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EDITORIAL

Editorial: Special issue on risk sharing

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Abstract
The special issue on risk sharing contains 12 papers. This editorial introduces these papers by briefly discussing their contents and contributions.

1. Risk sharing in a rapidly changing world

Risk sharing is one of the foundational principles of actuarial science: by pooling exposures and spreading losses, societies transform economic risks – as well as their associated uncertainty – into quantitatively manageable events. Yet the 2020s have reminded us that the landscape of risks – and of the mechanisms that absorb them – is evolving at an unprecedented pace. Climate-induced catastrophes occur with rising frequency and severity; demographic shifts compound longevity and social security risks; cyber threats materialise across interconnected digital infrastructures; and financial markets propagate shocks globally within seconds.

Against this backdrop, the design of efficient, equitable and resilient risk-sharing arrangements has become more urgent than ever. In this environment, classical actuarial models are being supplemented – and at times challenged – by new methodologies and technologies. Analytical tools such as game theory, robust optimisation, machine learning and distributed ledger technologies are integrated into frameworks of insurance, pensions and financial regulation at an increasingly speed. This blending of traditional theory with modern computation reflects a broader trend: risk sharing, both as theoretical concepts and practical mechanisms, must evolve to stay relevant.

This special issue responds to this momentum. Our aim is two-fold:

- To showcase conceptual advances that extend the theoretical foundations of risk sharing under uncertainty,
- To highlight innovations in practice that translate theory into mechanisms capable of protecting households, firms and societies under real-world constraints.

2. Overview of accepted contributions

The 12 articles collected in this special issue reflect the diversity and evolution of contemporary risk-sharing research. They span topics such as economic models under uncertainty, fairness and heterogeneity in insurance and retirement systems, technological innovation, and applied actuarial frameworks. Together, these contributions enrich several strands of literature and point to new directions for both theoretical and practical developments in modern financial and insurance contexts.

This editorial introduces these papers, highlighting their key insights, how they relate to one another and how they contribute to the broader, developing discourse on risk sharing.

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3. Equity and fairness in longevity and retirement risk sharing

Longevity heterogeneity across socioeconomic groups raises long-standing questions about fairness in pooled retirement products. Early work such as Yaari (1965) formalised the welfare gains from annuitisation in frictionless markets, while more recent studies – for example Piggott *et al*. (2005), Sabin (2010) and Milevsky and Salisbury (2015) – demonstrate how product design can mitigate inequities caused by mortality gradients. Against this backdrop, the four papers in this category push the frontier by uniting modern mortality modelling with redistributive or Pareto-efficient mechanisms.

Aragona *et al*. (2025) advance the concept of Group Self-Annuitisation (GSA) by incorporating explicit redistribution rules to offset socioeconomic mortality gaps. Building on the two-cohort design of Piggott *et al*. (2005), they generalise the framework to an arbitrary number of groups and derive closed-form transfer rates that preserve incentive compatibility while satisfying actuarial fairness principles similar to those discussed in the tontine literature (Chen and Rach, 2023).

Kang *et al*. (2025) explore fairness in tontine schemes using mortality models with long-range dependence. Integrating the Volterra framework with an innovative fairness measure (the f-value), they propose a hybrid optimisation model that accounts for age and wealth heterogeneity. Their analysis complements the constant relative risk aversion results of Sabin (2010) and the load-adjusted tontine pricing in Chen *et al*. (2021), showing that path-dependent survival dynamics can either amplify or dampen perceived inequities in payout profiles.

Hanbali *et al*. (2025) introduce time-varying Pareto-optimal designs for annuities. Their heuristic tackles a challenging optimisation problem and addresses the dynamic nature of annuity markets. By allowing benefit adjustments reminiscent of the 'money-back-in-expectation' constraint analysed by Milevsky and Salisbury (2025), the authors provide a tractable bridge between the static optimal-retirement-income model of Yaari (1965) and the adaptive 'tonuity' concept of Chen *et al*. (2019).

Tao *et al*. (2025) formulate a continuous-time stochastic control model for dynamic risk exchange among heterogeneous agents and derive explicit Pareto-optimal rules for investments, consumption and enter-entity transfers. Their framework generalises the classical sharing results of Borch (1960) and extends the continuous-time risk-exchange prototype of Iwaki (2002) by allowing risk and wealth to co-evolve over an entire planning horizon. When applied to a Target Benefit Pension (TBP) plan, the mechanism demonstrates how calibrated transfer schedules can absorb longevity-driven funding shocks without breaching solvency bounds, thereby complementing recent TBP design studies by Wang and Lu (2019) and Zhu *et al*. (2021).

