On Newtonian Induction

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This article examines Newton's method of induction and its connection to methodological atomism. The article argues that Newton's Rule III for the Study of Natural Philosophy is a criterion for isolating the primary qualities of the atomic parts; in other words, it interprets Rule III as a transductive inference. It is shown that both the standard inductive and invariance interpretations of Rule III can be subsumed under the transductive view, although the invariance criterion is reinterpreted; by qualities "that cannot be intended or remitted," Newton means qualities that are invariant under changes in the configuration of the atomic parts.

1. Introduction. According to Newton, science should not include any hypothesis whatever its merit might be; rather, scientific propositions ought to be gathered from the phenomenon by induction (1999, 943). Following various criticisms of induction, the most central one being David Hume's, many philosophers of science adopted a hypothetico-deductive (HD) account of science or some anti-inductivist line of thinking. Some philosophers argued that Newton's supposed inductivism is meant to express doubt about unsubstantiated hypotheses (see, e.g., Hanson 1970, 31–33) or that his official commitment to inductivism does not match his actual practice (see Stein 1970, 267; 1990, 217; Lakatos 1978, 212). The upshot of this revisionist trend is to claim that Newton's science must have relied on hypotheses and conjectures.

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In the last few decades, Newtonian scholarship has done much to undermine the HD reading of Newton's method (see Dorling 1973; Glymour and Stalker 1980; Smith 2002a, 2002b; Belkind 2012; Harper 2012). The main thrust of this scholarship might be summarized as showing that Newton made extensive use of demonstrative induction (DI). A DI argument takes an observational generalization and uses another universal claim to derive a further universal claim. Thus, a DI argument is a deductively valid argument. Nevertheless, a DI argument is still inductive in character, since it is an inference from the observed phenomenon to the immediate cause of that phenomenon.

Newton's application of a DI form of reasoning is perhaps one of the most sophisticated ones in the history of science. For example, in Book III of the *Principia*, Newton uses Kepler's Area Law (that the radius from the central body to each satellite covers equal areas in equal amounts of time) and Kepler's Harmonic Rule (that the period of the planets' orbits and the radius of the orbit are related as $T^2 \propto R^3$) to deduce the nature of the gravitational force—that it is a centripetal, inverse-square law. Thus, Newton uses a mathematical form of reasoning to derive the nature of the cause (i.e., the gravitational force) from its observed effects (Kepler's Laws).[1] Two important premises in this derivation, other than Euclid's postulates, are Newton's First and Second Laws of Motion. Newton can perhaps defend the claim that he has deduced the nature of gravity from Kepler's Laws, but it is not clear what justifies his bold assertion that gravitation is *universal*, that is, that gravity holds for every possible combination of bodies.

There is one methodological rule that might explain how such a universalization is possible, and that is Newton's Rule III for the Study of Natural Philosophy:

> **Rule III** The qualities of bodies that cannot be intended or remitted [i.e., qualities that cannot be increased or diminished] and that which belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally. (Newton 1999, 795)

Rule III purports to articulate a criterion for determining which qualities can be deemed universal. Presumably, once we are able to ascertain those qualities as universal, the mathematical laws articulating the effects of such qualities would acquire the status of laws of nature.

However, Rule III is notoriously difficult to interpret. We find a variety of readings of Rule III in the literature, which can be summarized as follows:

1. See Harper (1990, 2002, 2012) and Belkind (2012) for an account of these inferences.

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1. Rule III expresses commitment to naive *induction*.—According to this reading, Rule III argues that a property that is observed in all bodies can be universalized to all bodies whatsoever. So Rule III is simply an inductive procedure applied on a massive scale. Some version of this reading may be found in Burtt (1954, 219), Koyré (1965, 267ff.), Whewell (1971, 194), Torreti (1999, 73), Biener and Smeenk (2012), De Pierris (2012), and Ducheyne (2012, 117).
2. Rule III uses *invariance* as a criterion of universality.—According to this reading, Rule III universalizes those qualities that cannot be intended or remitted, that is, invariant qualities. According to this reading, Newton refers to the invariant nature of the mass parameter to show that it is an inherent and therefore a universal property of bodies. This reading of Rule III can be found in Finocchiaro (1974), McMullin (1978, 142), Okruhlik (1989), Hooker (1991), Janiak (2010, 95), and Harper (2012, 38).[2]
3. Rule III is a form of *transductive* reasoning.—According to this reading, Rule III argues that there is an inference from the observable qualities to the qualities of their ultimate atomic parts. The atomic parts are not accessible to the senses because they are too small to leave any traces on them. Thus, a transductive argument follows a more radical inferential transition than induction; it is an inference from qualities of observable bodies to bodies that are in principle unobservable. We find the assimilation of Rule III to transduction in Mandelbaum (1966, 84) and McGuire (1967, 1968a, 1968b, 1970).

These three interpretations of Rule III are not necessarily exclusive. Perhaps Newton meant to combine more than one criterion to form Rule III. But a few commentators have argued that reading Rule III as articulating a transductive form of reasoning cannot cohere with the invariance criterion. For example, McGuire has suggested that in addition to the inference to the qualities of the atomic parts, Rule III relies on the invariance criterion for highlighting the nature of these qualities as essential.[3] Janiak (2010, 93–

2. There are other interpretations of the intension/remission criterion. For example, Biener and Smeenk (2012, 118) think of the intension/remission criterion as describing a quality that cannot be taken away from bodies. Ducheyne (2012) also thinks that there are two ways to interpret this criterion: one takes the criterion to be assimilating an invariant quality to an essential quality, while another interprets the criterion as associating invariance with a quality that cannot be taken away from bodies.

