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Reflection of an electron beam by a photon mirror

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Abstract. A new configuration for the laser accelerator is proposed, which is inspired by the relativistic photon mirror effect. The material mirror is replaced here by an intense laser pulse, acting as a photon mirror for the incoming charged particles. A sufficient condition for particle reflection at such a photon mirror is established and three types of particle trajectories are described. A snow-plow acceleration regime is identified and quantitatively defined. Production of intense radiation bursts by the charged particle beam during the reflection process is also demonstrated.

1. Introduction

The possibility of accelerating particles by intense laser–plasma interactions is a subject of considerable interest and active research around the world [1]. In particular, it is now well understood, both from theory [2] and experiments [3, 4], that electrons can be accelerated to high energies by the electrostatic wakefields produced by intense laser pulses in a plasma. Accelerating electric fields of 100 GV m^{-1} are experimentally achieved with this method, which is four orders of magnitude larger than the accelerating fields in present-day linac accelerators of the conventional type.

In the present paper we propose a new configuration for the laser accelerator, which is inspired by the relativistic photon mirror effect. It is well known that the photon frequency can be considerably up-shifted by reflection at a relativistically moving mirror. This effect is described as due to a double Doppler shift [5].

However, we can reverse this relativistic mirror concept, and consider reflection of an incident electron beam by an intense laser pulse, propagating in plasma or in vacuum. For high pulse intensities, most of the electrons can be reflected by the moving electromagnetic perturbation, and the laser pulse can be seen as a kind of relativistic photon mirror, which is now reflecting charged particles.

Here, the charged particle dynamics will be considered, and the phase space characterization of the various types of trajectories will be given. We will determine sufficient conditions for electron reflection to take place and the energy gain of the particles being reflected by the photon mirror. Secondary radiation processes due to particle acceleration upon reflection will also be considered. We will estimate the intensity and duration of the radiation burst emitted by an electron beam reflected

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by an intense laser pulse. Possible use of the photon mirror configuration to single-step particle accelerators, or as a source of primary electrons for a multi-staged accelerator will be discussed.

McKinstrie and Startsev [6, 7] have recently published work along similar lines, where the charged particle behavior in the field of an intense laser pulse is studied. Here we return to the same problem by considering three new aspects. First, we give a consistent two-dimensional analytical description of the charged particle trajectories. Second, we discuss the various types of trajectories and establish a quantitative criterion for snow-plow acceleration. Finally, we discuss the radiation processes associated with the charged particle reflection at the photon mirror.

2. Particle equations of motion

We consider a test particle, with charge q and mass m, submitted to vector and scalar potentials $\mathbf{A}(\mathbf{r}, t)$ and $\phi(\mathbf{r}, t)$. It is appropriate to start with the particle Hamiltonian, written in the form

$$h(\mathbf{r}, \mathbf{p}, \tau) = \sqrt{1 + [\mathbf{p} - \mathbf{a}(\mathbf{r}, t)]^2} + \varphi(\mathbf{r}, \tau), \tag{1}$$

where the normalized potentials are

$$\mathbf{a} = \frac{q\mathbf{A}}{mc}, \quad \varphi = \frac{q\phi}{mc^2}, \tag{2}$$

and m is the rest mass of the particle, and c the velocity of light in vacuum.

The time variable used here is $\tau = ct$, and the normalized particle canonical momentum $\mathbf{p}$ is defined by

$$\mathbf{p} = \gamma\boldsymbol{\beta} + \mathbf{a}, \tag{3}$$

where $\boldsymbol{\beta} = \mathbf{v}/c$ is the normalized velocity of the particle and γ is the relativistic gamma factor:

$$\gamma = (1 - \beta^2)^{-1/2} = h - \varphi. \tag{4}$$

The particle dynamics can then be determined by the canonical equations

$$\frac{d\mathbf{r}}{d\tau} = \frac{\partial h}{\partial \mathbf{p}}, \quad \frac{d\mathbf{p}}{d\tau} = -\frac{\partial h}{\partial \mathbf{r}}. \tag{5}$$

Let us restrict our analysis to potentials of the form

$$\mathbf{a}(\mathbf{r}, t) = \mathbf{a}(\mathbf{r} - \boldsymbol{\beta}_g\tau), \quad \varphi(\mathbf{r}, t) = \varphi(\mathbf{r} - \boldsymbol{\beta}_g\tau), \tag{6}$$

where $\boldsymbol{\beta}_g = \mathbf{v}_g/c$, and $\mathbf{v}_g$ is the group velocity of an intense laser pulse associated with the potential perturbations. This simplified form of space and time dependence is valid as long as the laser pulse envelope remains nearly constant during the interaction time with the particle.