Zhou *et al*. (2025) extend the theory of risk sharing in mortality pooling products by incorporating stochastic and cohort-correlated mortality dynamics. Motivated by the empirical evidence that mortality rates are both random and correlated across groups, the authors propose a novel 'joint expectation rule' that preserves actuarial fairness while accounting for these complexities. Their model generalises and analytically compares established risk-sharing rules – including the proportional, regression and alive-only rules – under systematic mortality shocks. Through extensive simulation and analytical characterisation, they demonstrate how factors such as account balances, pool size and inter-cohort correlation shape the distribution of mortality credits. Building on foundations laid by Denuit (2019) and Fullmer and Sabin (2018), this paper provides a unifying extension to settings with stochastic and dependent mortality. The results offer design insights into dynamic pooling structures with heterogeneous and evolving membership and show that the choice of sharing rule crucially affects the equity and stability of retirement income under longevity risk.

4. Peer-to-peer and decentralised risk sharing

Recent years have witnessed a surge of interest in decentralised or peer-to-peer (P2P) insurance. Foundational work on comonotone *conditional-mean* risk-sharing (Denuit and Dhaene, 2012; Denuit *et al*., 2022) shows how pooling can reduce losses in convex order, yet those rules are computationally demanding. To retain tractability, later studies adopted *linear* allocation rules – variance minimisation

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in Feng (2023), mutual-aid efficiency in Abdikerimova and Feng (2022), while still pursuing fairness and participation incentives.

Yang and Wei (2025) revisit this variance-based strand and deliver the first theoretical proof that within a very broad admissible class, total variance is minimised when linear sharing is applied to *residual* (mean-centred) risks rather than the raw losses. Their result unifies earlier heuristic insights in Feng (2023), clarifies why residualisation preserves actuarial fairness and echoes the 'risk-anonymity' principle of Jiao *et al*. (2022). By adding market-oriented constraints – variance reduction and γ-retention consistency – the paper extends the optimisation framework of Abdikerimova and Feng (2022) and proves robustness against model estimation error.

Boonen *et al*. (2025) revisit the optimal design of P2P insurance by synthesising ideas from both reinsurance and mutual-aid literatures. While prior models often treated P2P pooling in isolation or assumed homogeneous agents (e.g., Abdikerimova and Feng, 2022), their study introduces a hybrid framework that integrates traditional insurance for catastrophic losses with decentralised cost-sharing for moderate claims via a common deposit fund. Within a mean-variance setting, they derive closed-form solutions for the optimal allocation of both fixed (premium) and variable (refundable deposit) costs under heterogeneity. The analysis highlights a trade-off between fixed and variable contributions driven by risk aversion and expected loss, offering a unified characterisation of 'buyer' and 'seller' types within the pool. Their fairness condition extends classical actuarial principles to dual-cost settings, while their individual-rationality constraints ensure robust participation incentives, generalising earlier contributions in altruistic and mutual-aid risk sharing.

5. Risk sharing and reinsurance design under ambiguity

Ambiguity, sometimes also called model uncertainty in actuarial science and risk management, has been a focal point in modern economic theory (Wakker, 2010). Ambiguity concerns how decisions can be made without full probabilistic or statistical modelling of the underlying risks. In the recent decade, developments are made on how ambiguity affects the outcome of risk sharing and reinsurance design.

Landriault *et al*. (2025) address model uncertainty in reinsurance design by considering worst-case reinsurance strategies under likelihood-ratio ambiguity. Their analysis bridges robust optimisation and insurance design, offering concrete guidance for insurers in adversarial environments. Builiding on the likelihood-robust framework of Wang *et al*. (2016) and its extension in Liu *et al*. (2022), the paper generalises the classic min-max approach of Scarf *et al*. (1957) to the reinsurance setting. It further connects to the growing literature on distributionally robust reinsurance – for example, Hu *et al*. (2015). By characterising distortion risk measures for which the insurer's optimal ceded-loss function remains unchanged under worst-case scenarios, the authors deliver ready-to-implement rules that close an important gap between theory and practice.

Ghossoub *et al*. (2025) move beyond the quadratic-loss paradigm. Extending the settings of distortion risk measures in Liu (2020) and quantile-based preferences in Embrechts *et al*. (2018), they characterise Pareto-optimal P2P allocations when each agent evaluates risk through a *robust* (set-valued) distortion functional. Their comonotone representation theorem generalises classical Borch rules to settings with heterogeneous beliefs and tail-sensitive preferences, linking to ambiguity-averse models such as Chateauneuf *et al*. (2000) and Dana (2004).

6. Innovations in risk models and risk measures

Several papers develop new models or measures that can be used in risk sharing problems. They either extend classic models to new types of risks or introduce new models inspired by technological innovations in the insurance market.