3. Okruhlik (1989) criticizes McGuire's reading since Newton argues that gravity is not an essential quality, given that the weight of a body depends on the distance to the central body. Thus, according to Okruhlik, since gravity cannot be inherent or essential, it also cannot be a primary quality. In Okruhlik's view, Newton was not being very coherent in his methodological remarks.

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95), however, argues that Newton could not have taken gravity to be essential, given the variance of its intensity, and so it cannot be a primary quality. Janiak claims that we should think of Rule III as incorporating two distinct criteria, one empirical (naive induction) and one conceptual (invariance). The invariance criterion applies only to the standard list of primary qualities; the empirical criterion applies to gravity as well.

In this article I argue that there is no genuine conflict between the three readings, as long as we properly understand what Newton meant by his intension/remission criterion. I aim to show that the transductive inference is in fact grounded by the intension/remission criterion.

According to the account developed here, Newton is committed to thinking about material bodies as composed of ultimate parts that are all similar. While atomism, or the corpuscular thesis, is traditionally associated with a predetermined list of primary qualities (i.e., size, shape, and motion), at Newton's hand the atomist thesis becomes a methodological requirement. The list of primary qualities is open ended, allowing experience to determine which qualities can be attributed to the atomic parts. I will term this Newtonian way of thinking "methodological atomism," to differentiate it from standard atomist theories. However, interpreting Rule III as a transductive inference requires a revision in our understanding of the invariance criterion. Newton's invariance criterion does not connect general invariance with universality; rather, Newton is thinking of a very specific form of invariance, namely, invariance under reconfiguration of the atomic parts.

In section 2 I reconstruct and provide some textual evidence for the transductive reading of Rule III. In section 3 I consider some reasons to doubt the existing readings of Rule III. In section 4 I argue that we ought to revise our understanding of the invariance criterion in Rule III and understand it as invariance under changes of configuration of the atomic parts. In section 5 I consider how the reading offered here coheres with the list of qualities deemed universal by Newton. I conclude in section 6.

2. Some Evidence in Support of the Transductive Reading. Before examining the coherence of the various readings of Rule III, we should take a look at the textual evidence for the transductive reading. After articulating Rule III, Newton adds the following explication:

> For the qualities of bodies can be known only through experiments; and therefore qualities that square with experiments universally are to be regarded as universal qualities; and qualities that cannot be diminished cannot be taken away from bodies. Certainly idle fancies ought not to be fabricated recklessly against the evidence of the experiments, nor should we depart from the analogy of nature, since nature is always simple and ever consonant with itself. The extension of bodies is known to us only through

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> our senses, and yet there are bodies beyond the range of these senses; but because extension is found in all sensible bodies, it is ascribed to all bodies universally. We know by experience that some bodies are hard. Moreover, because the hardness of the whole arises from the hardness of its parts, we justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies. That all bodies are impenetrable we gather not by reason but by our senses. We find those bodies that we handle to be impenetrable, and hence we conclude that impenetrability is a property of all bodies universally. That all bodies are movable and persevere in motion or in rest by means of certain forces (which we call forces of inertia) we infer from finding these properties in the bodies that we have seen. The extension, hardness, impenetrability, mobility, and force of inertia of the whole arise from the extension, hardness, impenetrability, mobility, and force of inertia of each of the parts; and thus we conclude that every one of the least parts of all bodies is extended, hard, impenetrable, movable, and endowed with a force of inertia. And this is the foundation of all natural philosophy. (1999, 795–96)

Newton asserts that Rule III allows him to infer from the observable composite bodies the qualities of the ultimate parts of matter. He argues that we universally find certain qualities to exist in observable bodies and that we should not depart from the analogy of nature. The analogy of nature is an analogy between various scales of nature: the very large, the medium sized, and the microscopic. While the atomic parts are inaccessible to our senses, it is reasonable to conclude that they conform to the analogy of nature. For example, the laws of Euclidean geometry are isometric; the same laws govern the quality of extension at all observable scales. Thus, it is reasonable to assume that, by analogy, the same laws governing extension in the observable realm will also hold for the unobservable, atomic parts.

The nature of the inference is fully brought out when Newton discusses the quality of hardness: "because the hardness of the whole arises from the hardness of its parts, we justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies." There is first the claim that the hardness of composite bodies is reduced to the hardness that characterizes their undivided, atomic parts. We therefore conclude that all parts of observable matter are hard. The further inference characterizes atoms in general. Thus, we conclude that all material bodies are made of hard atoms—that is, all material bodies are hard. Newton argues that the analogy of nature holds for extension, impenetrability, hardness, mobility, and force of inertia and that he has inferred from qualities of observed bodies the qualities of the atomic parts. He claims that this inference grounds the entire enterprise of natural philosophy; eventually he will argue that the same inference applies to gravity.

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Further support for the transductive reading can be found in Cotes's preface to the *Principia*. Cotes was the editor of the second edition, and he took upon himself to provide an overall view of Newton's achievement. After rehearsing the central features of the argument, Cotes claims that the evidence shows that gravity is a primary quality like any other primary quality (Newton 1999, 392–93). The choice is either to accept gravity as a primary quality or deny the status of primary to all the qualities that are deemed primary in the mechanistic tradition.