We can make a canonical transformation from the pair of variables $(\mathbf{r}, \mathbf{p})$, to the new ones $(\mathbf{r}', \mathbf{p}')$, by using the generating function

$$F(\mathbf{r}, \mathbf{p}', \tau) = (\mathbf{r} - \boldsymbol{\beta}_g\tau) \cdot \mathbf{p}'. \tag{7}$$

The new canonical variables are determined by the relations

$$\mathbf{p} = \frac{\partial F}{\partial \mathbf{r}} = \mathbf{p}', \quad \mathbf{r}' = \frac{\partial F}{\partial \mathbf{p}'} = \mathbf{r} - \boldsymbol{\beta}_g\tau, \tag{8}$$

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and the resulting new Hamiltonian is

$$h' = h + \frac{\partial F}{\partial \tau} = \gamma + \varphi - \boldsymbol{\beta}_g \cdot \mathbf{p}'. \tag{9}$$

It is important to note that this new Hamiltonian is a constant of motion, because it is independent of the time variable τ. In explicit form, we can write

$$h'(\mathbf{r}', \mathbf{p}') = \sqrt{1 + [\mathbf{p}' - \mathbf{a}(\mathbf{r}')]^2} + \varphi(\mathbf{r}') - \boldsymbol{\beta} \cdot \mathbf{p}'. \tag{10}$$

By comparing these last two equations, we can also conclude that

$$\gamma = \sqrt{1 + [\mathbf{p}' - \mathbf{a}(\mathbf{r}')]^2}. \tag{11}$$

At this point it is useful to introduce the parallel and perpendicular motion, such that

$$\mathbf{r}' = \eta \hat{b} + \mathbf{r}'_\perp, \quad \mathbf{p}' = p\hat{b} + \mathbf{p}'_\perp, \tag{12}$$

where $\hat{b} = \boldsymbol{\beta}_g / |\beta_g|$ is the unit vector parallel to the direction of propagation of the intense laser pulse.

The dynamical problem is considerably simplified if we assume that the fields are independent of the perpendicular coordinates. This assumption is valid if the transverse dimensions of the laser pulse are larger than the area of interaction with the test particle. Let us then use $\mathbf{a}(\mathbf{r}') = \mathbf{a}(\eta)$ and $\varphi(\mathbf{r}') = \varphi(\eta)$. We can then immediately recognize that the perpendicular momentum $\mathbf{p}_\perp$ is a new constant of motion, $\mathbf{p}_\perp(\tau) = \mathbf{p}_{\perp 0} =$ constant, because

$$\frac{d\mathbf{p}_\perp}{d\tau} = -\frac{\partial h'}{\partial \mathbf{r}'_\perp} = 0. \tag{13}$$

Using (3), we can determine the perpendicular velocity of the charged particle [6]

$$\boldsymbol{\beta}_\perp = \gamma^{-1}[\mathbf{p}_{\perp 0} - \mathbf{a}_\perp(\eta)] = \frac{\mathbf{p}_{\perp 0} - \mathbf{a}_\perp(\eta)}{\sqrt{1 + [p\hat{b} + \mathbf{p}_{\perp 0} - \mathbf{a}(\eta)]^2}}. \tag{14}$$

We are then left with the study of the parallel motion, which can be carried out by using the parallel canonical equations

$$\frac{d\eta}{d\tau} = \frac{\partial h''}{\partial p}, \quad \frac{dp}{d\tau} = -\frac{\partial h''}{\partial \eta}, \tag{15}$$

where the parallel Hamiltonian h'' is determined by

$$h''(p, \eta) = \sqrt{1 + [p\hat{b} + \mathbf{p}_{\perp 0} - \mathbf{a}_\perp(\eta)]^2} + \varphi(\eta) - \beta_g p. \tag{16}$$

Before starting the detailed discussion of these equations, it should be noted that we have identified two constants of motion, namely

$$I_1 = h''(p, \eta), \quad \mathbf{I}_2 = \mathbf{p}_{\perp 0}. \tag{17}$$

3. Reflection at the photon mirror

Let us assume a purely transverse field

$$\varphi(\eta) = 0, \quad \mathbf{a}_\parallel(\eta) = 0. \tag{18}$$

This is valid for laser pulses propagating in vacuum, as well as for pulses propagating in a plasma, if we neglect the influence of the wakefield, and if the pulse is not