Müller (2025) revisits the classical insurance paradigm by considering infinite-mean risk models. Building on seminar insights by Fama (1965) and Samuelson (1967) on stable distributions, subsequent

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evident of the non-diversifiable trap in Ibragimov *et al*. (2009) and Hofert and Wüthrich (2012), and the more recent development in Chen *et al*. (2025b) and Chen and Shneer (2025), Müller unifies these strands through a general skewness-based criterion, showing that any distribution more skewed than a Cauchy may thwart the benefits of pooling. In contrast to the conventional belief that pooling benefits the risk risk-averse, his work illustrates how heavy-tailed laws such that Pareto or Fréchet can trigger adverse selection into risky pools, extending the observations made by Chen *et al*. (2025a). This broad generalisation deepens our understanding of risk pooling in catastrophic scenarios and highlights when diversification may backfire.

Laudagé *et al*. (2025) propose *multi-asset return risk measures* (MARRMs), a natural generalisation of the *return risk measures* of Bellini *et al*. (2018). Their framework marries the relative-performance perspective of RRMs with the capital-adequacy logic of *multi-asset risk measures* (MARMs) developed by Frittelli and Scandolo (2006) and rigorously analysed by Farkas *et al*. (2015). In doing so, they offer a unified, multiplicative risk-sharing interpretation that complements the additive inf-convolution paradigm of classical MARMs.

Lin and Zeng (2025) study how *unobservable* access to advanced digital technology shapes competition in insurance markets. Their model combines search-cost frictions–à la Ellison and Wolitzky (2012) – with asymmetric information, extending the technology heterogeneity arguments of Eling and Lehmann (2018) and the cost-based competition framework of Eling *et al*. (2022). By showing that technology adoption can *lower* an insurer's market share when accessibility is hidden from customers, they reconcile mixed empirical findings on InsurTech penetration and echo classic insights on information asymmetry in insurance (e.g., Rothschild and Stiglitz, 1978). The paper thus links the digital-divide narrative in industrial economics to actuarial questions of market structure and welfare under imperfect information.

7. Conclusion

This special issue highlights the multifaceted nature of risk sharing today. From theoretical refinements under extreme risks and ambiguity to applied designs of equitable retirement systems, and from decentralised insurance markets to innovations in risk measurement and technology, the contributions herein showcase the vitality and relevance of this field.

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References

Abdikerimova, S. and Feng, R. (2022) Peer-to-peer multi-risk insurance and mutual aid. *European Journal of Operational Research*, **299**(2), 735–749.

Bellini, F., Laeven, R.J. and Rosazza Gianin, E. (2018) Robust return risk measures. *Mathematics and Financial Economics*, **12**(1), 5–32.

Borch, K. (1960) Reciprocal reinsurance treaties. *ASTIN Bulletin: The Journal of the IAA*, **1**(4), 170–191.

Chateauneuf, A., Dana, R.-A. and Tallon, J.-M. (2000) Optimal risk-sharing rules and equilibria with choquet-expected-utility. *Journal of Mathematical Economics*, **34**(2), 191–214.

Chen, A., Guillen, M. and Rach, M. (2021) Fees in tontines. *Insurance: Mathematics and Economics*, **100**, 89–106.

Chen, A., Hieber, P. and Klein, J.K. (2019) Tonuity: A novel individual-oriented retirement plan. *ASTIN Bulletin: The Journal of the IAA*, **49**(1), 5–30.

Chen, A. and Rach, M. (2023) Actuarial fairness and social welfare in mixed-cohort tontines. *Insurance: Mathematics and Economics*, **111**, 214–229.

Chen, Y., Embrechts, P. and Wang, R. (2025a) Risk exchange under infinite-mean pareto models. *Insurance: Mathematics and Economics*, **124**, 103131.

Chen, Y., Embrechts, P. and Wang, R. (2025b) Technical note—an unexpected stochastic dominance: Pareto distributions, catastrophes, and risk exchange. *Operations Research*, **73**(3), 1336–1344.

Chen, Y. and Shneer, S. (2025) Risk aggregation and stochastic dominance for a class of heavy-tailed distributions. *ASTIN Bulletin, to appear*.

Dana, R.-A. (2004) Ambiguity, uncertainty aversion and equilibrium welfare. *Economic Theory*, **23**(3), 569–587.

https://doi.org/10.1017/asb.2025.10063 Published online by Cambridge University Press