The atomistic background to Rule III is also evident from its prehistory. Some intriguing revisions occurred between the first and second editions of the *Principia*, which mostly came in response to severe criticisms of the nonmechanical nature of Newton's gravity. Rule III is not present in the first edition of the *Principia* (1687): instead we find the following hypothesis:

> **Hypothesis III** Every body can be transformed into a body of any other kind and successively take on the intermediate degrees of qualities. (Newton 1999, Cohn's introduction, 202)

Newton here appears to be committed to an atomistic conception, according to which all parts of matter are of the same kind. According to this view, the difference between kinds of body can only be the result of a body's transformation. For example, it should be possible to take a piece of wood and reconfigure the parts of the body—that is transform it—so as to produce a piece of iron. Newton does not seem to give significance to the shapes or contingent properties of the atoms in explaining how qualities of composite bodies are determined, but he seems to argue that no quality (other than the ones to be found in the atomic parts) is inseparable from the body. Any such quality can be made to diminish its amplitude or degree and assume successively the quality's intermediate degrees until it is no longer present in the body. For example, assume that we have a body with the color white. Given the atomistic thesis, the quality of being white belongs to the composite body. When we manipulate the configuration of the parts—for instance, take the body and melt it or pound it to reduce it to powder—we may, through the transformation of the body, introduce various manifestations of the color white. Furthermore, it may be possible to slowly transform the body and diminish the intensity of the white color until the body no longer has the quality of being white; that is, it becomes transparent. The suggestion is that while bodies appear to have various sensible qualities, such as the color of wood, the smoothness of metal, the acidity of lemon juice, these qualities are partly the result of the contingent configuration of the parts and can be separated from bodies as a result of a process of reconfiguration.

The transductive reading of Rule III becomes plausible when we place Rule III in its proper historical and textual context. Newton articulates his

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commitment to atomism and the transmutation hypothesis in Hypothesis III. While revising the first edition of the *Principia*, Newton recognizes that his commitment to atomism amounts to a hypothesis. Thus, without relinquishing the methodological force of the atomist thesis, he replaces it with a rule for revealing the primary qualities of the atomic parts. Instead of committing himself to the atomistic hypothesis, he explains how one can use empirical evidence to reveal the qualities of the atomic parts, if such atomic parts exist.

The most natural reading of Rule III, given the atomistic background articulated in Hypothesis III and his explication of Rule III is to take it as a transductive inference. But if the transduction reading is the most natural one, it remains for us to determine whether there is any role for induction or the invariance criterion in explaining the nature of the transductive inference.

3. Problems with Traditional Readings of Rule III. In the next section I argue that a transductive reading of Rule III requires a revision in our understanding of Newton's invariance criterion. Before articulating the argument, it serves us to consider the tensions each reading of Rule III raises. Both the inductive and the invariance interpretations of Rule III have the strange consequence that they cannot explain why Newton thinks it is legitimate to replace a commitment to atomism, articulated in Hypothesis III, with an inductive rule and an invariance criterion, expressed in Rule III. The transductive reading fits more comfortably with the prehistory of Rule III.

Another problem with assimilating Rule III to naive induction is that it largely ignores the invariance criterion. Some commentators attempt to reduce the invariance criterion to the claim that a quality cannot be taken away from bodies. This would amount to saying that the quality is universally present in all observed bodies, thereby substantiating the premise of the inductive inference. However, it would be odd to think of the invariance criterion as simply another articulation of the inductive criterion.

The main benefit of the invariance interpretation is that it brings to bear Newton's skillful use of mathematical methods for analyzing physical quantities. It is the invariant nature of mass, for example, in various physical and chemical processes that makes this quality a very good candidate for an inherent quality that exists in all bodies. The reading's drawback is that it is difficult to make it cohere with the list of qualities that are mentioned in Rule III. For example, given that the force of inertia depends on the body's velocity, it is clearly not an invariant quality of the body; moreover, it would be strange if one could think of it as essential in any way.[4] More important, how can Newton think that invariance is a mark of the universal nature of gravity, given that its intensity is not invariant? In response to this worry one often finds the claim that gravity is proportional to the body's mass

4. See Finocchiaro (1974, 68) for raising this issue with the force of inertia.

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and so is a function of an invariant parameter. If a force law depends on invariant parameters of bodies, so the argument goes, the force is a universal force. But this is a strange reading of the wording of Rule III that focuses on the invariance of the quality, not the invariance of another quality that causes it.

The benefit of assimilating Rule III to a transductive inference is that it is much more faithful to Newton's text and Rule III's historical context. The doctrine of transmutability is essentially a commitment to atomism, and so it is reasonable to read Rule III as differentiating primary from secondary qualities. Nevertheless, there is a problem with showing how gravity can be considered primary. It is ordinarily understood during the seventeenth century that primary qualities are essential and inherent to bodies. However, after explicating Rule III, Newton asserts that despite applying Rule III to gravity he does not take gravity to be essential: "Yet I am by no means affirming that gravity is essential to bodies. By inherent force I mean only the force of inertia. This is immutable. Gravity is diminished as bodies recede from the earth" (Newton 1999, 796). This remark is quite puzzling since Newton claims that the invariance criterion implies that gravity is a universal quality and then, in the explication of the rule, he argues that gravity is not essential, since it changes its intensity when bodies recede from the earth. This is puzzling under both the invariance and the transductive interpretations. If gravity's intensity is not invariant, why does Rule III apply to gravity? Moreover, if gravity is not essential, how is it primary?

It is clear that each of the available readings of Rule III has serious drawbacks. The suggestion in what follows is to combine elements of these various readings into one coherent interpretation that will remove the faults of each reading.

