of revision when no conditionals are involved, $P_A(C) = P(C/A)$. So we can keep $P(A > C) = P(C/A)$, even though we must deny that in general $P_A(\psi) = P(\psi/A)$. Exploring the details of the best way to do this is outside the scope of this paper.[10]

7. Conclusion. In the tradition inspired by Goodman ([1947] 1955), counterfactual conditionals have provided the strongest impetus to search for a theory other than the truth-functional one. But indicative conditionals pose their own significant challenges to the material conditional. The Ramsey Test promises insight, but the triviality results present an obstacle to using principles of belief revision to constrain the logic of the conditional. Indeed, the variety of triviality results seems to present several obstacles.

I have argued that several of the most important triviality results have more in common than is at first apparent, thanks to a triviality proof based on a shared formulation of the Ramsey Test. A consequence of the proof is that in order to escape triviality without making conditionals indexical, iterated conditionals fail to obey modus ponens. The phenomenon about iterated conditionals also holds on any reasonable Ramsey Test theory that avoids triviality by making conditionals indexical to belief states.

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for each sequence s of heads and tails, the statement that s occurs, and also the statement that some sequence will occur. For each s, P assigns 0 to that sequence occurring, so the statement that s does *not* occur will be in K. But the statement that some sequence s occurs is assigned 1 by P, so it is in K. Because P is a probability, there is no ϕ such that both $P(\phi) = 1$ and $P(\neg\phi) = 1$; hence K is consistent. Nevertheless, not all the members of K can be true. So it follows regardless of AT that there are not three pairwise incompatible statements where the *truth* of each is jointly compatible with the truth of every member of K.

10. McGee 1989 proposes a theory that endorses $P(A > \psi) = P_A(\psi)$ even where ψ is a conditional, while denying $P(A > \psi) = P(\psi/A)$; this divorces the Ramsey Test (about probability revision) from AT except in the case of simple conditionals $A > C$. See Hajek and Hall 1994 for a survey of further negative and positive results for AT, including those (such as a construction by Van Fraassen) that keep AT within a single probability but divorce it from the revision-oriented Ramsey Test.

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