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tightly focused. In this case, the parallel Hamiltonian will be reduced to

$$h''(p,\eta) = \sqrt{\nu^2 + p^2 + g(\eta)} - \beta_g p, \tag{19}$$

where $\nu^2 = 1 + p_{\perp 0}^2$ is a constant, and

$$g(\eta) = a^2(\eta) - 2\mathbf{p}_{\perp 0} \cdot \mathbf{a}(\eta). \tag{20}$$

In a plasma, this should be reduced to $g(\eta) = 1/2a^2(\eta)$, after a time average [6, 8]. However, for single-cycle pulses in vacuum, the above expression of $g(\eta)$ should be maintained. If necessary, we can specify the form of the laser pulse, by using

$$\mathbf{a}(\eta) = \mathbf{a}_0 f(\eta), \tag{21}$$

where $\mathbf{a}_0$ determines the maximum laser field amplitude, and $f(\eta)$ is a form function, giving the precise shape of the laser pulse. For instance, a gaussian pulse will be described by

$$f(\eta) = \exp(-k_b^2 \eta^2), \tag{22}$$

so that we can write

$$g(\eta) = a_0^2 (e^{-2k_b^2\eta^2} - \alpha e^{-k_b^2\eta^2}), \tag{23}$$

with

$$\alpha = 2\frac{\mathbf{p}_{\perp 0} \cdot \mathbf{a}_0}{|a_0|^2}. \tag{24}$$

Let us call h_0 the value taken by the first invariant $I_1 = h''(p,\eta)$ for a particular trajectory. Solving (19) for the parallel momentum p, we can obtain an expression for the particle trajectory,

$$p(\eta) = \frac{h_0 \beta_g}{1-\beta_g^2}\left\{1 \pm \sqrt{1 - \frac{1-\beta_g^2}{h_0^2 \beta_g^2}[\nu^2 + g(\eta) - h_0^2]}\right\}. \tag{25}$$

In order to understand this result, let us discuss the topology of the parallel phase space (p,η). First, we define a condition for *trajectory reversal* in phase space, which can be seen as a kind of trajectory reflection in the moving frame associated with the laser pulse. Note, however, that such a moving frame is not explicitly considered, because we have used a simple canonical transformation which is similar but not identical to a Lorentz transformation into the moving frame. Trajectory reversal implies that, at some point, the derivative of $p(\eta)$ will diverge,

$$\frac{dp}{d\eta} \to \pm\infty. \tag{26}$$

From (25), we get

$$h_0^2 \beta_g^2 = (1-\beta_g^2)[\nu^2 + g(\eta) - h_0^2]. \tag{27}$$

This means that trajectory reversal in phase space will occur at a position $\eta = \eta_r$ such that

$$g(\eta_r) = \frac{h_0^2}{1-\beta_g^2} - \nu^2. \tag{28}$$

However, from the definition of the function $g(\eta)$, we can easily see that we always have $g(\eta) \leqslant a_0^2(1-\alpha)$. So, introducing the relativistic gamma factor associated with the laser pulse, $\gamma_g = (1-\beta_g^2)^{-1/2}$, we can state that trajectory reversal in phase space will only occur if we verify the inequality

$$a_0^2(1-\alpha) \geqslant h_0^2 \gamma_g^2 - \nu^2. \tag{29}$$

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This condition can be rewritten as $h_0 \leqslant h_r$, where the critical value for Hamiltonian is determined by

$$h_r = \gamma_{\rm g}^{-1}\sqrt{\nu^2 + a_0^2(1-\alpha)}. \tag{30}$$

For purely one-dimensional motion, $p_{\perp 0} = 0$, we would simply have $\alpha = 0$ and $\nu^2 = 1$. Let us now establish the condition for *particle reflection* by the laser pulse. When reflection occurs, we can identify the laser pulse with a photon mirror. Assuming that, initially, we have $p < 0$, reflection implies that, at some point, we will observe $p = 0$. According to the solution (25), this implies that, at some point $\eta = \eta_x$, we will verify

$$g(\eta_x) = h_0^2 - \nu^2 = 1. \tag{31}$$

The sufficient condition for particle reflection will then be

$$a_0^2(1-\alpha) \geqslant h_0^2 - \nu^2 = 1. \tag{32}$$

This means that $h_0 \leqslant h_x$, where the new critical value for the trajectory invariant I_1 is

$$h_x = h_r\gamma_{\rm g} = \sqrt{\nu^2 + a_0^2(1-\alpha)}. \tag{33}$$

Obviously, we will always have $h_x > h_r$, because the laser pulse (or photon mirror) gamma factor is always larger than one, $\gamma_{\rm g} > 1$. So, we can establish three types of trajectories for particles colliding with a photon mirror, which can be characterized in terms of their initial energy, or, equivalently, in terms of their invariant $I_1 \equiv h_0$.