4. The Nature of Invariance in Newton's Invariance Criterion and the Character of Newtonian Inductivism. It is clear from drafts of the second edition of the *Principia* (1713) that Hypothesis III can be taken to be a predecessor to Rule III, which appears in the second edition. It is also clear from the drafts that Newton had erased the word "Hypothesis" where Hypothesis III was written and introduced the word "Rule" (McGuire 1968b, 235). He also articulated various versions of this rule, ending with the final version inserted in the second edition. That they are intended to provide similar methodological functions is made clear by the fact that Rule III is cited at the same place within Book III, Proposition VII, Corollary II of the *Principia*, where previously Hypothesis III was cited.[5]

5. Further evidence that he viewed Rule III as a reformulation of Hypothesis III is a draft of Corollary II written after the first edition, where Newton argued that the fact that grav-

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If Hypothesis III is a predecessor to Rule III, how is Newton transitioning from qualities that can assume all intermediary degree to qualities that cannot be intended or remitted? McMullin (1978, 7), for example, thinks that this generates a conflict between Hypothesis III and Rule III. Supposedly Hypothesis III allows for a primary quality such as solidity to change its intensity until it is no longer present in the body, rendering the quality nonprimary. But this is only an apparent problem if we take Hypothesis III to refer to secondary qualities that are separable from bodies and Rule III as referring to primary, inseparable qualities. I agree with Brading (2012, 28–29) that the conflict between Hypothesis III and Rule III is only apparent. But contrary to Brading I do not think that Rule III adds a new invariance criterion; rather, Newton is merely shifting focus from the possibility of transformations (the existence of variant and separable secondary qualities) to the qualities that make such transformations possible (the existence of invariant primary qualities).

In Hypothesis III, Newton is committed to the notion that the real difference between bodies of distinct kinds amounts to differences of configuration. Boyle's technical term for the notion of configuration is "texture" (see Boyle 1999, 315–16). According to Boyle the texture of a body includes the *order* and *posture* of the different corpuscles—that is, order consists of the relative positions between the parts, and posture includes the spatial orientation of each of the parts. Thus, for Newton whenever we find a quality of a composite body, which has an intensity that varies as a result of changing the body's texture, the quality can assume every degree of its intensity, until a body is transformed into a different kind of body. When Newton considers invariant qualities in Rule III, he is merely shifting focus from the qualities that are separable from bodies to qualities that exist in the atomic parts—that is, qualities that are inseparable.[6]

There is textual evidence that the transition from Hypothesis III to Rule III is a transition from secondary to primary qualities. We can actually see the transition in one of Newton's drafts to Rule III, titled "Axiom 4":

> Qualities which are intended (and remitted) such as motion and rest, heat and cold, wet and dry, light and darkness, . . . these things do not come in

ity is a quality that cannot be remitted is a property of all bodies, according to Hypothesis III. See McGuire (1968b, 234).

6. We are not using the distinction between primary and secondary qualities in Descartes's or Locke's sense, who claim that ideas of primary qualities resemble those qualities while ideas of secondary qualities do not. In the context of Rule III, the distinction amounts to that between fundamental and derived; the primary qualities are those that form the basis of a scientific theory (and are present in all the atomic parts), and secondary qualities are those derived from the fundamental qualities of the parts, together with the body's texture.

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> for consideration here. The things which cannot be intended and remitted such as (bulk) impenetrability (solidity) and motion . . . and that inertia which causes a resistance to motion and to changes of motion (solidity and extension) are usually considered to be the properties of all bodies. And the reason is because a quality which cannot be remitted cannot be taken away (from the whole); and on the other hand that which can be taken away, if it were to be taken away from some parts of the whole, it could be remitted in the whole. Impenetrability indeed is usually described as the essence of bodies and is hence attributed to all of them; but the essential properties of bodies do not become known by the light of nature. We gather (only) from the senses the fact that the bodies we touch (all tangible things) are impenetrable and we (conclude that) we attribute to all bodies alike, no less to the heavenly bodies and to bodies imperceptible to the senses than to those bodies which we touch: and that too with an argument which we think so strong (that nothing in the whole nature of things) that this quality is taken to be the essence of bodies and is therefore considered to be the firmest foundation of all philosophy. Therefore the axiom on the strength of which we gather this property of bodies must not be repudiated. Of the same kind is the argument for inertia and (mobility and force of resistance) of matter. (McGuire 1968b, 237)

We see in Axiom 4 an intermediate formulation that connects Hypothesis III with Rule III and mentions both variant and invariant qualities. It is natural to assume that this draft to Rule III is a stage in the change of focus in articulation from secondary to primary qualities. Another important thing to notice is that Newton connects invariance with a part whole relation. He says that a quality that cannot be remitted cannot be taken away "from the whole." By this he seems to mean that given that various attempts to transform a composite entity (say, by manipulations of force) are unable to increase or diminish the intensity of the entity's quality implies that it is inseparable from the whole. The implication of this invariance is that a quality that is invariant in the whole is the direct sum of the qualities that are present in the parts, and so it cannot be diminished when the whole is transformed. Similarly, Newton says that if the quality were separable from the parts, it would be diminished in the whole. Thus, the notion of inseparability is shown to be intimately tied to the invariance of a quality in the whole.

In this draft for Rule III Newton describes the type of inference he is considering. Newton argues that "the essential properties of bodies do not become known by the light of nature." That is, unlike Descartes and other speculative philosophers who inquire after the essence of bodies by way of some conceptual criterion, the purpose of the invariance criterion is to determine the basic qualities of the atomic parts through experience. Because the invariance criterion is based on our experience with transformations of texture,

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the list of primary qualities might grow in tandem with our understanding of the phenomenon. Newton is therefore articulating an empiricist criterion for primary qualities. In Rule III Newton drops altogether any positive mention of essential qualities, after realizing that Rule III is a rule for revealing primary qualities but is not a rule for uncovering the inherent or essential qualities of matter (more on this in the next section).