Denuit, M. (2019) Size-biased transform and conditional mean risk sharing, with application to p2p insurance and tontines. *ASTIN Bulletin: The Journal of the IAA*, **49**(3), 591–617.
Denuit, M. and Dhaene, J. (2012) Convex order and comonotonic conditional mean risk sharing. *Insurance: Mathematics and Economics*, **51**(2), 265–270.
Denuit, M., Dhaene, J. and Robert, C.Y. (2022) Risk-sharing rules and their properties, with applications to peer-to-peer insurance. *Journal of Risk and Insurance*, **89**(3), 615–667.
Eling, M., Jia, R., Lin, J. and Rothschild, C. (2022) Technology heterogeneity and market structure. *Journal of Risk and Insurance*, **89**(2), 427–448.
Eling, M. and Lehmann, M. (2018) The impact of digitalization on the insurance value chain and the insurability of risks. *The Geneva Papers on Risk and Insurance-Issues and Practice*, **43**(3), 359–396.
Ellison, G. and Wolitzky, A. (2012) A search cost model of obfuscation. *The RAND Journal of Economics*, **43**(3), 417–441.
Embrechts, P., Liu, H. and Wang, R. (2018) Quantile-based risk sharing. *Operations Research*, **66**(4), 936–949.
Fama, E.F. (1965) Portfolio analysis in a stable paretian market. *Management science*, **11**(3), 404–419.
Farkas, W., Koch-Medina, P. and Munari, C. (2015) Measuring risk with multiple eligible assets. *Mathematics and Financial Economics*, **9**(1), 3–27.
Feng, R. (2023) Decentralized insurance. In *Decentralized Insurance: Technical Foundation of Business Models*, pp. 119–139. Springer.
Frittelli, M. and Scandolo, G. (2006) Risk measures and capital requirements for processes. *Mathematical Finance*, **16**(4), 589–612.
Fullmer, R.K. and Sabin, M.J. (2018) Individual tontine accounts. *Fullmer, RK, & Sabin, MJ (2019). Individual Tontine Accounts. Journal of Accounting and Finance*, **19**(8).
Hofert, M. and Wüthrich, M.V. (2012) Statistical review of nuclear power accidents. *Asia-Pacific Journal of Risk and Insurance*, **7**(1). https://doi.org/10.1515/2153-3792.1157.
Hu, X., Yang, H. and Zhang, L. (2015) Optimal retention for a stop-loss reinsurance with incomplete information. *Insurance: Mathematics and Economics*, **65**, 15–21.
Ibragimov, R., Jaffee, D. and Walden, J. (2009) Nondiversification traps in catastrophe insurance markets. *The Review of Financial Studies*, **22**(3), 959–993.
Iwaki, H. (2002) An economic premium principle in a continuous-time economy (special issue on theory, methodology and applications in financial engneering). *Journal of the Operations Research Society of Japan*, **45**(4), 346–361.
Jiao, Z., Kou, S., Liu, Y. and Wang, R. (2022) An axiomatic theory for anonymized risk sharing. arXiv preprint arXiv:2208.07533.
Liu, F., Mao, T., Wang, R. and Wei, L. (2022) Inf-convolution, optimal allocations, and model uncertainty for tail risk measures. *Mathematics of Operations Research*, **47**(3), 2494–2519.
Liu, H. (2020) Weighted comonotonic risk sharing under heterogeneous beliefs. *ASTIN Bulletin: The Journal of the IAA*, **50**(2), 647–673.
Milevsky, M.A. and Salisbury, T.S. (2015) Optimal retirement income tontines. *Insurance: Mathematics and Economics*, **64**, 91–105.
Milevsky, M.A. and Salisbury, T.S. (2025) The riccati tontine: How to satisfy regulators on average. *The Geneva Risk and Insurance Review*, **50**, 72–102.
Piggott, J., Valdez, E.A. and Detzel, B. (2005) The simple analytics of a pooled annuity fund. *Journal of Risk and Insurance*, **72**(3), 497–520.
Rothschild, M. and Stiglitz, J. (1978) Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. In *Uncertainty in Economics*, pp. 257–280. Elsevier.
Sabin, M.J. (2010) Fair tontine annuity. Available at SSRN: https://ssrn.com/abstract=1579932.
Samuelson, P.A. (1967) General proof that diversification pays. *Journal of Financial and Quantitative Analysis*, **2**(1), 1–13.
Scarf, H.E., Arrow, K. and Karlin, S. (1957) A min-max solution of an inventory problem. Technical report, Rand Corporation Santa Monica.
Wakker, P.P. (2010) *Prospect Theory: For Risk and Ambiguity*. Cambridge: Cambridge UniversityPress.
Wang, S. and Lu, Y. (2019) Optimal investment strategies and risk-sharing arrangements for a hybrid pension plan. *Insurance: Mathematics and Economics*, **89**, 46–62.
Wang, Z., Glynn, P.W. and Ye, Y. (2016) Likelihood robust optimization for data-driven problems. *Computational Management Science*, **13**(2), 241–261.
Yaari, M.E. (1965) Uncertain lifetime, life insurance, and the theory of the consumer. *The Review of Economic Studies*, **32**(2), 137–150.
Zhu, X., Hardy, M. and Saunders, D. (2021) Fair transition from defined benefit to target benefit. *ASTIN Bulletin: The Journal of the IAA*, **51**(3), 873–904.

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