(1) For incident particles with high energies, corresponding to large values of the trajectory invariant $h_0 > h_x$, there is transmission through the laser pulse, and no photon mirror effect is observed.

(2) For moderate energy values, such that $h_r < h_0 < h_x$, particle reflection occurs, with a significant energy gain, as indicated below, but the laser pulse will always overcome the reflected particles, in such a way that no trajectory reversal takes place in the parallel phase space (p, η).

(3) Finally, for very low particle energies, such that $h_0 < h_r$, both particle reflection and trajectory reversal take place, meaning that the reflected particles will move ahead of the laser pulse, which corresponds to what is sometimes called the snow-plow effect.

Let us now make use of the invariance $h''(p, \eta) = h_0$, in order to calculate the energy gain of the reflected particles. When the incident particles move far away from the photon mirror, well before the interaction process takes place, at $\eta \to -\infty$, we can assume an initial parallel momentum $p = -p_0 < 0$, and $g(\eta) = 0$. So, from (19) we can establish

$$h_0 = \sqrt{\nu^2 + p_0^2} + \beta_{\rm g}p_0 = \gamma_0 + \beta_{\rm g}\sqrt{\gamma_0^2 - \nu^2}, \tag{34}$$

where we have used the initial gamma factor γ_0, defined by $\gamma_0^2 = \nu^2 + p_0^2$. In the other asymptotic limit, $\eta \to \infty$, which is valid for a long time after interaction with the mirror took place, we have again $g(\eta) = 0$, but a new momentum $p = p_1 > 0$. We can now write

$$h_0 = \sqrt{\nu^2 + p_1^2} - \beta_{\rm g}p_1 = \gamma_1 - \beta_{\rm g}\sqrt{\gamma_1^2 - \nu^2}, \tag{35}$$

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where γ_1 is the final relativistic gamma factor. Equating these two expressions for h_0, we obtain a relation between the initial and the final values of the particle energy

$$\Delta\gamma \equiv \gamma_1 - \gamma_0 = \beta_g\left[\sqrt{\gamma_1^2 - \nu^2} + \sqrt{\gamma_0^2 - \nu^2}\right]. \tag{36}$$

This is an implicit result for the energy gain. The explicit solution is

$$\gamma_1 = \gamma_g^2\left[\gamma_0\left(1 + \beta_g^2\right) - 2\beta_g\sqrt{\gamma_0^2 - \nu^2}\right] \tag{37}$$

which can be represented in a more revealing way in two extreme conditions. First, for highly energetic particles, such that $\gamma_0, \gamma_1 \gg \nu^2$, we can write $\Delta\gamma \simeq \beta_g(\gamma_1 + \gamma_0)$, which leads to a simple and very interesting result,

$$\gamma_1 = \gamma_0\frac{1 + \beta_g}{1 - \beta_g} \simeq 4\gamma_0\gamma_g^2. \tag{38}$$

This shows that the energy gain of energetic particles reflected by the ponderomotive force associated with an intense laser pulse is formally identical to the frequency shift of a photon after suffering a double Doppler shift at a relativistic mirror. In the other extreme case, we can consider very low energy particles, or particles initially at rest, with $\gamma_0 = 1$. From the expressions (36) and (37), we can now derive

$$\Delta\gamma = \beta_g\sqrt{(1 + \Delta\gamma)^2 - \nu^2} \tag{39}$$

and, for purely one-dimensional motion, such that $\nu^2 = 1$, we simply get $\Delta\gamma = 2\gamma_g^2$. So, in both cases we have an energy enhancement factor of order γ_g^2.

Two important points should be stated. As a result of our discussion, no dependence on the exact shape of the photon mirror is observed in the threshold for the different regimes, in the maximum energy gain for 'reflection' out of the mirror. Furthermore, this reflection can be made tunable in a plasma because the group velocity of the laser pulse, in the linear regime, leads to $\gamma_g = \omega_0/\omega_p$: by tuning the background plasma density it should be possible to not only select the reflected particles but also to tune their energy, as observed in (36), (37) and (39).

4. Radiation burst

During the process of reflection by a photon mirror, the charged particles are strongly decelerated and then accelerated, which means that they can emit secondary radiation. We can envisage two types of radiation processes. First, for purely one-dimensional motion, the secondary radiation emitted by the charged particle is simply a sum of two consecutive bremsstrahlung processes with the deceleration and the acceleration stages. Second, for oblique incidence, such that $p_{\perp 0} \neq 0$, we can have a different kind of emission process, which is due to curvature, and looks like an instantaneous synchrotron emission. In this case, the intensity of the emitted radiation can be much higher and will have a component in the perpendicular direction.