Despite the connection between Hypothesis III and Rule III, they are not equivalent formulations of the same idea. One way to think about the transition from Hypothesis III to Rule III is to place it relative to the context of Newton's response to criticisms that his gravity is nonscientific. Newton's theory of gravity seems to be committed to action at a distance and does not adhere to the prevalent mechanistic paradigm. As a result, Newton's theory received severe criticisms from luminaries such as Leibniz and Huygens. In response, Newton recognized that he needs to form a sharper distinction between hypotheses and scientific propositions and argue that no hypothesis, including the mechanistic hypothesis, should be used to undermine a scientific proposition derived from the phenomenon. Thus, Newton proceeded to eliminate explicit commitments to hypotheses from the *Principia*. One of the revisions included discarding Hypothesis III and replacing it with a methodological rule. The transition might be described as follows: Hypothesis III commits Newton to a strong atomistic thesis. To weaken his hypothetical commitment to atomism, Newton has turned it into a conditional statement: if all parts of matter are alike, we will discover in our experience universal qualities that are directly amassed from the ultimate parts of matter. The empirical evidence that such qualities exist is that there are qualities present in every experiment performed, and they are invariant under changes of texture.

The recasting of atomism as methodological atomism explains Newton's remarks in Rule III about the divisible nature of particles. After explaining how Rule III underwrites the status of extension, impenetrability, hardness, force of inertia, and mobility as universal qualities, Newton adds the following statements:

> Further, from phenomena we know that the divided, contiguous parts of bodies can be separated from one another, and from mathematics it is certain that the undivided parts can be distinguished into smaller parts by our reason. But it is uncertain whether those parts which have been distinguished in this way and not yet divided can actually be divided and separated from one another by the forces of nature. But if it were established by even a single experiment that in the breaking of a hard and solid body, any undivided particle underwent division, we should conclude by the force of this third rule not only that divided parts are separable but also that undivided parts can be divided indefinitely. (Newton 1999, 796)

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While Rule III articulates a rule for revealing the primary qualities of the ultimate parts of matter, whether those ultimate parts are divisible remains to be determined. Newton argues that if we come across a particle that is impenetrable and hard (and solid) and are able to conduct an experiment showing that the particle is divisible, we have as a matter of fact uncovered the divisible nature of impenetrable parts of matter. By the force of Rule III, which allows us to universalize qualities of the seemingly noncomposite parts of matter, we should think that all particles are divisible. Unlike Hypothesis III, which seems to be committed to the existence of atoms, the methodological atomism of Rule III is open to the possibility that experience would show that atoms do not exist. What we can conclude from applying Rule III is that there are universal qualities that describe all parts of matter.

The type of invariance described in Rule III is distinct from the kind of invariance involved in finding the same parameter under all changes in relations. The invariance Newton is concerned with is invariance under the active reconfiguration of the body's parts. Such invariance does not necessarily presuppose that the quality at stake is inherent since it may be that a quality that is invariant under changes of texture is still a relative or a noninherent quality (i.e., determined by a set of relations to parts that are not part of the body). Thus, the reading provided here is distinct from the standard reading that assimilates Rule III to a general invariance criterion.

We might summarize Newton's inference as follows:

> All observed bodies have a quality Q.
>
> The quantity describing the intensity of Q is invariant under transformations of texture (i.e., reconfiguration of the parts).
>
> Therefore, all bodies have a quality Q.

The inference described by Rule III is therefore a particular kind of inductive inference. It is an inference that requires a wider basis than most inductive procedures because it relies on a quality that is found in all observed bodies. But the inference is not a simple inductive inference since it incorporates two distinct steps. From the fact that all observed bodies have quality Q, and the quantity describing the intensity of Q is invariant under transformations of texture, Newton concludes that all ultimate parts of observed matter have quality Q. Let us call this type of inference the "transductive" inference from the observed composites to their unobserved, ultimate parts. The second inductive step derives from the fact that all ultimate parts of observed matter have quality Q the claim that all parts of matter have quality Q. Let us call this step "universal induction." From the fact that all parts of matter have quality Q, Newton concludes that all bodies have quality Q, assuming here

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that a primary quality Q of any composite body is a direct sum of the qualities present in the parts.

5. Newton's List of Primary Qualities. It remains to be seen how the reading just articulated is consistent with Newton's list of universal qualities, namely, extension, hardness, impenetrability, mobility, force of inertia, and gravity. When considering the invariance of a quality, we need to consider the quantities that represent those qualities. In the case of extension, we may think of the geometric quantity of volume, which represents the volume or space a body takes up. It is clear that while a body may increase or decrease in volume (as, e.g., in the case of a gas that expands or contracts) such changes in volume are ordinarily the result of the body's empty regions and pores increasing or diminishing. But the body itself, pores excluded, has a volume that is a mathematical sum of the volumes of the parts. One may consider this additive nature of volume as perhaps the defining characteristic of Euclidean geometry. A central presupposition for Euclidean geometry is the notion that when segments of lines are added, the magnitude attributed to the composite line is a sum of the magnitudes attributed to the parts. A similar assumption holds for the entire volume attributed to a composite body as a function of the volumes attributed to the parts. Thus, it seems to be an essential assumption in Euclidean geometry that the volume of a body will not change as a result of a mere reconfiguration of the parts, as long as we do not include pores and empty regions in the volume's calculation. This implies that the extensions attributed to the body's ultimate parts are analogous to the extension we attribute to observable composite bodies, and so extension is a universal quality.

