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TRACE FORMULAS IN NONCOMMUTATIVE GEOMETRY

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Trace formulas appear in many forms in noncommutative geometry (NCG). In the first part of this thesis, we obtain results for asymptotic expansions of trace formulas such as heat trace expansions by adapting the theory of Multiple Operator Integration to NCG. More broadly, this construction provides a natural language for operator integrals in NCG, which systematises and simplifies operator integral arguments throughout the literature. Towards this end, we construct a functional calculus for abstract pseudodifferential operators and generalise Peller's construction of multiple operator integrals to this abstract pseudodifferential calculus. In the process, we obtain a noncommutative Taylor formula.

In the second part of the thesis, we shift our attention to Dixmier trace formulas. First, we provide an approximation of the noncommutative integral for spectrally truncated spectral triples in the Connes–Van Suijlekom paradigm of operator system spectral triples. Our approximation has a close link to Quantum Ergodicity, which we will use to state an NCG analogue of the fundamental result that ergodic geodesic flow implies quantum ergodicity. Furthermore, we provide a Szegő limit theorem in NCG. Next, we provide a Dixmier trace formula for the density of states, a measure originating in solid-state physics that can be associated with an operator on a geometric space. We first provide this formula in the setting of discrete metric spaces and then in the setting of manifolds of bounded geometry. The latter leads to a Dixmier trace formula for Roe's index on open manifolds.

Some of this research has been published in [1–4].

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