We will explore here the radiation process occurring solely due to the deceleration/acceleration stages in the one-dimensional configuration. In general, the full calculation of the radiation spectrum, and the radiated power can only be achieved for particular scenarios. We will employ the model of radiation by impulsive scattering [9]. In fact, the upper limit of the significant range of radiated frequencies

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is cut off at $\omega \sim 1/T$, where T is the acceleration/deceleration time. T is also the duration of the radiation burst for a single charged particle. For times shorter than T, the acceleration process can be considered instantaneous and the radiation spectral distribution is given by [9]

$$\frac{dE}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2 c^3}\left|\mathbf{n} \times \left(\frac{\mathbf{v}_\mathrm{i}}{1-\mathbf{n}\cdot\mathbf{v}_\mathrm{i}/c} - \frac{\mathbf{v}_\mathrm{f}}{1-\mathbf{n}\cdot\mathbf{v}_\mathrm{f}/c}\right)\right|^2 \tag{40}$$

where $\mathbf{v}_{\mathrm{i(f)}}$ is the initial (final) velocity, $\mathbf{n}$ is the radiation collection direction, and $d\Omega$ is the solid angle element.

For deceleration, $|\mathbf{v}_\mathrm{i}| = |\mathbf{v}_0| = \beta_0 c$ and $|\mathbf{v}_\mathrm{f}| = 0$, which leads, after integration over all angles of (40), to

$$\frac{dE}{d\omega} = \frac{2e^2}{\pi c^3}\left(\frac{1}{2\beta_0}\log\frac{1+\beta_0}{1-\beta_0} - 1\right) \tag{41}$$

for frequencies up to $\omega \sim 4c/l_\mathrm{mirror}$, estimated from the typical crossing time of the charged particle over the mirror thickness l_mirror, and assuming $\beta_0 \sim 1$. For the acceleration phase ($|\mathbf{v}_\mathrm{i}| = 0$ and $|\mathbf{v}_\mathrm{f}| = |\mathbf{v}_1| = \beta_1 c$), the charged particle will radiate in the opposite direction with the energy radiated at the frequency ω given by

$$\frac{dE}{d\omega} = \frac{2e^2}{\pi c^3}\left(\frac{1}{2\beta_1}\log\frac{1+\beta_1}{1-\beta_1} - 1\right) \tag{42}$$

for frequencies up to $\omega \sim c/(\gamma_\mathrm{g}^2 l_\mathrm{mirror})$, a much lower frequency range than in the deceleration stage.

Radiation will be emitted essentially along the angle $\theta \simeq 1/\gamma_{0(1)}$, which for relativistic velocities is very close to the direction of the charged particle velocity, but the frequency of the emitted radiation will be low, when compared with the frequency of the electromagnetic field in the photon mirror. However, a significant radiated energy can be achieved since all the electrons scattered in the photon mirror will radiate. Again, we observe a significant number of parameters that allow for tunability not only of the features of the radiated spectrum, but also of the total radiated energy.

5. Conclusions

We have shown here that an intense laser pulse, propagating in vacuum or in an under-dense plasma, can reflect charge particles (electrons, for instance) and act as a photon mirror. A sufficient condition for particle reflection at a photon mirror was established and three types of particle trajectory were identified. Two different regimes of particle reflection can exist. One, where the reflected particle is overtaken by the laser pulse, and the other where the reflected particle moves ahead of the pulse. The last case can be identified with snow-plow acceleration.

Particle energy gain was calculated, and shown to be similar to the frequency up-shift of photons reflected by a moving mirror. The double Doppler shift is also observed for reflected particles. A clear symmetry between charged particles and photons is then established. The intense laser pulses, acting on charged particles as relativistic photon mirrors, can move very close to the speed of light in vacuum (especially if the beams are not strongly focused), and the double Doppler energy shift can be of considerable importance. For instance, if we assume a primary electron beam with an initial mean energy $E_\mathrm{i} \sim 5$ MeV, interacting with a peta-watt

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laser pulse, and considering that the group velocity of the laser pulse can attain a relativistic gamma factor of $\gamma_g \geqslant 200$, we can expect a reflected beam with final mean energies around $E_f \sim 1$ TeV. So, a tera-electronvolt particle accelerator could eventually be achieved, in a single step, by using the photon mirror configuration.

The production of intense radiation bursts, by the electron beam during the reflection process, was also demonstrated. This secondary radiation is essentially due to two consecutive bremsstrahlung processes, for purely one-dimensional particle motion, and to instantaneous synchrotron emission, for oblique incidence. Such radiation processes can eventually be pertinent to the generation of tunable radiation sources in the tera-hertz to infra-red domain.

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