Hardness and impenetrability seem to be associated with a body and make the body distinct from space itself. Notice that Newton is not mentioning mass, or quantity of matter, as the universalized quality because for him the mass parameter is merely a quantity, not the qualitative aspect of bodies that make them impenetrable or hard. This point perhaps ought to be emphasized because commentators often ignore the actual list of qualities mentioned by Newton, which includes hardness and impenetrability but not mass; nevertheless, commentators list mass as the universal quality. The difference between a quantity and a quality is that a quantity may indicate a measure attributed to a quality, but it does necessarily represent the quality itself, given that the quantity attributed to the body depends also on the number of bodies present or the magnitude of a composite body. Thus, the mass of a composite body does not indicate solely the inherent quality of a body but is oftentimes amassed from the measures representing the inherent qualities of the body's ultimate parts. If we divide a composite body into two, it is often the case that nothing inherently changes in that composite body: we simply separate out one part of the aggregate from another. However, the mass, or

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quantity of matter of the composite body, will be divided into two distinct quantities of matter attributed to the parts. Thus, it is a misreading of Newton's words when commentators take the mass parameter itself to be a quality since the mass parameter is nothing but a measure, a magnitude representing a certain intensity associated with a quality, or a measure associated with an aggregate of bodies that have the quality.

The claim that hardness and impenetrability are universal qualities is implied by the fact that the mass parameter is observed in all bodies and is invariant under transformations of texture (i.e., mass is conserved). We can conclude from this feature of the mass parameter that impenetrability and hardness are universal qualities. Given that the quantity that represents the magnitude of bodies behaves like an extensive quantity, analogous to temporal duration and extension, it is divisible into the ultimate parts of matter.

Mobility is the power of bodies to move from one place to another, and force of inertia compels a body to move in a straight line. Both of these qualities are reflected in the quantity associated with movable bodies, namely, the quantity of motion (Newton's term for momentum). The quantity of motion of a composite body is always the sum of the quantities of motion attributed to the parts. Thus, there is a strong indication that changes in the configuration of the parts do not correlate with changes in the overall quantity of motion (assuming a particular distribution of quantities of motion in the parts). The divisible nature of quantity of motion is also implied by the fact that the quantity of motion of a solid body is proportional to the quantity of matter. This implies that each part of the solid body carries its own quantity of motion: that is, each part of the body has both mobility and force of inertia.

The fact that quantity of motion is proportional to quantity of matter renders mobility and force of inertia universal qualities. But this is not because mass is an inherent parameter and causes the force of inertia. Rather, the fact that quantity of motion is proportional to the mass parameter is mathematical evidence that quantity of motion arises from the quantities of motion attributed to the ultimate parts of matter, not from the particular configuration of the parts. This is a subtle point because the traditional invariance view is unable to make sense of the force of inertia being a universal quantity. The force of inertia can be intended or remitted as the result of changes in the body's velocity.[7]

7. To make sense of how the force of inertia is explained by Rule III, Janiak (2010, 95) argues that the force of inertia is simply mass, given that Definition III of the *Principia* associates the force of inertia with a body's mass. But this is a confused reading of Definition III, in which Newton asserts that the force of inertia is proportional to the body and is the inertia of the body. The force of inertia is also determined by the body's velocity since it can be thought of as the body's impetus. Moreover, identifying force of inertia with mass is odd, since Newton lists four universal qualities, hardness, impenetrability, mobility, and force of inertia. It would be odd because it would make three of these qualities refer to mass.

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If one views the quantity of motion as a vector quantity equal to the product of an inherent mass parameter and velocity, one will not be able to understand how Rule III applies to force of inertia. But consider Newton's explication of quantity of motion: "The motion of the whole is the sum of the motions of the individual parts, and thus if a body is twice as large as another and has equal velocity, there is twice as much motion, and if it has twice the velocity there is four times as much motion" (1999, 404). Newton does not clearly distinguish between the extension of the body and its mass, but he thinks that quantity of motion behaves partly like an extensive quality, given that it is proportional to the "size" of the body. Thus, we can think of the quantity of motion of a composite body as being amassed from the quantities of motion in the ultimate parts of bodies. The quantity of motion can be intended or remitted, but it cannot be intended or remitted as a result of changes in texture (in a closed system). The consequence is that mobility and force of inertia are universal qualities present in all material bodies.

Rule III therefore underwrites the universality of the force of inertia; at the same time, it also supports the universality of the law that governs this quality, namely, the conservation of quantity of motion. In other words, Rule III underwrites the claim that the conservation of quantity of motion is a law of nature. Newton does not mention explicitly that the conservation of quantity of motion is a law of nature. However, assuming that we can define force as a change in quantity of motion, and assuming that Newton models his forces with impulse forces, we can demonstrate that the three laws of motion are logically equivalent to the conservation of quantity of motion. Therefore, Rule III explains why Newton thinks he is justified in taking the three laws of motion as laws of nature.

Finally we get to the quality of gravity. It is the purpose of Rule III, I contend, to show that gravity is a primary quality like any other primary quality. Newton's explication of Rule III continues as follows:

> Finally, if it is universally established by experiments and astronomical observations that all bodies on or near the earth gravitate toward the earth, and do so in proportion to the quantity of matter in each body, and that the moon gravitates toward the earth in proportion to the quantity of its matter, and that our sea in turn gravitates towards the moon, and that all planets gravitate toward one another, and that there is a similar gravity of comets toward the Sun, it will have to be concluded by this third rule that all bodies gravitate toward one another. Indeed, the argument from phenomena will be even stronger for universal gravity than for the impenetrability of bodies, for which, of course, we have not a single experiment, and not even an observation, in the case of the heavenly bodies. Yet I am by no means affirming that gravity is essential to bodies. By inherent force I mean only the force of inertia. Gravity is diminished as bodies recede from the earth. (1999, 796)

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Notice that Newton highlights two properties of gravity: that gravity is present in all observable bodies and that it is proportional to quantity of matter. This, according to him, suggests that criteria articulated in Rule III apply to gravity as well. Under the traditional invariance reading of Rule III, this is a baffling claim. It is one thing to claim that quantity of matter is universal, given that it is invariant. But why should we think that a force proportional to quantity of matter is a universal quality?[8]

A more charitable reading connects the claim about gravitation being proportional to quantity of matter to a claim about the divisible nature of the gravitational force. Take, for example, the empirical fact that all bodies on the surface of the earth experience the same gravitational acceleration. Take a body with a certain mass of 1 kilogram. It experiences a gravitational acceleration of $g = 9.8[m]/[s]^2$. If we divide the body into two parts, each of which is 1/2 kilogram, each part experiences the exact same gravitational acceleration as the composite whole. This implies that the gravitational force must be twice as large in the composite body as the gravitational forces operating on the parts. Divide the body further and further into parts; since each divided part has the same gravitational acceleration as the whole, one may conclude that the gravitational force is proportional to the quantity of matter that is found in each of the parts. The gravitational force is therefore exactly divisible in proportion to the masses found in the parts. If we divide a body further until we reach the atomic parts, the force of gravity will divide itself until we reach the force of gravity that operates on each atomic part. The important fact about gravity is that its intensity does not vary as a result of reconfiguring the atomic parts or changing the body's texture.

To see how Rule III is used by Newton, consider the steps he takes before arguing that gravity is universal. After deriving the centripetal and inverse-square nature of the force of gravity, Newton considers the significance of the invariance of gravitational acceleration. Because gravitational acceleration for bodies is constant at a particular distance from the central body, Newton concludes in Book III, Proposition VI, that the force is proportional to the quantity of matter. The next step, taken in Corollary 1 to Proposition VI,

8. In response to this difficulty, Ducheyne (2012, 17) argues that Newton did not equate the nature of gravity with the invariance criterion articulated in Rule III. Ducheyne claims that Newton distinguishes between qualities that cannot be intended or remitted, in the sense of being invariant, and qualities that cannot be intended or remitted, in the sense of it being impossible for them to be "taken away." Ducheyne is basing this reading on a quote from one of the drafts of Rule III in which Newton says that "their gravity in proportion to their matter is not intended or remitted in the same region of the earth by any variety of forms and therefore it cannot be taken away" (cited in Ducheyne 2012, 117). But Ducheyne's quote itself suggests that the invariance criterion amounts in the case of gravity to the idea that gravity does not change its intensity as a result of "any variety of forms," by which Newton means varieties of the form or texture of bodies.

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is to derive the conclusion that a certain type of invariance applies to the force of gravity:

> Corollary 1. Hence, the weights of bodies do not depend on their forms and textures. For if the weights could be altered with the forms, they would be, in equal matter, greater or lesser according to the variety of forms, entirely contrary to experience. (Newton 1999, 809)

Thus, in Corollary 1 (still present in the second edition), Newton says that because gravity is proportional to quantity of matter, it does not change its intensity as a result of changes in form or texture. The notion of invariance relative to changes in form connects us to Hypothesis III and the transmutation thesis associated with it. Unlike the secondary qualities that change their intensity while a body transforms, gravity remains invariant. Also, Newton equates form with the notion of "texture," which connects us directly to Boyle's technical term for the configuration of the parts. Thus, Corollary 1 brings out explicitly the type of invariance that Newton has in mind, which is the idea that the weights are invariant under changes of the configuration of the parts.

In Corollary 2 to Proposition VI, Newton repeats the claim that gravity is proportional to quantity of matter and then argues that "this is a quality of all bodies on which experiments can be performed and therefore by rule 3 is to be affirmed of bodies universally" (1999, 809). So Corollary 2, the only place where Newton cites Rule III, comes right after Corollary 1, which is the proper context for applying Rule III to gravity.[9]

The invariance of gravity under changes of texture is in direct conflict with mechanical theories of ether. When we imagine gravity to be a product of the friction created by an ether, we must suppose that the configuration of a body, its particular shape, whether its surface is smooth or coarse, whether it is dense or porous, would affect the intensity of gravity. Newton's *Principia* is also a study of the theory of fluids that demonstrates how bodies of varying shapes, volumes, and textures would move differently through a fluid medium.

The second part of Corollary 2 is as follows:

9. Harper (2012) explains how Proposition VI, Corollary 1, is very much aligned with the notion that gravity does not change as a result of changes in form or texture (271). He then does not correlate Corollary 1 with what is said in Corollary 2, which is that the fact that gravity is proportional to quantity of matter implies, by Rule III, that gravity is universal. Harper argues that Newton has in mind the invariant parameter $Q(x) = w(x)/m(x)$, which is the ratio between weight and mass at a particular distance from a central body. He then goes on to argue that this parameter is invariant and is therefore universal. But Harper's reading does not explain how Corollary 1 and Corollary 2 are related or why the particular invariance mentioned in Corollary 1 is relevant to universalization.

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> If the aether of any other body whatever either were entirely devoid of gravity or gravitated less in proportion to the quantity of matter, then, since (according to the opinions of Aristotle, Descartes, and others) it does not differ from other bodies except in the form of its matter, it could by change of its form be transmuted by degrees into a body of the same condition as those that gravitate the most in proportion to the quantity of their matter; and, on the other hand, the heaviest bodies, through taking on by degrees of the form of the other body, could by degrees lose their gravity. And accordingly the weights would depend on the forms of bodies and could be altered with the forms, contrary to what has been proved in corol. 1. (Newton 1999, 809)

Newton here describes what would happen if the motion through the ether were to bring about the phenomenon of gravity. An ether theory of gravity would allow us to change the form of a body (say, by making it less dense and more porous) to produce varying degrees of ether pressure. By mere changes in form, one could transmute the body until it no longer experiences gravity or take a nongravitating body and transform it, until it acquires the full gravitational pull. Thus, from the fact that changes in form or changes in texture do not produces changes in the intensity of gravity, it follows that gravity cannot be produced by fluid pressures and instead arises from the gravitational forces operating on the parts.

Harper (2012, 273) discusses Corollaries 3 and 4 of Proposition VI, which is related to Cotes's objection to Newton that Newton has not shown empirically that all parts of matter have the same density. Corollary 4 is stated as a hypothetical suggesting that if all parts of matter have the same density, empty regions of space must be assumed given that different bodies have different specific gravities. At the end of Corollary 3 Newton asks whether the fact that density changes as a result of changes in texture (e.g., in condensation and rarefaction) does not imply that it is a secondary quality, and therefore removable from bodies. On the one hand, density behaves like a secondary quality, while on the other hand, quantity of matter does not. The problem would be resolved if it could be shown that all atomic parts have the same density (Corollary 4), generating a direct link between impenetrability and density. Biener and Smeenk (2012) have reconstructed the correspondence between Cotes and Newton and have shown how Cotes has drawn Newton's attention to this unsubstantiated assumption. They describe the tension between the geometric conception of matter, which thinks of material bodies as impenetrable regions of space, and the dynamic conception, which derives the measure of quantity of matter from the dynamic interactions of a body. The dynamic measures of quantity of matter could potentially lead to the ascription of varying densities to impenetrable regions of space of equal size.

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Cotes has convinced Newton that he cannot show that all atoms have the same density. Since Newton has no proof that all full bodies without any pores have the same density, he cannot be certain of the transmutation hypothesis articulated in Hypothesis III. Thus, other than the motivation to loosen his hypothetical commitment to atomism stated above, Newton could have been pushed to make the change to Rule III on the force of Cotes's objection. Granted that these are relevant motivations, they do not undermine Newton's overall approach, namely, that one can distinguish between secondary qualities of bodies, which vary whenever the texture of the body changes, and primary qualities that are invariant.[10]

This revision in interpreting the invariance criterion enables us to view gravity as primary and nonessential or noninherent. It is primary, in the sense that it can be universalized to all atomic parts. However, it is not essential because the force of gravity that operates on each part varies in its intensity as the parts recede from the central gravitating body. Thus, Newton's innovation is in articulating an empirical criterion for distinguishing primary from secondary qualities without this criterion necessarily demonstrating the essential nature of the quality.

One difficulty with this view is that it seems to suggest that a world containing a single atom might not have the gravitational quality. If the gravitational force is necessarily derived from relations between bodies, it cannot be present in a single atom if that atom is the only one existing. Newton has two interpretive options to deal with such a case. One option is to argue that Rule III shows gravity to be a universal quality, except for the case of a universe containing a single atom, in which gravity does not exist. This option takes the universality of gravity to lack the metaphysical grounds that make it a truly universal quality. Another interpretive option would argue that the universality of gravity implies that a universe containing a single atom is ruled out as physically impossible. Given that gravity is universal, one may require that each atomic part would experience a gravitational pull, so that any single atom requires at least one other atom to exit. Either of these interpretive options seems counterintuitive, but the best way to accommo-

10. Smeenk and Schliesser (2013) claim that the shift from Hypothesis III to Rule III perhaps reflects Newton's complete withdrawal of his commitment to atomism. However, Smeenk and Schliesser seem to conflate questions about a recondite issue within a broad atomistic framework with questions about the atomistic framework as a whole. Whether all atomic parts are exactly alike, or whether in fact ultimate parts are always divisible, does not change the general tenor of the argument, namely, that some qualities stem directly from the qualities present in the ultimate parts of matter. In fact, as Smeenk and Schliesser point out themselves, all the evidence from the *Opticks* and from Newton's followers shows that he entertained a belief in atomism (maybe we should call it Newton's corpuscularianism) all throughout his scientific career.

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date everything that Newton says, especially the claim that gravity is both primary and nonessential, compels us to adopt one of these two options.

6. Conclusion. Recent scholarship regarding Newton's inductive procedures has emphasized his use of DI arguments and has demonstrated the argumentative strategies that distance Newton from the HD account. However, emphasizing the role of DI arguments requires further analysis of the method by which a phenomenal law can be elevated to a law of nature, without which the DI arguments do not have any hinges on which to turn. Newtonian induction combines DI arguments with a distinct inductive procedure for revealing primary or universal qualities. Newton's methodological atomism is based on the idea that those qualities that are derived from qualities of the atomic parts have an intensity that does not depend on a body's transformation of texture. Once we have reduced a quality to qualities of the atomic parts, we can generalize to all atoms and to all composite bodies. Laws that describe the nature of primary qualities are taken to be universal laws of nature.

One might argue that methodological atomism is a hypothesis, and a false one at that. Not all parts of matter are the same, and there are good reasons for thinking that the analogy of nature fails. I cannot enter this important debate given the limited scope of the article, except to say that the universal nature of extension, inertial forces, and gravity is still part of the foundation of contemporary physics, despite the radically different theories that we have about these qualities. So, perhaps in the physical sciences methodological atomism has its modern variations. The significance of this article was to uncover the distinct nature of Newton's inductive procedure and how it differs from naive induction.

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